Computer Graphics: Assignment 04

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1 Euler Angles and even more Transformations

$$\psi \quad R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$\theta \quad R_y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\phi \quad R_z(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$R_{zyz} = R_z(\psi)\, R_y(\theta)\, R_z(\phi)$$

$$= R_z(\psi) \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta\cdot\cos\phi & -\cos\theta\sin\phi & \sin\theta \\ \sin\phi & \cos\phi & 0 \\ -\sin\theta\cdot\cos\phi & \sin\theta\cdot\sin\phi & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} 0.7071 & 0 & -0.7071 \\ 0 & -1 & -0 \\ -0.7071 & 0 & -0.7071 \end{bmatrix} = \begin{bmatrix} \cos\psi\cdot\cos\theta\cdot\cos\phi - \sin\phi\sin\psi & -\cos\theta\cdot\cos\psi\cdot\sin\phi - \sin\psi\cdot\cos\phi & \sin\theta\cdot\cos\psi \\ \cos\theta\cdot\sin\phi\cdot\cos\psi + \sin\phi\cdot\cos\psi & -\cos\theta\cdot\sin\phi\sin\psi + \cos\phi\cdot\cos\psi & \sin\theta\cdot\sin\psi \\ -\sin\theta\cdot\cos\phi & \sin\theta\cdot\sin\phi & \cos\theta \end{bmatrix}$$

$$\cos\theta = -0.7071 \Rightarrow \underline{\underline{\theta}} = \cos^{-1}(-0.7071) = (134.9994505)^\circ$$

$$\underline{\underline{\theta \approx 135^\circ}}$$

$$-\sin\theta\cdot\cos\phi = -0.7071 \Rightarrow \cos\phi = \frac{0.7071}{\sin(134.99\ldots)} = 0.99998082344$$

$$\phi = \cos^{-1}(0.9999\ldots) = \underline{\underline{(0.35483273)^\circ}}$$

$$\sin\theta\cdot\cos\psi = -0.7071 \Rightarrow \cos\psi = \frac{-0.7071}{\sin(134.99\ldots)} = -0.9999806196$$

$$\psi = (179.6451\ldots)^\circ \approx \underline{177^\circ}$$

(2)

$$R = R_z(\phi)\cdot R_y(\theta)\, R_x(\psi) = \begin{pmatrix} 0.7500 & -0.6495 & -0.1250 \\ 0.4330 & 0.6250 & -0.6495 \\ 0.5000 & 0.4330 & 0.7500 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta\cdot\cos\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\psi\sin\theta\cdot\cos\phi + \sin\psi\sin\phi \\ \cos\theta\cdot\sin\phi & \sin\psi\sin\theta\cdot\sin\phi + \cos\psi\cos\phi & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi \\ -\sin\theta & \sin\psi\cos\theta & \cos\psi\cos\theta \end{pmatrix}$$

$$\Rightarrow \quad -\sin\theta = 0.5 \quad \Rightarrow \quad \theta = -30^\circ$$

$$\frac{\sin\psi\cdot\cancel{\cos\theta}}{\cos\psi\,\cancel{\cos\theta}} = \tan\psi = \frac{0.433}{0.75} = 0.57733 \Rightarrow \psi = \underline{\underline{29.99^\circ}} \approx \underline{\underline{30^\circ}}$$

$$\frac{\cos\theta\cdot\sin\phi}{\cos\theta\cdot\cos\phi} = \tan\phi = \frac{0.433}{0.75} = 0.57733 \Rightarrow \phi \approx \underline{\underline{30^\circ}}$$

5.1-1

5.1-2

The sun is in the coordinate center, rotates around itself (y axis) with arbitrary angle, earth rotates around the original y axis around coordinate center with arbitrary angle, then there has to be a rotation about the new, by 23.5 rotated, y-axis the moon rotates around the a translated and rotating y-axis...

- Set *sun* to coordinate center
- PushMatrix()
  - Rotate *sun* about angle ϕ_{sun} around y-axis
- popMatrix()
- PushMatrix()
  - Rotate *earth* about $\frac{360}{365}$ around y-axis
  - Translate *earth* and *moon* about $dist_{earth-sun}$
  - PushMatrix()
    * Rotate *earth* about 23.5 around z-axis

    * Rotate *earth* about ϕ_{earth} around y-axis
  - PopMatrix()
  - PushMatrix()
    * Rotate *moon* about $\frac{360}{12}$ around y-axis
    * Translate *moon* about $dist_{moon-earth}$
  - PopMatrix()
- popMatrix()