**Governing Equation**

For, *one-dimensional, steady state, conductive* heat transfer,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right) + q = 0$$

**Parameters**

$$k = 1000\ \mathrm{W\ (m.K)^{-1}}$$

$$q = 5095\ \mathrm{kW\ m^{-3}}$$

$$L = 0.5\ \mathrm{m}$$

$$A = 10 \times 10^{-3}\ \mathrm{m^2}$$

$$T_{\mathrm{A}} = 100℃$$

$$T_{\mathrm{B}} = 500℃$$

**Discretisation**

Dividing $L$ in to 5 equal control volumes, results in nodal distances, $\delta x$, of 0.1 m.

Now, over a control volume, for nodes 2,3 & 4, the governing equation:

$$\int_{\Delta V}\frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right)\mathrm{d}V + \int_{\Delta V} q\ \mathrm{d}V = \int_{A}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right)\mathrm{d}A + \int_{\Delta V} q\ \mathrm{d}V = 0$$

$$\left[\left(kA\frac{\mathrm{d}T}{\mathrm{d}x}\right)_e - \left(kA\frac{\mathrm{d}T}{\mathrm{d}x}\right)_w\right] + q\Delta V = 0 \qquad (1)$$

$$\left[\left(kA\frac{T_E - T_P}{\delta x}\right)_e - \left(kA\frac{T_P - T_W}{\delta x}\right)_w\right] + qA\delta x = 0$$

$$\frac{kA}{\delta x}T_E - \frac{kA}{\delta x}T_P - \frac{kA}{\delta x}T_P + \frac{kA}{\delta x}T_W + qA\delta x = 0$$

$$\left(2\frac{kA}{\delta x}\right)T_P = \left(\frac{kA}{\delta x}\right)T_W + \left(\frac{kA}{\delta x}\right)T_E + qA\delta x$$

So that,

| $a_W$ | $a_E$ | $a_P$ | $S_P$ | $S_u$ |
|---|---|---|---|---|
| $\frac{kA}{\delta x}$ | $\frac{kA}{\delta x}$ | $2\frac{kA}{\delta x}$ | 0 | $qA\delta x$ |
| 100 | 100 | 200 | 0 | 5,095 |

For $i = 2,3$ or 4

$$200T_i = 100T_{i-1} + 100T_{i+1} + 5095 \qquad (2)$$

Now, over a control volume, for node 1, equation 1 becomes:

$$\left[\left(kA\frac{T_E - T_P}{\delta x}\right)_e - \left(kA\frac{T_P - T_A}{0.5\delta x}\right)_w\right] + qA\delta x = 0$$

$$\frac{kA}{\delta x}T_E - \frac{kA}{\delta x}T_P - \frac{kA}{0.5\delta x}T_P + \frac{kA}{0.5\delta x}T_A + qA\delta x = 0$$

$$\left(\frac{kA}{\delta x} + \frac{kA}{0.5\delta x}\right)T_P = \left(\frac{kA}{\delta x}\right)T_E + qA\delta x + \frac{kA}{0.5\delta x}T_A$$

So that,

| $a_W$ | $a_E$ | $a_P$ | $S_P$ | $S_u$ |
|---|---|---|---|---|
| 0 | $\frac{kA}{\delta x}$ | $3\frac{kA}{\delta x}$ | $-2\frac{kA}{\delta x}$ | $qA\delta x + 2\frac{kA}{\delta x}T_A$ |
| 0 | 100 | 300 | −200 | 25,095 |

$$300T_1 = 100T_2 + 5095 + 200T_A \tag{3}$$

Now, over a control volume, for node 5, equation 1 becomes:

$$\left[\left(kA\frac{T_B - T_P}{0.5\delta x}\right)_e - \left(kA\frac{T_P - T_W}{\delta x}\right)_w\right] + qA\delta x = 0$$

$$\frac{kA}{0.5\delta x}T_B - \frac{kA}{0.5\delta x}T_P - \frac{kA}{\delta x}T_P + \frac{kA}{\delta x}T_W + qA\delta x = 0$$

$$\left(\frac{kA}{\delta x} + \frac{kA}{0.5\delta x}\right)T_P = \left(\frac{kA}{\delta x}\right)T_W + qA\delta x + \frac{kA}{0.5\delta x}T_B$$

So that,

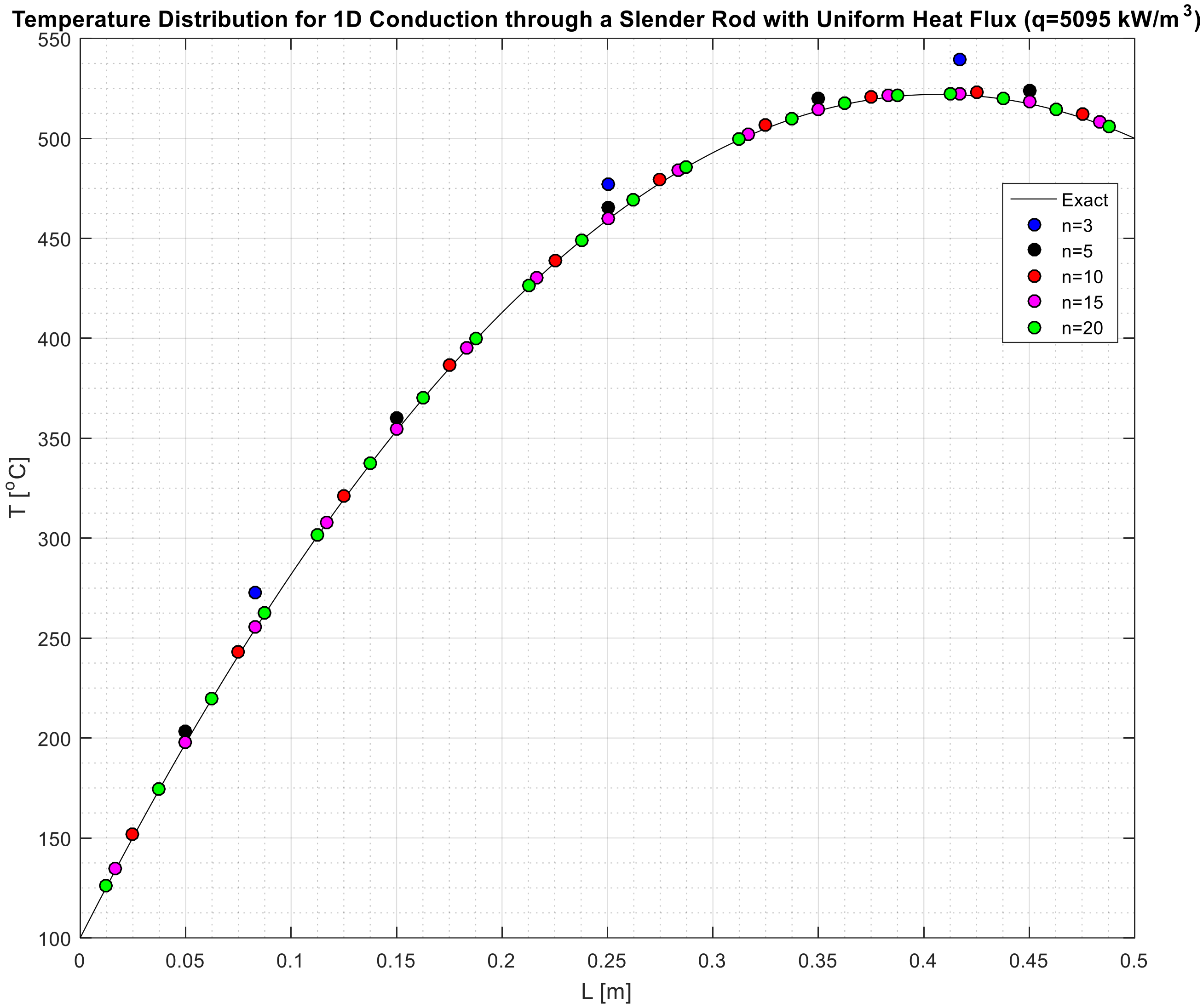
| $a_W$ | $a_E$ | $a_P$ | $S_P$ | $S_u$ |
|---|---|---|---|---|
| $\frac{kA}{\delta x}$ | 0 | $3\frac{kA}{\delta x}$ | $-2\frac{kA}{\delta x}$ | $qA\delta x + 2\frac{kA}{\delta x}T_B$ |
| 100 | 0 | 300 | −200 | 105,095 |

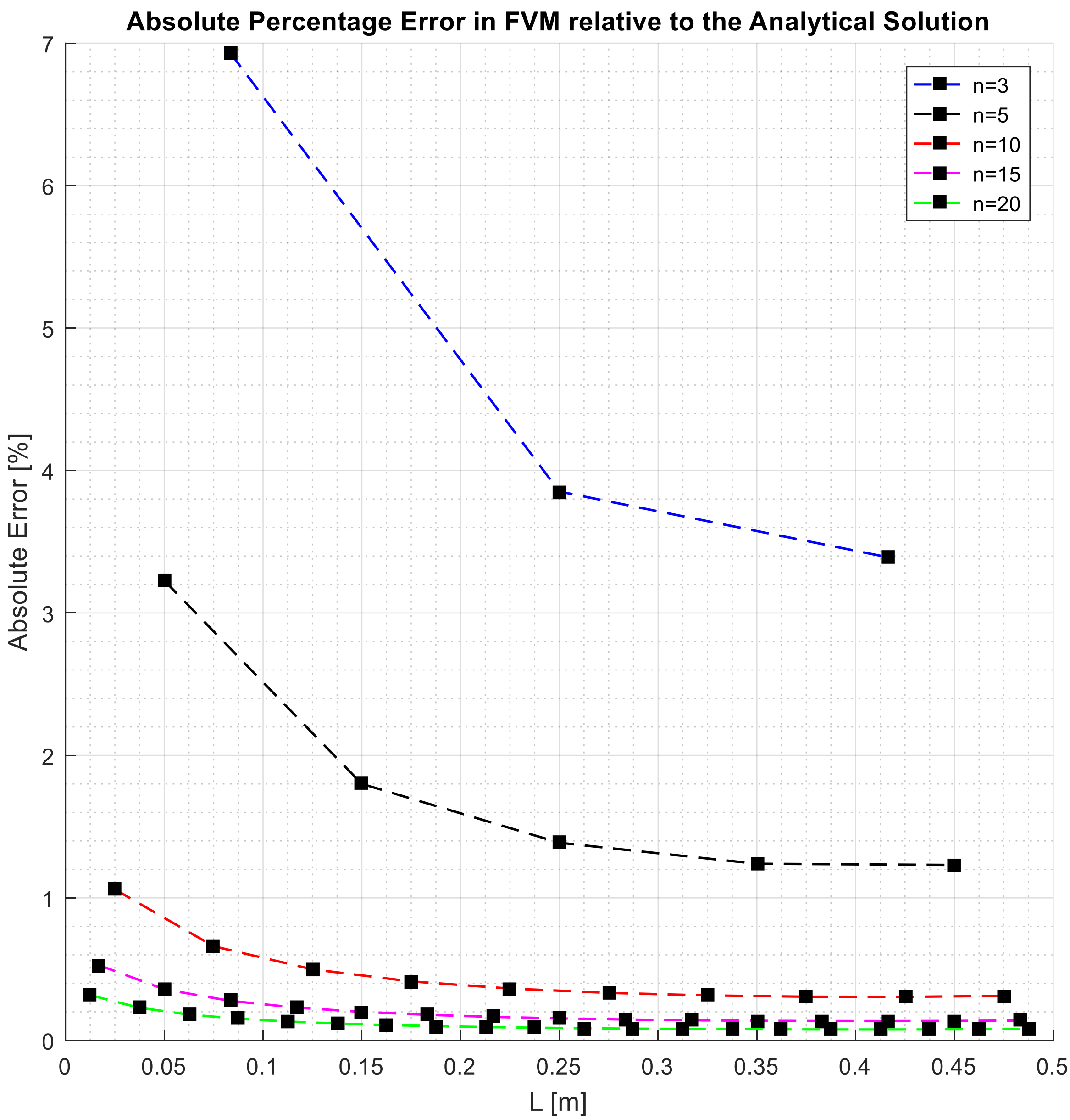
$$300T_5 = 100T_4 + 5095 + 200T_B \tag{4}$$

Temperatures at nodes 1-5 can be evaluated by solving the following equations, where the coefficient matrix is formed from noting the respective coefficients in equations 2-4;

$$\begin{bmatrix} 300 & -100 & 0 & 0 & 0 \\ -100 & 200 & -100 & 0 & 0 \\ 0 & -100 & 200 & -100 & 0 \\ 0 & 0 & -100 & 200 & -100 \\ 0 & 0 & 0 & -100 & 300 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 25,095 \\ 5,095 \\ 5,095 \\ 5,095 \\ 105,095 \end{bmatrix}$$

A similar procedure for 3, 10, 15 and 20 nodes is done and the solutions are plotted against the exact (analytical) solution, as well as the absolute percentage errors for each case.


Temperature Distribution for 1D Conduction through a Slender Rod with Uniform Heat Flux (q=5095 kW/m^3)
T [°C]
L [m]
Exact
n=3
n=5
n=10
n=15
n=20
550
500
450
400
350
300
250
200
150
100
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5




The error is seen to decrease with the increase in the number of nodes and the numerical solution approaches the exact analytical solution.