

$$E^A(At) = ?$$

1	0	0
1	2	1
-1	-1	0

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & 0 & 0 \\ -1 & \lambda-2 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda-1)(\lambda-2)(\lambda+1) = (\lambda-1)^3$$
 by rule of undetermined coefficients

$$\text{mvs}[\text{adj}(\lambda I - A)] = \text{NWD}[\text{adj} \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}] =$$

$$\text{adj}(\lambda I - A) = \begin{bmatrix} (\lambda-2)(\lambda+1) & -(\lambda+1) & 1-2 \\ 0 & \lambda(\lambda-1) & 1-\lambda \\ 0 & \lambda-1 & (\lambda-1)(\lambda-2) \end{bmatrix}^T$$

$$A = (\lambda-1)^2 \quad \lambda_1 = 1 \quad k_1 = 2 \quad s = 1$$

$$P(A) = \sum_{j=1}^2 \sum_{i=1}^k f_{ji} \lambda_{ji} z_{ji}$$

$$P(A) = f_{(1)}^0 z_{11} + f_{(1)}^1 z_{12} + f_{(1)}^2 z_{13}$$

$$w = [1, \lambda-1]^T \quad z_{11} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$I = z_{11} \cdot \quad z_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

$$A - I = z_{12}$$

$$e^{tA} = e^t z_{11} + t e^t z_{12}$$

$$e^{tA} = \begin{bmatrix} e^t & 0 & 0 \\ t e^t & e^t + t e^t & t e^t \\ -t e^t & -t e^t & e^t - t e^t \end{bmatrix}^T$$

$$e^{tA} = ? \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix} \quad \lambda I - A = \begin{bmatrix} \lambda-1 & 0 & 0 \\ -1 & \lambda-2 & -1 \\ 1 & 1 & \lambda \end{bmatrix}$$

$$\det(\lambda I - A) = (\lambda-1)(\lambda-2)(\lambda) + (\lambda-1) = (\lambda-1)^3$$

$$1 \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{matrix} x_1 = -x_2 - x_3 \\ x_2 = a \\ x_3 = b \end{matrix} \quad 1h_n = \begin{bmatrix} -a-b \\ +a \\ +b \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -a-b \\ a \\ b \end{bmatrix} = \quad \begin{matrix} 0 = -a-b \Rightarrow b = -a \\ x_3 = b - x_2 - x_1 \\ x_2 = c \\ x_3 = d \end{matrix}$$

$$b=1 \quad a=-1 \quad 1h_1^2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad b=0 \quad a=1$$

$$1h_1^1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad 1h_2^1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad J = T^{-1} A T$$

$$T = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$J = [1] \oplus \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad e^{Jt} = \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^t & -te^t \\ 0 & 0 & e^t \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^t & -te^t \\ 0 & 0 & e^t \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} e^t & 0 & 0 \\ te^t & te^t + e^t & te^t \\ -te^t & -te^t & e^t - te^t \end{bmatrix}$$