COMP9319 Web Data Compression and Search

BWT, MTF and Pattern Matching

BWT

- Burrows–Wheeler transform (BWT) is an algorithm used to prepare data for use with data compression techniques such as bzip2.
- It was invented by Michael Burrows and David Wheeler in 1994 at DEC SRC, Palo Alto, California.
- It is based on a previously unpublished transformation discovered by Wheeler in 1983.

A simple example

Input:

#BANANAS

All rotations

```
#BANANAS
S#BANANA
AS#BANAN
NAS#BANA
ANAS#BAN
NANAS#BA
ANANAS#B
BANANAS#
```

Sort the rows

```
#BANANAS
ANANAS#B
ANAS#BAN
AS#BANAN
BANANAS#
NANAS#BA
NAS#BANA
S#BANANA
```

Output

#BANANAS

ANANAS#B

ANAS#BAN

AS#BANAN

BANANAS#

NANAS#BA

NAS#BANA

S#BANANA

Now the inverse, for decoding…

Input:

S

B

N

N

#

A

A

A

First add

S

B

N

N

#

A

A

A

Then sort

```
#
A
A
A
B
N
N
S
```

Add again

```
S#
BA
NA
NA
#B
AN
AN
AS
```

Then sort

```
#B
AN
AN
AS
BA
NA
NA
S#
```

Then add

```
S#B
BAN
NAN
NAS
#BA
ANA
ANA
AS#
```

Then sort

#BA
ANA
ANA
AS#
BAN
NAN
NAS
S#B

Then add

S#BA
BANA
NANA
NAS#
#BAN
ANAN
ANAS
AS#B

Then sort

```
#BAN
ANAN
ANAS
AS#B
BANA
NANA
NAS#
S#BA
```

Then add

```
S#BAN
BANAN
NANAS
NAS#B
#BANA
ANANA
ANAS#
AS#BA
```

Then sort

```
#BANA
ANANA
ANAS#
AS#BA
BANAN
NANAS
NAS#B
S#BAN
```

Then add

S#BANA
BANANA
NANAS#
NAS#BA
#BANAN
ANANAS
ANAS#B
AS#BAN

Then sort

```
#BANAN
ANANAS
ANAS#B
AS#BAN
BANANA
NANAS#
NAS#BA
S#BANA
```

Then add

```
S#BANAN
BANANAS
NANAS#B
NAS#BAN
#BANANA
ANANAS#
ANAS#BA
AS#BANA
```

Then sort

```
#BANANA
ANANAS#
ANAS#BA
AS#BANA
BANANAS
NANAS#B
NAS#BAN
S#BANAN
```

Then add

```
S#BANANA
BANANAS#
NANAS#BA
NAS#BANA
#BANANAS
ANANAS#B
ANAS#BAN
AS#BANAN
```

Then sort (???)

```
#BANANAS
ANANAS#B
ANAS#BAN
AS#BANAN
BANANAS#
NANAS#BA
NAS#BANA
S#BANANA
```

Implementation

Do we need to represent the table in the encoder?

No, a single pointer for each row is needed.

Just points to the beginner position is enough.

BWT(S)

```
function BWT (string s)
  create a table, rows are all possible
      rotations of s
  sort rows alphabetically
  return (last column of the table)
```

InverseBWT(S)

function inverseBWT (string s)

create empty table

L = length(s)

repeat $\boxed{\text{length(s)}}$ times

insert s as a column of table before first column of the table // first insert creates first column

sort rows of the table alphabetically

return (row that ends with the 'EOF' character)

Move to Front (MTF)

- Reduce entropy based on local frequency correlation
- Usually used for BWT before an entropy-encoding step
- Author and detail:
  - Original paper at cs9319/papers
  - http://www.arturocampos.com/ac_mtf.html

Example: abaabacad

Symbol	Code	List
a	0	abcde…..
b	1	bacde…..
a	$\boxed{1}$	abcde…..
a	0	abcde…..
b	1	bacde…..
a	1	abcde…..
c	2	cabde…..
a	1	acbde…..
d	3	dacbe…..

To transform a general file, the list has 256 ASCII symbols.

BWT compressor vs ZIP

ZIP (i.e., LZW based)

BWT+RLE+MTF+AC

File Name	Raw Size	PKZIP Size	PKZIP Bits/Byte	BWT Size	BWT Bits/Byte
bib	111,261	35,821	2.58	29,567	2.13
book1	768,771	315,999	3.29	275,831	2.87
book2	610,856	209,061	2.74	186,592	2.44
geo	102,400	68,917	5.38	62,120	4.85
news	377,109	146,010	3.10	134,174	2.85
obj1	21,504	10,311	3.84	10,857	4.04
obj2	246,814	81,846	2.65	81,948	2.66

From http://marknelson.us/1996/09/01/bwt/

Other ways to reverse BWT

Consider L=BWT(S) is composed of the symbols $V_0 \ldots V_{N-1}$, the transformed string may be parsed to obtain:

- The number of symbols in the substring $V_0 \ldots V_{i-1}$ that are identical to V_i.
- For each unique symbol, V_i, in L, the number of symbols that are lexicographically less than that symbol.

按字典序.

Example

B - ranking

sorted by lexicographically

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

????????]

Last column

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

First column

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

???????A]

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

?????NA]

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

????ANA]

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

???NANA]

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

??ANANA]

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

?BANANA]

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

[BANANA]

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

[BANANA]

Position	Symbol	# Matching
0	B	0
1	N	0
2	N	1
3	[	0
4	A	0
5	A	1
6	]	0
7	A	2

Occ / Rank

Symbol	# LessThan
A	0
B	3
N	4
[	6
]	7

C [ ]

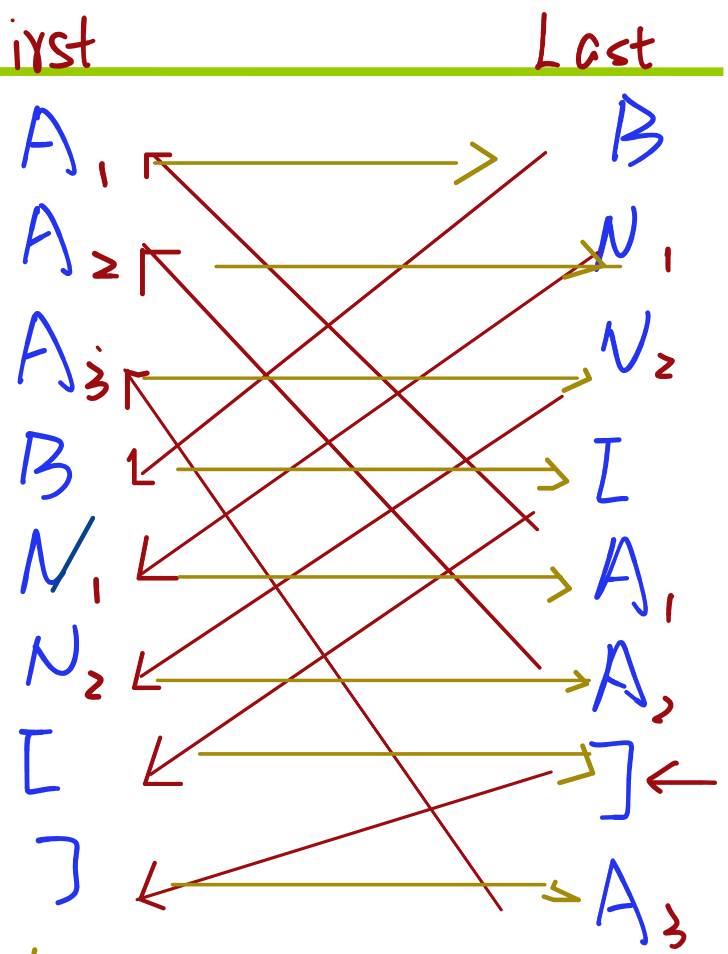
An illustration

A
A
A
B
N
N
[
]

First

B
N
N
[
A
A
]
A

Last



the order is

[B A_1 N_1 A_2 N_2 A_3]

A]

A
A
A
B
N
N
[
]

B
N
N
[
A
A
]
A

NA]

-A NA

=A NB

A

B

N A

N B

[

]

B

N

N

[

A -

A =

]

A

ANA]

A	B
A	N
A	N
B	[
N	A
N	A
[	]
]	A

NANA]

A	B
A	N
A	N
B	[
N	A
N	A
[	]
]	A

ANANA]

A	B
A	N
A	N
B	[
N	A
N	A
[	]
]	A

BANANA]

A	B
A	N
A	N
B	[
N	A
N	A
[	]
]	A

[BANANA]

A	B
A	N
A	N
B	[
N	A
N	A
[	]
]	A

Dynamic BWT ?

Instead of reconstructing BWT, local reordering from the original BWT.

Details:

Salson M, Lecroq T, Léonard M and Mouchard L (2009). "A Four-Stage Algorithm for Updating a Burrows–Wheeler Transform". Theoretical Computer Science 410 (43): 4350.

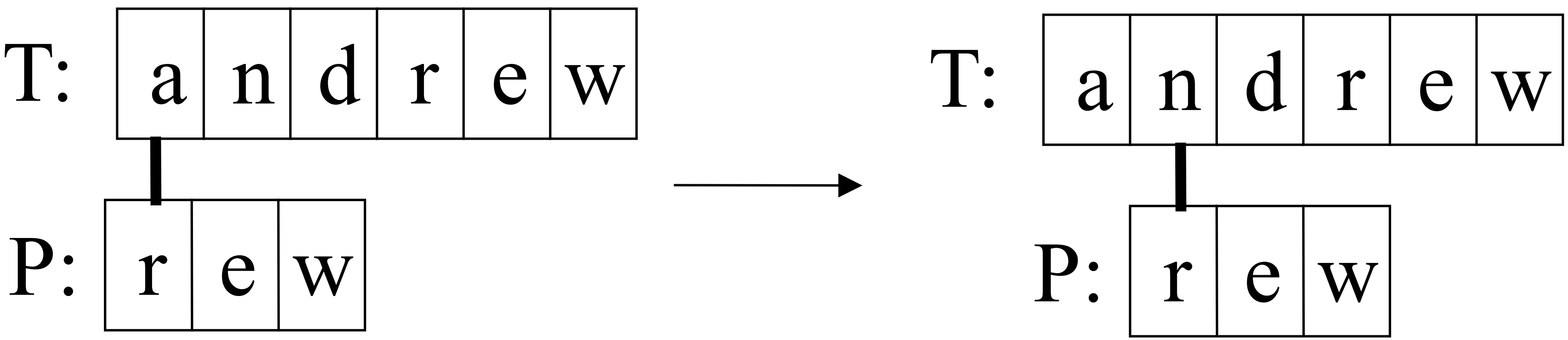
Search

What is Pattern Matching?

- Definition:
  - given a text string T and a pattern string P, find the pattern inside the text
    - T: “the rain in spain stays mainly on the plain”
    - P: “n th”

The Brute Force Algorithm

- Check each position in the text T to see if the pattern P starts in that position



each time only moves 1 char

P moves 1 char at a time through T

. . . .

Analysis

- Brute force pattern matching runs in time O(mn) in the worst case.
- But most searches of ordinary text take O(m+n), which is very quick.

continued

- The brute force algorithm is fast when the alphabet of the text is large
  - e.g. A..Z, a..z, 1..9, etc.

Since the occurence of failure is frequent.

- It is slower when the alphabet is small
  - e.g. 0, 1 (as in binary files, image files, etc.)

continued

- Example of a worst case:
  - T: "aaaaaaaaaaaaaaaaaaaaaaaaaah"
  - P: "aaah"

$\rightarrow O(3n)$

- Example of a more average case:
  - T: "a string searching example is standard"
  - P: "store"

The KMP Algorithm

- The Knuth-Morris-Pratt (KMP) algorithm looks for the pattern in the text in a *left-to-right* order (like the brute force algorithm).

- But it shifts the pattern more intelligently than the brute force algorithm.

continued

Summary

- If a mismatch occurs between the text and pattern P at P[j], what is the *most* we can shift the pattern to avoid wasteful comparisons?

Summary

- If a mismatch occurs between the text and pattern P at P[j], what is the *most* we can shift the pattern to avoid wasteful comparisons?

very important

- *Answer*: the largest prefix of P[0 .. j-1] that is a suffix of P[1 .. j-1]

Example

T: *a b a c a a b a c c a b a c a b a a b b*

1 2 3 4 5 6

P: *a b a c a b*

7 a b a c ab

a b a c a b

0 1 2

8 9 10 11 12

a b a c a b

0 1 2 3

13

a b a c a b

14 15 16 17 18 19

a b a c a b

-2 -1 0 1 2 3 4 5

k	0	1	2	3	4	5
F(*k*)	-1	0	0	1	0	1

KMP Advantages

- KMP runs in optimal time: O(m+n)
  - very fast
- The algorithm never needs to move backwards in the input text, T
  - this makes the algorithm good for processing very large files that are read in from external devices or through a network stream

KMP Disadvantages

- KMP doesn’t work so well as the size of the alphabet increases
  – more chance of a mismatch (more possible mismatches) make j ← F[j] so many times
  – mismatches tend to occur early in the pattern, but KMP is faster when the mismatches occur later

The Boyer-Moore Algorithm

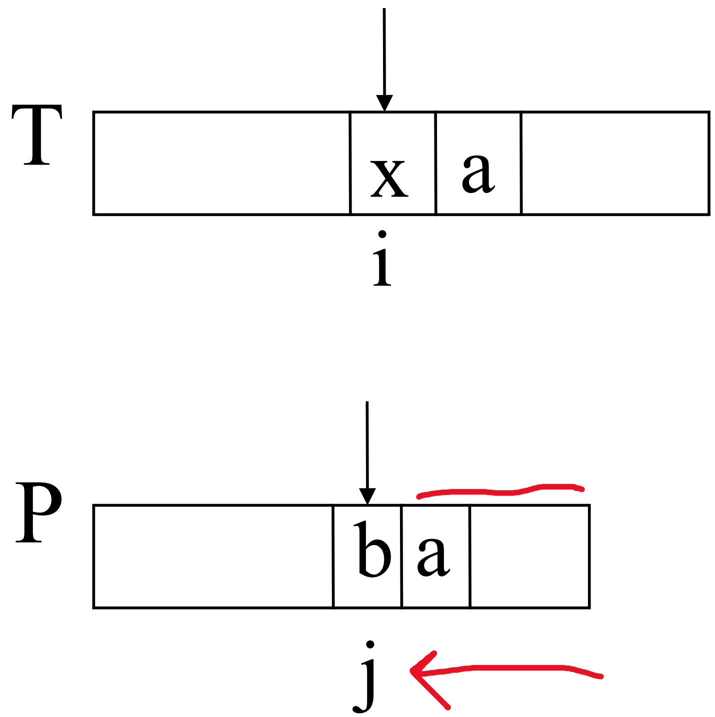
- The Boyer-Moore pattern matching algorithm is based on two techniques.

- 1. The *looking-glass* technique
  - find P in T by moving *backwards* through P, starting at its end

From right to left

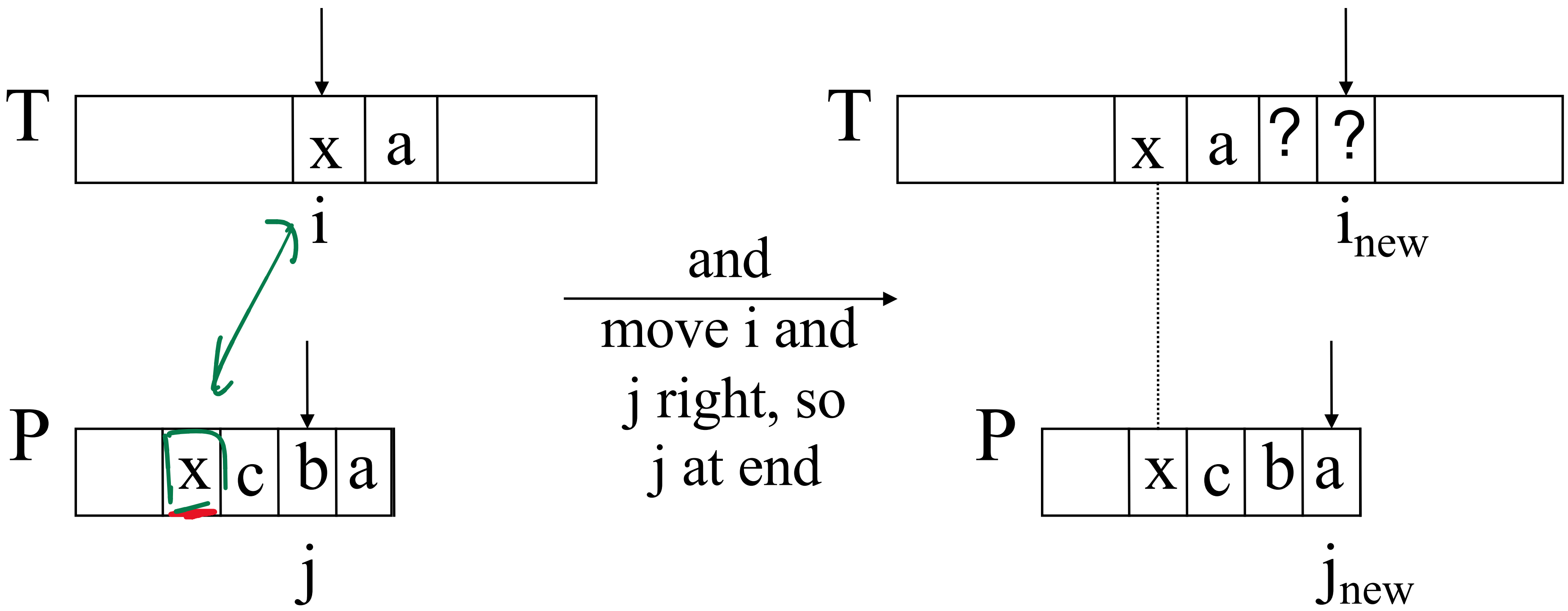
- 2. The *character-jump* technique
  - when a mismatch occurs at T[i] == x
  - the character in pattern P[j] is not the same as T[i]

- There are 3 possible cases.



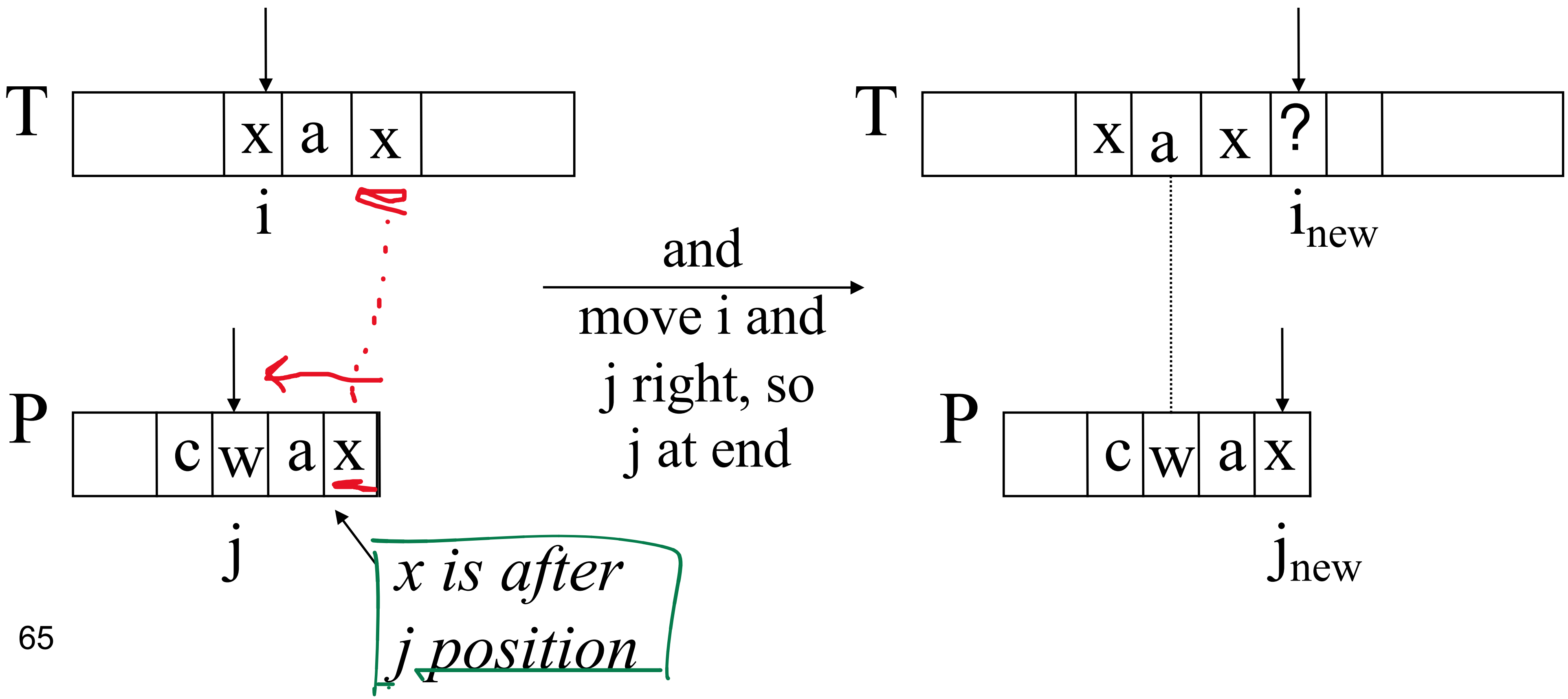
Case 1

- If P contains x somewhere, then try to *shift P* right to align the last occurrence of x in P with T[i].



Case 2

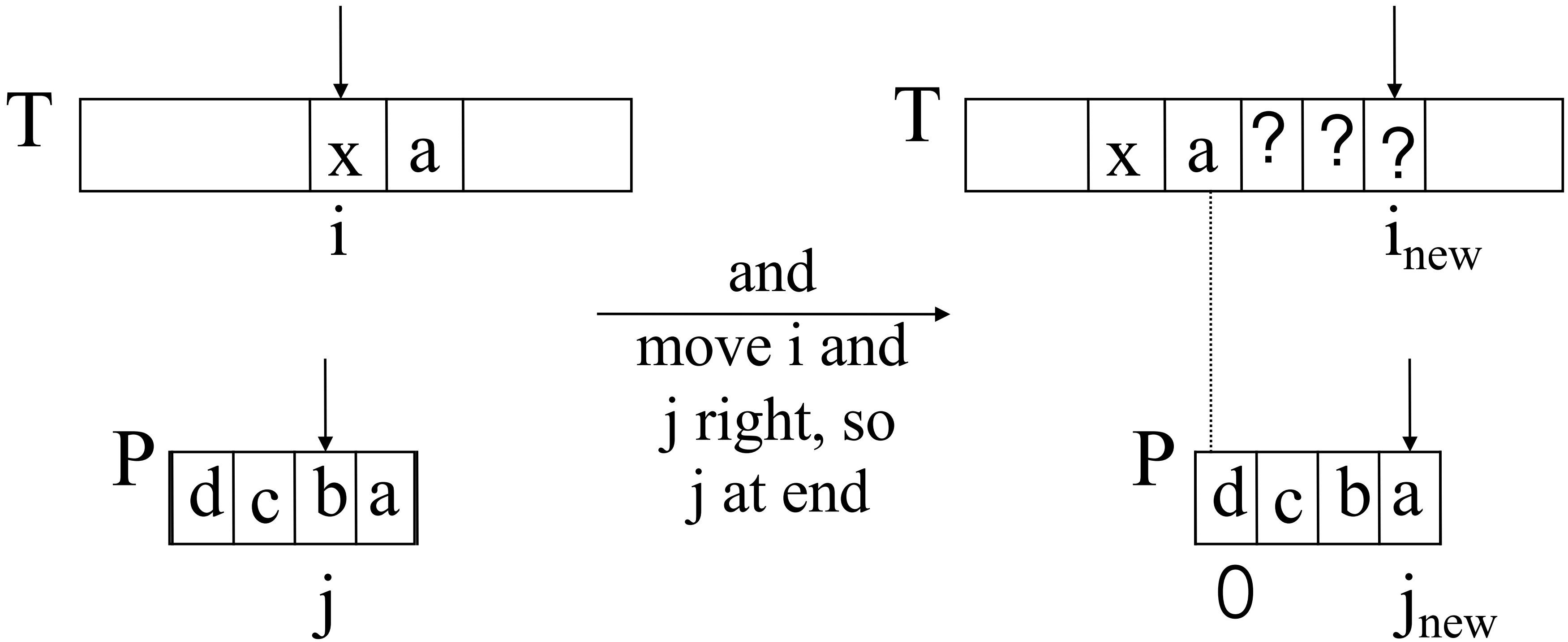
- If P contains x somewhere, but a shift right to the last occurrence is *not* possible, then *shift P* right by 1 character to T[i+1].



character (x) in the pattern doesn't have corresponding

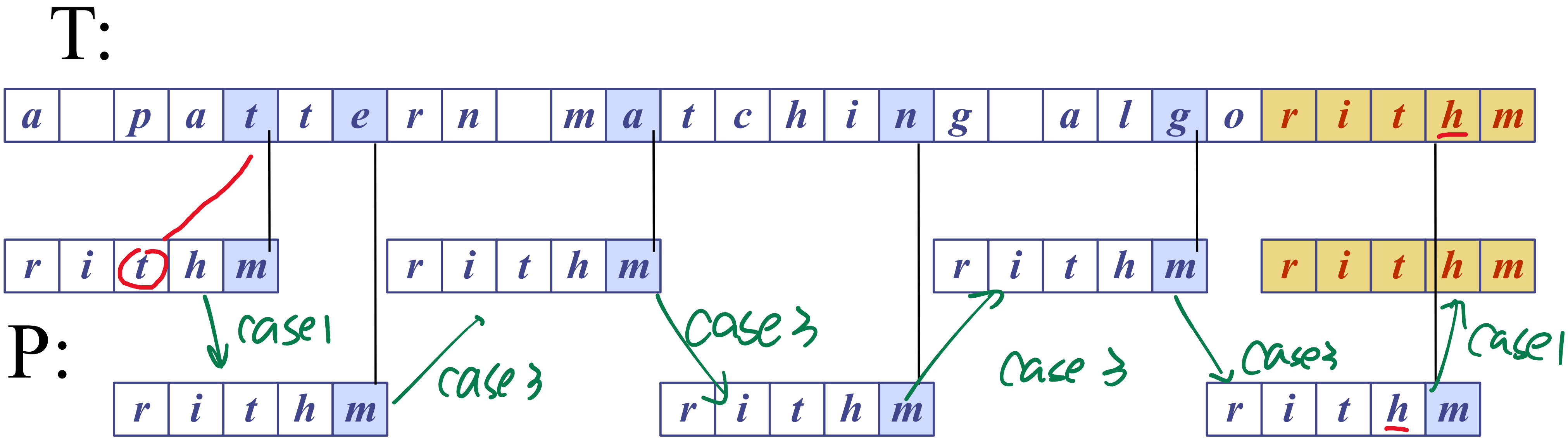
Case 3

- If cases 1 and 2 do not apply, then *shift* P to align P[0] with T[i+1].



No x in P

Boyer-Moore Example (1)



Last Occurrence Function

- Boyer-Moore's algorithm preprocesses the pattern P and the alphabet A to build a last occurrence function L()
  - L() maps all the letters in A to integers

- L(x) is defined as: // x is a letter in A
  - the largest index i such that P[i] == x, or
  - -1 if no such index exists

$$L(x) = \begin{cases} \max_{i \in S} i, & S \notin \phi \\ -1 & S \in \phi \end{cases} \quad \text{while } S = \{i \mid P[i] = x\}$$

L() Example

- A = {a, b, c, d}
- P: "abacab"

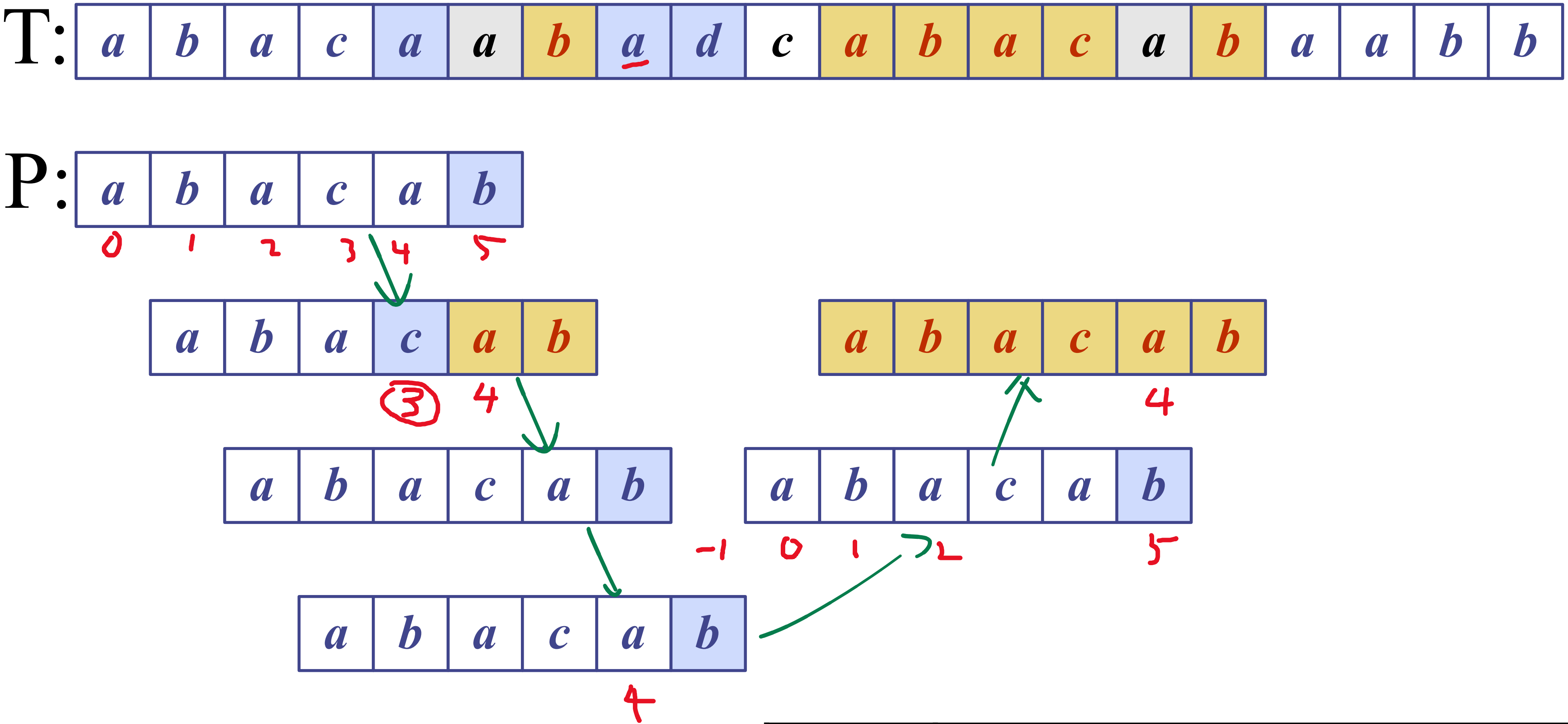
P

a	b	a	c	a	b
0	1	2	3	4	5

x	***a***	***b***	***c***	***d***
***L*(*x*)**	4	5	3	-1

L() stores indexes into P[]

Boyer-Moore Example (2)



x	a	b	c	d
L(x)	4	5	3	–1

Analysis

- Boyer-Moore worst case running time is O(nm + A)

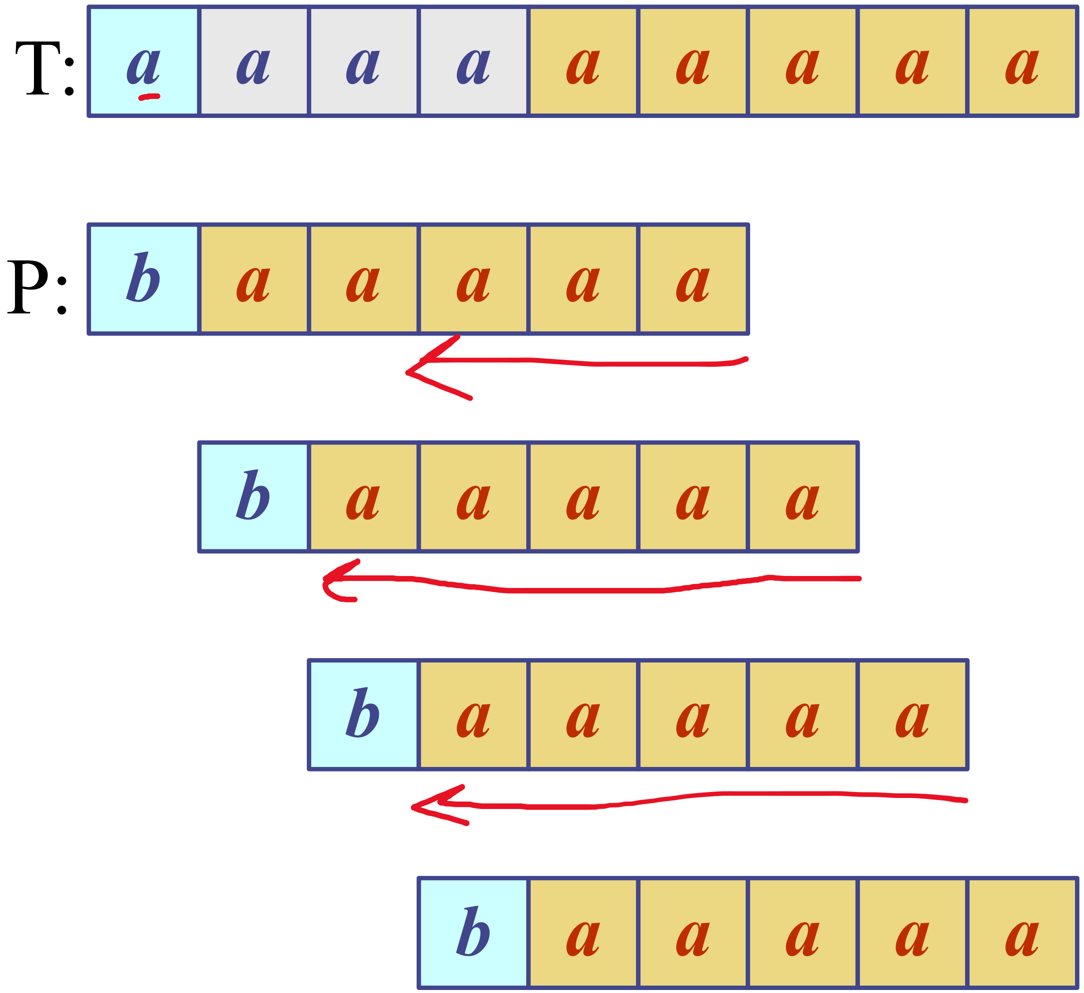
  O(mn+A) time complexity.

- But, Boyer-Moore is fast when the alphabet (A) is large, slow when the alphabet is small.
  - e.g. good for English text, poor for binary

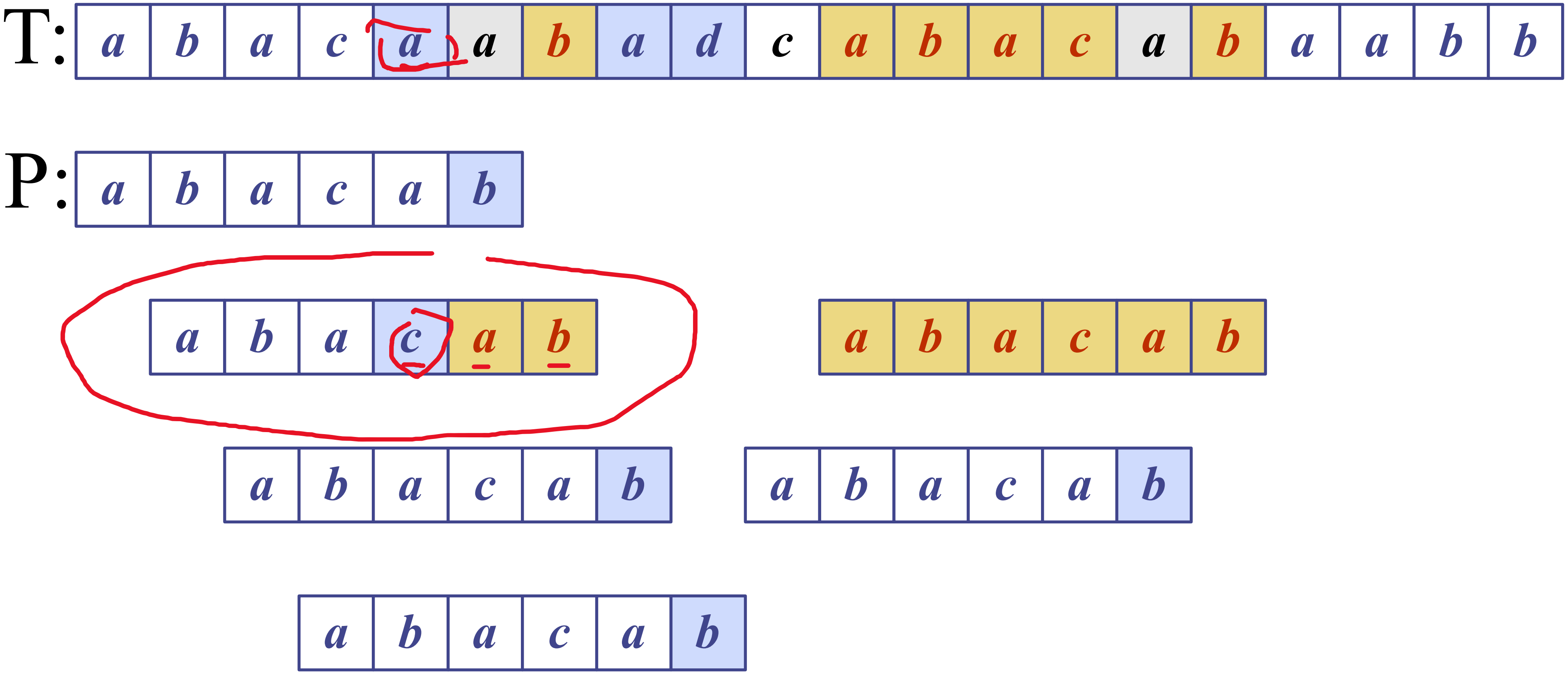
- Boyer-Moore is *significantly faster than brute force* for searching English text.

Worst Case Example

- T: "aaaaa...a"
- P: "baaaaa"



Boyer-Moore Example (2)



x	a	b	c	d
$L(x)$	4	5	3	–1

Boyer-Moore: Good suffix rule

If t is the longest suffix of P that matches T in the current position, then P can be shifted so that the previous occurrence of t in P matches T. In fact, it can be required that the character before the previous occurrence of t be different from the character before the occurrence of t as a suffix. If no such previous occurrence of t exists, then the following cases apply:

- Find the smallest shift that matches a prefix of the pattern to a suffix of t in the text
- If there's no such match, shift the pattern by n (the length of P)

Boyer-Moore: Good suffix rule

- Consider the example in the paper:

. . . . . . YXABC . . . . .

-2 -1 ABCXXXABC

- P = ABCXXXABC
- 0 1 2 3 4 5 6 7 8

- -6 -5 -4 -3 -2 -1 -3 -2 7

Boyer-Moore: Good suffix rule

- Consider the example in the paper:

_AEYX

ABY....

- P = ABYXCDEYX
- 012345678
- -9 -8 -7 -6 -5 -4 1 -2 7

Boyer-Moore: Good suffix rule

- Consider the examples in the paper:
- ABCXXXABC
- ABYXCDEYX

- -6 -5 -4 -3 -2 -1 -3 -2 7
- -9 -8 -7 -6 -5 -4 1 -2 7

Boyer-Moore: Good suffix rule

- Another example:

xcab

-2 -1 a b acab

- abacab
- 0 1 2 3 4 5

- -4 -3 -2 -1 -2 4

Boyer-Moore Example (3)

T:	a	b	a	c	a	a	b	a	d	c	a	b	a	c	a	b	a	a	b	b

3

P:	a	b	a	c	a	b

a	b	a	c	a	b

a	b	a	c	a	b

a	b	a	c	a	b

a	b	a	c	a	b

3

4 b

-1
-2

Good suffix rule

-4 -3 -2 -1 -2 4

x	a	b	c	d
L(x)	4	5	3	–1

KMP & BM

- Please refer to the original papers (available at WebCMS) for the details of the algorithms
- Most text processors use BM for “find” (& “replace”) due to its good performance for general text documents

like

Linux-grep

Move to Front (MTF)

- Reduce entropy based on local frequency correlation
- Usually used for BWT before an entropy-encoding step
- Author and detail:
  - Original paper at cs9319/papers
  - http://www.arturocampos.com/ac_mtf.html

Example: abaabacad

Symbol	Code	List
a	0	abcde…..
b	1	bacde…..
a	1	abcde…..
a	0	abcde…..
b	1	bacde…..
a	1	abcde…..
c	2	cabde…..
a	1	acbde…..
d	3	dacbe…..

second

2

1

3

To transform a general file, the list has 256 ASCII symbols.

Example: abaaabbbccddddcc

Symbols: abaaabbbccddddcc
Codes (in ASCII binary): 01100001, 01100010, 01100001, 01100001, ..., 01100100, 01100011, 01100011
Codes (in ASCII dec): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100, 99, 99

Example: abaaabbbccddddcc

Symbols: abaaabbbccddddcc
Codes (in ASCII binary): 01100001, 01100010, 01100001, 01100001, ..., 01100100, 01100011, 01100011
Codes (in ASCII dec): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100, 99, 99

Recall that Shannon's entropy reaches the max when there is max uncertainly, i.e., equal probability, like the example above (4 "97"s, 4 "98"s, 4 "99"s, 4 "100"s).

e.g., Entropy H = 2.00

$$H = -\sum_x p(x) \log p(x)$$

$$= -4 \times \frac{1}{4} \log \frac{1}{4} = \log_2 4 = 2 \text{ bits.}$$

Example: abaaabbbccddddcc

Symbols: abaaabbbccddddcc
Codes (in ASCII binary): 01100001, 01100010, 01100001, 01100001, ...,
01100100, 01100011, 01100011
Codes (in ASCII dec): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100,
100, 99, 99

List

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	..	..	..	..	..	a	b	c	d	e	f	...	..

Codes (in ASCII): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100,
99, 99
Codes (in MTF): 97

Example: abaaabbbccddddcc

Symbols: abaaabbbccddddcc
Codes (in ASCII binary): 01100001, 01100010, 01100001, 01100001, ..., 01100100, 01100011, 01100011
Codes (in ASCII dec): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100, 99, 99

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	..	..	..	..	..	a	b	c	d	e	f	...	..

Codes (in ASCII): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100, 99, 99
Codes (in MTF): 97

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	..	..	..	..	..	a	b	c	d	e	f	...	..

move to front

Example: abaaabbbccddddcc

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	a	..	..	..	..	..	b	c	d	e	f	...	..

Codes (in ASCII): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100, 99, 99

Codes (in MTF): 97, 98

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	b	a	..	..	..	..	..	c	d	e	f	...	..

Example: abaaabbbccddddcc

List

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	b	a	..	..	..	..	..	c	d	e	f	...	..

Codes (in ASCII): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100, 99, 99

Codes (in MTF): 97, 98, 1

List

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	b	a	..	..	..	..	..	c	d	e	f	...	..

Example: abaaabbbccddddcc

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	a	b	..	..	..	..	..	c	d	e	f	...	..

Codes (in ASCII): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100, 99, 99

Codes (in MTF): 97, 98, 1, 0,

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	a	b	..	..	..	..	..	c	d	e	f	...	..

Example: abaaabbbccddddcc

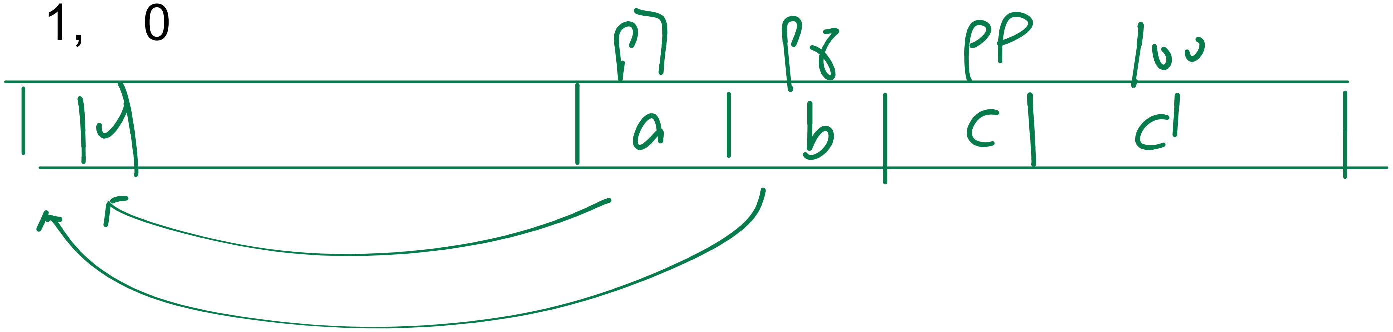
dynamic

<u>List</u>

Index: 0 1 2 3 4 97 98 99 100 101 102 ... 255
Value: a b .. .. c d e f ... ..

Codes (in ASCII): 97, 98, 97, 97, 97, 98, 98, 98, 99, 99, 100, 100, 100, 100, ~~99, 99~~

Codes (in MTF): 97, 98, 1, 0, 0, 1, 0, 0, 99, 0, 100, 0, 0, 0, 1, 0



Example: MTF decoding

List

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	..	..	..	..	..	a	b	c	d	e	f	...	..

Codes (in MTF): 97, 98, 1, 0, 0, 1, 0, 0, 99, 0, 100, 0, 0, 0, 1, 0

Symbols: **a**, **b** a.

List

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	b	a	..	..	..	..	..	c	d	e	f	...	..

Example: MTF decoding

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	b	a	..	..	..	..	..	c	d	e	f	...	..

Codes (in MTF): 97, 98, 1, 0, 0, 1, 0, 0, 99, 0, 100, 0, 0, 0, 1, 0

Symbols: **a**, **b, a**

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	a	b	..	..	..	..	..	c	d	e	f	...	..

Example: MTF decoding

List

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	a	b	..	..	..	..	..	c	d	e	f	...	..

Codes (in MTF): 97, 98, 1, 0, 0, 1, 0, 0, 99, 0, 100, 0, 0, 0, 1, 0

Symbols: **a**, **b, a, a**

List

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	a	b	..	..	..	..	..	c	d	e	f	...	..

Example: MTF decoding

<u>List</u>

Index:	0	1	2	3	4	97	98	99	100	101	102	...	255
Value:	a	b	..	..	..	..	..	c	d	e	f	...	..

Codes (in MTF): 97, 98, 1, 0, 0, 1, 0, 0, 99, 0, 100, 0, 0, 0, 1, 0

Symbols: **a**, **b, a, a, a, b, b, b, c, c, d, d, d, d, c, c**

Example: MTF decoding

List

Index: 0 1 2 3 4 97 98 99 100 101 102 ... 255

Value: a b .. .. c d e f ... ..

Codes (in MTF): 97, 98, 1, 0, 0, 1, 0, 0, 99, 0, 100, 0, 0, 0, 1, 0

Symbols: **a, b, a, a, a, b, b, b, c, c, d, d, d, d, c, c**

The distribution of symbols is changed, with more *local* references (1 "97", 1 "98", 1 "99", 1 "100", 9 "0"s, 3 "1"s). => Reduced entropy

H = 1.92 $H(X) = -\left(4 \times \frac{1}{16}\log_2\frac{1}{16} + \frac{9}{16}\log_2\frac{9}{16} + \frac{3}{16}\log_2\frac{3}{16}\right)$

$= 1.92$ bits

BWT compressor vs ZIP

ZIP (i.e., LZW based)

BWT+RLE+MTF+AC

File Name	Raw Size	PKZIP Size	PKZIP Bits/Byte	BWT Size	BWT Bits/Byte
bib	111,261	35,821	2.58	29,567	2.13
book1	768,771	315,999	3.29	275,831	2.87
book2	610,856	209,061	2.74	186,592	2.44
geo	102,400	68,917	5.38	62,120	4.85
news	377,109	146,010	3.10	134,174	2.85
obj1	21,504	10,311	3.84	10,857	4.04
obj2	246,814	81,846	2.65	81,948	2.66

From http://marknelson.us/1996/09/01/bwt/