

Lecture 5:

Sequence Methods

Part 2: Discriminative Models

Maximum Entropy Markov Taggers

- MEMMs (Maximum Entropy Markov Models) are discriminative, log-linear models for sequence labeling:

$$\begin{aligned} p(y_1 \dots y_m | x_1 \dots x_m) &= \prod_{i=1}^m p(y_i | y_1 \dots y_{i-1}, x_1 \dots x_m) \\ &= \prod_{i=1}^m p(y_i | y_{i-1}, x_1 \dots x_m) \end{aligned}$$

$$p(y_i | y_{i-1}, x_1 \dots x_m) = \frac{e^{w \cdot f(x_1 \dots x_m, i, y_{i-1}, y_i)}}{\sum_{y'} e^{w \cdot f(x_1 \dots x_m, i, y_{i-1}, y')}}$$

Feature/History Representation

- ▶ We have some input domain \mathcal{X} , and a finite label set \mathcal{Y} . Aim is to provide a conditional probability $p(y \mid x)$ for any $x \in \mathcal{X}$ and $y \in \mathcal{Y}$.
- ▶ A **feature** is a function $f : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}$
(Often **binary features** or **indicator functions** $f : \mathcal{X} \times \mathcal{Y} \rightarrow \{0, 1\}$).
- ▶ Say we have m features f_k for $k = 1 \dots m$
 \Rightarrow A **feature vector** $f(x, y) \in \mathbb{R}^m$ for any $x \in \mathcal{X}$ and $y \in \mathcal{Y}$.

Feature/History Representation

Alternative notation:
 f is a feature function
from “*histories*” and
target label

- ▶ \mathcal{X} is the set of all possible histories of form $\langle t_{-2}, t_{-1}, x_{[1:n]}, i \rangle$
- ▶ $\mathcal{Y} = \{\text{NN}, \text{NNS}, \text{Vt}, \text{Vi}, \text{IN}, \text{DT}, \dots\}$
- ▶ We have m features $f_k : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}$ for $k = 1 \dots m$

t : the current tag
 t_{-1}, t_{-2} : previous tags

For example:

$$f_1(\textcolor{green}{h}, \textcolor{red}{t}) = \begin{cases} 1 & \text{if current word } x_i \text{ is base and } t = \text{Vt} \\ 0 & \text{otherwise} \end{cases}$$

$$f_2(\textcolor{green}{h}, \textcolor{red}{t}) = \begin{cases} 1 & \text{if current word } x_i \text{ ends in ing and } t = \text{VBG} \\ 0 & \text{otherwise} \end{cases}$$

...

$$f_1(\langle \text{JJ}, \text{DT}, \langle \text{Hispaniola}, \dots \rangle, 6 \rangle, \textcolor{red}{\text{Vt}}) = 1$$

$$f_2(\langle \text{JJ}, \text{DT}, \langle \text{Hispaniola}, \dots \rangle, 6 \rangle, \textcolor{red}{\text{Vt}}) = 0$$

A trigram
MEMM

Feature Set in Ratnaparkhi's MEMM Tagger

- ▶ Word/tag features for all word/tag pairs, e.g.,

$$f_{100}(h, t) = \begin{cases} 1 & \text{if current word } x_i \text{ is base and } t = Vt \\ 0 & \text{otherwise} \end{cases}$$

- ▶ Spelling features for all prefixes/suffixes of length ≤ 4 , e.g.,

$$f_{101}(h, t) = \begin{cases} 1 & \text{if current word } x_i \text{ ends in ing and } t = \text{VBG} \\ 0 & \text{otherwise} \end{cases}$$

$$f_{102}(h, t) = \begin{cases} 1 & \text{if current word } x_i \text{ starts with pre and } t = \text{NN} \\ 0 & \text{otherwise} \end{cases}$$

Feature Set in Ratnaparkhi's MEMM Tagger

► Contextual Features, e.g.,

$$f_{103}(h, t) = \begin{cases} 1 & \text{if } \langle t_{-2}, t_{-1}, t \rangle = \langle \text{DT}, \text{JJ}, \text{Vt} \rangle \\ 0 & \text{otherwise} \end{cases}$$

$$f_{104}(h, t) = \begin{cases} 1 & \text{if } \langle t_{-1}, t \rangle = \langle \text{JJ}, \text{Vt} \rangle \\ 0 & \text{otherwise} \end{cases}$$

$$f_{105}(h, t) = \begin{cases} 1 & \text{if } \langle t \rangle = \langle \text{Vt} \rangle \\ 0 & \text{otherwise} \end{cases}$$

$$f_{106}(h, t) = \begin{cases} 1 & \text{if previous word } x_{i-1} = \textit{the} \text{ and } t = \text{Vt} \\ 0 & \text{otherwise} \end{cases}$$

$$f_{107}(h, t) = \begin{cases} 1 & \text{if next word } x_{i+1} = \textit{the} \text{ and } t = \text{Vt} \\ 0 & \text{otherwise} \end{cases}$$

Full Feature Set in Ratnaparkhi's Tagger

- ▶ We can come up with practically any questions (*features*) regarding history/tag pairs.
- ▶ For a given history $h \in \mathcal{X}$, each label in \mathcal{Y} is mapped to a different feature vector

$f(\langle \text{JJ, DT, } \langle \text{Hispaniola, } \dots \rangle, 6 \rangle, \text{Vt}) = 1001011001001100110$

$f(\langle \text{JJ, DT, } \langle \text{Hispaniola, } \dots \rangle, 6 \rangle, \text{JJ}) = 0110010101011110010$

$f(\langle \text{JJ, DT, } \langle \text{Hispaniola, } \dots \rangle, 6 \rangle, \text{NN}) = 0001111101001100100$

$f(\langle \text{JJ, DT, } \langle \text{Hispaniola, } \dots \rangle, 6 \rangle, \text{IN}) = 0001011011000000010$

MLE in MEMMs

- Recall the model:

$$Pr(y|x) = \prod_{i=1}^N \prod_{j=1}^{n(i)} Pr(y_j^{(i)} | y_{j-1}^{(i)}, x^{(i)}) = \prod_{i=1}^N \prod_{j=1}^{n(i)} \frac{\exp(f(j, y_j^{(i)}, y_{j-1}^{(i)}, x^{(i)})^T \cdot w)}{Z(y_{j-1}^{(i)}; w)}$$

- The partition function: (locally normalized)

$$Z(y_{j-1}^{(i)}; w) = \sum_{y_j} \exp(f(j, y_j, y_{j-1}^{(i)}, x^{(i)})^T \cdot w)$$

- Log-likelihood (where $(x^{(i)}, y^{(i)})$ is the training data):

$$LL(w) = \sum_{i=1}^N \sum_{j=1}^{n(i)} \left[f(j, y_j^{(i)}, y_{j-1}^{(i)}, x^{(i)})^T \cdot w - \log(Z(y_{j-1}^{(i)})) \right]$$

- Gradient for w : (T is the set of labels)

$$\frac{\partial LL}{\partial w_k} = \sum_{i=1}^N \sum_{j=1}^{n(i)} \left[f_k(j, y_j^{(i)}, y_{j-1}^{(i)}, x^{(i)}) - \sum_{y' \in T} Pr(y_j = y' | y_{j-1}^{(i)}, x^{(i)}) \cdot f_k(j, y_j = y', y_{j-1}^{(i)}, x^{(i)}) \right]$$

Inference in MEMMs

- ▶ Define n to be the length of the sentence
- ▶ Define

$$r(t_1 \dots t_k) = \prod_{i=1}^k q(t_i | t_{i-2}, t_{i-1}, x_{[1:n]}, i)$$

- ▶ Define a dynamic programming table

$\pi(k, u, v)$ = maximum probability of a tag sequence ending
in tags u, v at position k

that is,

$$\pi(k, u, v) = \max_{\langle t_1, \dots, t_{k-2} \rangle} r(t_1 \dots t_{k-2}, u, v)$$

Inference in MEMMs (Viterbi)

Base case:

$$\pi(0, *, *) = 1$$

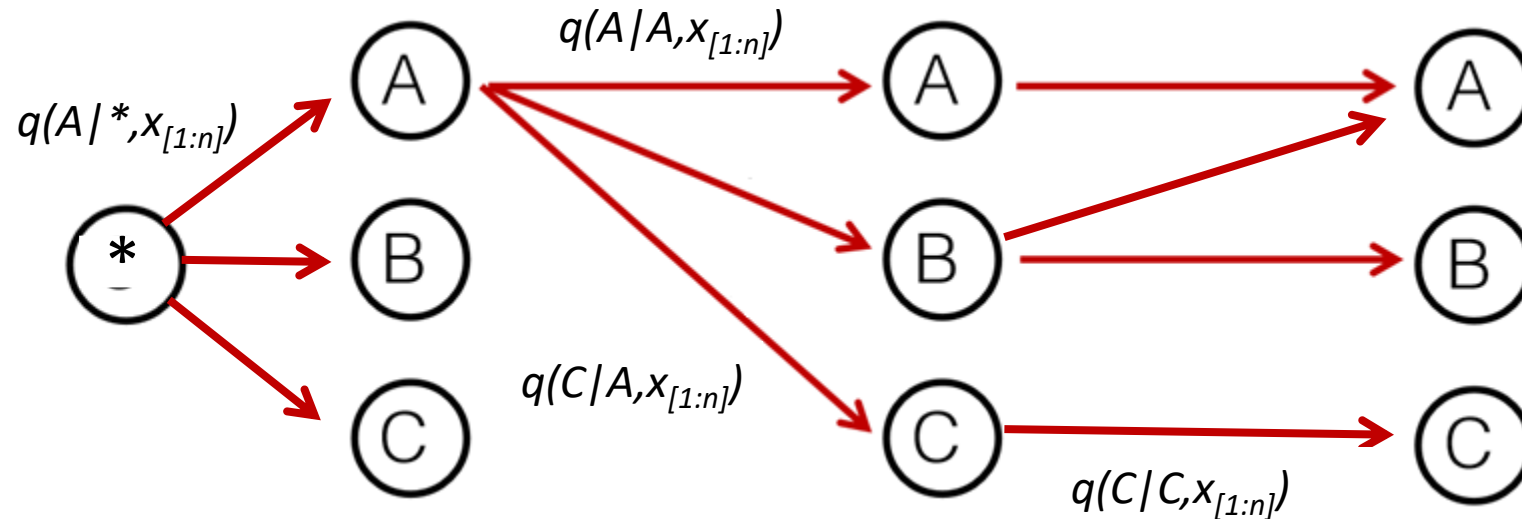
Recursive definition:

For any $k \in \{1 \dots n\}$, for any $u \in \mathcal{S}_{k-1}$ and $v \in \mathcal{S}_k$:

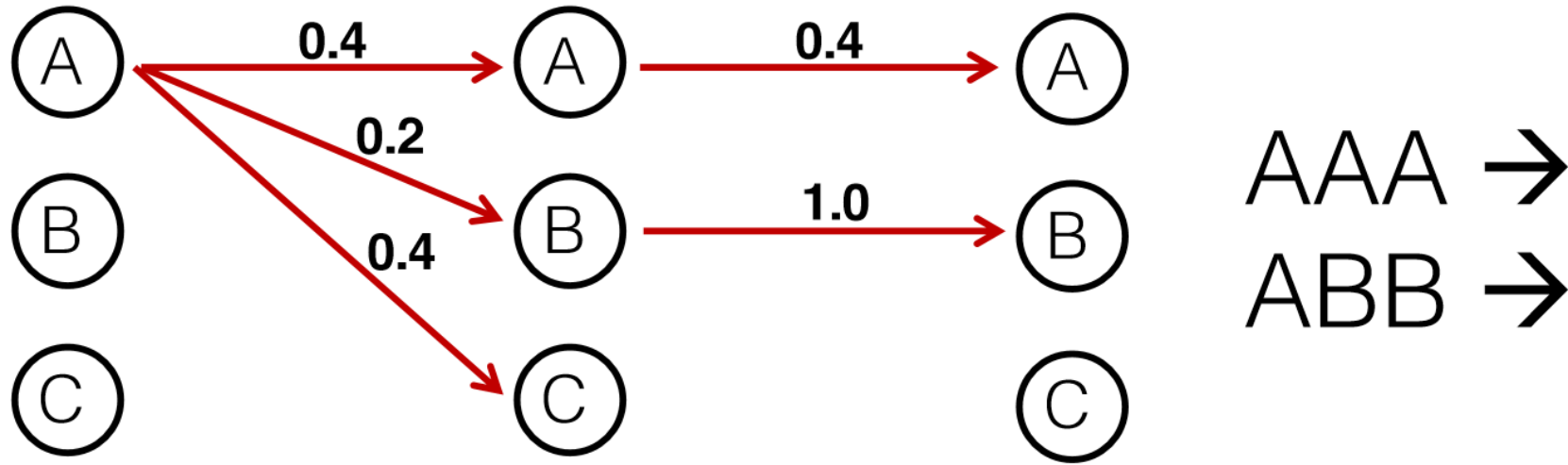
$$\pi(k, u, v) = \max_{t \in \mathcal{S}_{k-2}} (\pi(k-1, t, u) \times q(v|t, u, \mathbf{x}_{[1:n]}, k))$$

where \mathcal{S}_k is the set of possible tags at position k

Reminder: The Paths on a Grid Representation

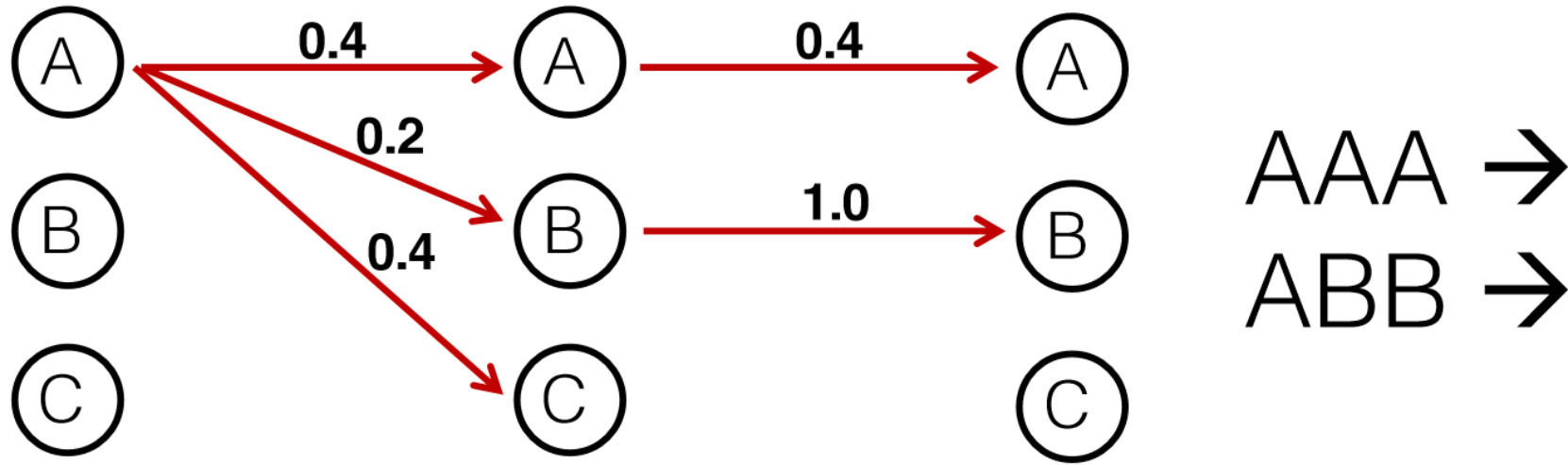


The Label Bias



- We have so far considered locally-normalized models
 - Namely, the scores (probability term) of each transition must sum up to 1
- This assumption may lead to a bias towards labels that have high-probability transitions, even if rarely observed

The Label Bias



- This happens in real-life settings
 - For instance, in NER, consider a label which is rare, but tends to have long names (e.g., the “continue” category of nobility names or books by E. Lockhart)

Books by E. Lockhart

The Boyfriend List: 15 Guys, 11 Shrink Appointments, 4 Ceramic Frogs and Me, Ruby Oliver (Ruby Oliver, #1)

The Boy Book: A Study of Habits and Behaviors, Plus Techniques for Taming Them (Ruby Oliver, #2)

The Treasure Map of Boys: Noel, Jackson, Finn, Hutch, Gideon—and me, Ruby Oliver (Ruby Oliver, #3)

Real Live Boyfriends: Yes. Boyfriends, Plural. If My Life Weren't Complicated, I Wouldn't Be Ruby Oliver

The Boyfriend Quartet: 15 Boys, 43 Lists, 120 Footnotes, and Too Many Panic

Attacks to Count, All in Four Novels about Ruby Oliver

Globally-normalized Models

- Newer, higher-powered discriminative sequence models
- Do not decompose training into independent local regions
 - Thus avoiding the label bias
- Often take a very long time to train - require repeated inference on training set

Sequence Conditional Random Fields (CRFs)

- CRFs are similar to MEMMs, but the normalization is global.
- This solves the label bias: the scores of the next state given the current one need not sum up to 1
- That is, given sequences $(x^{(1)}, x^{(2)}, \dots, x^{(N)})$ and labels $(y^{(1)}, y^{(2)}, \dots, y^{(N)})$, we define:

$$Pr(y|x) = \prod_{i=1}^N Pr(y^{(i)}|x^{(i)}) = \prod_{i=1}^N \frac{\prod_{j=1}^{n(i)} \exp(f(j, y_j^{(i)}, y_{j-1}^{(i)}, x^{(i)})^T \cdot w)}{Z(x^{(i)}; w)}$$

$$Z(x^{(i)}; w) = \sum_y \prod_{j=1}^{n(i)} \exp(f(j, y_j, y_{j-1}, x^{(i)})^T \cdot w)$$

Estimating w

- We use maximum likelihood estimation (sometimes with l_2 regularization):

$$LL(w) = \sum_{i=1}^N \left[\sum_{j=1}^{n(i)} f(j, y_j^{(i)}, y_{j-1}^{(i)}, x^{(i)})^T \cdot w - \log(Z(x^{(i)}; w)) \right]$$

- No close formula. We use gradient-based methods:

$$\frac{\partial LL}{\partial w_k} = \sum_{i=1}^N \left[\sum_{j=1}^{n(i)} f_k(j, y_j^{(i)}, y_{j-1}^{(i)}, x^{(i)}) - \sum_{y \in T^{n(i)}} Pr(y|x^{(i)}) \cdot \sum_{j=1}^{n(i)} f_k(j, y_j, y_{j-1}, x^{(i)}) \right]$$

CRFs: Gradient Computation

- The second term is more tricky to compute, as it sums over an exponential number of elements:

$$\sum_{y \in T^{n(i)}} Pr(y|x^{(i)}) \cdot \sum_{j=1}^{n(i)} f_k(j, y_j, y_{j-1}, x^{(i)}) =$$

$$\sum_{j=1}^{n(i)} \sum_{y \in T^{n(i)}} Pr(y|x^{(i)}) \cdot f_k(j, y_j, y_{j-1}, x^{(i)}) =$$

$$\sum_{j=1}^{n(i)} \sum_{y_j, y_{j-1}} \boxed{Pr(y_j, y_{j-1}|x^{(i)})} \cdot f_k(j, y_j, y_{j-1}, x^{(i)})$$

CRFs: Gradient Computation

- The marginals require dynamic programming to compute: (we should compute them for every pair of values for y_j and y_{j-1})

$$Pr(y_j, y_{j-1} | x^{(i)})$$

- We can solve this with dynamic programming

CRFs: Forward-Backward Algorithm

- Goal (for every a, b):

$$Pr(y_j = a, y_{j-1} = b | x^{(i)}) = \sum_{y \in T^{n(i)} : y_j = a, y_{j-1} = b} Pr(y | x^{(i)})$$

- Approach: dynamic programming

$$M_i(y, y') = \exp(f(i, y', y, x)^T \cdot w)$$

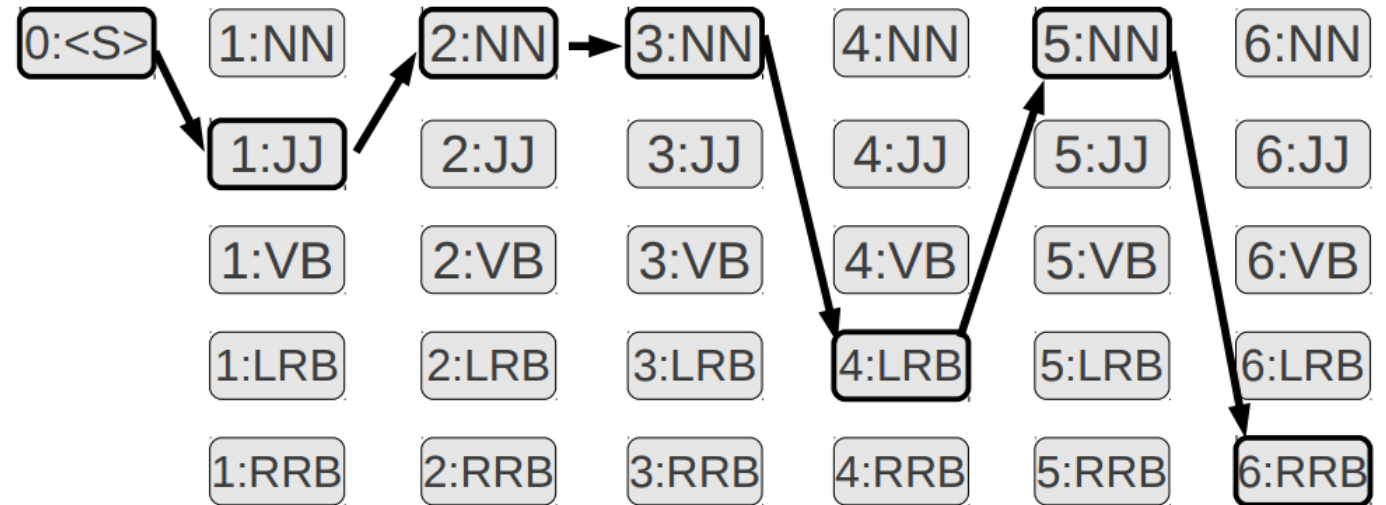
- Define:

$$\alpha_i(y) = \sum_{y_1, \dots, y_{i-1} = y} \prod_k M_k(y_{k-1}, y_k)$$

$$\beta_i(y) = \sum_{y_n, \dots, y_i = y} \prod_k M_k(y_k, y_{k+1})$$

Forward-Backward Algorithm in Viterbi Trellis

- The marginal $Pr(y_j, y_{j-1} | x^{(i)})$ is the sum over all paths that have the edge (y_{j-1}, y_j) between states $j-1$ and j
- Edge weights between y and y' is $M_i(y, y')$



CRFs: Forward-Backward Algorithm

- The parameters can be computed using dynamic programming with these formulas (for $i=1$, computing α , same for β with $i=n$)

$$\alpha_i(y) = \sum_{y'} M_i(y', y) \alpha_{i-1}(y') \quad \beta_i(y) = \sum_{y'} M_{i+1}(y, y') \beta_{i+1}(y')$$

- The marginal can now be computed as:

$$Pr(y_j, y_{j-1} | x^{(i)}) = \frac{M_j(y_j, y_{j-1}) \cdot \beta_j(y_j) \cdot \alpha_j(y_{j-1})}{Z(x)}$$

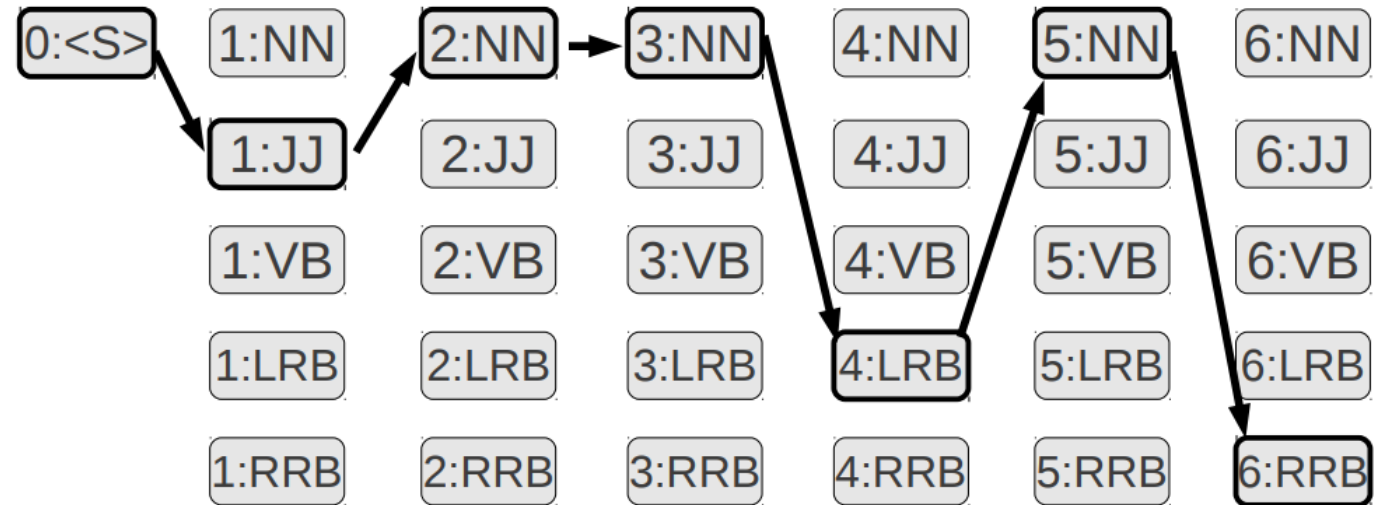
CRFs: Forward-Backward Algorithm

- Computing $Z(x)$ can be done by summing the numerator over all possible values for y_j and y_{j-1}
 - A practical solution since we need to compute the marginals for every value of y_j and y_{j-1} anyway
- It can also be done directly by employing dynamic programming, or using the computed $\alpha_{n+1}(y)$:

$$Z(x) = \sum_y \alpha_{n+1}(y)$$

CRFs: Inference

- Inference again uses the Viterbi algorithm
- The same idea as with MEMMs. Edge weights between a pair of tags y and y' is $M_i(y, y')$:



Some Accuracies on English Supervised POS Tagging

- Rough accuracies: (overall accuracy/unknown words)
 - Most frequent tag: ~90% / ~50%
 - Trigram HMM: ~95% / ~55%
 - TnT – Trigram HMM with better smoothing and handling of unknown words (Brants, 2000): 96.7% / 85.5%
 - Local MaxEnt: 96.8% / 86.8%
 - MEMM tagger: 96.8% / 86.9%
 - Structured Perceptron: 97.1% (we'll talk about this algorithm later in the course)
 - Cyclic tagger (multiplying two MEMMs) / CRFs: 97.2% / 89.0%
 - Inter-annotator agreement: ~98%

Domain Effects

- Accuracies degrade outside of domain
 - Up to triple error rate
 - Usually make the most errors on the things you care about in the domain (e.g. protein names)
- Open questions
 - How to effectively exploit unlabeled data from a new domain (what could we gain?)
 - How to best incorporate domain lexica in a principled way (e.g. UMLS specialist lexicon, ontologies)

Other Applications of CRFs

- CRFs have been used extensively in NLP, Vision and Computational Biology
- A few references: (#citations is a tricky measure, but the paper that introduced CRFs has about 13K of them..)
 - Named Entity Recognition (e.g., identifying protein names in biology papers)
 - Chinese Word Segmentation
 - Prediction of pitch accents
 - Semantic role labeling
 - Shallow Parsing
 - Syntactic Parsing

See section 2.7 in this CRF tutorial:

<http://homepages.inf.ed.ac.uk/csutton/publications/crftut-fnt.pdf>