Sample Space and Probability

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Contents

1 Sets (Quick Review)

Set, element, empety set $\emptyset$, finit set, countably finit set, uncountable set, sub set, equal, universal set Ω

1.1 Set opeartions:

1. Complement of a set S, with respect it the universe Ω, denoted by S^c
2. Union of two sets S, T, $S \cup T$
3. Intersection of two sets S, T, $S \cap T$
4. Union of several, $\bigcup_{n=1}^{\infty} S_n = S_1 \cup S_2 \cup \ldots$
5. Intersection of several, $\bigcap_{n=1}^{\infty} S_n = S_1 \cap S_2 \cap \ldots$
6. Sets are Disjoint if they share no element
7. A collection of sets is a partition of set S, if they are disjoint and the union of them are S

1.2 The Algrebra of Sets:

De Morgan's laws:

$$\left(\bigcup_n S_n\right)^c = \bigcap_n S_n^c$$

$$\left(\bigcap_n S_n\right)^c = \bigcup_n S_n^c$$

2 Probabilistic Models

Elements of a Probabilistic Model
- The sample space Ω, the set of all possiable outcomes
- The probability law, which assigns any event A a non-negative number $P(A)$

2.1 Choosing an Appropriate Sample Space

The element of the sample space should be distinct and *mutually exclurive*, and the sample space should be collectively exhaustive.

2.2 Probability Axioms

1. **(Nonnegativity)** $P(A) \geq 0$, for every event A
2. **(Additivity)** A, B are disjoint, then $P(A \cup B) = P(A) + P(B)$
3. **(Normalization)** $P(\Omega) = 1$

2.3 Discrete Models

e.g. The toss of a coin several times *Like {HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}(3 times) and the probability stuff*

Discrete Probability Law

The sample space $S = \{s_1, s_2, s_3, ..., s_n\}$ consists of finite number of elements, we have:

$$P(S) = P(\{s_1, s_2, s_3, ..., s_n\}) = P(s_1) + P(s_2) + P(s_3) + ... + P(s_n)$$

Discrete Uniform Probability Law

Ii the outcomes are equally likely, then the Probability of any single outcome A becomes:

$$P(A) = \frac{\text{number of elements of } A}{n}$$

2.4 Continuous Models

Like throughing a dart on a certian area or sth else ...

2.5 Properties of Probability Laws

1. If $A \in B$, then $P(A) \leq P(B)$
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
3. $P(A \cup B) \leq P(A) + P(B)$
4. $P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$

3 Conditional Prabability

Conditional probability provides us with a way to reason about the outcome of an ekperiment, based on **parcial information**. (The experiment is done and the only have some parcial information about it.)

e.g. The experiment involving two successive rolls of a die, you are toled that the sum of the two rolls are 9. What's the probability of the first roll is a 6?

In precise terms, the conditional probability is when we know the is with in a given event B, we wish to know the probability of the event A. We call this *conditional probability of A given* B, denoted by $P(A \mid B)$

Defenition$_{conditional\ probability}$: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$