University of Science, VNU-HCM
Faculty of Information Technology

Data Structure and Algorithm

M-way Tree
BTree

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HCM City

Outline

- **M-way Tree**
- B-Tree

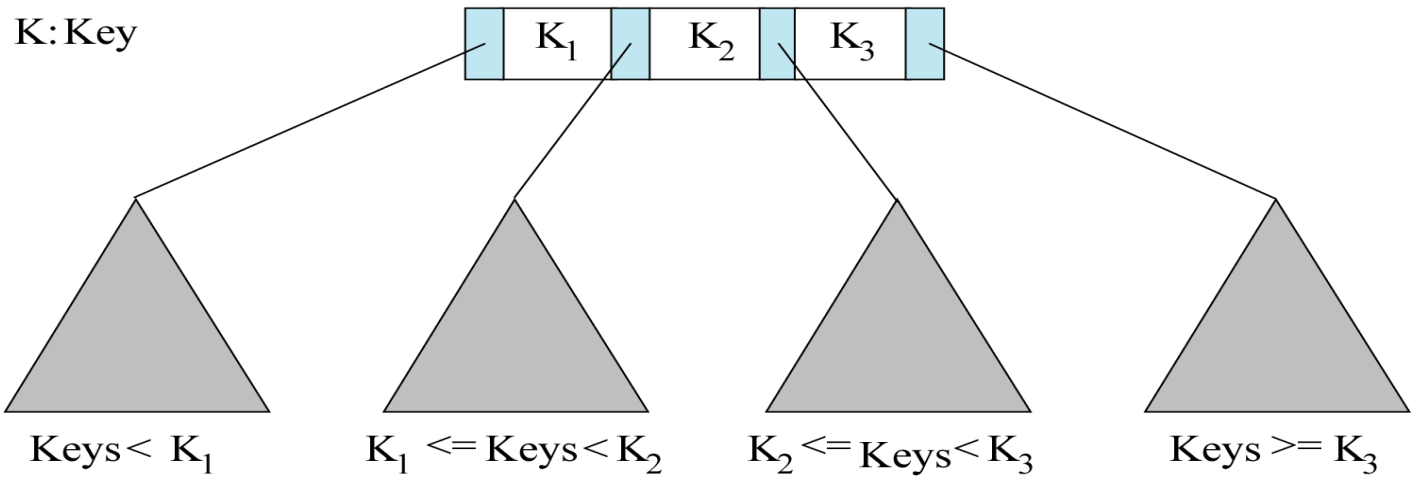
About binary tree

- Binary tree: each node has 1 data value and 2 branches
- Some disadvantages:
  - Each stored value need at least two more memory (left pointer, right pointer)
  - Tree can be very high if number of values is large.
  - Implement a balanced binary search tree is complex.

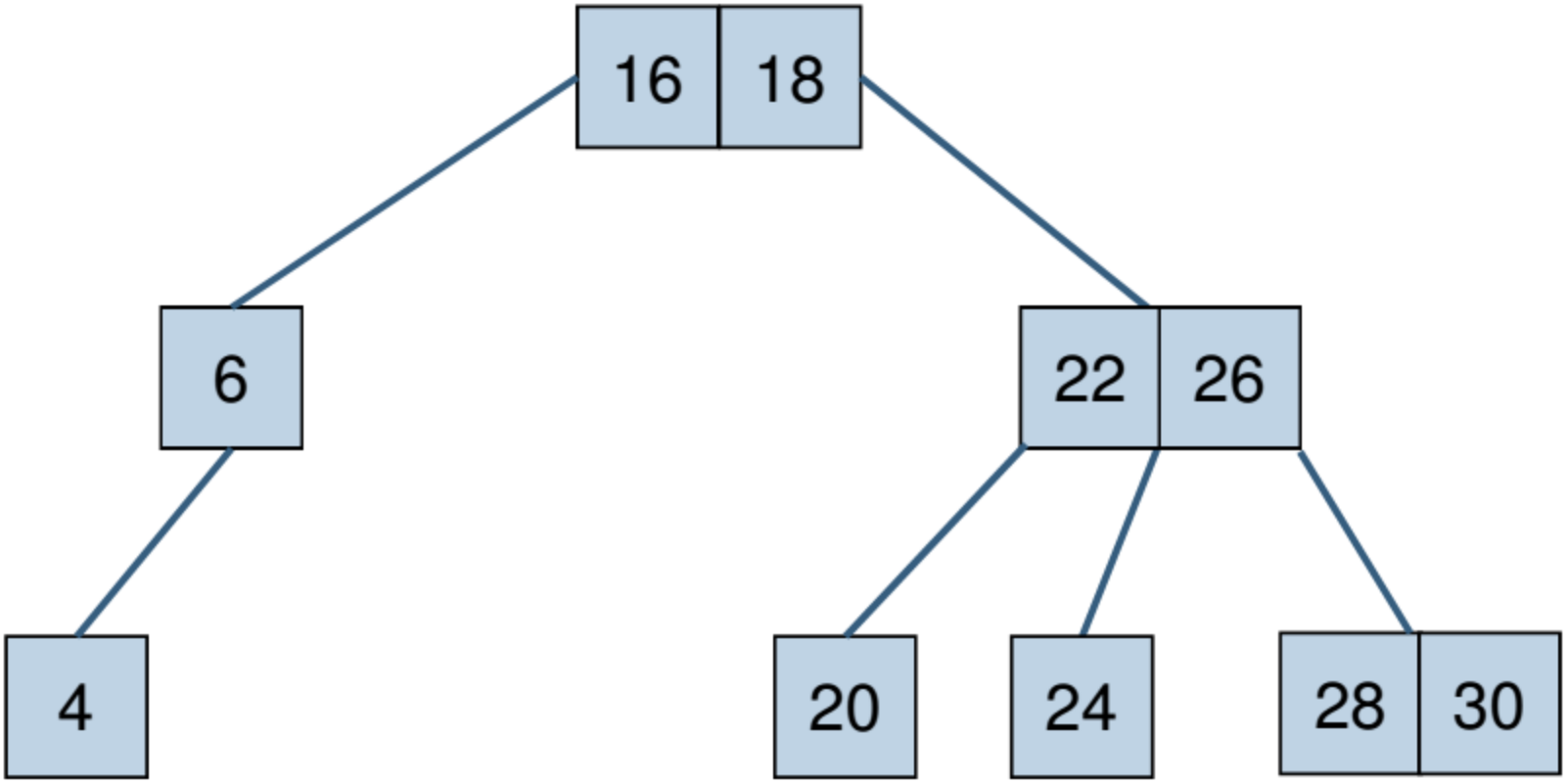
Multi-way Tree

- The multi-way tree (m-way tree) is a tree:
  - Each node contains 1 to m-1 keys with distinct values
  - The keys in each node are ordered (ascending).
  - A node with k keys will have k + 1 subtree, the subtree can be empty.
  - The i^{th} subtree ($0 \le i \le k$) of the node contains the keys such that:

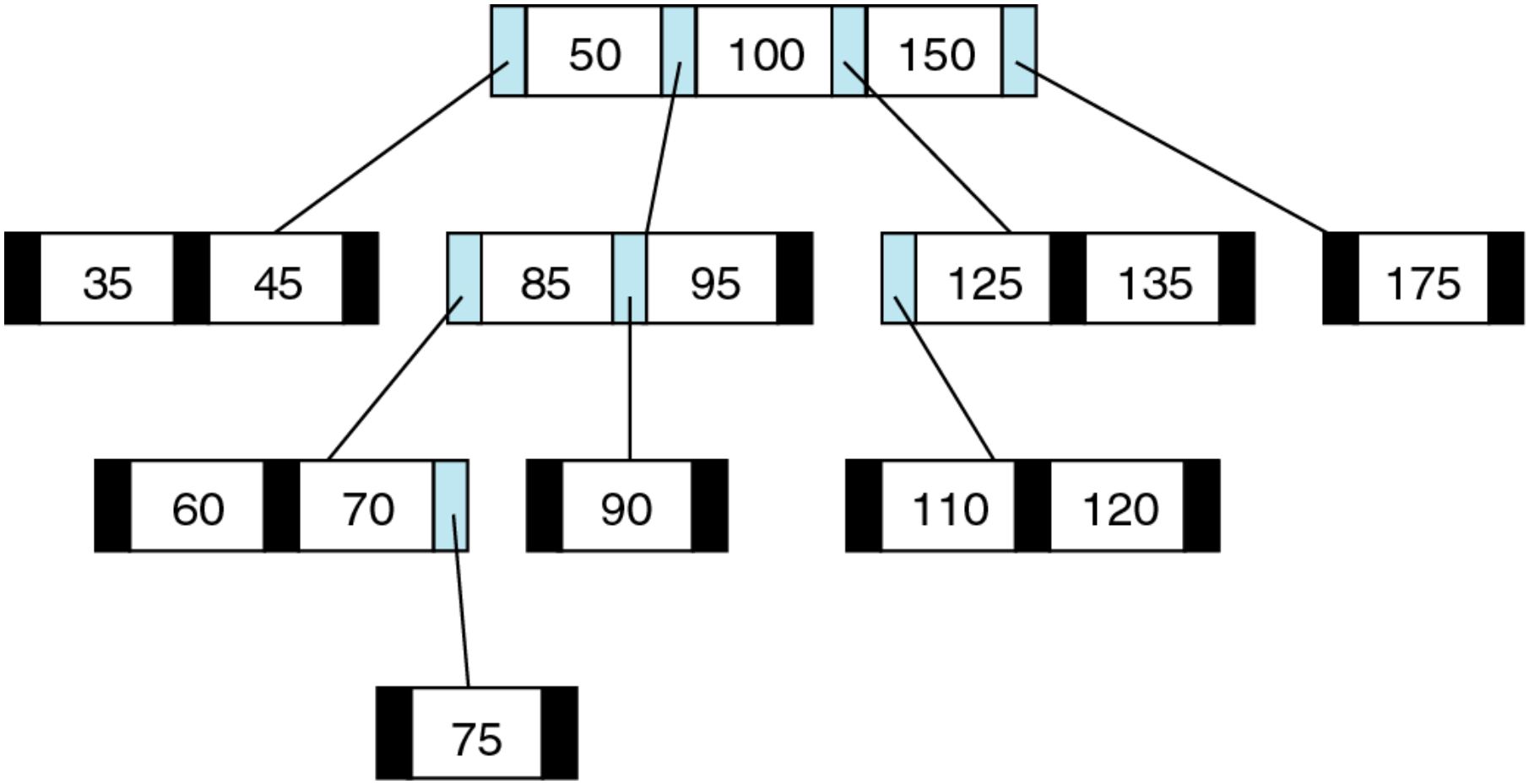
$$v_i \le v \le v_i + 1$$



3-way tree example



4-way tree example

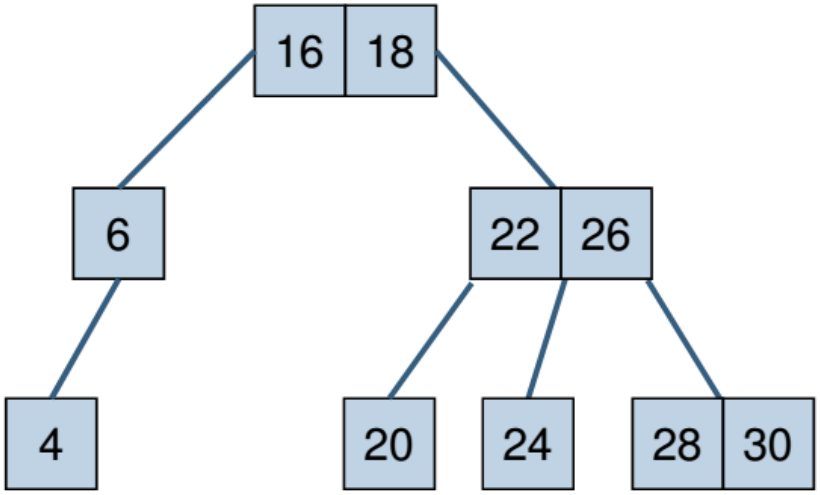


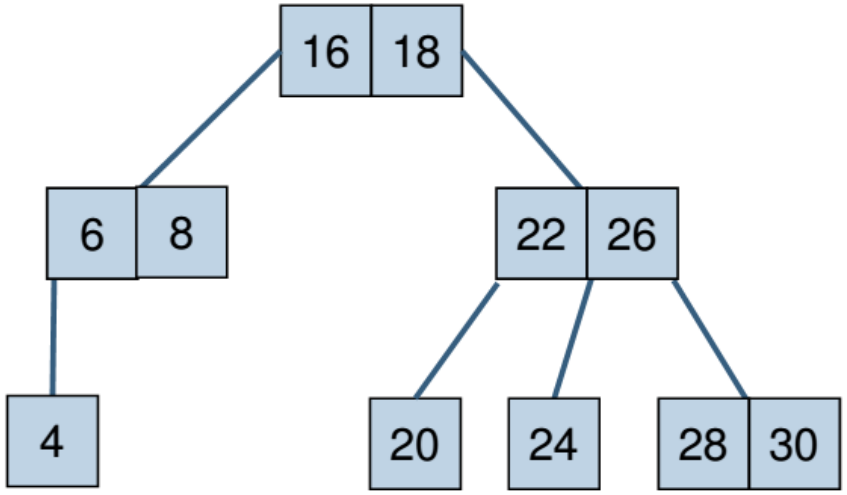
Insert new element

- Insert a key v into the tree:
  - Travese the tree until it finds an empty node
  - Case 1: If the parent node still has slot: add the key v to the parent node at this slot.
  - Case 2: If the parent node is full: create a new node and add the key v to it.

Insert new element

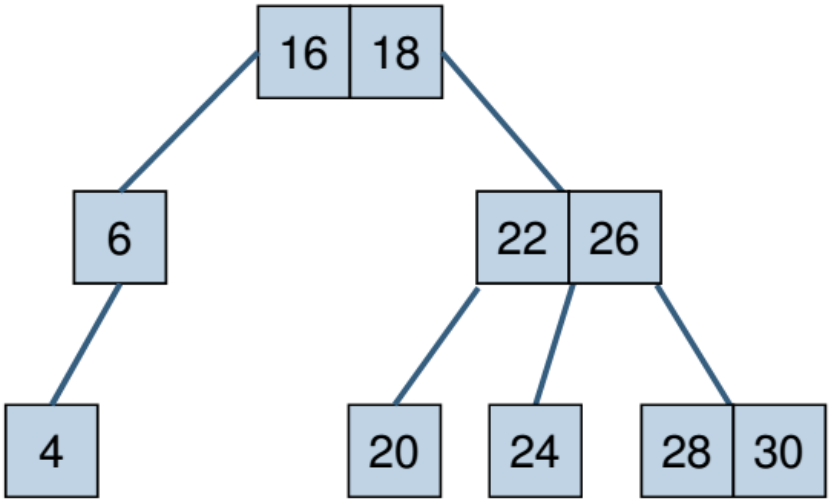
- Insert 8 to 3-way tree (case 1)

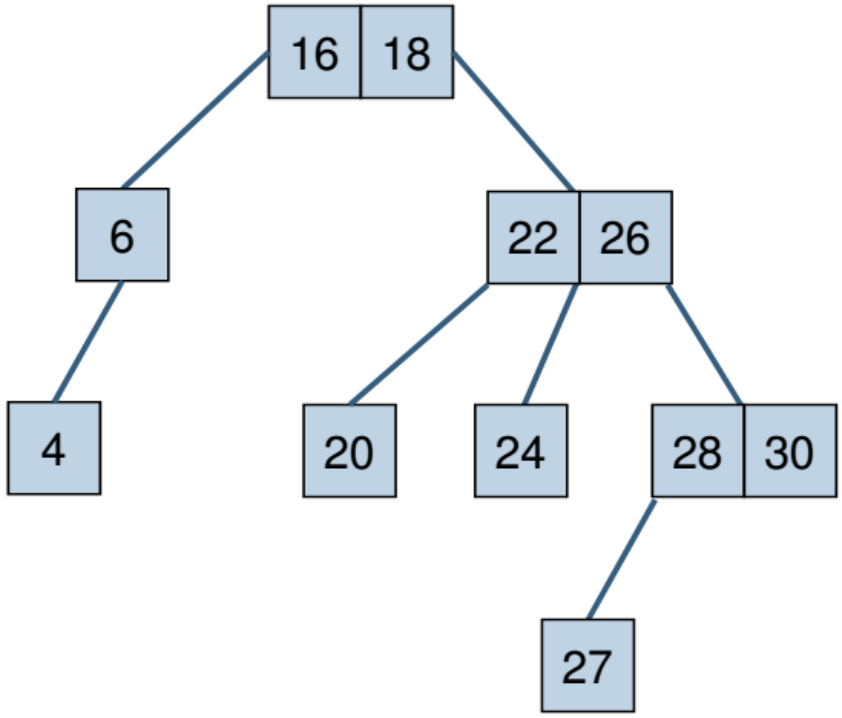




Insert new element

- Insert 27 to 3-way tree (case 2):



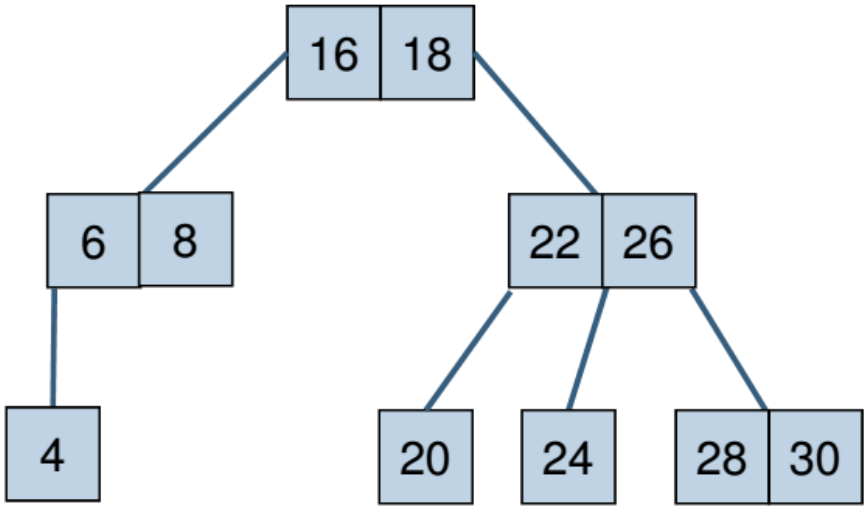


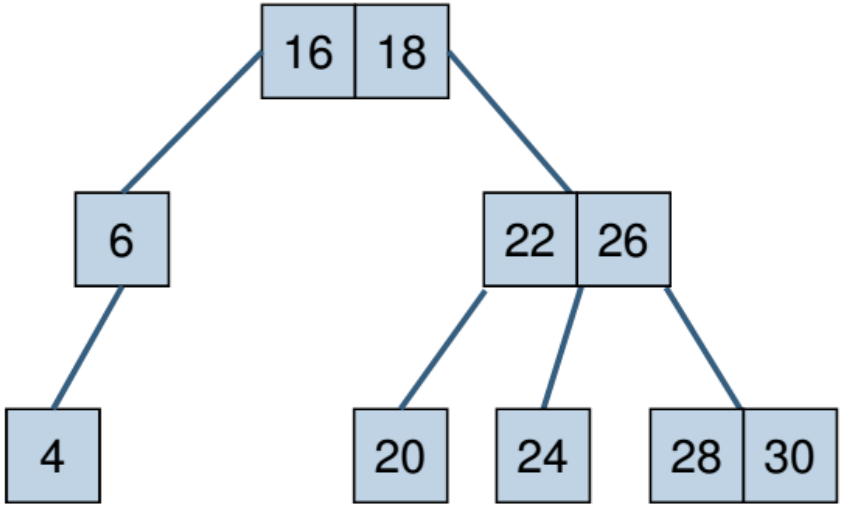
Delete an element

- Delete a key v from the tree:
  - Case 1: If v has no child (between 2 empty subtree) then just delete v.
  - Case 2: If v has any child, replace v with largest element in the left subtree of v or smallest element in the right subtree of v.

Delete an element

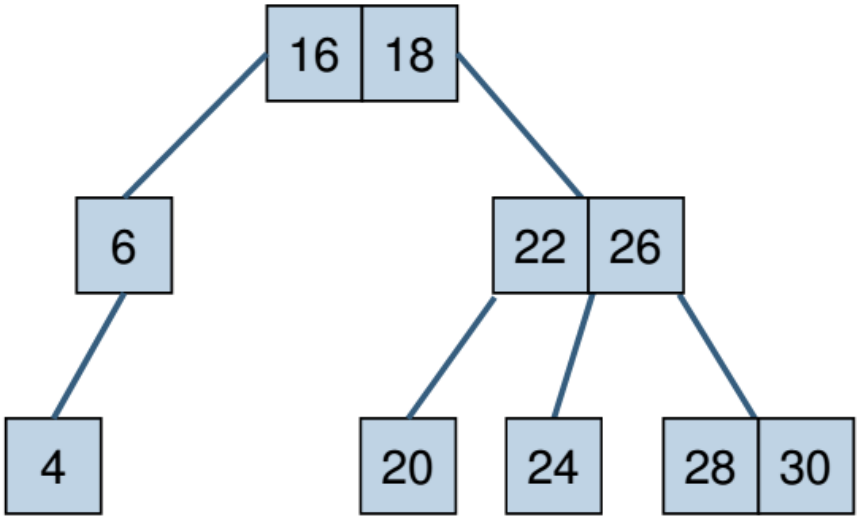
- Delete 8 (case 1)

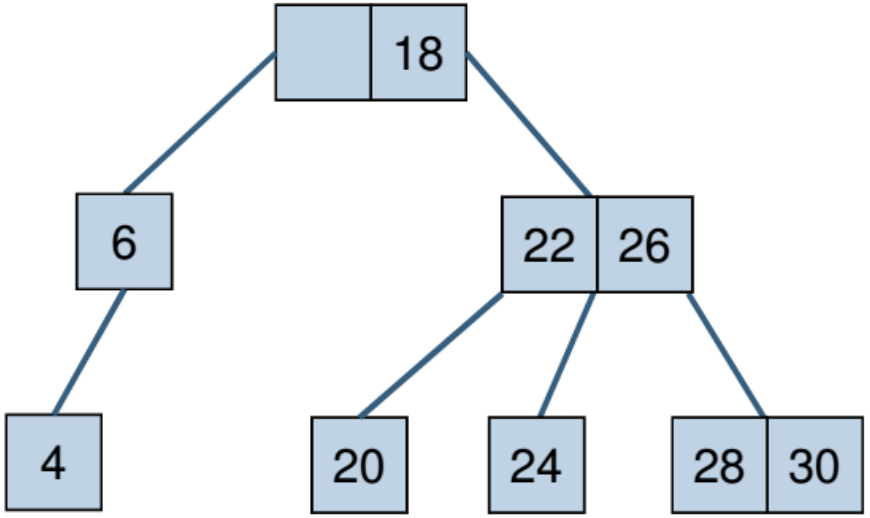




Delete an element

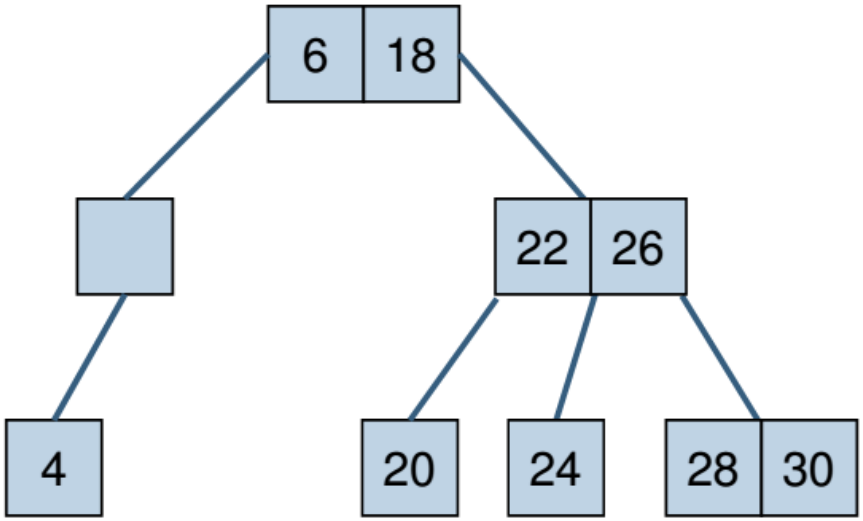
- Delete 16 (case 2):

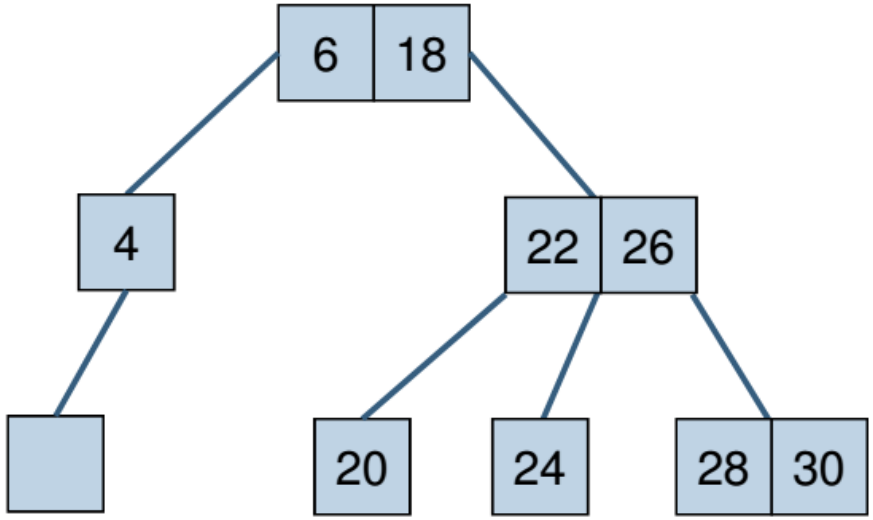




Delete an element

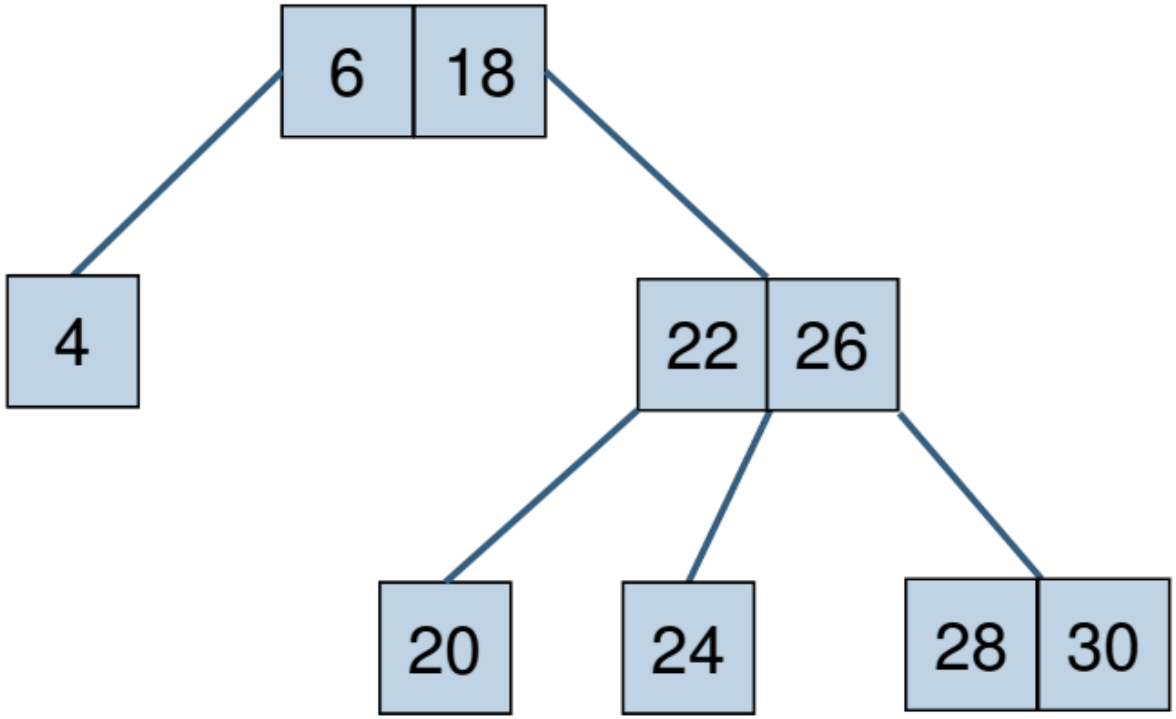
- Delete 16 (case 2):





Delete an element

- Delete 16 (case 2):

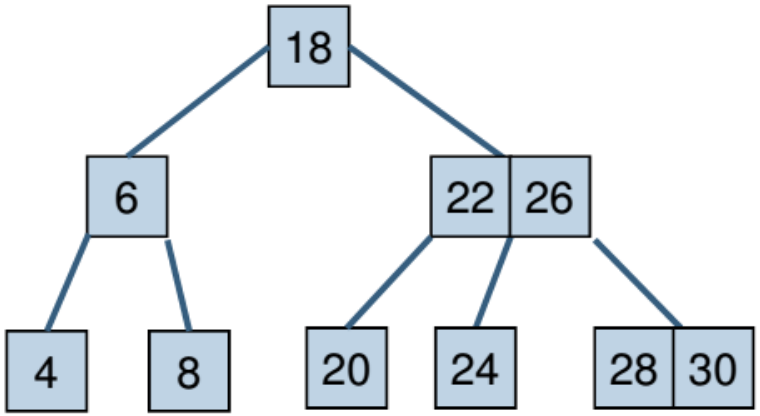


Outline

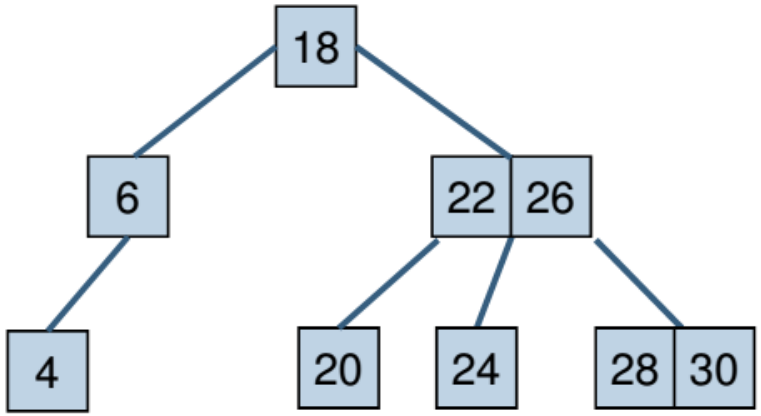
- M-way Tree
- **B-Tree**

Balanced m-way Tree: B-Tree

- B-Tree is a m-way tree which satisfies:
  - Root node has at least 1 key
  - Branch nodes have at least [(m-1) / 2] +1 subtree
    - i.e. have at least [(m-1) / 2] keys
  - All empty nodes belong to the same level



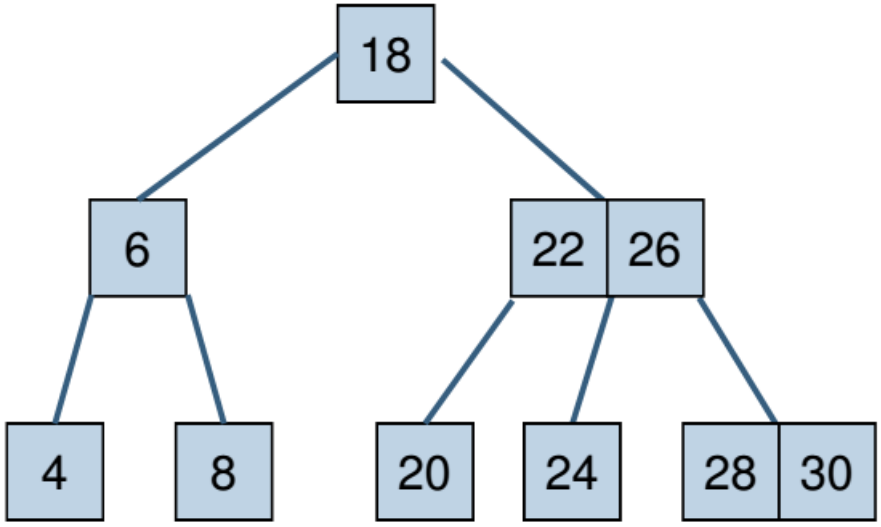
B-Tree



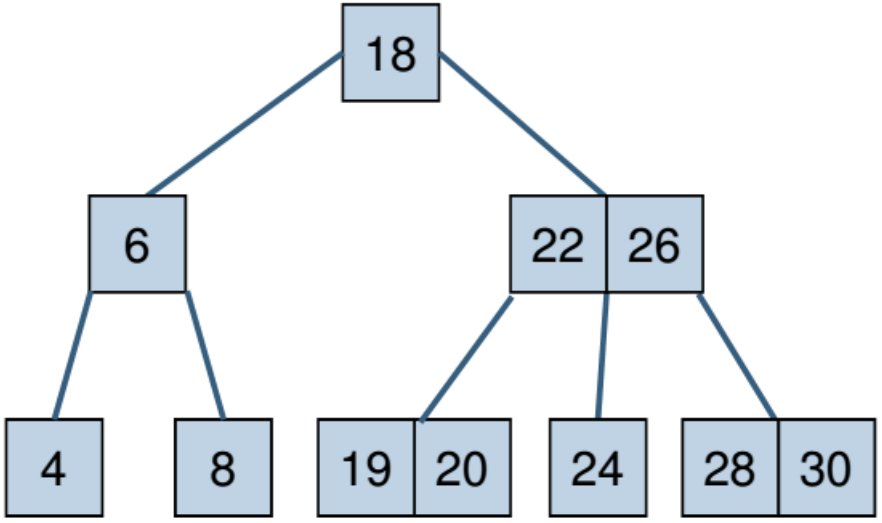
Not a B-Tree

Insert new element in B-Tree

- Insert a key v to the B-tree
  - Add v to a leaf node
  - If the leaf node is full: splits the leaf node in half and moves the middle element onto the parent node.

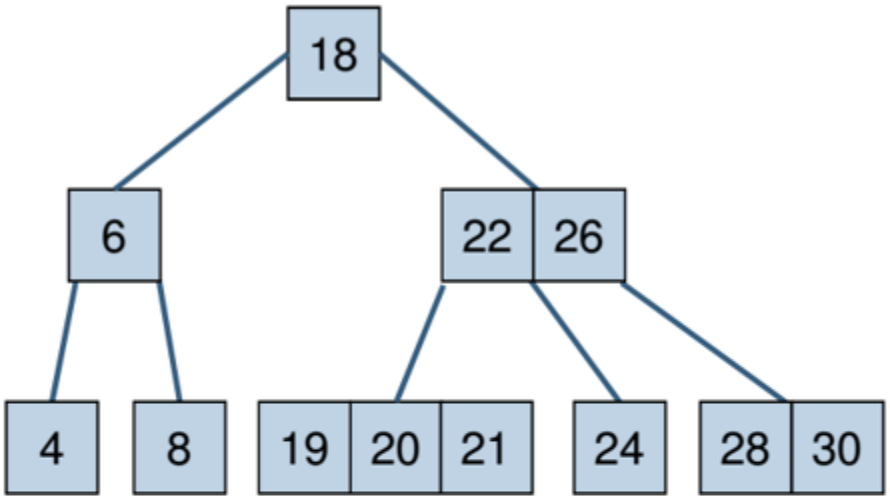


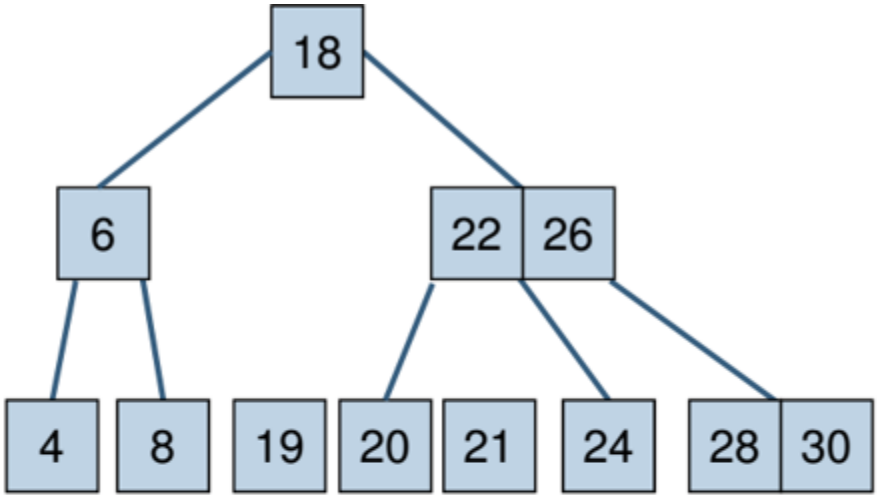
B-Tree

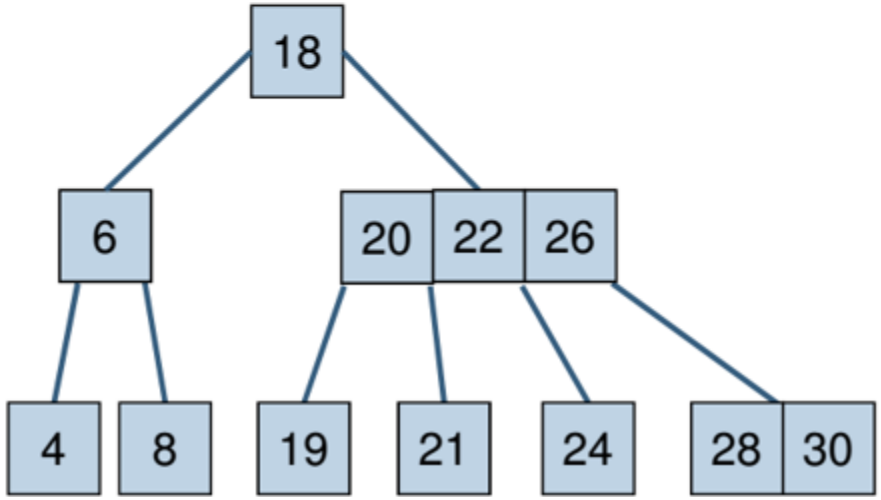


Insert value 19

Example for 3-way tree



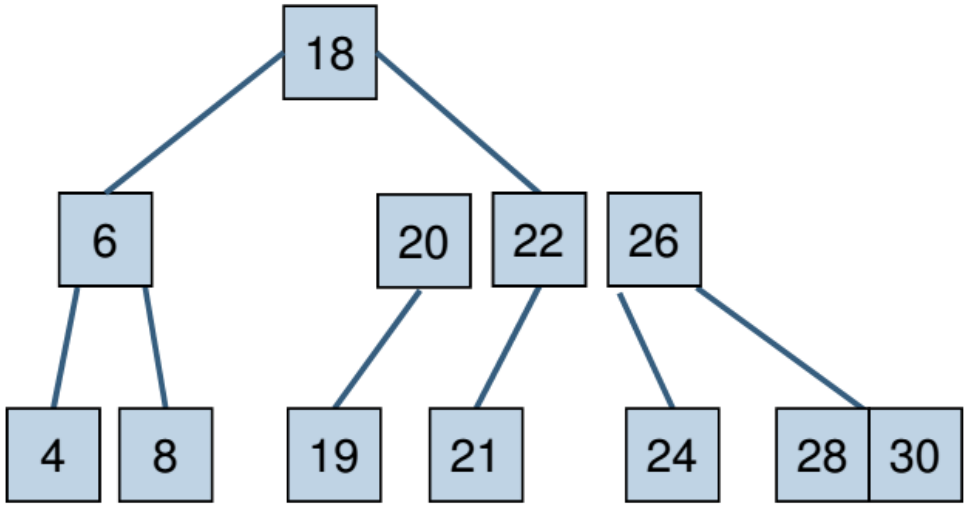


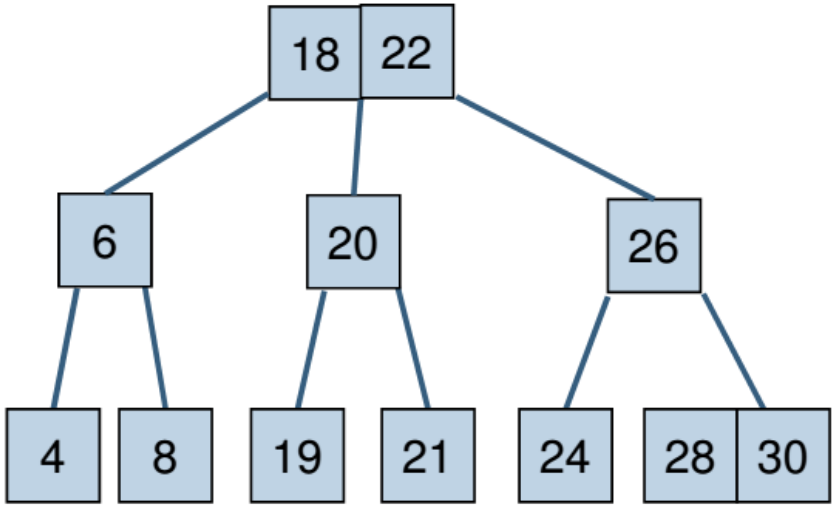


Adding key 21 at leaf node cause it full, so splitting the leaf node in half and moving the middle element onto the parent node.

Example

- After middle node move to parent node:





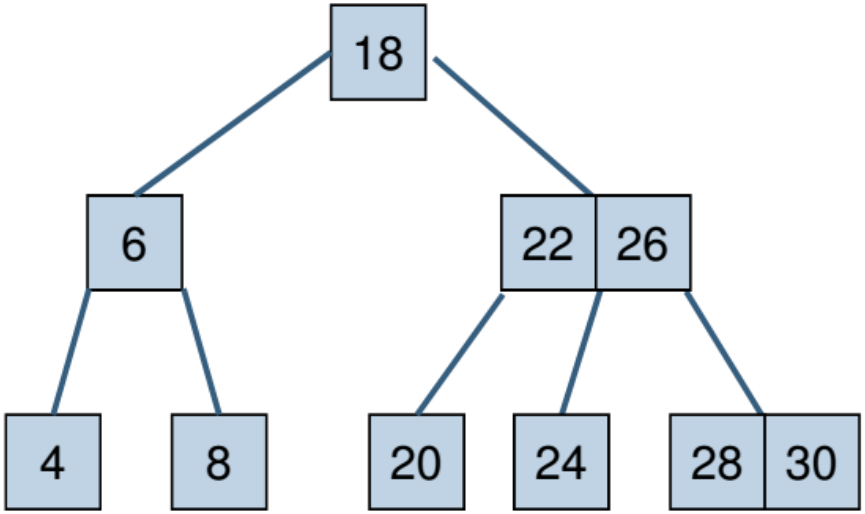
Parent node is full, so repeat same action

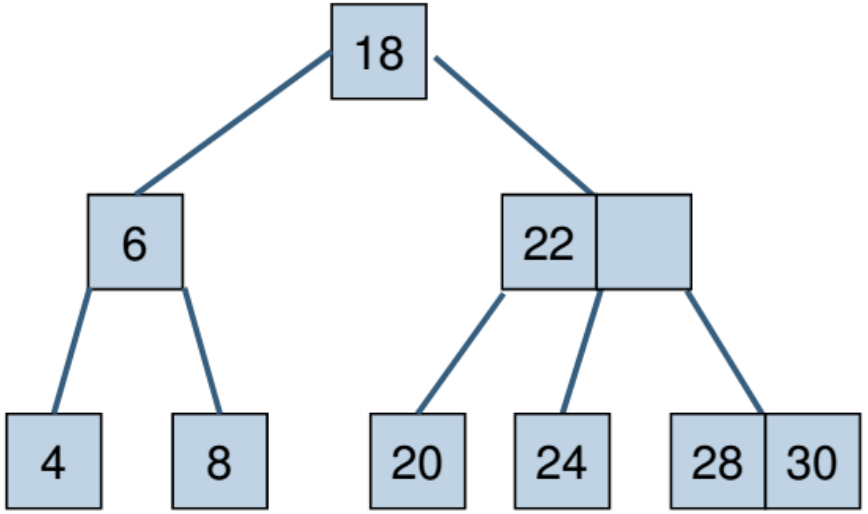
Delete an element from B-Tree

- Delete a key v from the tree
  - Do the same as a m-way tree.
  - If a node has less than [(m-1) / 2] keys
    - Borrow 1 key from the adjacent sibling node if the sibling node have enough key, or
    - Merge with an adjacent sibling node if the sibling node does not have enough key and a corresponding key from the parent node.

Example

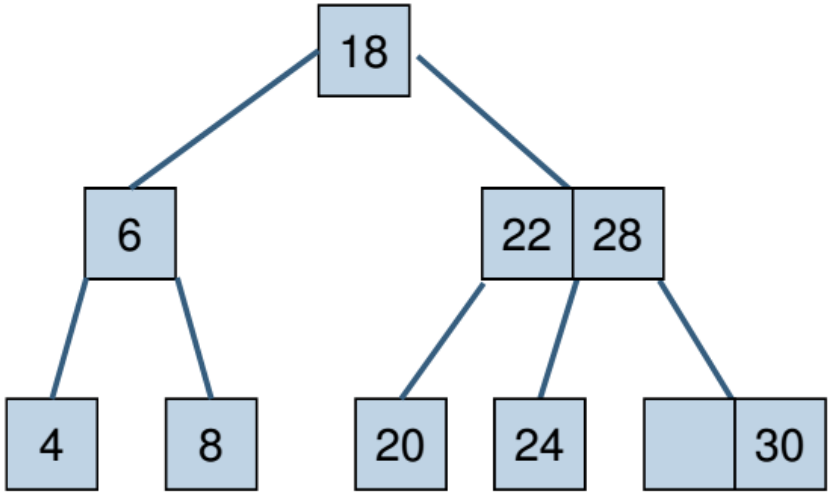
- Delete value 26:

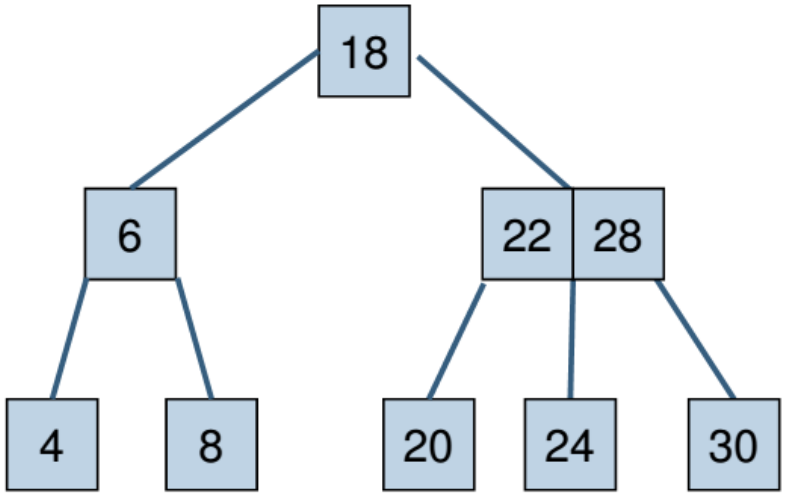




Example

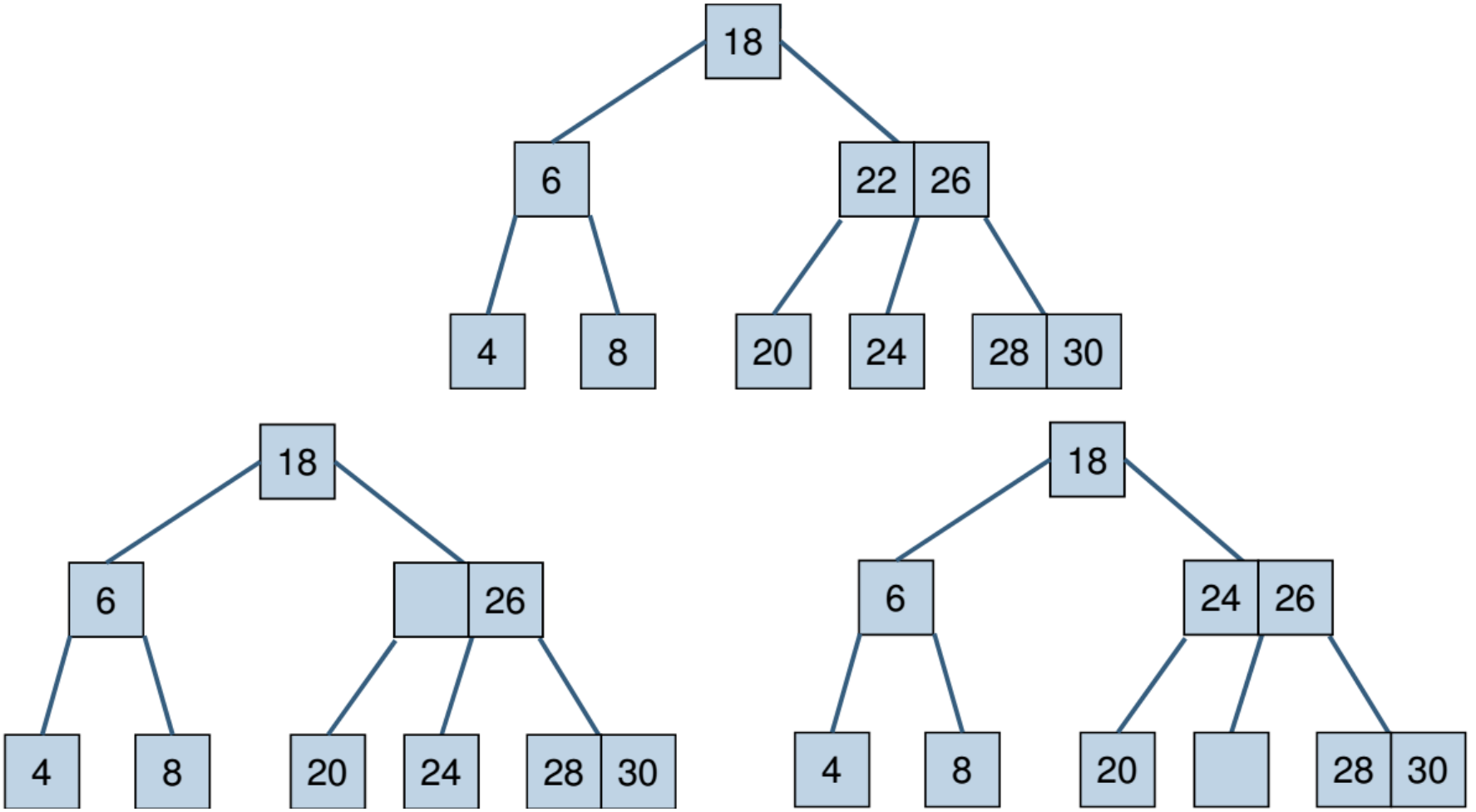
- To delete value 26, replace with the smallest value on the left subtree





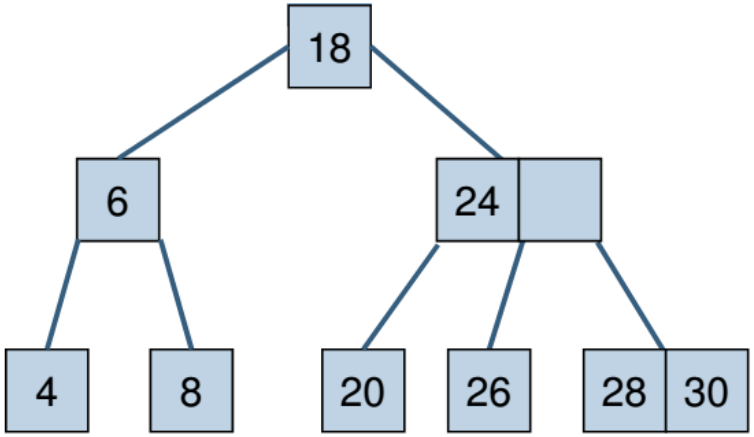
Example

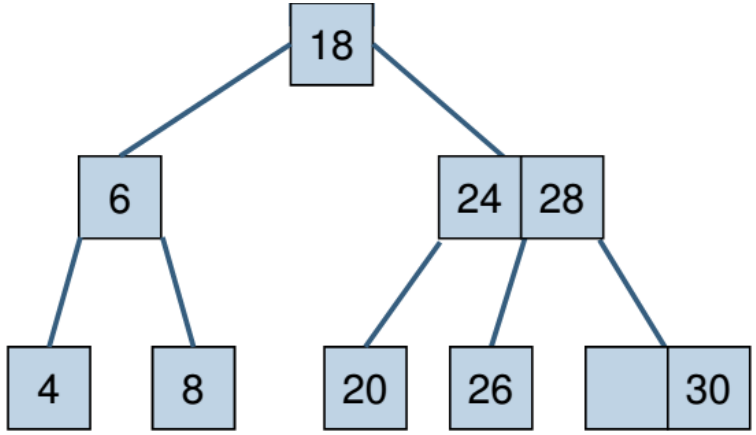
- How to delete value 22?

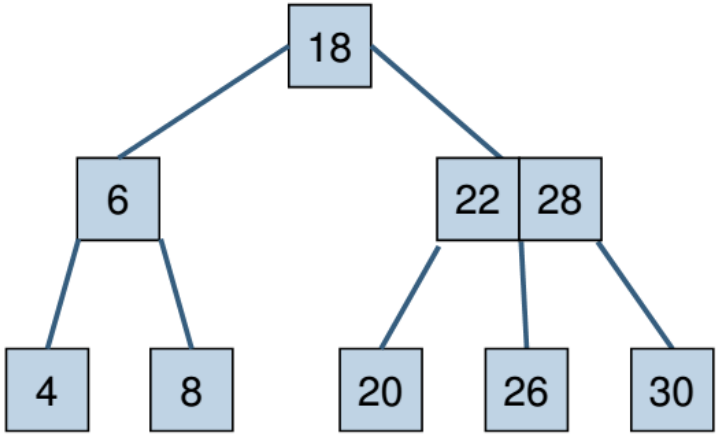


Example

- After delete 22, a node is not enough key, so adjust it (case 1)

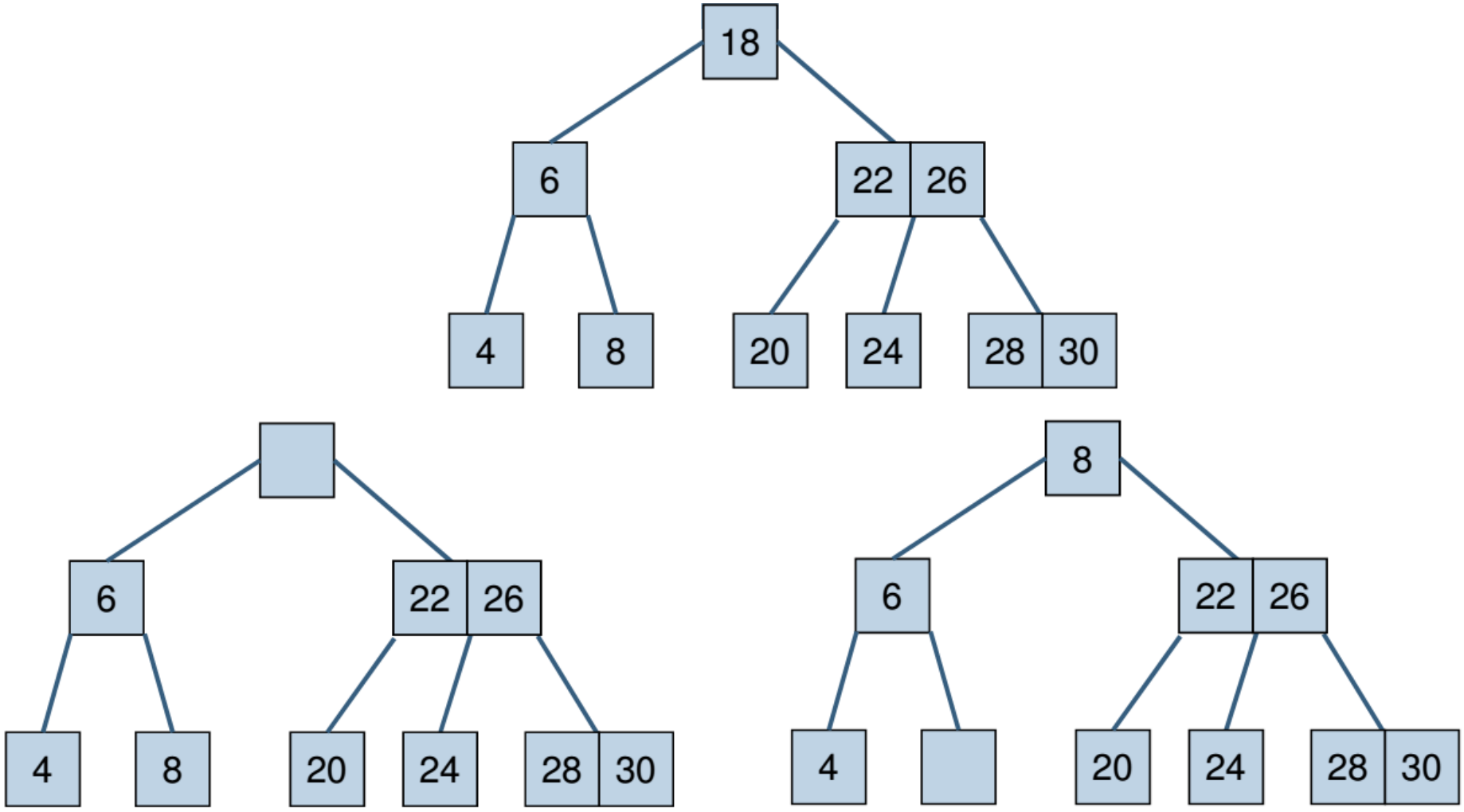




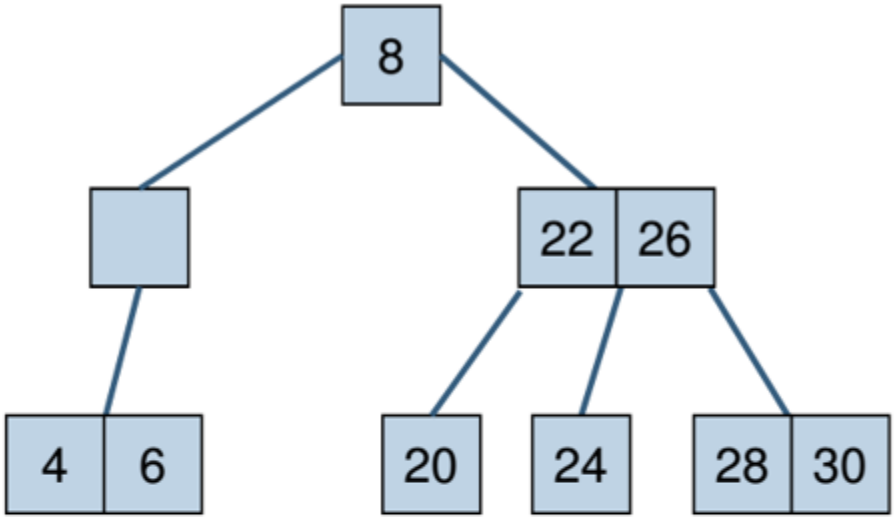


Example

- How to delete 18?

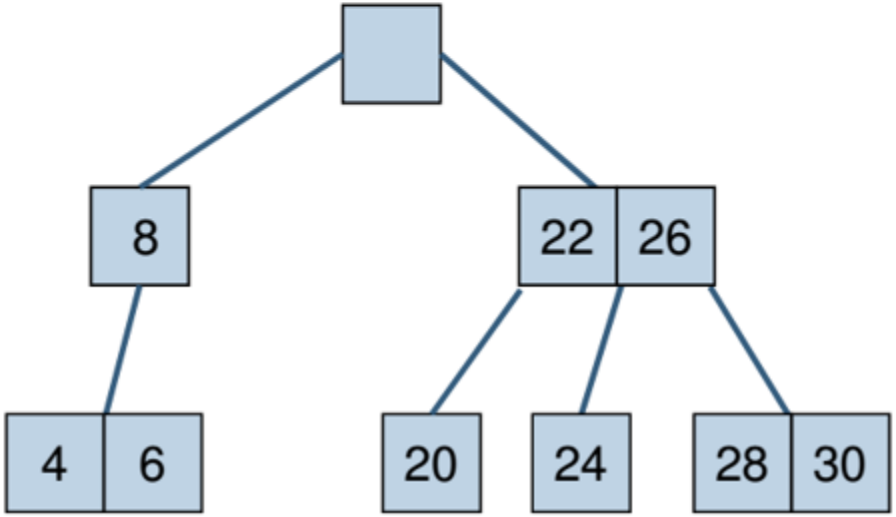


Example



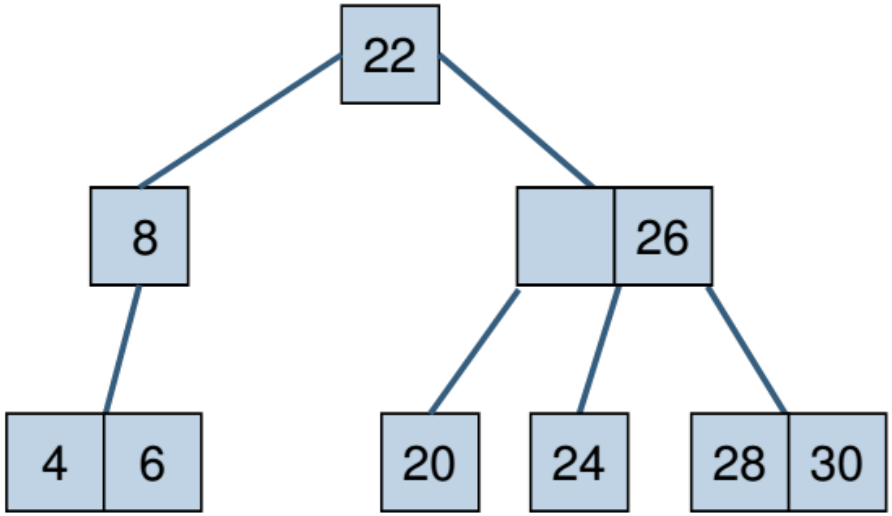
Merge with sibling sandwiching their separator taken off from their parent

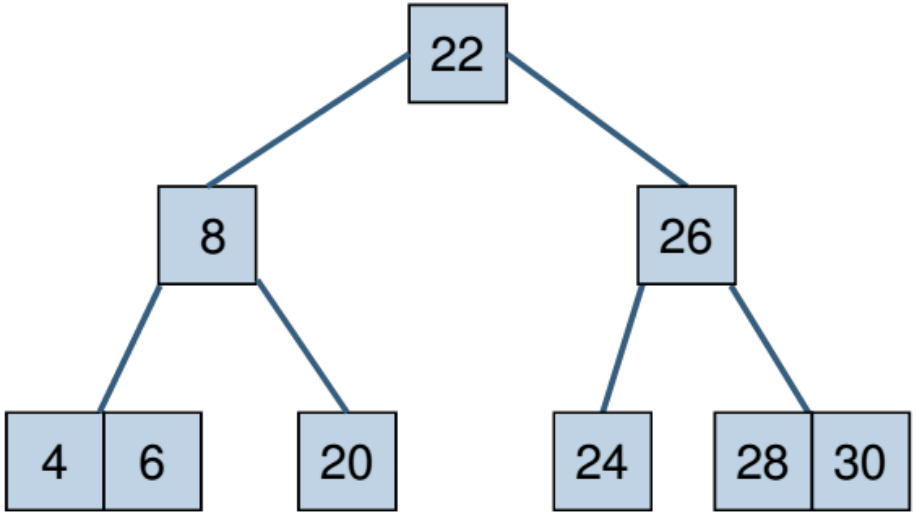
Rebalance the parent because the parent has not enough key.



Example

- Balance nodes:





Exercises

- Let's create a 5-way B-Tree with the following data in turn:

  3, 7, 9, 23, 45, 1, 5, 14, 25, 24, 13, 11, 8, 19, 4, 31, 35, 56, 2, 6, 12.

- Delete the following keys:

  4, 5, 7, 3, 14.

The End.