wyznaczyć e^At metodą (gr. A) reprezentacji jordanowskiej / (gr. B) rozkładu spektralnego:

$$A = \begin{vmatrix} 6 & 2 & 2 \\ -2 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

Metoda Spektralna

$e^{At} = ?$ $\quad A = \begin{bmatrix} 6 & 2 & 2 \\ -2 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $\quad \det(\lambda I - A) = (\lambda-4)^2(\lambda-2)^{(1)}$

$[\lambda I - A] = \begin{bmatrix} \lambda-6 & -2 & -2 \\ 2 & \lambda-2 & 0 \\ 0 & 0 & \lambda-2 \end{bmatrix}$ czy (1) jest minimalny

$NWD \deg [\lambda I - A]| = \begin{bmatrix} (\lambda-2)^2 & & \\ & & \\ & 4 & \end{bmatrix}$ już mnożeniem 2 elementy które się wyglądzie pierwsze więc jest minimalny

$\begin{cases} \lambda_1 = 2 & k_1 = 1 \\ \lambda_2 = 4 & k_2 = 2 \end{cases} \quad s = 2$

$$f(A) = \sum_{i=1}^{s=2} \sum_{j=1}^{k} f^{(j-1)}(\lambda_i) Z_{ij} = f(\lambda_1 = 2) Z_{11} + f(\lambda_2 = 4) Z_{21} + f'(\lambda_2 = 4) Z_{22}$$

$W = [I, \lambda - 4, (\lambda-4)^2]^T$

$$\begin{cases} I = Z_{11} + Z_{21} \\ (A - 4I) = -2 Z_{11} + Z_{22} \\ (A - 4I)^2 = 4 Z_{11} \end{cases} \Rightarrow$$

$$Z_{11} = \frac{1}{4}\begin{bmatrix} 2 & 2 & 2 \\ -2 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Z_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Z_{22} = \begin{bmatrix} 2 & 2 & 2 \\ -2 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$e^{tA} = e^{2t} Z_{11} + e^{4t} Z_{21} + t e^{4t} Z_{22}$$

$$e^{tA} = \begin{bmatrix} e^{4t} + 2te^{4t} & 2te^{4t} & 2te^{4t} \\ -2te^{4t} & e^{4t} - 2te^{4t} & -e^{2t} + e^{4t} - 2te^{4t} \\ 0 & 0 & e^{2t} \end{bmatrix}$$

Metoda Rozkładu Jordana

$e^{At} = ?$ Jordan

$$\begin{bmatrix} 6 & 2 & 2 \\ -2 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \det(\lambda I - A) = (\lambda-4)^2(\lambda-2)$$

$$\lambda_1 = 2: \quad \begin{bmatrix} -4 & -2 & -2 \\ 2 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}_1h_1^1 = \begin{bmatrix} 0 \\ -a \\ a \end{bmatrix} \qquad a^2 \neq 0$$

$$\lambda_2 = 4 \quad \begin{bmatrix} -2 & -2 & -2 \\ 2 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}_1h_2^1 = \begin{bmatrix} b \\ -b \\ 0 \end{bmatrix} \qquad b^2 \neq 0$$

veriga ${}_1h_2^2$:

$$\begin{bmatrix} -2 & -2 & -2 \\ 2 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b \\ -b \\ 0 \end{bmatrix} \qquad {}_1h_2^2 = \begin{bmatrix} c \\ \frac{b}{2} - c \\ 0 \end{bmatrix}$$

$a=1 \quad b=2 \quad c=1$

$-2x_1 + 2x_2 = b \Rightarrow x_2 = \frac{b}{2} - x_1$

$x_1 = c$

$c^2 + \left(\frac{b}{2} - c\right)^2 \neq 0$

$$J = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{bmatrix}$$

$e^{At} = T e^{JA} T^{-1}$ potreba e^{JA}

$J = [2] \oplus \begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$

$e^{JA} = [e^{2t}] \oplus \begin{bmatrix} e^{4t} & te^{4t} \\ 0 & e^{4t} \end{bmatrix}$

$$e^{JA} = \begin{bmatrix} e^{2t} & 0 & 0 \\ 0 & e^{4t} & te^{4t} \\ 0 & 0 & e^{4t} \end{bmatrix}$$

$T \quad e^{Jt} \quad T^{-1}$

Input:

$$\begin{pmatrix} 0 & 2 & 1 \\ -1 & -2 & 0 \\ 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} e^{2t} & 0 & 0 \\ 0 & e^{4t} & t\,e^{4t} \\ 0 & 0 & e^{4t} \end{pmatrix} . \begin{pmatrix} 0 & 0 & 1 \\ 0 & -0.5 & -0.5 \\ 1 & 1 & 1 \end{pmatrix}$$

Result:

$$\begin{pmatrix} 2e^{4t}t + e^{4t} & 2e^{4t}t & 2e^{4t}t \\ -2e^{4t}t & e^{4t} - 2e^{4t}t & -2e^{4t}t - e^{2t} + e^{4t} \\ 0 & 0 & e^{2t} \end{pmatrix}$$