

Computer Graphics: Assignment 04

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5.1 Euler Angles and even more Transformations

$$\begin{aligned}
 \psi \quad R_x(\phi) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \\
 \theta \quad R_y(\phi) &= \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \\
 \phi \quad R_z(\phi) &= \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

①

$$\begin{aligned}
 A_{xyz} &= R_z(\psi) R_y(\theta) R_x(\phi) \\
 &= R_z(\psi) \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta \cdot \cos\phi & -\cos\theta \cdot \sin\phi & \sin\theta \\ \sin\phi & \cos\phi & 0 \\ -\sin\theta \cdot \cos\phi & \sin\theta \cdot \sin\phi & \cos\theta \end{bmatrix} \\
 \begin{bmatrix} 0.7071 & 0 & -0.7071 \\ 0 & -1 & 0 \\ -0.7071 & 0 & -0.7071 \end{bmatrix} &= \begin{bmatrix} \cos\psi \cdot \cos\theta \cdot \cos\phi - \sin\psi \cdot \sin\phi & -\cos\theta \cdot \cos\psi \cdot \sin\phi - \sin\psi \cdot \cos\phi & \sin\theta \cdot \cos\psi \\ \cos\theta \cdot \sin\phi \cdot \cos\psi + \sin\theta \cdot \sin\psi & -\cos\theta \cdot \sin\phi \cdot \sin\psi + \cos\phi \cdot \cos\psi & \sin\theta \cdot \sin\psi \\ -\sin\theta \cdot \cos\phi & \sin\theta \cdot \sin\phi & \cos\theta \end{bmatrix} \\
 \cos\theta &= -0.7071 \Rightarrow \underline{\underline{\theta = \cos^{-1}(-0.7071) = (134.9996505)^\circ}} \\
 &\quad \underline{\underline{\theta \approx 135^\circ}} \\
 -\sin\theta \cdot \cos\phi &= -0.7071 \Rightarrow \cos\phi = \frac{0.7071}{\sin(134.99)} = 0.99998082344 \\
 \phi &= \cos^{-1}(0.9999...) = \underline{\underline{(0.35483273)^\circ}} \\
 \sin\theta \cdot \sin\psi &= -0.7071 \Rightarrow \cos\psi = \frac{-0.7071}{\sin(134.99...)} = -0.9999805196 \\
 \psi &= (179.6451)^\circ \approx \underline{\underline{177^\circ}}
 \end{aligned}$$

②

$$R = R_z(\phi) \cdot R_y(\theta) \cdot R_x(\psi) = \begin{pmatrix} 0.7500 & -0.6495 & -0.1250 \\ 0.4330 & 0.6250 & -0.6495 \\ 0.5000 & 0.4330 & 0.7500 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta \cdot \cos\phi & \sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi & \cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi \\ \cos\theta \cdot \sin\phi & \sin\psi \sin\theta \sin\phi + \cos\psi \cos\phi & \cos\psi \sin\theta \sin\phi - \sin\psi \cos\phi \\ -\sin\theta & \sin\psi \cos\theta & \cos\psi \cos\theta \end{pmatrix}$$

$$\Rightarrow -\sin\theta = 0.5 \Rightarrow \theta = -30^\circ$$

$$\frac{\sin\psi \cdot \cos\theta}{\cos\psi \cos\theta} = \tan\psi = \frac{0.433}{0.75} = 0.57733 \Rightarrow \psi = \frac{29.99^\circ}{\approx 30^\circ}$$

$$\frac{\cos\theta \cdot \sin\phi}{\cos\theta \cdot \cos\phi} = \tan\phi = \frac{0.433}{0.75} = 0.57733 \Rightarrow \phi \approx 30^\circ$$

5.1-1

- pushMatrix
 - Translate V_i by $-V_1$, such that V_1 is at coordinate center.
 - Rotate V_i by 45°
- popMatrix()
- Translate V_i by $+V_1$ (old)

5.1-2

The sun is in the coordinate center, rotates around itself (y axis) with arbitrary angle ϕ_{sun} , earth rotates around the original y axis (not the rotating one) around coordinate center with angle $\frac{360}{365}^\circ$, then there has to be a rotation about the new, by 23.5° rotated, y-axis by ϕ_{earth} , the moon rotates around the translated and rotating y-axis (but in x-z-layer, not the rotated axes).

- Set *sun*, *earth* and *moon* to coordinate center.
- PushMatrix()
 - Rotate *sun* about angle ϕ_{sun} around y-axis
- popMatrix()

- PushMatrix()
 - Rotate *earth* about $\frac{360}{365}^\circ$ around y-axis
 - Translate *earth* and *moon* about $dist_{earth-sun}$
 - PushMatrix()
 - * Rotate *earth* about 23.5° around z-axis
 - * Rotate *earth* about ϕ_{earth} around y-axis
 - PopMatrix()
 - PushMatrix()
 - * Rotate *moon* about $\frac{360}{12}^\circ$ around y-axis
 - * Translate *moon* about $dist_{moon-earth}$
 - PopMatrix()
- popMatrix()