Computer Graphics: Assignment 04

Lina Gundelwein, Letitia Parcalabescu, Anushalakshmi Manila

November 16, 2016

2. Analytical Geometry

1. Equation for sphere: $||\mathbf{x} - M||^2 = r^2$
   Equation for ray: $\mathbf{x} = \mathbf{o} + t\mathbf{d}$
   Combine and plug in values:

$$
\begin{aligned}
&||\mathbf{o} + t\mathbf{d} - \mathbf{M}||^2 = r^2 \\
&\mathbf{d}^2 t^2 + 2\mathbf{d}(\mathbf{o} - \mathbf{M})t + |\mathbf{o} - \mathbf{M}|^2 - r^2 = 0 \\
&\rightarrow a = \mathbf{d}^2 = 2,\ b = 2\mathbf{d}(\mathbf{o} - \mathbf{M}) = -20,\ c = |\mathbf{o} - \mathbf{M}|^2 - r^2 = 42 \\
&t_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{20 \pm \sqrt{400 - 4 \cdot 2 \cdot 42}}{2 \cdot 2} \\
&t_1 \approx 7 \rightarrow \mathbf{x}_1 = \begin{pmatrix} 7 \\ 7 \\ 1 \end{pmatrix} \\
&t_2 \approx 3 \rightarrow \mathbf{x}_2 = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \leftarrow \text{this one is visible to the observer}
\end{aligned}
$$

2. Vector notation for a plane: $(\mathbf{p} - \mathbf{p}_0) \cdot n = 0$

   Point on plane: $\mathbf{p}_0 = \frac{\mathbf{n}}{|\mathbf{n}|} \cdot 3 = \begin{pmatrix} \frac{3}{\sqrt{2}} \\ \frac{3}{\sqrt{2}} \\ 0 \end{pmatrix}$

   Substitute equation for a line into equation for the plane and calculate intersection:

$$
\begin{aligned}
t &= \frac{\mathbf{n} \cdot (\mathbf{p}_0 - \mathbf{o})}{\mathbf{n} \cdot \mathbf{d}} = \frac{3}{\sqrt{2}} \\
\rightarrow \mathbf{x}_3 &= \begin{pmatrix} \frac{3}{\sqrt{2}} \\ \frac{3}{\sqrt{2}} \\ 1 \end{pmatrix}
\end{aligned}
$$

3. Calculate normal on triangle:

$n = (B - A) \times (C - A) = \begin{pmatrix} 36 \\ 36 \\ 36 \end{pmatrix} \Rightarrow$ normalized: $n_0 = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$

Calculate intersection of ray with plane in which the triangle lies:

$$t = \frac{\mathbf{n} \cdot (\mathbf{p}_0 - \mathbf{o})}{\mathbf{n} \cdot \mathbf{d}} = \frac{\mathbf{n} \cdot (A - \mathbf{o})}{\mathbf{n} \cdot \mathbf{d}} = 2.5$$
$$\rightarrow \mathbf{x}_4 = \begin{pmatrix} 2.5 \\ 2.5 \\ 1 \end{pmatrix}$$

Parametric plane equation: $A + s(B - A) + t(C - A)$
Check whether x_4 lies on the triangle which is the case if $s \geq 0$, $t \geq 0$ and $s + t \leq 1$:

$$A + s(B - A) + t(C - A) = x_4$$
$$\Rightarrow \text{ Leads to three equations:}$$
$$-6s - 6t + 6 = 2.5$$
$$6s = 2.5 \rightarrow s = \frac{5}{12} \geq 0$$
$$6t = 1 \rightarrow t = \frac{1}{6} \geq 0$$
$$s + t = \frac{7}{12} \leq 1$$

x_4 lies on the triangle!