Rotating Wave Approximation for QOC

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$$H_{lab} = H_0 + \sum_{m}^{M} \Omega_m \cos(\omega_m t + \phi_m) \sum_{S} g\sigma^x \tag{1}$$

$$\begin{aligned} H_{lab} = H_0 + \big[&\Omega_{D0}\cos(\omega_{D0}t + \phi_{D0}) \\ &+\Omega_{D1}\cos(\omega_{D1}t + \phi_{D1}) \\ &+\big(\Omega_{U0}\cos(\omega_{U0}t + \phi_{U0}) \\ &+\Omega_{U1}\cos(\omega_{U1}t + \phi_{U1})\big)\big] \\ &*\sigma^x \end{aligned} \tag{2}$$

$$\begin{aligned} H_{rot} = H_0 + \frac{dR}{dt}\big[&\Omega_{D0} + e^{-iR}\cos(\omega_{D0}t + \phi_{D0}) \\ &+\Omega_{D1}\cos(\omega_{D1}t + \phi_{D1}) \\ &+\Omega_{U0}\cos(\omega_{U0}t + \phi_{U0}) \\ &+\Omega_{U1}\cos(\omega_{U1}t + \phi_{U1})\big] \\ &*\sigma^x e^{iR} \end{aligned} \tag{3}$$

$$H_{rot} = H_0 + \frac{dR}{dt} + \frac{1}{2}\sum_{m=1}^{2} \Omega_m \{\cos(\phi_m)\sigma^x - \sin(\phi_m)\sigma^y\} \tag{4}$$

$$\tag{5}$$

For the transformation we require that:

$$\begin{aligned} e^{-iR}(\cos{(\omega_m t + \phi_m)})\sigma^x - \sin{(\omega_m t + \phi_m)}\sigma^y)e^{iR} \\ = \cos\phi_m\sigma^x - \sin\phi_m\sigma^y \end{aligned} \tag{6}$$

With $m = D0, D1$, we only rotate on these two axis
Then when we perform the transformation it is just applied separately to each drive and control

Following from that we have the requirement

$$\begin{aligned} \cos(\theta)\sigma^x + \sin(\theta)\sigma^y \\ = \cos(\omega_m t)\sigma^x + \sin(\omega_m t)\sigma^y \end{aligned} \tag{7}$$

We have four fields, D0, D1, U0, U1 Note we don't transform the U control fields

0.1 Finding R

$$[R, \sigma^x] = \omega t(i\sigma^y) \tag{8}$$

$$\tag{9}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} c_{D0} \\ c_{D1} \end{bmatrix} = \begin{bmatrix} \omega_1 t \\ \omega_2 t \end{bmatrix} \tag{10}$$

$$c_{D0} + c_{D1} = \omega_1 t \tag{11}$$

$$c_{D0} + c_{D1} = \omega_2 t \tag{12}$$

This shouldn't be right though, because it implies that $\omega_1 = \omega_2$

We have states $|00\rangle$ through $|22\rangle$, and can transition from 01, 02, 03 to 11, 12, 13, (we are kind of ignoring the 1-2 transition, because we are using the general sigmax and hoping it counts for both, but)NOTE: This might be a place to come back and switch to a linear combination if it isn't working

what if we want to do linear combination? The worry is that there aren't different drives, it's the same

Let's test this. First does it fit into (6)?

0.2 Figuring out R from pdf

Use 3.13 for braket definition of pauli matrices

$$R = \sum c_s \sigma_s^z \tag{13}$$

$$[R, \sigma_{nn'}^x] = \sum c_{mm'} [\sigma_{mm'}^z, \sigma_{nn'}^x] \tag{14}$$

$$[\sigma_{mm'}^z, \sigma_{nn'}^x] = [(|m\rangle \langle m| - |m'\rangle \langle m'|), (|n\rangle \langle n'| + |n'\rangle \langle n|)] \tag{15}$$

Now we notice that there are a number of cases, depending on whether m, m', n, and n' are equal to or different from each other. Fischer lays out the evaulation for each of these options.

The commutator simplifies to

$$[\sigma_{mm'}^z, \sigma_{nn'}^x] = \begin{cases} +2i\sigma_{nn'}^y & m = n, m' = n' \\ +i\sigma_{nn'}^y & m = n, m' \neq', \\ & \text{or } m \neq n, m' = n' \\ -i\sigma_{nn'}^y & m = n' or m' = n \\ 0 & m \neq n, m' \neq n' \end{cases} \tag{16}$$

Given this, we can note that $[R, \sigma_{nn'}^x]$ is always proportional to $i\sigma_{nn'}^y$

$$[R, \sigma_{nn'}^x = \theta_{nn'} i \sigma_{nn'}^y] \tag{17}$$

For some constant $\theta_{nn'}$, and we look at eq (3.19) from Fischer to evaluate 3.31

$$e^{-iR} \sigma_{nn'}^x e^{iR} = \cos{(\theta_{nn'})} \sigma_{nn'}^x + \sin{(\theta_{nn'})} \sigma_{nn'}^y \tag{18}$$

note that the i from eq 3.19 cancels out with the i from (17)

Now we equate the right hand side of (18) with the right hand side of (F:3.31) to get

$$\cos{(\theta_{nn'})} \sigma_{nn'}^x + \sin{(\theta_{nn'})} \sigma_{nn'}^y = \cos{(\omega t)} \sigma_{nn'}^x + \sin{(\omega t)} \sigma_{nn'}^y \tag{19}$$

Our goal is to satisfy equation 3.29 from Fischer, which is the one in which $e^{-iR}\{cos - sin\}e^{iR} = cos - sin$

So thus in order to satisfy equation 3.29 we must satisfy (19). This only holds if $\theta_{nn'} = \omega t$ for all $(n, n')S$

$$[R, \sigma^x_{nn'}] = \omega t (i\sigma^y_{nn'}) \tag{20}$$

We use the expansion (21) for R, and the commutations from (16), we obtain a matrix equation to solve for the coefficients.

$$R = \sum c_s \sigma^z_s \tag{21}$$

$$[\sigma^z_{mm'}, \sigma^x_{nn'}] = \begin{cases} +2i\sigma^y_{nn'} & m = n, m' = n' \\ +i\sigma^y_{nn'} & m = n, m' \neq', \\ & \text{or } m \neq n, m' = n' \\ -i\sigma^y_{nn'} & m = n' or m' = n \\ 0 & m \neq n, m' \neq n' \end{cases} \tag{22}$$

$$[R, \sigma^x_{nn'}] = \omega t (i\sigma^y_{nn'}) \tag{23}$$

Thus:

$$\{[c_{mm'}\sigma^z_{mm'}, \sigma^x_{nn'}] = i\omega t \sigma^y_{nn'}\} \forall (m, m') in S \forall (n, n') \in S \tag{24}$$

And we can solve using (16)

So let's take the case where we have 2 drive channels, and either general σ^x or $\sigma^x_{01} + \sigma^x_{12}$

For both options, we have 9 states, and since we have two separate control fields which effect orthogonal state transitions, the derivation can proceed at least temporarily without taking into account the multiple control fields.

Let's follow the single field derivation first, because that is the first step either way.

for the $|00\rangle \to |01\rangle$ transition we obtain the following equation:

potential error: we ignore skipping between 0 and 2 (00) and (02), because not really allowed quick reference

- 00 – 0
- 01 – 1
- 02 – 2
- 10 – 3
- 11 – 4
- 12 – 5
- 20 – 6
- 21 – 7
- 22 – 8

POtential error: We ignore reversals, cause always reversible? Seems like they are just negative of each other?

NOTE: we move from the two index basis to single, so $|00\rangle$ becomes $|0\rangle$ and $|01\rangle$ becomes $|1\rangle$ thus we can easily show $|0\rangle \to |1\rangle$ as 01

$$c_{00,01}(2i\sigma^y_{00,01}) + c_{00,10}(i\sigma^y_{00,01}) + c_{01,02}(-i\sigma^y_{00,01}) + c_{01,11}(-i\sigma^y_{00,01}) = \omega t(i\sigma^y_{00,01}) \tag{25}$$

$$c_{00,10}(2i\sigma^y_{00,10}) + c_{00,01}(i\sigma^y_{00,10}) + c_{10,20}(-i\sigma^y_{00,10}) + c_{10,11}(-i\sigma^y_{00,10}) = \omega t(i\sigma^y_{00,01}) \quad \omega t \tag{26}$$

Now convert to

$$c_{01}(2i\sigma^y_{01}) + c_{03}(i\sigma^y_{01}) + c_{12}(-i\sigma^y_{01}) + c_{14}(-i\sigma^y_{01}) = \omega t(i\sigma^y_{01}) \tag{27}$$

We do the math in python, see `rwa_system_solver.py`

So now that we have the system of equations we need to solve so with M being the matrix of values from the commutations

$$M[c_{nn'}] = [\omega_i t] \begin{cases} i = 1 & \text{for transitions on qubit 1} \\ i = 2 & \text{for transitions on qubit 2} \end{cases} \tag{28}$$

$$
\begin{aligned}
H(t) = H_0 + \Omega_{d0} D_0(t)\sigma_0^x + \Omega_{d1} D_1(t)\sigma_1^x \\
+\Omega_{d0} U_0^{0,1}(t)\sigma_0^x + \Omega_{d1} U_1^{1,0}(t)\sigma_1^x
\end{aligned}
\tag{29}
$$

$$
D_n(t) = e^{i\omega_n t} d(t) \qquad d(t) = amp \tag{30}
$$

$$
U_n^{a,b}(t) = e^{i\omega_b t} u(t) \qquad u(t) = amp \tag{31}
$$

OK IT SEEMS LIKE singular matrix for if we take 01 and 12 separately?