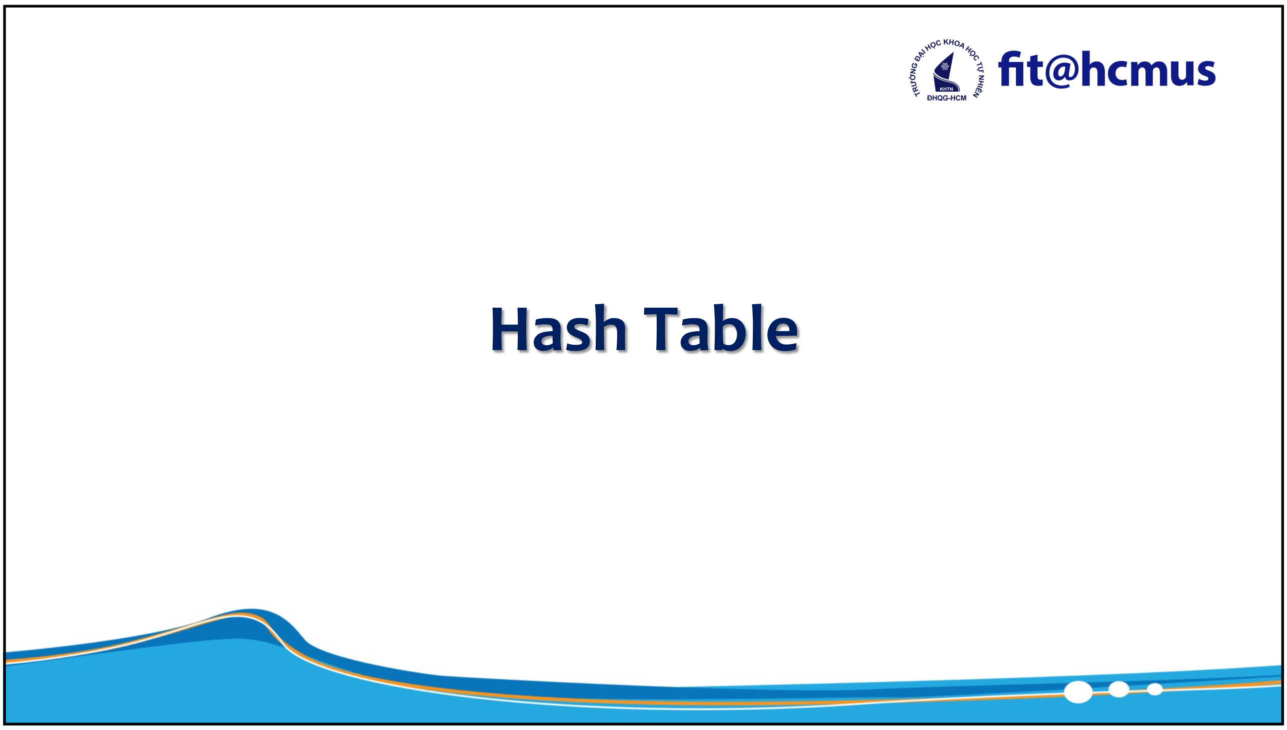

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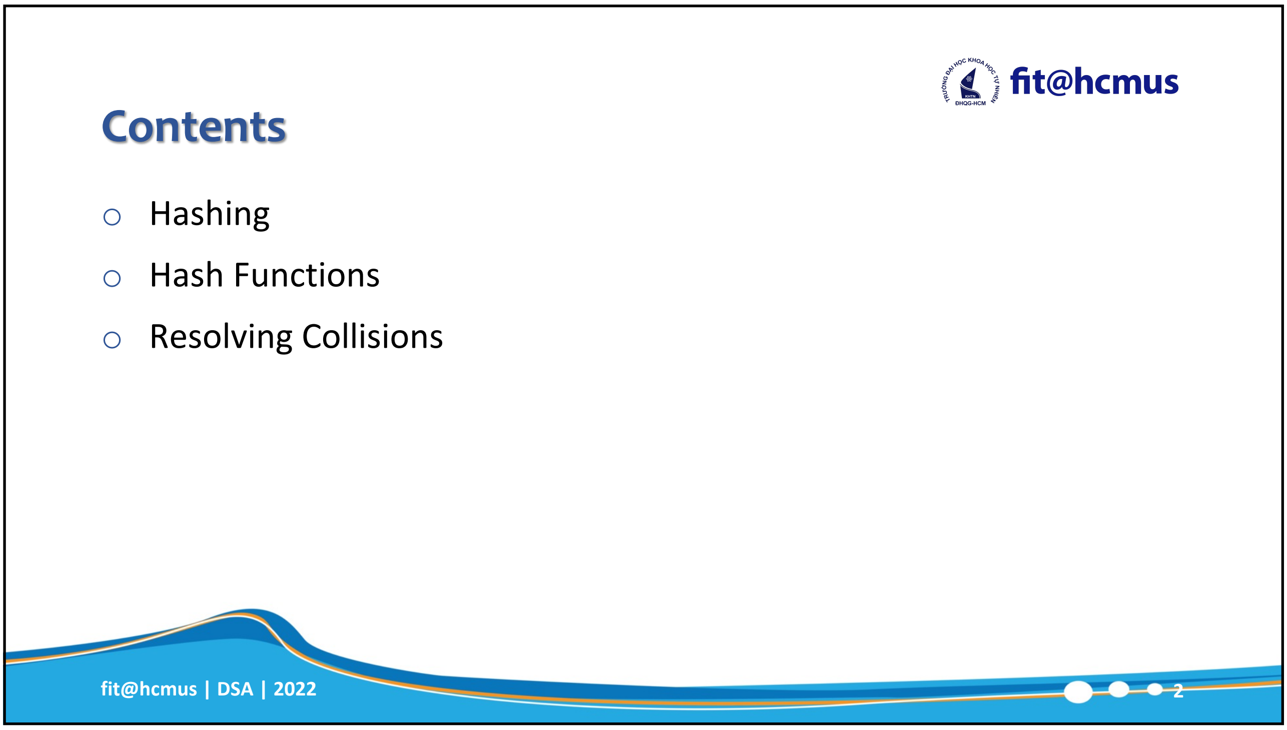
Hash Table

1



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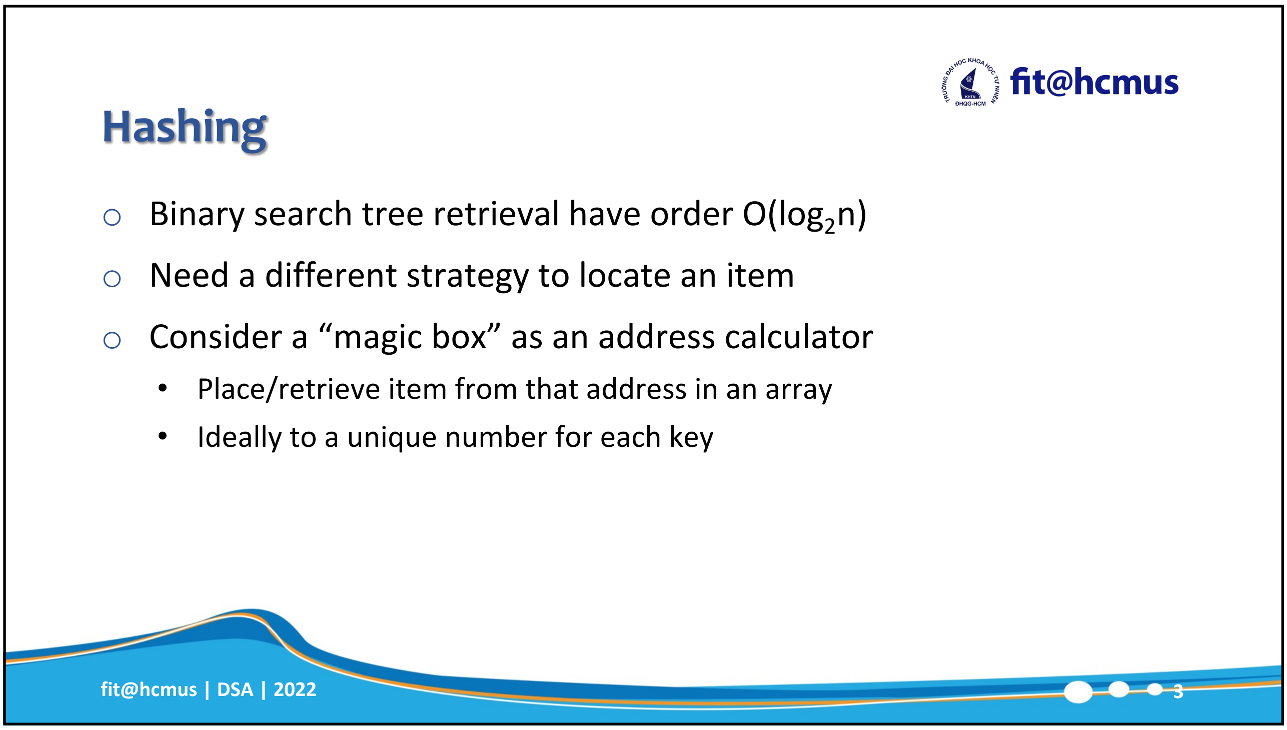
Contents

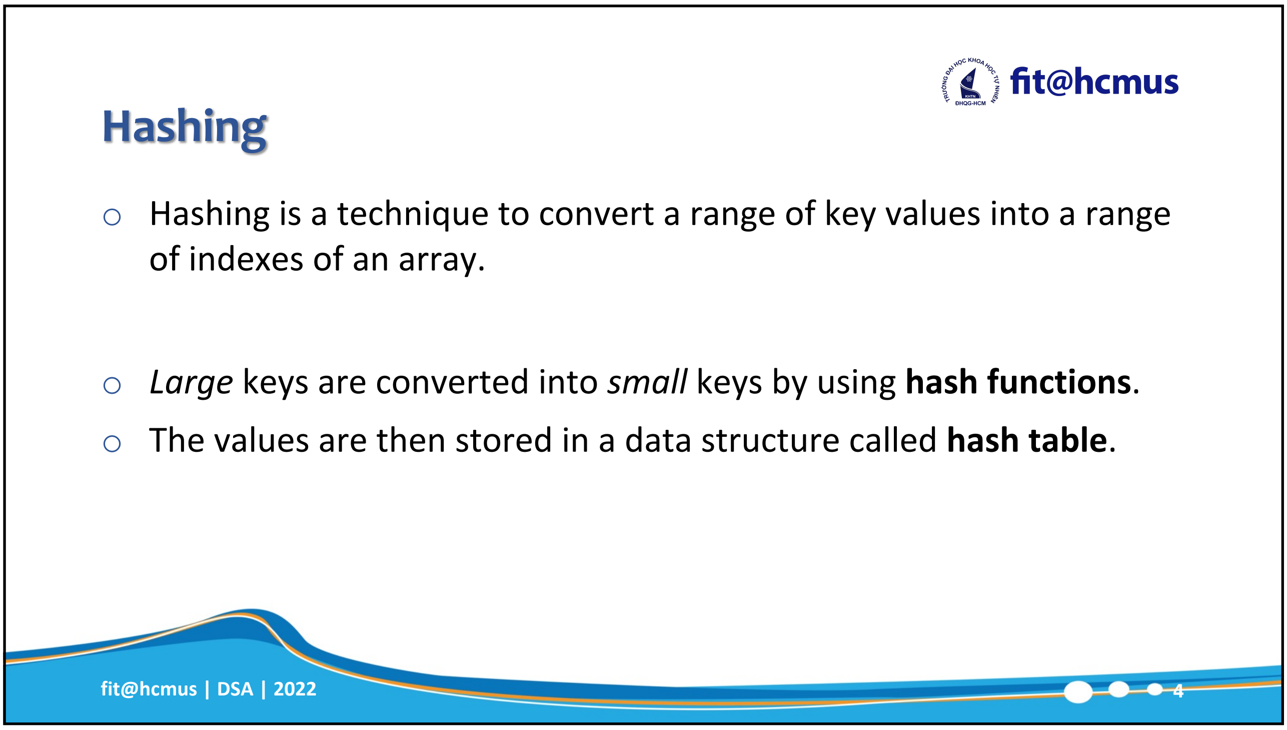
- ○ Hashing
- ○ Hash Functions
- ○ Resolving Collisions

2



Hashing

- Binary search tree retrieval have order O($\log_2 n$)
- Need a different strategy to locate an item
- Consider a “magic box” as an address calculator
  - Place/retrieve item from that address in an array
  - Ideally to a unique number for each key



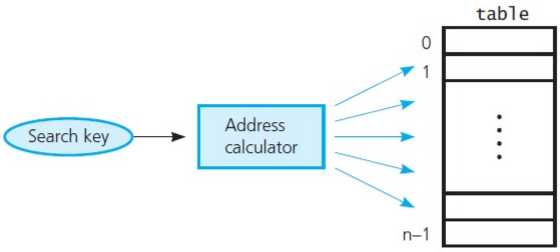
Hashing

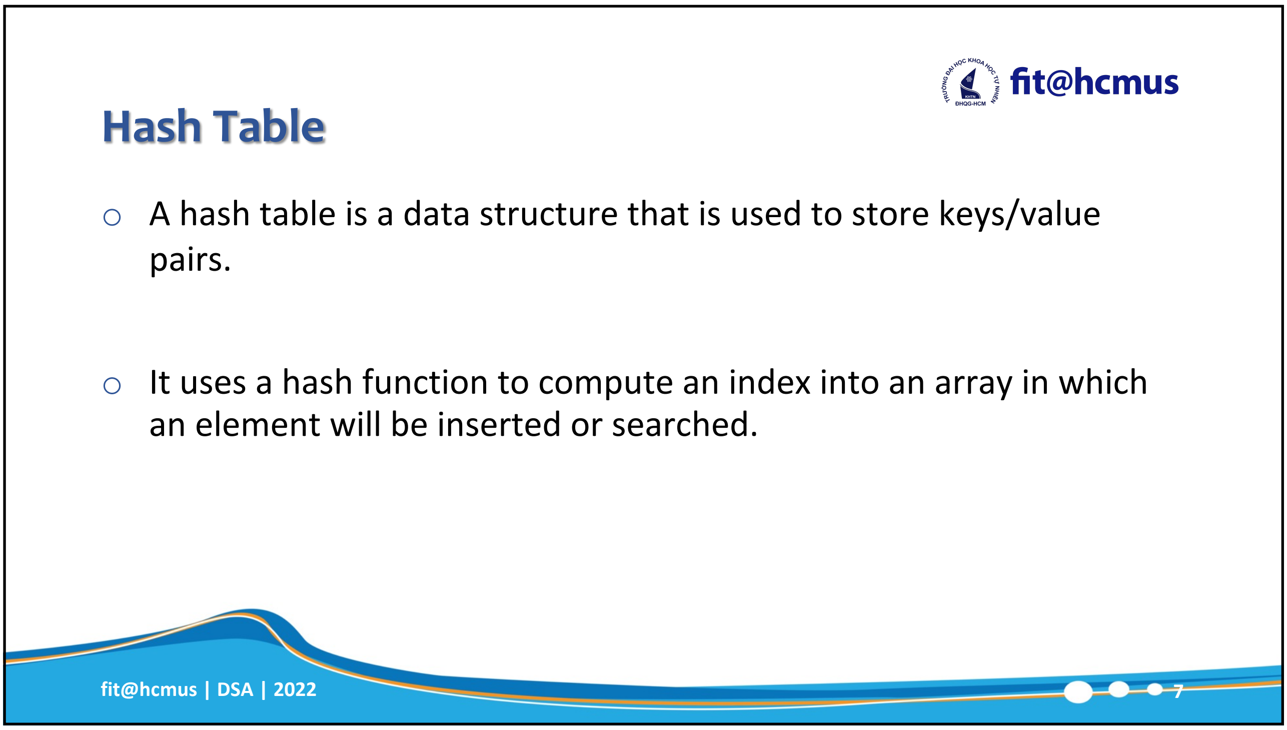
- Hashing is a technique to convert a range of key values into a range of indexes of an array.
- *Large* keys are converted into *small* keys by using **hash functions**.
- The values are then stored in a data structure called **hash table**.

Hashing

- Idea:
  - Distribute entries (key/value pairs) uniformly across an array.
  - Each element is assigned a key (converted key).
  - Using that key to access the element in O(1) time. (The hash function computes an index suggesting where an entry can be found or inserted.)

Hashing





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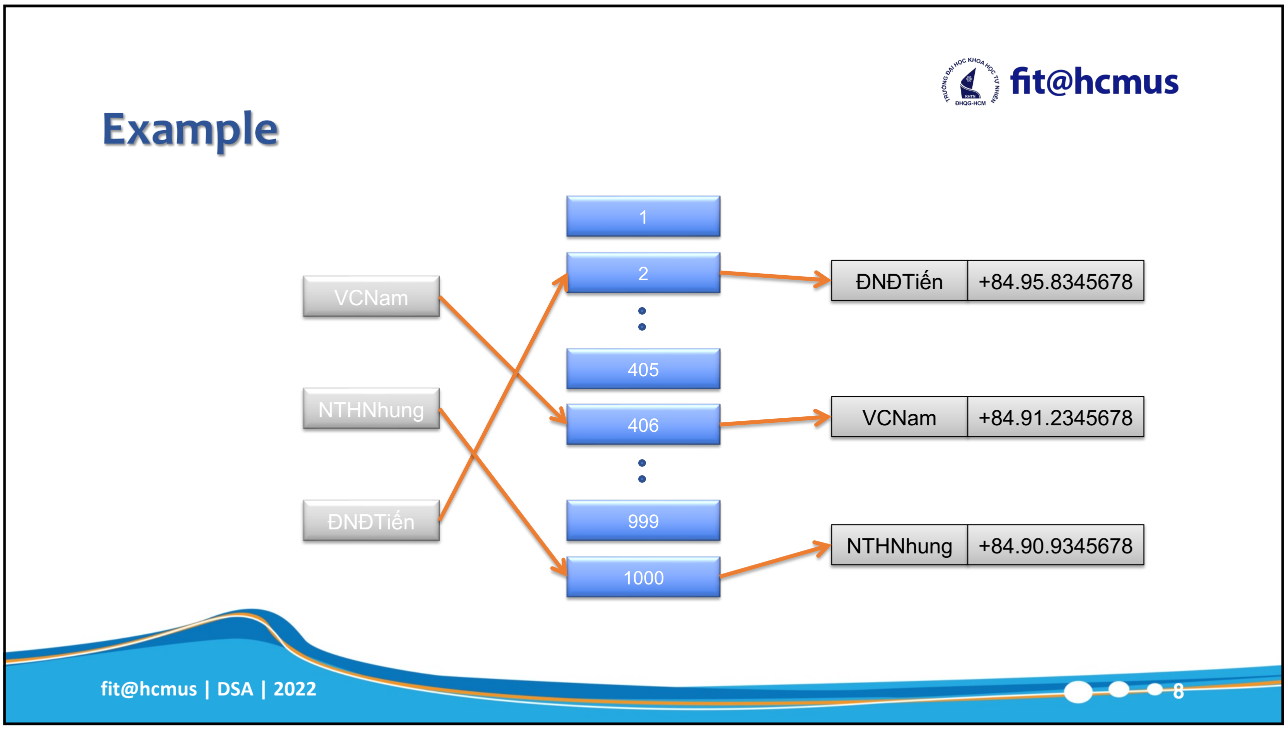
Hash Table

- ○ A hash table is a data structure that is used to store keys/value pairs.
- ○ It uses a hash function to compute an index into an array in which an element will be inserted or searched.



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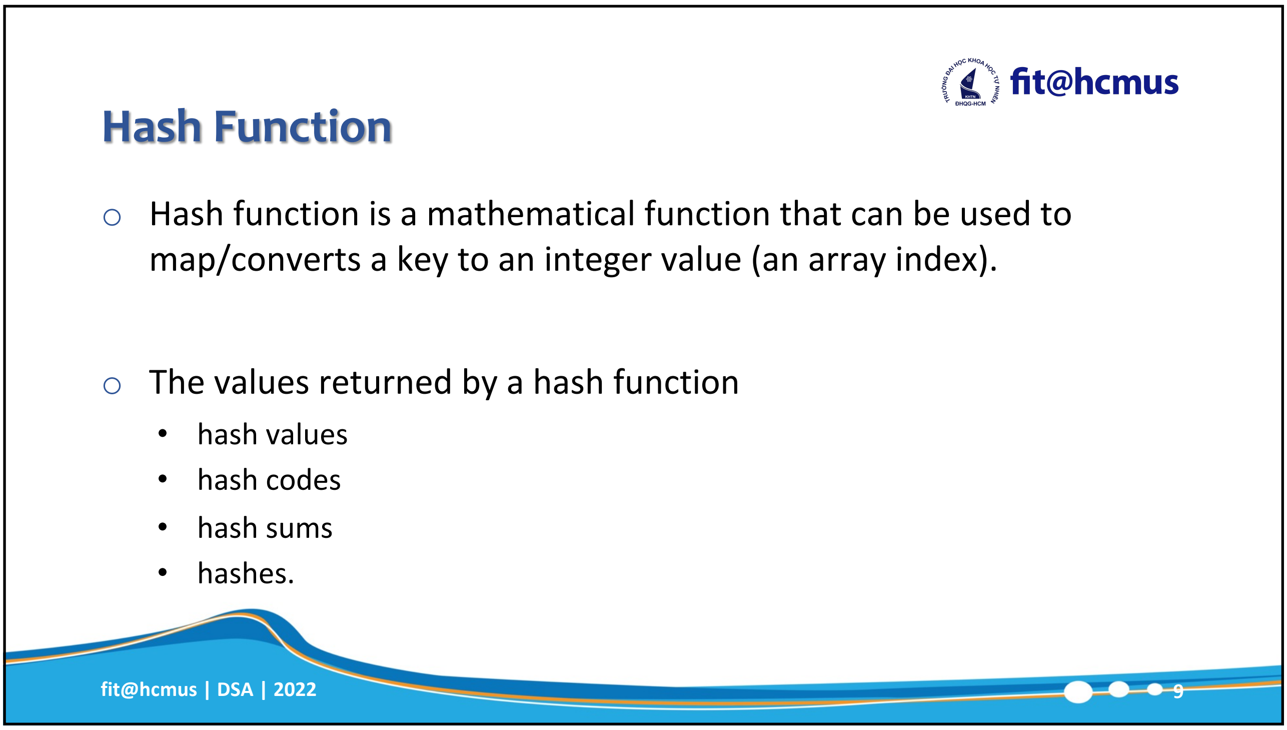
Example

VCNam
NTHNhung
ĐNĐTiến

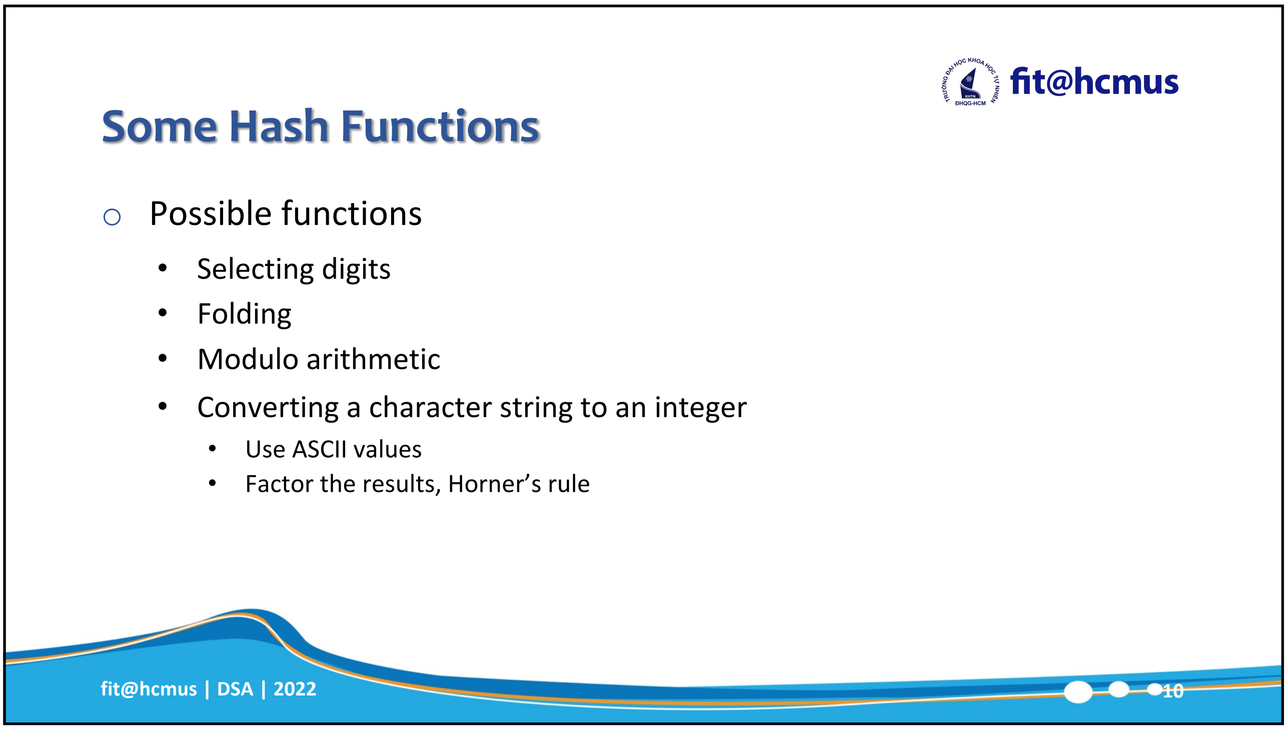
1
2
⋮
405
406
⋮
999
1000

ĐNĐTiến	+84.95.8345678
VCNam	+84.91.2345678
NTHNhung	+84.90.9345678



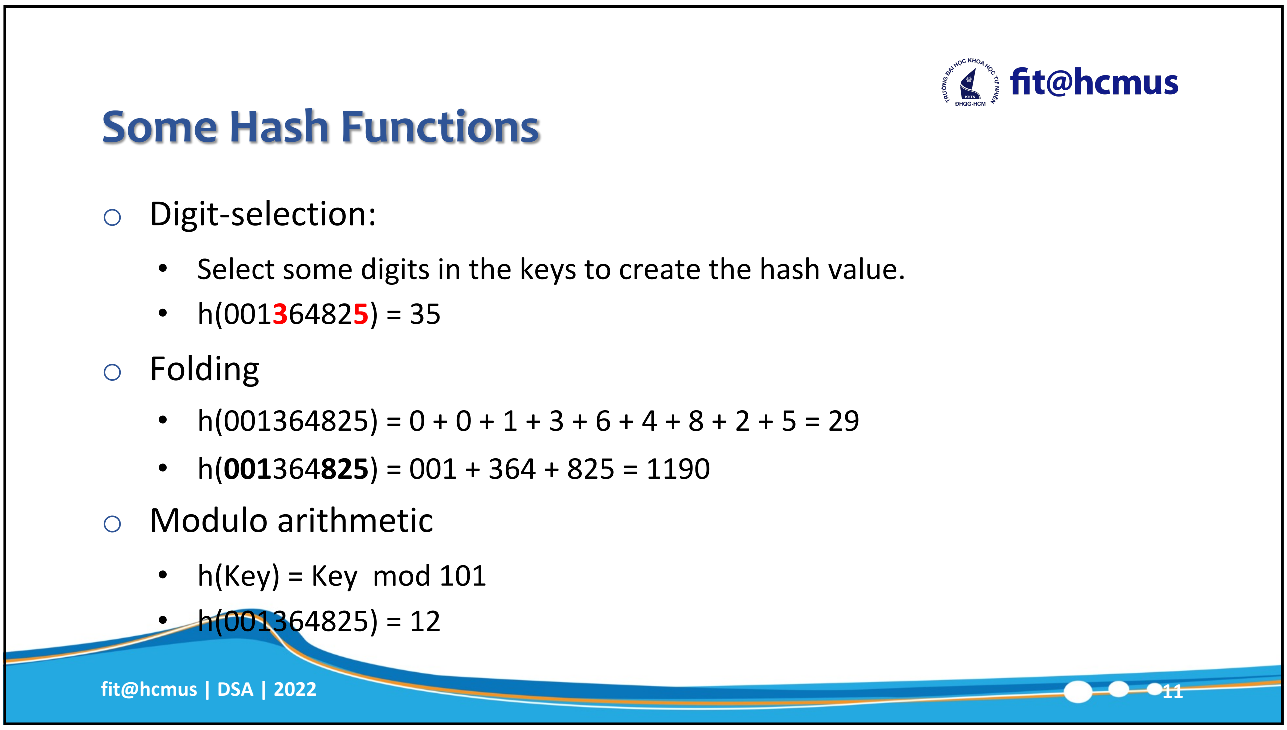
Hash Function

- Hash function is a mathematical function that can be used to map/converts a key to an integer value (an array index).
- The values returned by a hash function
  - hash values
  - hash codes
  - hash sums
  - hashes.



Some Hash Functions

- Possible functions
  - Selecting digits
  - Folding
  - Modulo arithmetic
  - Converting a character string to an integer
    - Use ASCII values
    - Factor the results, Horner’s rule



Some Hash Functions

- Digit-selection:
  - Select some digits in the keys to create the hash value.
  - h(001**3**6482**5**) = 35
- Folding
  - h(001364825) = 0 + 0 + 1 + 3 + 6 + 4 + 8 + 2 + 5 = 29
  - h(**001**364**825**) = 001 + 364 + 825 = 1190
- Modulo arithmetic
  - h(Key) = Key mod 101
  - h(001364825) = 12

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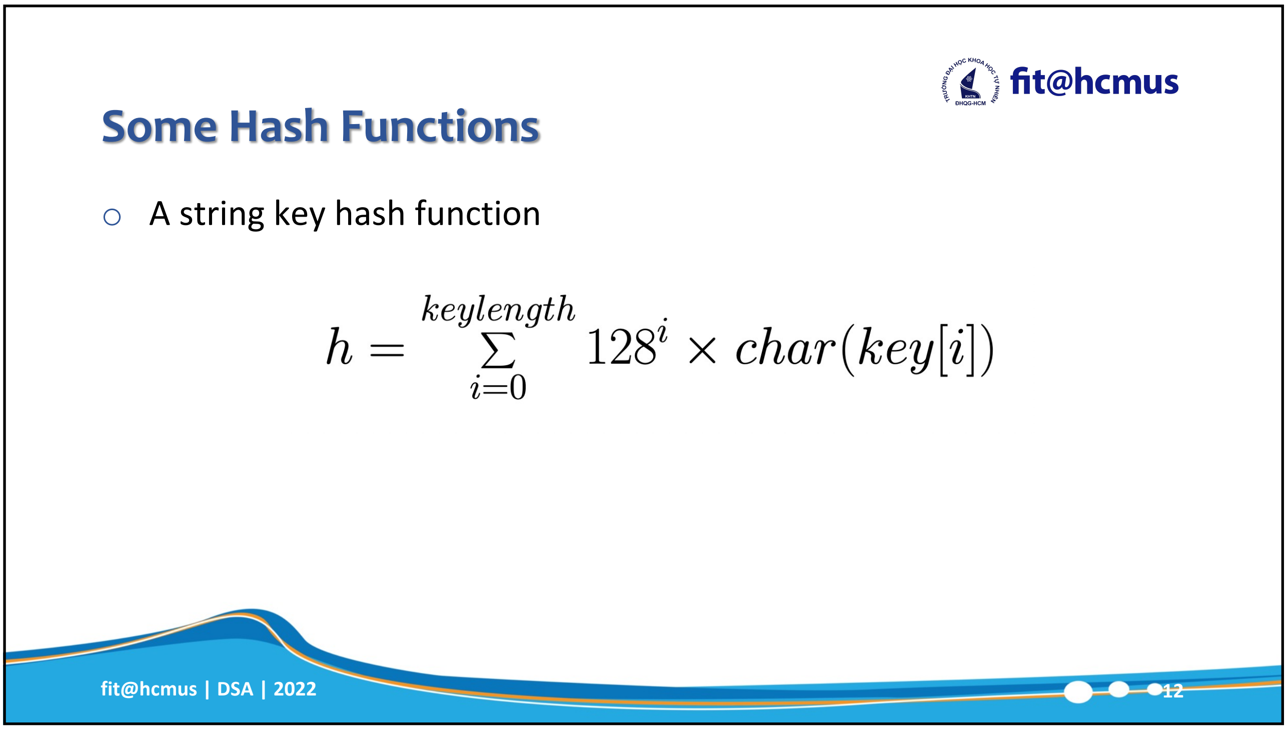

Some Hash Functions

- A string key hash function

$$h = \sum_{i=0}^{keylength} 128^{i} \times char(key[i])$$

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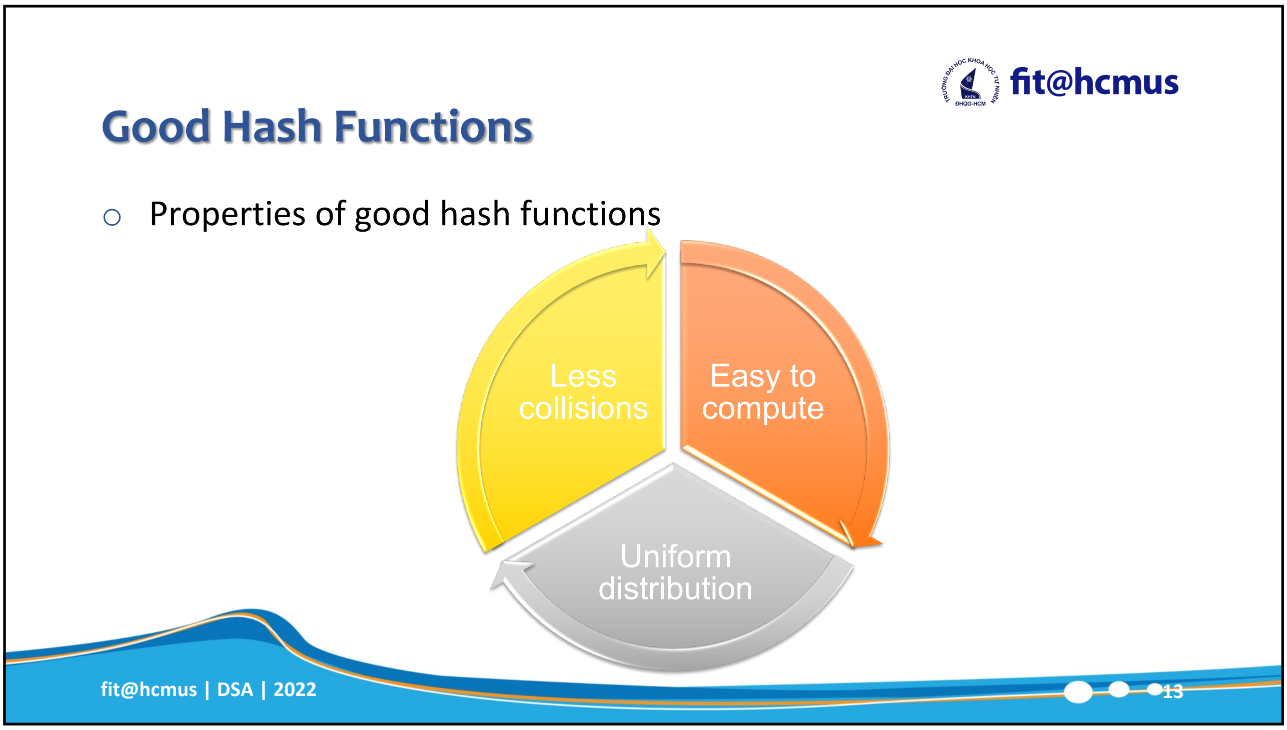

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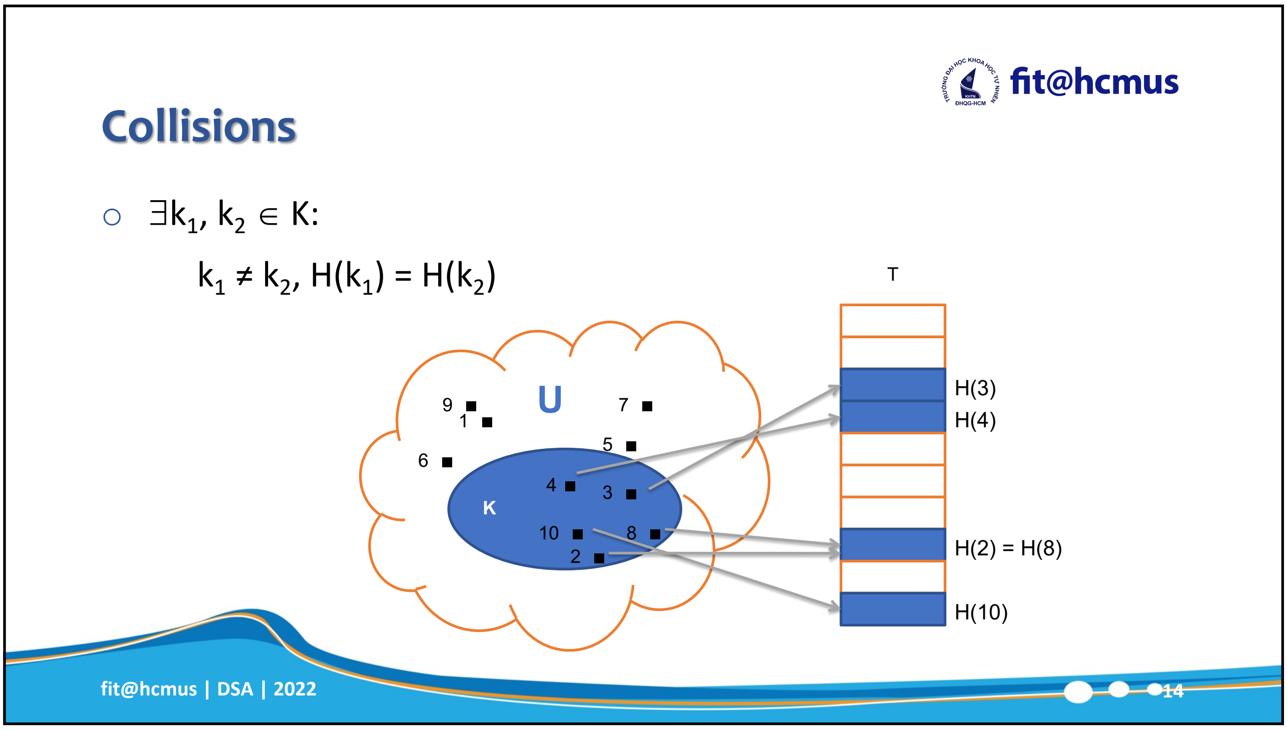
Good Hash Functions

- Properties of good hash functions

Less collisions
Easy to compute
Uniform distribution



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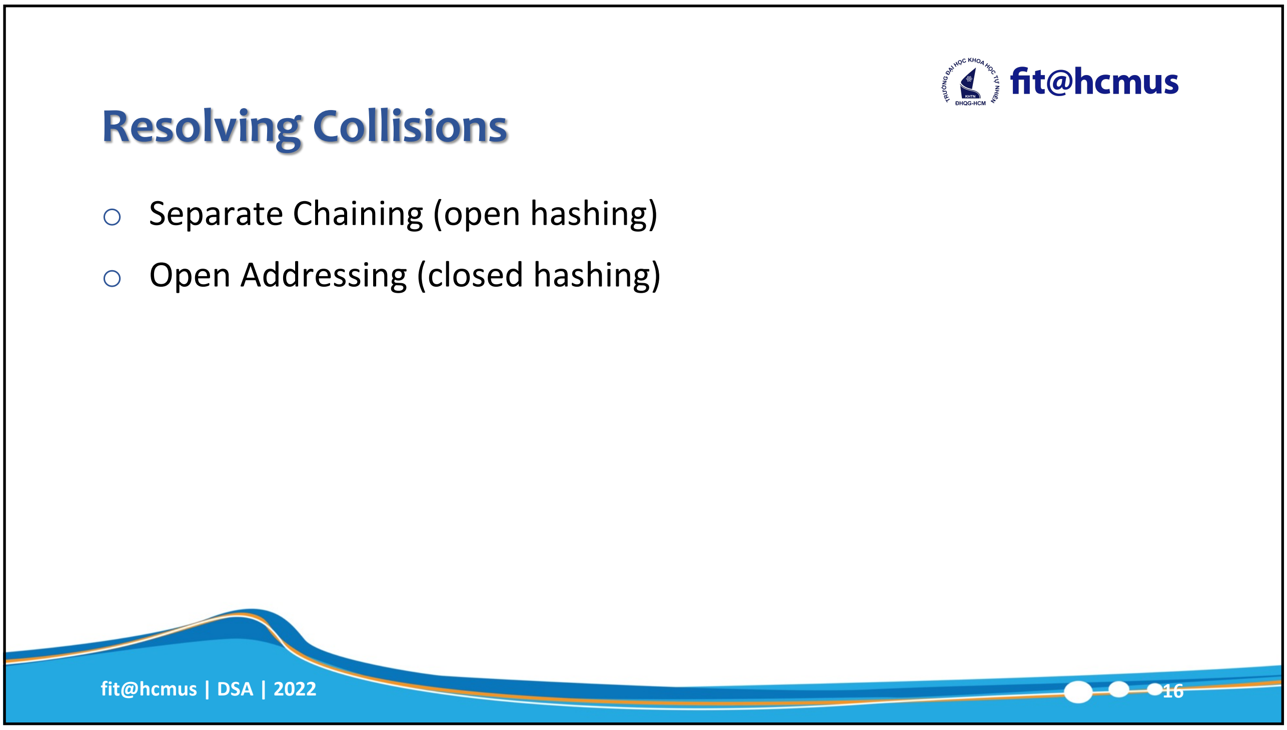
Collisions

- $\exists k_1, k_2 \in K$:

  $k_1 \neq k_2, H(k_1) = H(k_2)$

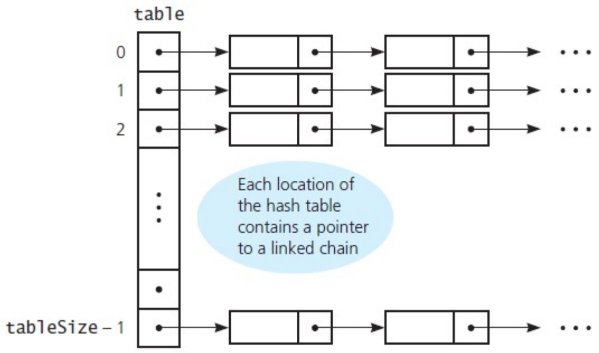
Resolving Collisions



Resolving Collisions

- ○ Separate Chaining (open hashing)
- ○ Open Addressing (closed hashing)

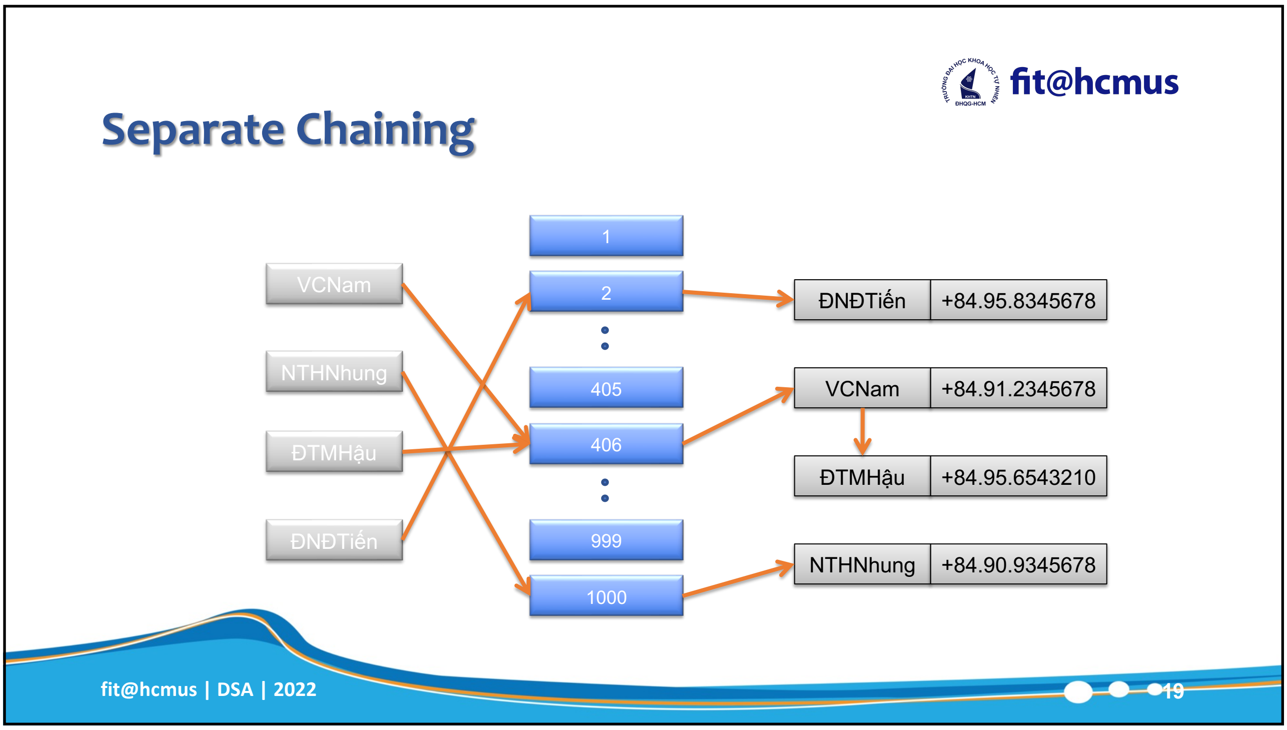
Separate Chaining



Each location of the hash table contains a pointer to a linked chain

Separate Chaining

- ○ Each hash location can accommodate more than one item
- ○ Each location is a “bucket” or an array itself
- ○ Alternatively, design the hash table as an array of linked chains (“*separate chaining*”).



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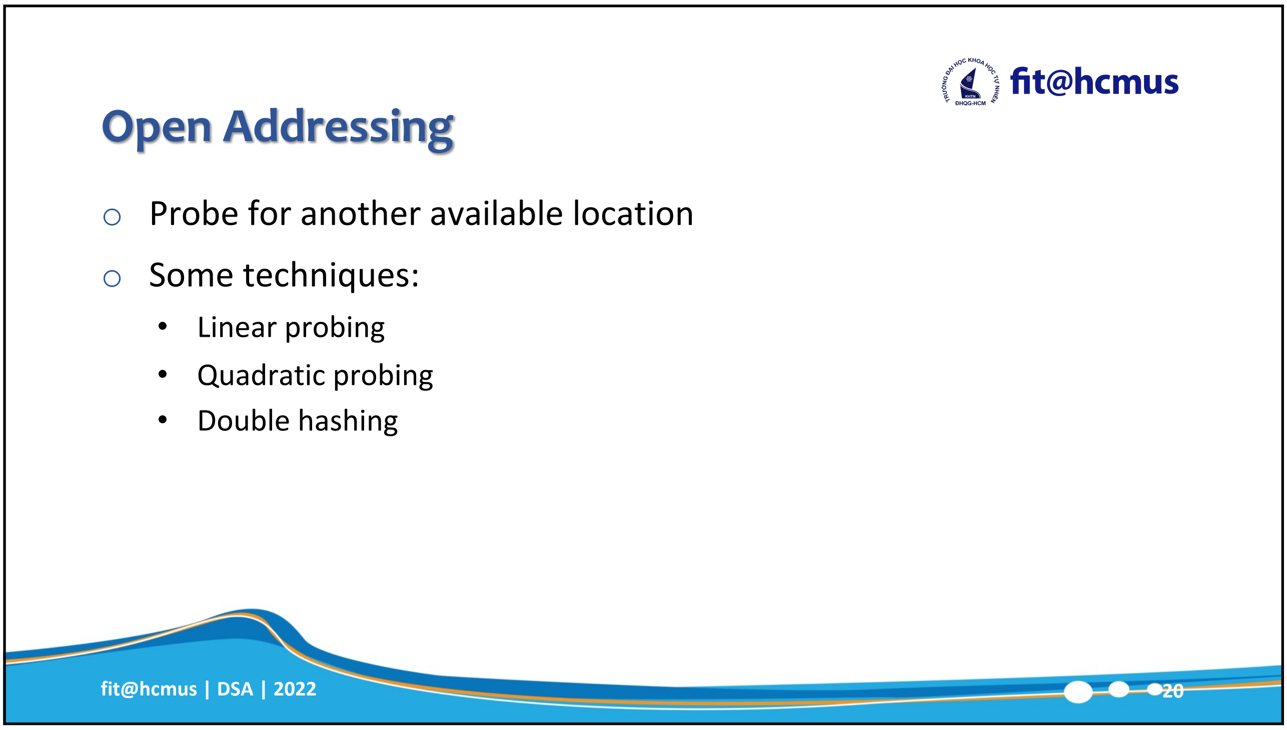
Separate Chaining

VCNam
NTHNhung
ĐTMHậu
ĐNĐTiến

1
2
405
406
999
1000

ĐNĐTiến	+84.95.8345678
VCNam	+84.91.2345678
ĐTMHậu	+84.95.6543210
NTHNhung	+84.90.9345678



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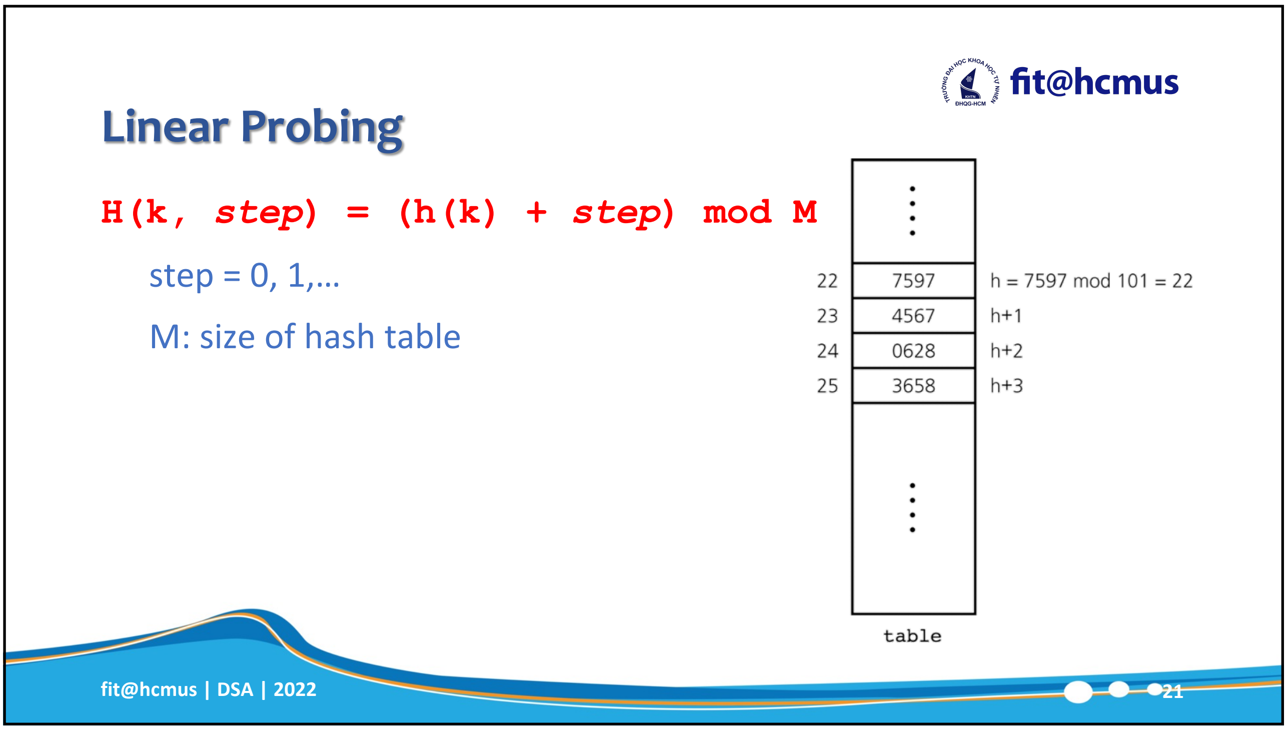
Open Addressing

- Probe for another available location
- Some techniques:
  - Linear probing
  - Quadratic probing
  - Double hashing



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Linear Probing

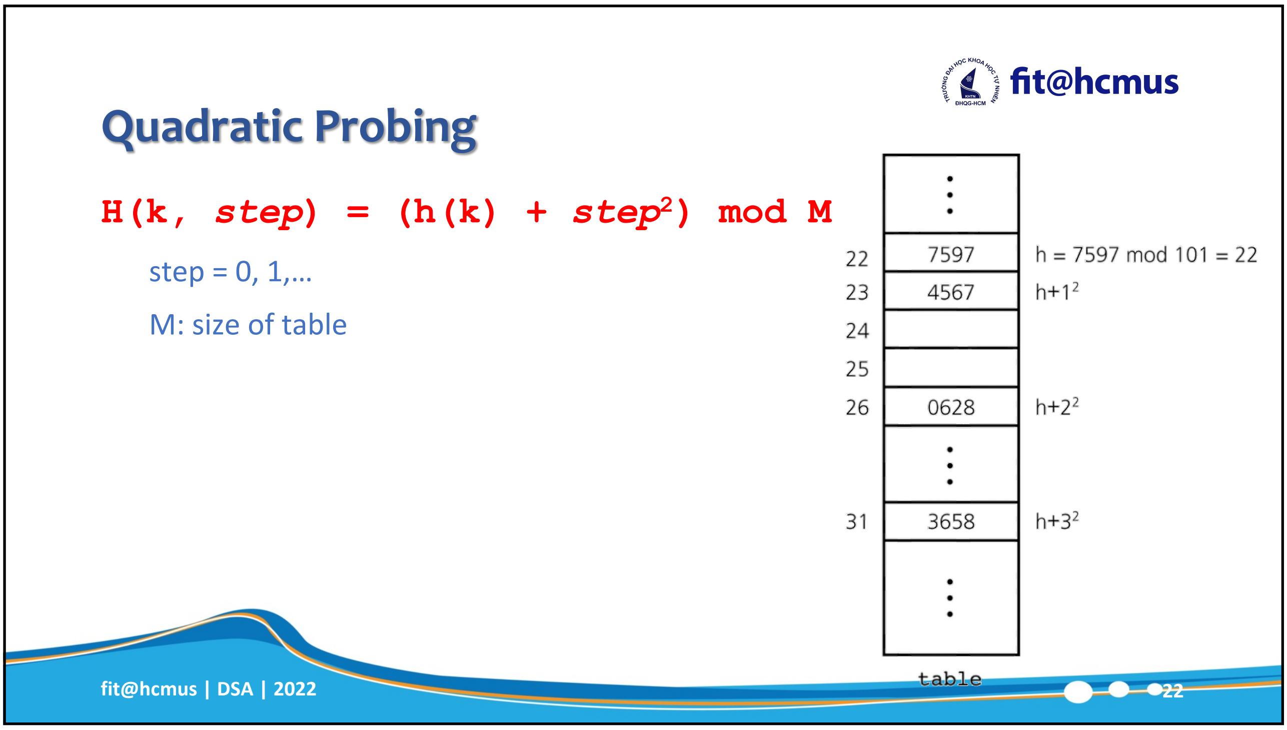
$$H(k, step) = (h(k) + step) \bmod M$$

step = 0, 1,…

M: size of hash table

	⋮	
22	7597	h = 7597 mod 101 = 22
23	4567	h+1
24	0628	h+2
25	3658	h+3
	⋮	

table



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Quadratic Probing

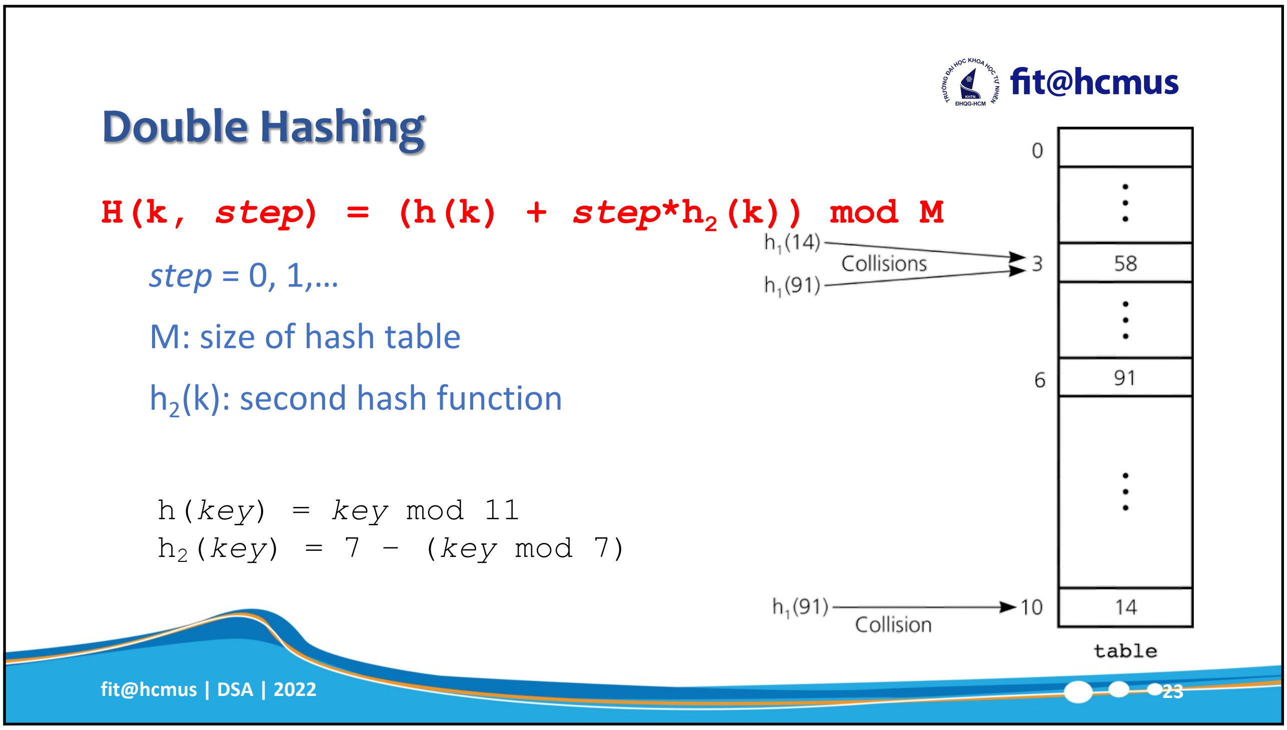
$$H(k, step) = (h(k) + step^2) \bmod M$$

step = 0, 1,…

M: size of table

	⋮	
22	7597	h = 7597 mod 101 = 22
23	4567	$h+1^2$
24		
25		
26	0628	$h+2^2$
	⋮	
31	3658	$h+3^2$
	⋮	

table



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Double Hashing

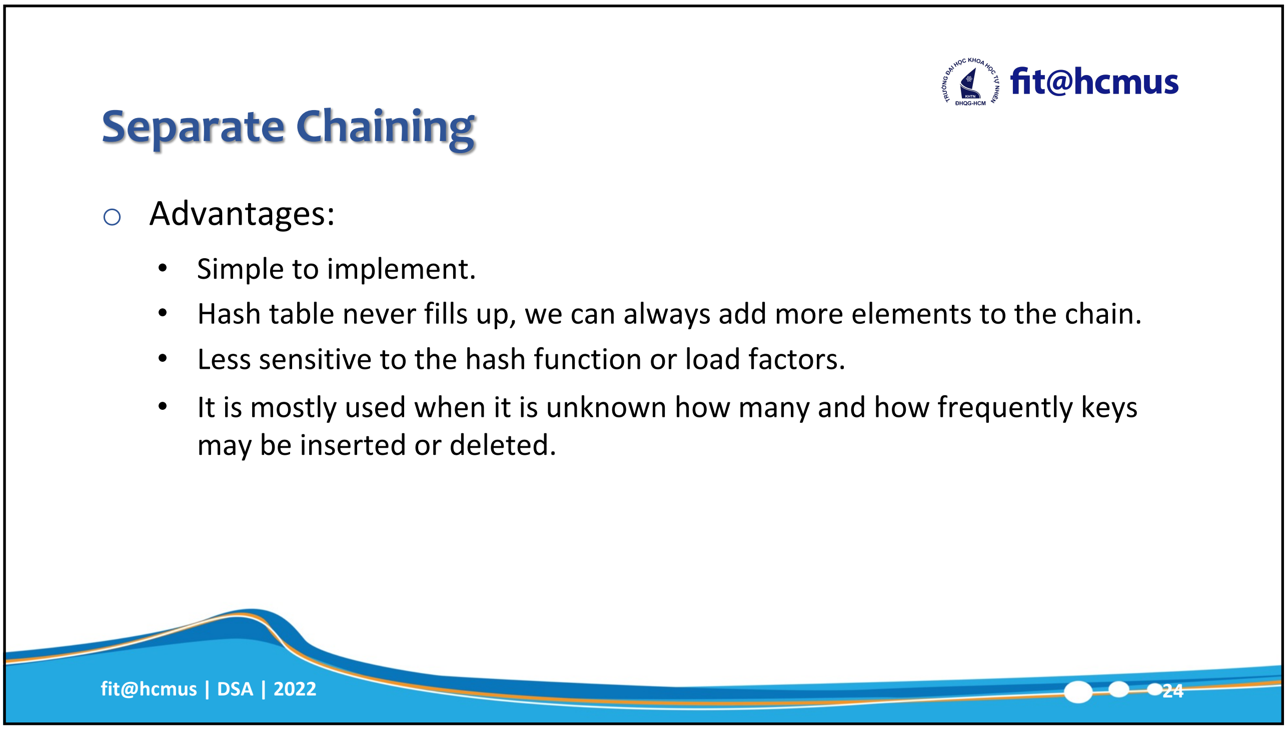
$H(k, step) = (h(k) + step * h_2(k)) \bmod M$

step = 0, 1,…

M: size of hash table

$h_2(k)$: second hash function

$h(key) = key \bmod 11$
$h_2(key) = 7 - (key \bmod 7)$



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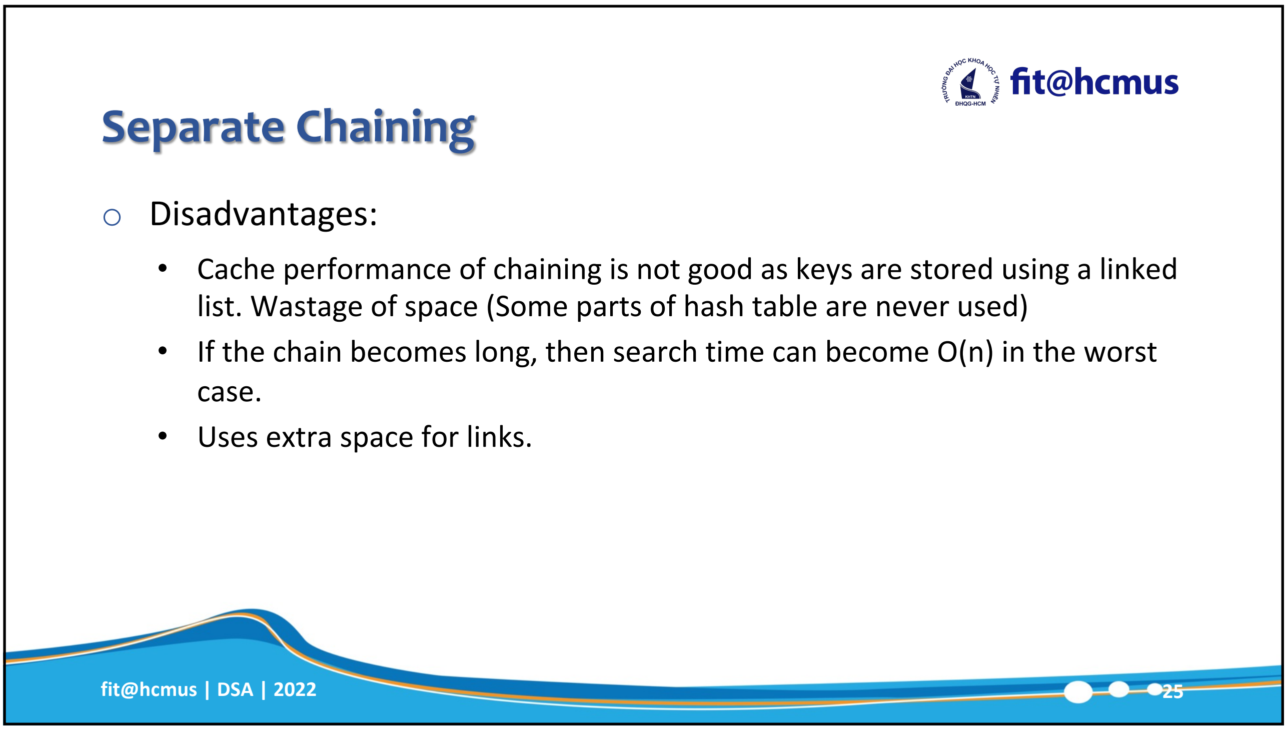
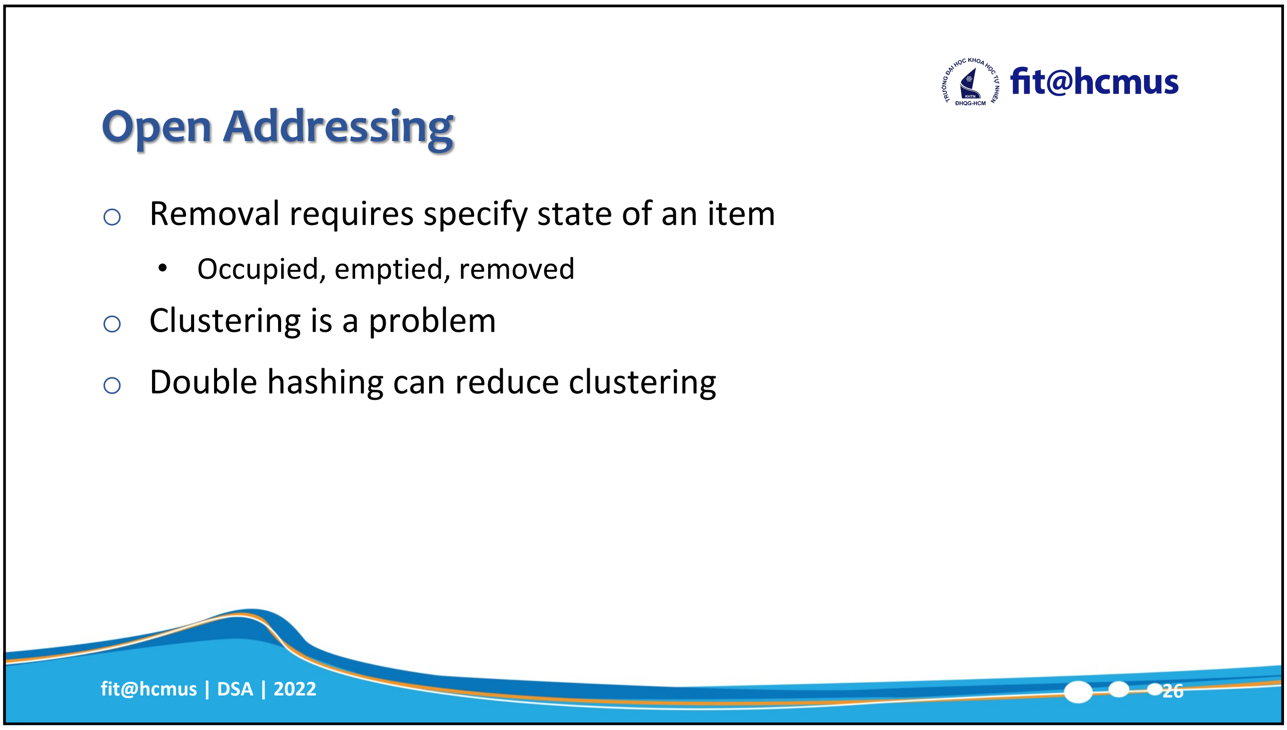
Separate Chaining

○ Advantages:
  - Simple to implement.
  - Hash table never fills up, we can always add more elements to the chain.
  - Less sensitive to the hash function or load factors.
  - It is mostly used when it is unknown how many and how frequently keys may be inserted or deleted.



Separate Chaining

- ○ Disadvantages:
  - Cache performance of chaining is not good as keys are stored using a linked list. Wastage of space (Some parts of hash table are never used)
  - If the chain becomes long, then search time can become O(n) in the worst case.
  - Uses extra space for links.



Open Addressing

- ○ Removal requires specify state of an item
  - Occupied, emptied, removed
- ○ Clustering is a problem
- ○ Double hashing can reduce clustering

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The Efficiency of Hashing

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The Efficiency of Hashing

- Efficiency of hashing involves the load factor alpha (α)

$$\alpha = \frac{\textit{Current number of table items}}{\textit{tableSize}}$$

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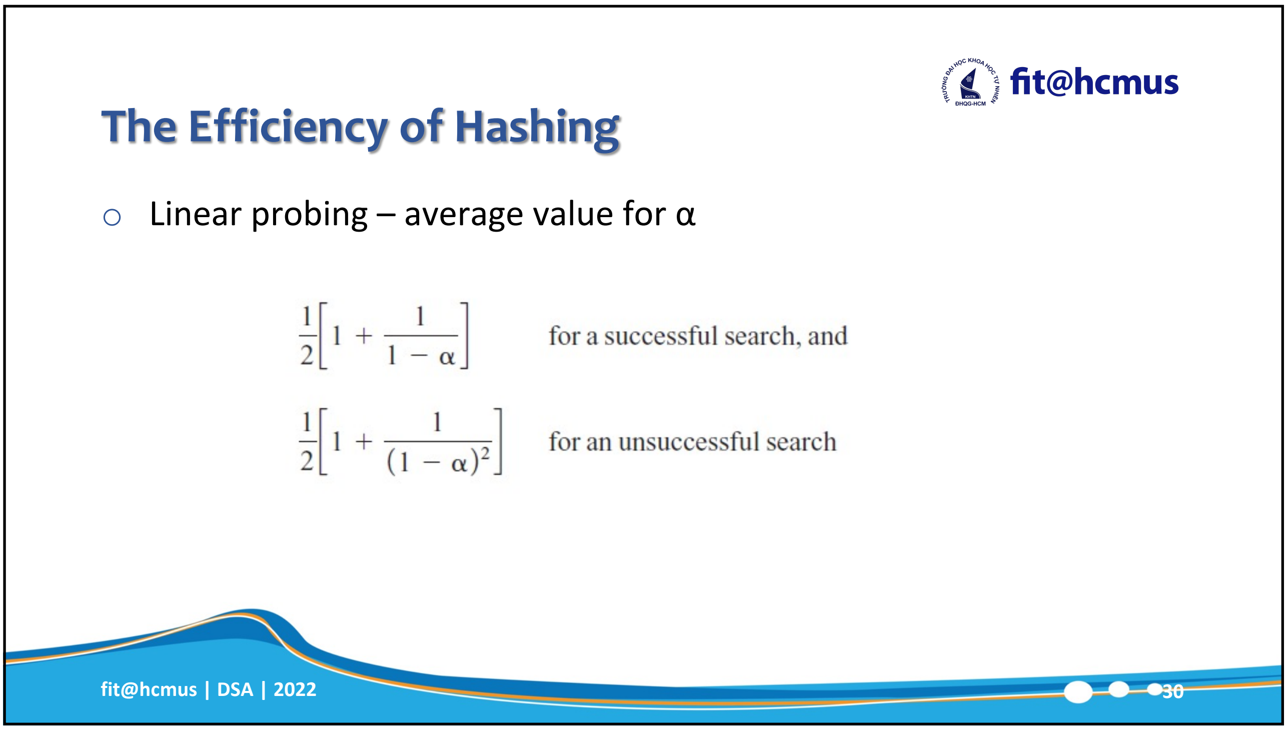

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The Efficiency of Hashing

- ○ Linear probing – average value for α

$$\frac{1}{2}\left[1 + \frac{1}{1 - \alpha}\right]$$ for a successful search, and

$$\frac{1}{2}\left[1 + \frac{1}{(1 - \alpha)^2}\right]$$ for an unsuccessful search

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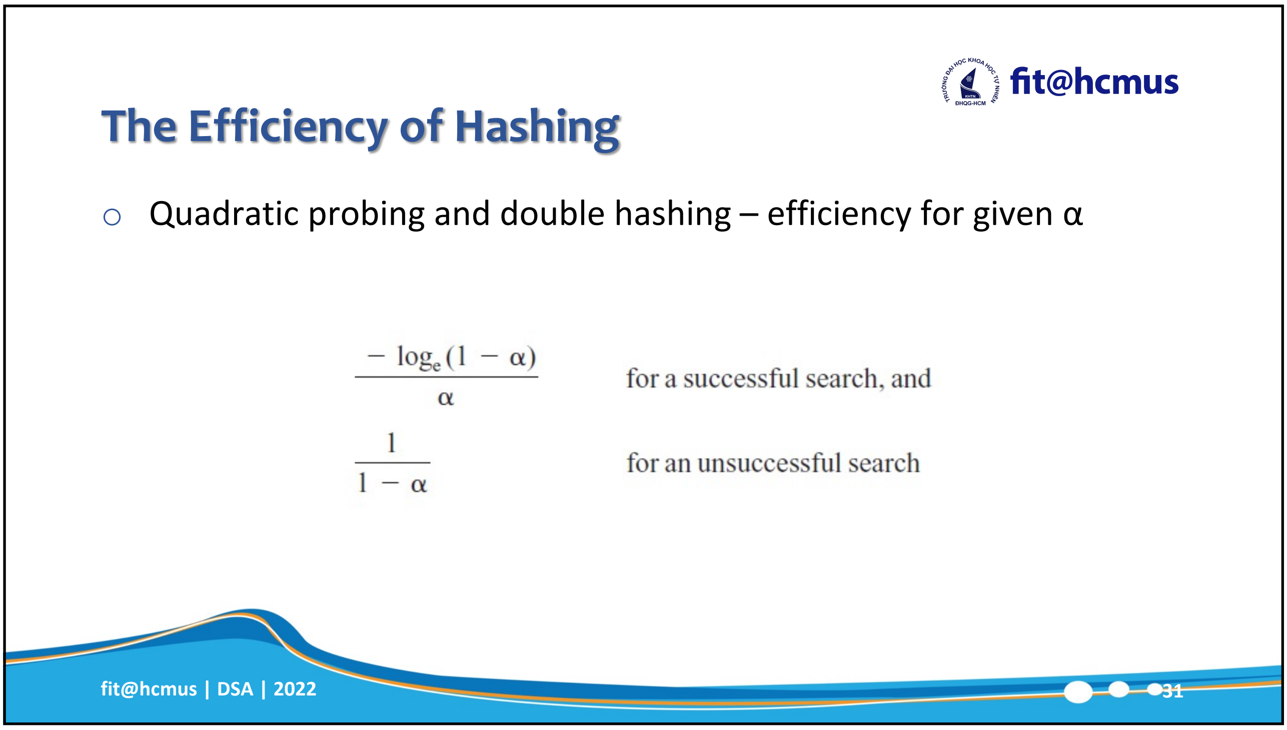

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The Efficiency of Hashing

- ○ Quadratic probing and double hashing – efficiency for given α

$$\frac{-\log_e(1 - \alpha)}{\alpha}$$ for a successful search, and

$$\frac{1}{1 - \alpha}$$ for an unsuccessful search

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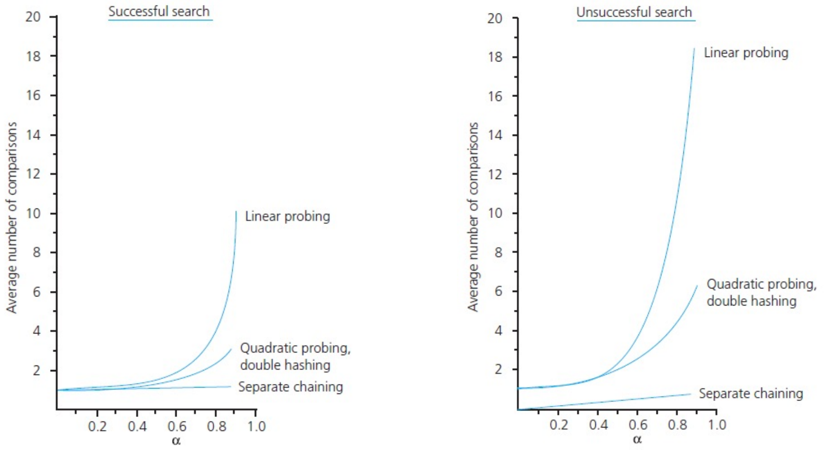
The Efficiency of Hashing

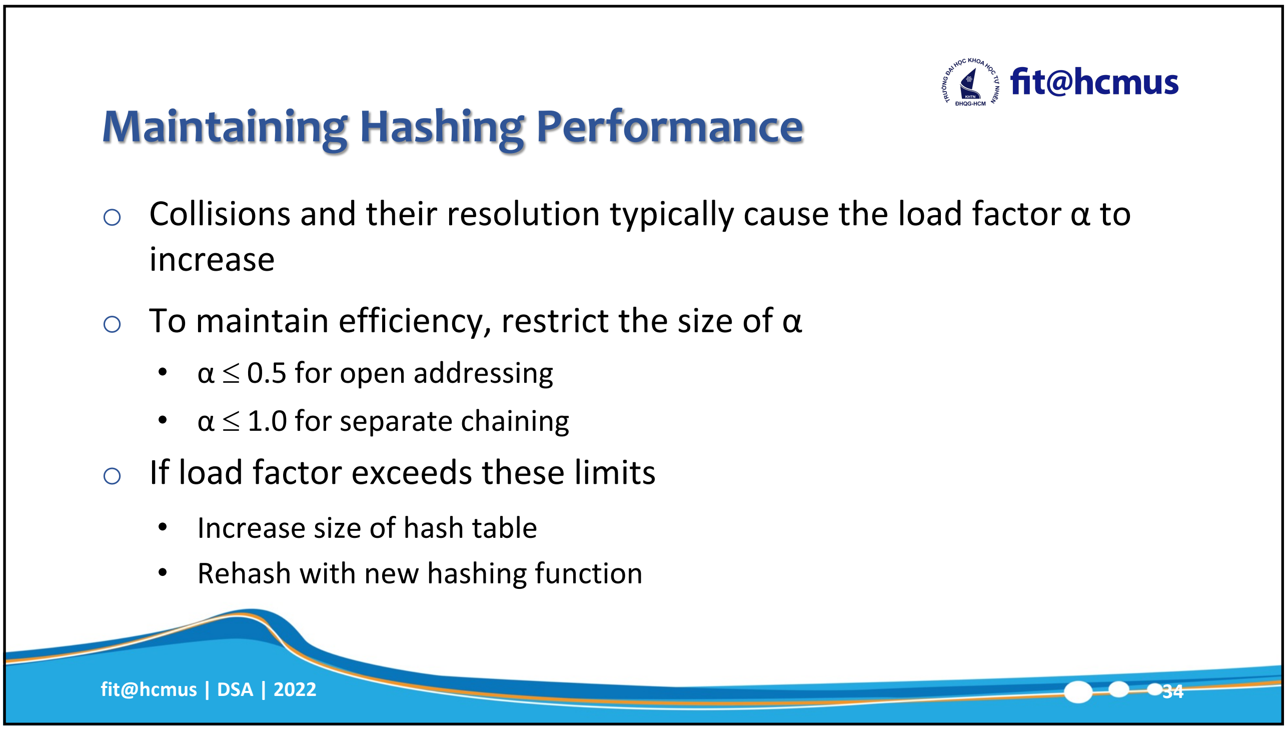
- Separate chaining – efficiency for given α

$1 + \frac{\alpha}{2}$ for a successful search, and

α for an unsuccessful search

The Efficiency of Hashing





Maintaining Hashing Performance

- Collisions and their resolution typically cause the load factor α to increase
- To maintain efficiency, restrict the size of α
  - α ≤ 0.5 for open addressing
  - α ≤ 1.0 for separate chaining
- If load factor exceeds these limits
  - Increase size of hash table
  - Rehash with new hashing function

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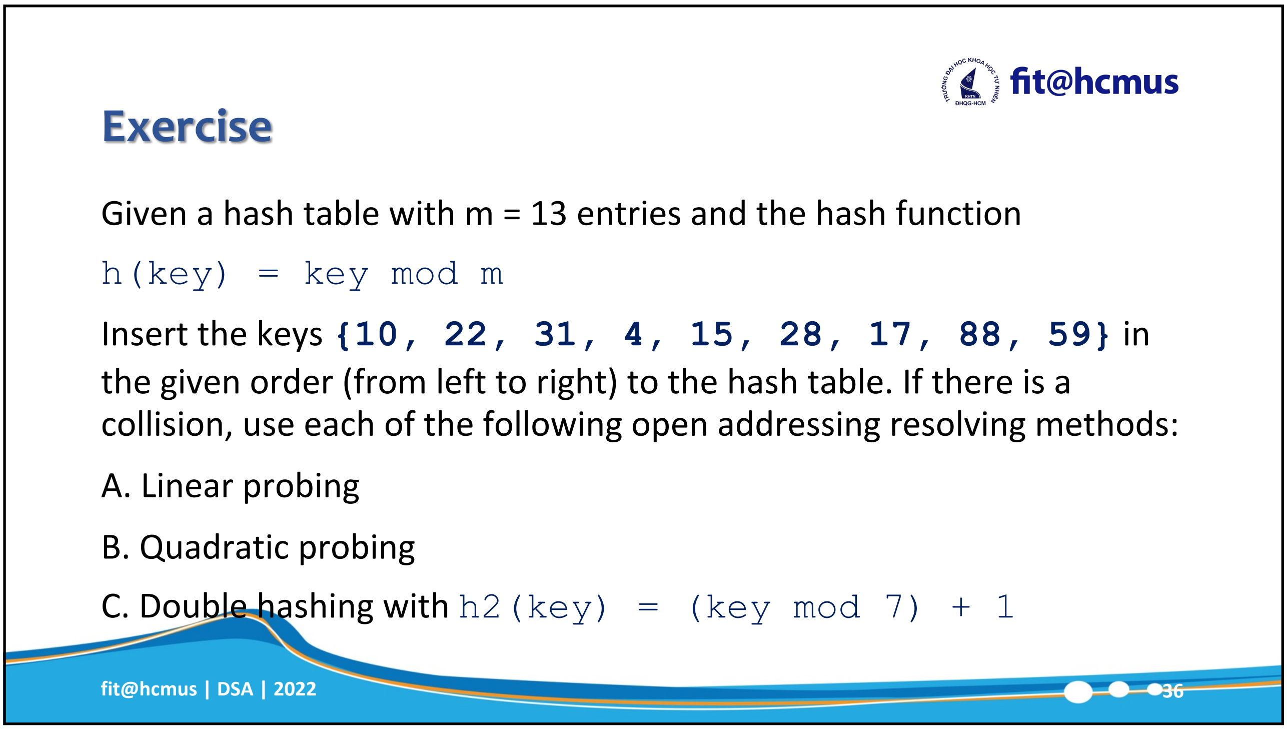

Exercise

Given a hash table with m = 13 entries and the hash function

`h(key) = key mod m`

Insert the keys **`{10, 22, 31, 4, 15, 28, 17, 88, 59}`** in the given order (from left to right) to the hash table. If there is a collision, use each of the following open addressing resolving methods:

A. Linear probing

B. Quadratic probing

C. Double hashing with `h2(key) = (key mod 7) + 1`

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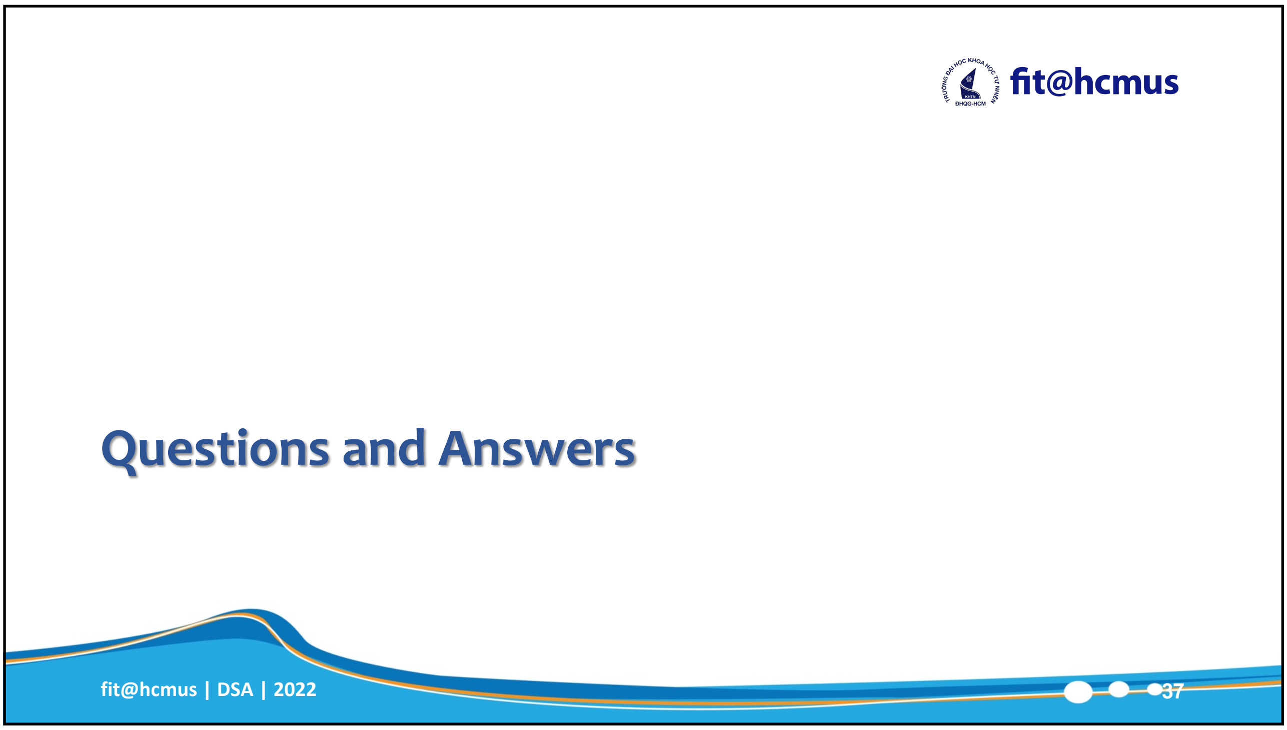

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Questions and Answers