

Trees

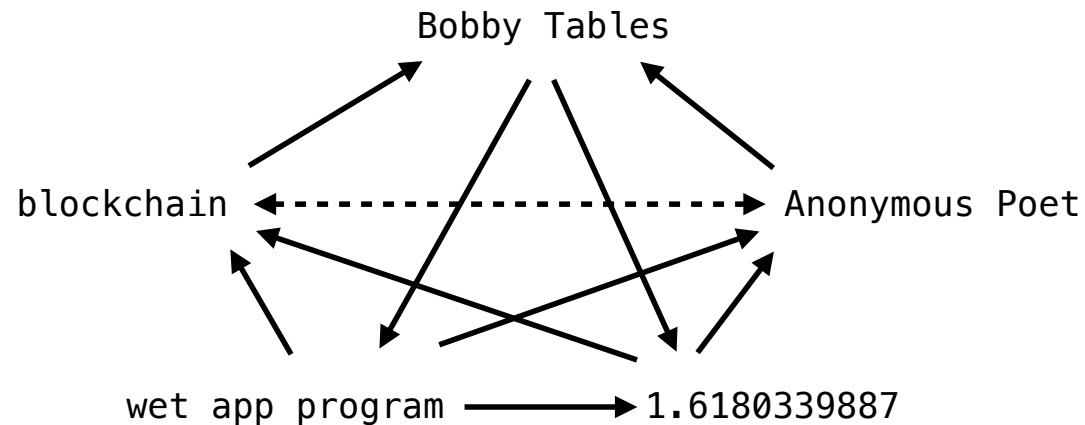
Announcements

Congratulations to the Winners of the Hog Strategy Contest

1st Place with 146 wins:

A five-way tie for first place!

"A submission scores a match point each time it has an expected win rate strictly above 50.0001%."



Congratulations to Timothy Guo, Shomini Sen, Samuel Berkun, Mitchell Zhen, Lucas Clark, Dominic de Bettencourt, Allen Gu, Alec Li, Aaron Janse

hog-contest.cs61a.org

Box-and-Pointer Notation

The Closure Property of Data Types

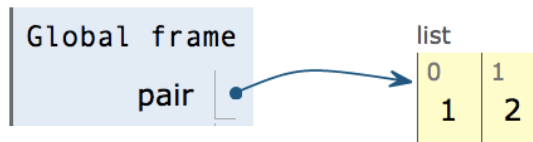
- A method for combining data values satisfies the *closure property* if:
The result of combination can itself be combined using the same method
- Closure is powerful because it permits us to create hierarchical structures
- Hierarchical structures are made up of parts, which themselves are made up of parts, and so on

Lists can contain lists as elements (in addition to anything else)

Box-and-Pointer Notation in Environment Diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element

Each box either contains a primitive value or points to a compound value

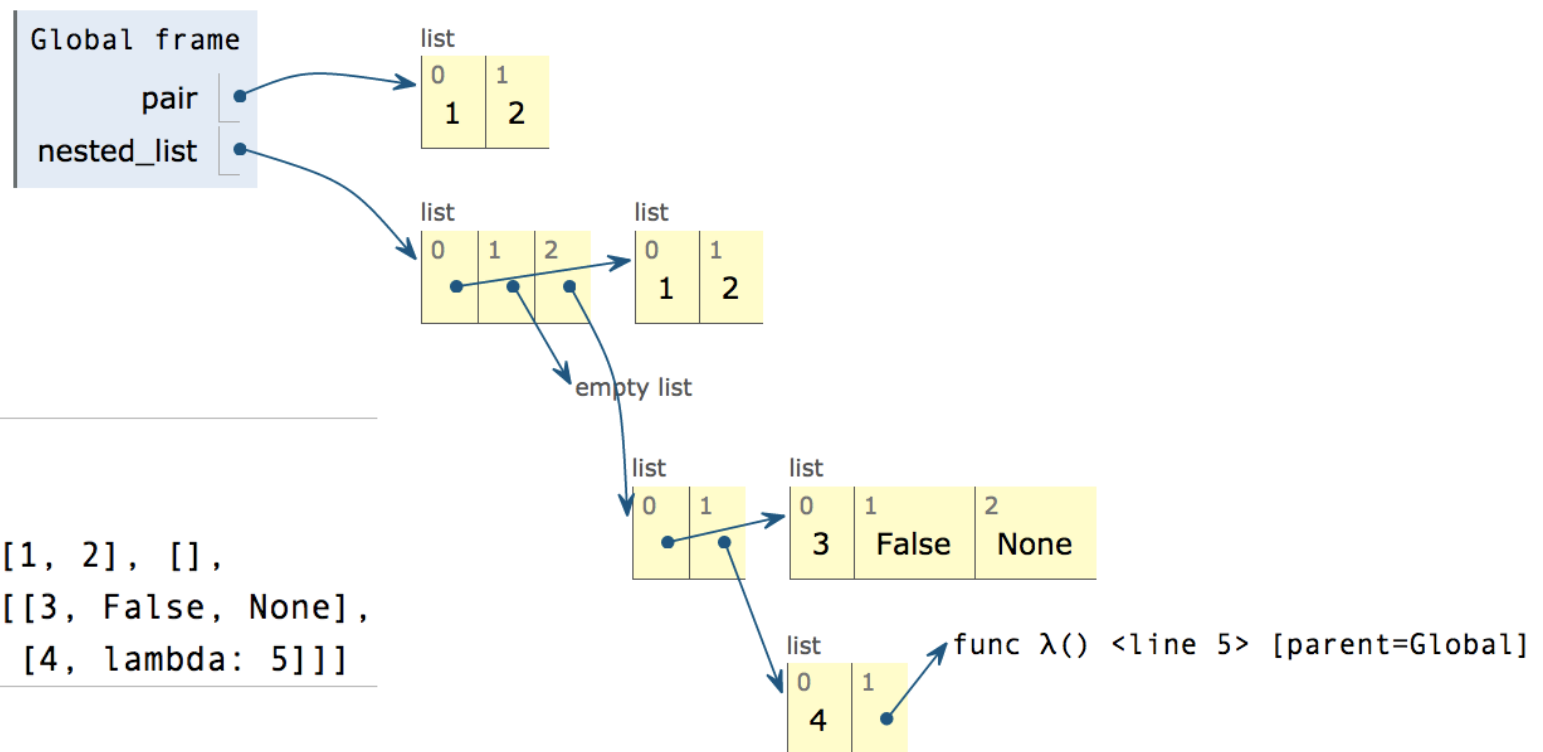


```
pair = [1, 2]
```

Box-and-Pointer Notation in Environment Diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element

Each box either contains a primitive value or points to a compound value

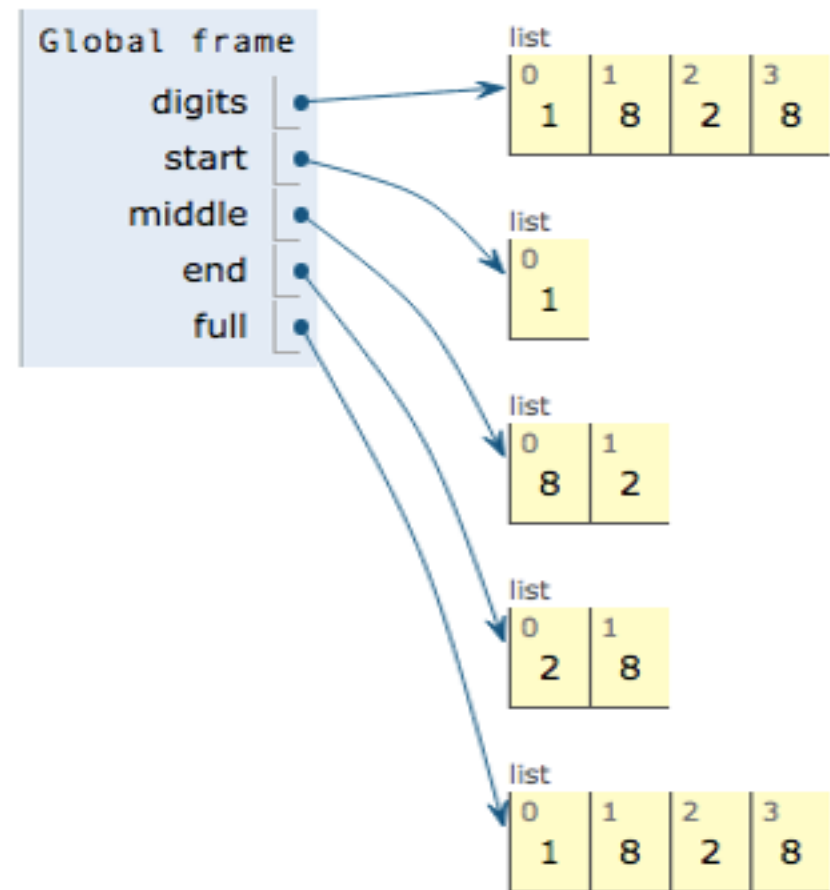


Slicing

(Demo)

Slicing Creates New Values

```
1 digits = [1, 8, 2, 8]
2 start = digits[:1]
3 middle = digits[1:3]
4 end = digits[2:]
→ 5 full = digits[:]
```



Processing Container Values

Sequence Aggregation

Several built-in functions take iterable arguments and aggregate them into a value

- `sum(iterable[, start])` -> value

Return the sum of a 'start' value (default: 0) plus an iterable of numbers.

- `max(iterable[, key=func])` -> value
`max(a, b, c, ...[, key=func])` -> value

With a single iterable argument, return its largest item.

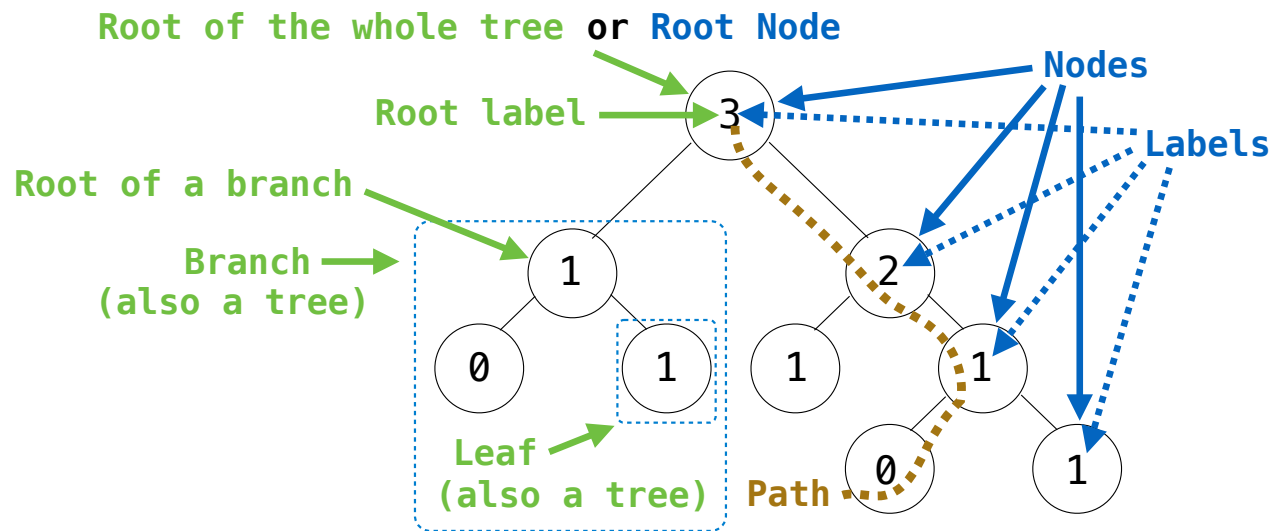
With two or more arguments, return the largest argument.

- `all(iterable)` -> bool

Return True if `bool(x)` is True for all values `x` in the iterable.
If the iterable is empty, return True.

Trees

Tree Abstraction



Recursive description (wooden trees):

A **tree** has a **root label** and a list of **branches**

Each **branch** is a **tree**

A **tree** with zero **branches** is called a **leaf**

A **tree** starts at the **root**

Relative description (family trees):

Each location in a tree is called a **node**

Each **node** has a **label** that can be any value

One node can be the **parent/child** of another

The top node is the **root node**

People often refer to labels by their locations: "each parent is the sum of its children"

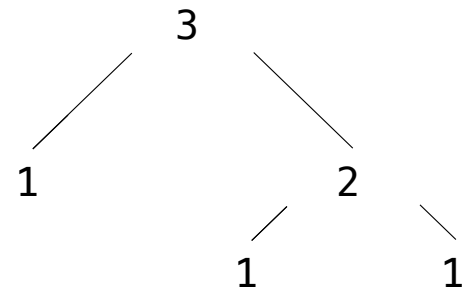
Implementing the Tree Abstraction

```
def tree(label, branches=[]):  
    return [label] + branches
```

```
def label(tree):  
    return tree[0]
```

```
def branches(tree):  
    return tree[1:]
```

- A **tree** has a root **label** and a list of **branches**
- Each branch is a tree



```
>>> tree(3, [tree(1),  
...         tree(2, [tree(1),  
...                 tree(1)])])  
[3, [1], [2, [1], [1]]]
```

Implementing the Tree Abstraction

```
def tree(label, branches=[]):  
    for branch in branches:  
        assert is_tree(branch)  
    return [label] + list(branches)
```

Verifies the tree definition

```
def label(tree):  
    return tree[0]
```

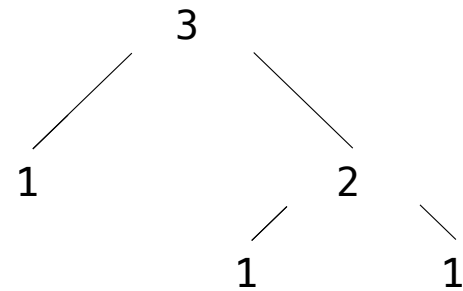
Creates a list from a sequence of branches

```
def branches(tree):  
    return tree[1:]
```

Verifies that tree is bound to a list

```
def is_tree(tree):  
    if type(tree) != list or len(tree) < 1:  
        return False  
    for branch in branches(tree):  
        if not is_tree(branch):  
            return False  
    return True
```

- A **tree** has a root **label** and a list of **branches**
- Each branch is a tree



```
>>> tree(3, [tree(1),  
...         tree(2, [tree(1),  
...                 tree(1)])])  
[3, [1], [2, [1], [1]]]
```

```
def is_leaf(tree):  
    return not branches(tree)      (Demo)
```

Tree Processing

(Demo)

Tree Processing Uses Recursion

Processing a leaf is often the base case of a tree processing function

The recursive case typically makes a recursive call on each branch, then aggregates

```
def count_leaves(t):  
    """Count the leaves of a tree."""  
    if is_leaf(t):  
        return 1  
    else:  
        branch_counts = [count_leaves(b) for b in branches(t)]  
        return sum(branch_counts)
```

(Demo)

Discussion Question

Implement `leaves`, which returns a list of the leaf labels of a tree

Hint: If you `sum` a list of lists, you get a list containing the elements of those lists

```
>>> sum([ [1], [2, 3], [4] ], [])
[1, 2, 3, 4]
>>> sum([ [1] ], [])
[1]
>>> sum([ [[1]], [2] ], [])
[[1], 2]
```

```
def leaves(tree):
    """Return a list containing the leaf labels of tree.

    >>> leaves(fib_tree(5))
    [1, 0, 1, 0, 1, 1, 0, 1]
    """
    if is_leaf(tree):
        return [label(tree)]
    else:
        return sum(List of leaf labels for each branch, [])
```

| | |
|--|--|
| <code>branches(tree)</code> | <code>[b for b in branches(tree)]</code> |
| <code>leaves(tree)</code> | <code>[s for s in leaves(tree)]</code> |
| <code>[branches(b) for b in branches(tree)]</code> | <code>[branches(s) for s in leaves(tree)]</code> |
| <code>[leaves(b) for b in branches(tree)]</code> | <code>[leaves(s) for s in leaves(tree)]</code> |

Creating Trees

A function that creates a tree from another tree is typically also recursive

```
def increment_leaves(t):  
    """Return a tree like t but with leaf labels incremented."""  
    if is_leaf(t):  
        return tree(label(t) + 1)  
    else:  
        bs = [increment_leaves(b) for b in branches(t)]  
        return tree(label(t), bs)  
  
def increment(t):  
    """Return a tree like t but with all labels incremented."""  
    return tree(label(t) + 1, [increment(b) for b in branches(t)])
```

Example: Printing Trees

(Demo)

Example: Summing Paths

(Demo)

Example: Counting Paths

Count Paths that have a Total Label Sum

```
def count_paths(t, total):  
    """Return the number of paths from the root to any node in tree t  
    for which the labels along the path sum to total.  
  
    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])])  
    >>> count_paths(t, 3) ◀  
    2  
    >>> count_paths(t, 4) ◀  
    2  
    >>> count_paths(t, 5)  
    0  
    >>> count_paths(t, 6)  
    1  
    >>> count_paths(t, 7) ◀  
    2  
    """  
    if label(t) == total:  
        found = 1  
    else:  
        found = 0  
  
    return found + sum([count_paths(b, total - label(t)) for b in branches(t)])
```

