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FACULTAD DE CIENCIAS EXACTAS Y NATURALES

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Mass dependence of the Baryonic Acoustic Oscillations in N-body simulations

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CHAPTER 1

Introduction

In the standard model of cosmology the universe was born in a big bang, a primordial singularity with very high energy and matter density that finally produced an expanding, isotropic and homogeneous Universe. From observations it has been found that this expansion is currently accelerating with time Hamuy (1996).

There are four components of the matter-energy content of the universe, dark and baryonic matter, radiation and dark energy. According to recent estimations, the last one accounts for around 70% of this content and is responsible for the accelerated expansion of the universe Ruiz-Lapuente (2010). The baryonic acoustic oscillations allows to study the nature of this expansion as it will be explained.

In the early universe the dark matter (DM) formed density fluctuations, causing baryonic matter to be unstable against gravitational perturbations. At this stage in the evolution of the universe the temperature was very high, allowing a coupling between baryonic matter and radiation through Thomson scattering. So the increase of baryonic matter in the DM density fluctuations not only caused an increase in density, but also radiation pressure against collapse. Therefore, an expanding wave centered in the fluctuation is caused because of the radiation pressure. This wave is the baryonic acoustic oscillation hereinafter BAO Hu & Sugiyama (1996), Eisenstein & Hu (1998).

Nevertheless, it is necessary to consider that the universe is expanding and this results in a temperature decrease. Therefore, when temperature is low enough, the baryonic matter and radiation decoupled, making BAO to stop expanding and leaving an imprint in the matter distribution. This is, a peak in the matter distribution that can be noticed in the correlation function. The distance that a BAO could have traveled by the time of decoupling is called sound horizon. This scale has been measured in the Cosmic Microwave Background as 146.8 ± 1.8Mpc, Komatsu et al. (2008).

Since BAO do not change in size after decoupling they can be used as a standard ruler. They allow to measure the Hubble parameter and angular diameter distance as a function of z, and this way to measure the rate of expansion at different times during the evolution of the universe. Hence, BAO is key to constraint dark energy parameters.

A way to study the imprint in the matter distribution associated to BAO signal is through the 2D point correlation function or the power spectrum that is its Fourier pair, Eisenstein et al. (2005), Tegmark et al. (2006). A peak due to the BAO appears in the correlation function but there are several issues to take into consideration. For example, the non-linear clustering smear out the BAO imprint causing a broadening of the peak Crocce & Scoccimarro (2007). These, among other problems, have to be taken into account when BAO are studied.

In the present work, we plan to study the BAO from numerical cosmological simulations. More precisely, the BAO will be studied trying to answer what are the changes of the BAO's properties with the change of the scale of the tracer halo population? is there any change in the position peak? is there any change in the width peak? In general, the question we want to answer is: Is there any dependence in the width and amplitude of the BAO signal with the tracer halo population? Answering this questions will lead not only to a better understanding of the physics of BAO but a better understanding of the accelerated expansion of the universe that still has so many questions to be answered.

CHAPTER 2

Cosmological Background

Cosmology is the branch of physics that studies the Universe as a whole, therefore, it attempts to explain its origin, evolution and structure at big scales. Hence, a coarse grained approximation is mandatory due to the scales considered, this is, several approximations are necessary in the endeavour of such a task.

In this search, two major points are considered. The first one is the cosmological principle, it assumes that on sufficiently large scales the Universe is homogeneous and isotropic. In this context, homogeneity can be understood like invariance under translation and isotropy like invariance under rotation. Then, this principle establishes that for the fundamental observers, the Universe should appear isotropic and homogeneous. Although the Universe is expanding, the distance among fundamental observers does not change with time, their reference system moves with the Universe expansion. Using these observers it is possible to synchronize clocks with a light pulse, the time measured by them is named cosmic time Longair (2008).

The overall isotropy and homogeneity have been found in observations of the cosmic microwave background radiation (CMB) and the sponge like structure of the distribution of galaxies. Until few years ago, all observations have agreed with this asseveration, however, recent evidence from Planck data have shown that anisotropies can appear at big scales Ade et al. (2015).

The sponge like structure refers to the fact that, nowadays at cosmological scales, big structures such as halos, sheets and voids forming a filamentary distribution are observed.

They are explained through the growth of initial seeds, small density perturbations at the early Universe that evolve due to gravitational instabilities. These processes are occurring in an expanding Universe. This latter affirmation is predicted by the theory of general relativity in which modern cosmology is based, a second important point to consider. Here, Einstein field equations (EFE) serve as a set of fundamental equations to study the evolution of the Universe at big scales. Fortunately, isotropy and homogeneity led to a simple form of these ones and hence a relatively simple mathematical treatment in cosmology may be developed. From EFE, Friedmman equations are obtained which provide a theoretical framework to study the Universe expansion Padmanabhan (1996).

A standard model in cosmology that takes into consideration the aspects exposed previously is ΛCDM, where additionally to an expanding universe, there is a dark energy component that accelerates its expansion. This is precisely the framework that is going to be used in this work. Here, ΛCDM stands for Λ cold dark matter.

In this chapter, several basic concepts in ΛCDM standard model are going to be introduced to finally lead to bayronic acoustic oscillations (BAO).

2.1 Robertson Walker Metric (RW)

As was mentioned before, observations of the Universe at big scales show that it is homogeneous and isotropic, at least as a good approximation, i.e., inhomogeneities appear also at big scales in the CMB. Nevertheless, it is taken as a postulate for ΛCDM cosmology . Let's see this in more detail

- Cosmological principle: *The Universe is homogeneous and isotropic at big scales.*

  In this context, homogeneous refers to that the fact that independently of where the reference system is located we are going to observe the same global patterns, i.e., the structure of the observed Universe is the same no matter the reference system used Padmanabhan (1996). On the other hand, isotropy establishes that regardless of the direction chosen, the same structure is going to be observed. Then, we are dealing with translational and rotational symmetry Padmanabhan (1996).

  These characteristics are observed on mega parsec scales, i.e., big scales. However, this is only valid for the actual epoch, the scale changes with time due to the expansion of the Universe.

- Weyl postulate : This one defines a set of observers that move along the geodesics, allowing to synchronize watches among different observers, hence a cosmic time could be defined. Therefore, the distance between galaxies can be measured at the same cosmic time.

As already stated the Universe is expanding. It was due to a research on nearby galaxies performed by Edwin Hubble that a redshift was found in most of the galaxies, i.e., they are moving away from us. Considering this movement, one could conclude we are in the center of the expansion. But this conclusion is wrong since the expansion Hubble law is valid independently where the coordinate system is defined.

A metric that satisfies homogeneity and isotropy and additionally contains a term that accounts for the Universe's expansion is the Robertson Walker metric. It is defined in general terms as $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$, where $g_{\mu\nu}$ is the metric tensor and uses coordinates $x^\alpha = \{ct, x, y, z\}$. The metric tensor takes the next form $g_{\mu\nu} = diag\{1, -\frac{a^2}{1-k^2} - a^2r^2, -a^2r^2\sin^2\theta\}$, and the metric is:

$$ds^2 = c^2dt^2 - a(t)^2\left[\frac{d^2r}{1-Kr^2} + r^2(d^2\theta + \sin^2\theta d^2\phi)\right], \tag{2.1}$$

the term a is the scale factor, it describes how the relative distance between two fundamental observers changes with time. The term K is the curvature constant for the current time and defines the Universe geometry. When $K = 0$ an euclidean metric is recovered leading to a flat universe. If $K = 1$ the Universe would be described by a spherical geometry and it would collapse because of its energy matter density content. And finally, $K = -1$ corresponds to a hyperbolic geometry where the Universe would be in accelerated expansion. The last case is precisely what it is observed of our Universe. In the figure 2.1 we show the behavior of different cosmologies depending on the K value and density content Longair (2008).

One important aspect to consider is that the geometry depends on the total energy-matter density content, Ω_o. This can be concluded from the definition of the curvature constant $K = H_o^2(\Omega_o - 1)/c^2$.

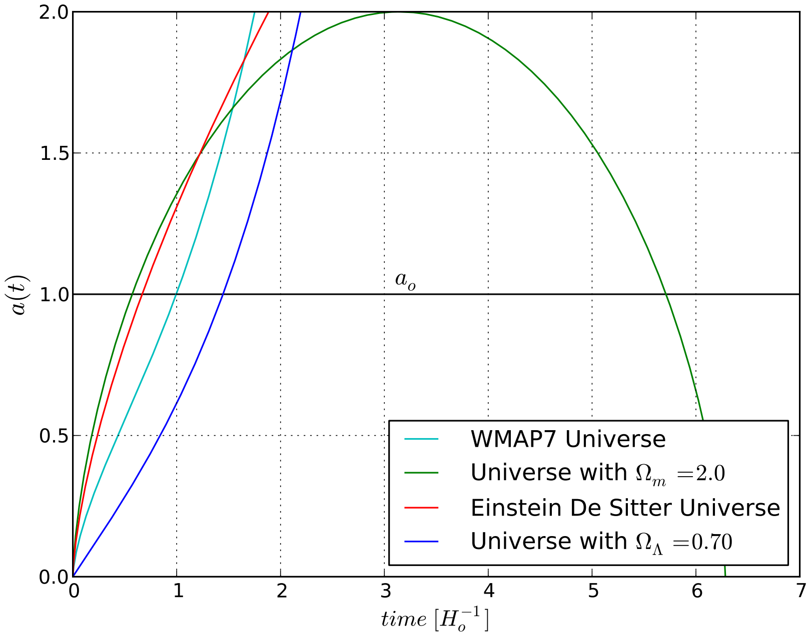

Figure 2.1: Scale factor as a time function. The Universe expansion for different density contributions. A closed Universe is obtained when $\Omega_m = \Omega_o > 1$ (see 2.6). Also, the PLANCK parameters show an accelerated expansion.

2.2 Hilbert-Einstein field equation

At big scales, gravitation dominates many of the physical phenomena occurring. To study the gravitational interaction is necessary the theory of general relativity (TGR). At smaller scales, the Newtonian gravitational theory might be used, where the Poisson equation offers a relation between the second derivative of the gravitational potential Φ and the source of the field ρ

$$\nabla^2\Phi = 4\pi G\rho,$$

this equation is obtained from TGR for low velocities and a weak gravitational field ($\Phi/c^2 << 1$) Padmanabhan (1996). A key equation of TGR is the Hilbert-Einstein field equation, a six independent component tensorial equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} + g_{\mu\nu}\Lambda, \tag{2.2}$$

in equation 2.2 the first term of the left side is Ricci's tensor (second derivatives of the metric tensor). The second one contains the scalar of curvature R that defines space-time geometry.

In the right side of the equation, the tensor energy-momentum $T_{\mu\nu}$ is present. It includes, as its name suggest, all the contributions to energy and momentum. In the last term, Λ is the cosmological constant that could be associated with the vacuum density term and is responsible for the accelerated expansion of the Universe.

Hence, the left side of the equation has associated geometry terms, while the right one, the ones associated with the matter and energy distribution. Then, it could be interpreted as if geometry is determined by the matter-energy content of the Universe, though, strictly speaking, the energy-momentum tensor depends in the metric tensor too Padmanabhan (1996).

There is an interesting case of this tensor when we are dealing with a perfect fluid, i.e., without viscosity. It shows that not only density causes curvature of space-time but also pressure. The Universe can be modeled with this particular shape of the energy-momentum tensor. It can be expressed as

$$T^{\mu}_{\sigma} = diag\{c^2\rho, -P, -P, -P\},$$

where ρ is the density and P is the fluid pressure.

There are several solutions to the Einstein field equation but not many in an analytical form. An analytical solution is Schwarzchild's solution that represents the metric of a static spherical mass. Other possible solution is the Kerr metric that corresponds to a rotating uncharged mass. The RW metric satisfies these equations too.

2.3 Friedmann equations

From HE field equations and the RW metric it is possible to propose cosmological models that give account for the observed dynamics in the Universe. In this direction, the components of the field equation can be taken, $\beta = \nu = 0$, time-time component, and $ii = 1, 2, 3$, from where one gets, for the scale factor:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right) + \frac{\Lambda c^2}{3}, \tag{2.3}$$

$$\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{c^2 K}{a^2} = 4\pi G\left(\rho - \frac{P}{c^2}\right) + \Lambda c^2, \tag{2.4}$$

where, it has been used the energy momentum tensor for an ideal fluid. The former expressions are the Friedmann equations, hereinafter FE, that are written in a form that can account for the Universe expansion. In equations 2.3 and 2.4 $a(t)$ is the scale factor that is set to one for the actual epoch, $a(t_o) = 1$, ρ is the total density (radiation plus matter density), P is the total pressure.

The equation 2.3 has the form of force equation and it can be partially deduced from newtonian mechanics (without the pressure and cosmological constant terms) Longair (2008). A most convenient and used form is obtained after algebraically manipulating them, a second form of FE that can be interpreted as an energy equation

$$H(t) = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(\rho + \frac{\Lambda c^2}{8\pi G}\right) - \frac{Kc^2}{a^2}, \tag{2.5}$$

where the first term in the right hand side is the potential energy. This equation also allows to define the Hubble parameter since $H(t) = \dot{a}^2/a^2$ and for the actual epoch its value is equal to $H(t_o) = H_o = 100h$ Km s^{-1} Mpc^{-1} where $h = 0.6774$ according to Planck measures Ade et al. (2015).

Additionally 2.5 can be expressed in terms of the critical density, i.e. the matter-energy density required for a flat Universe. Therefore, if the Universe has a bigger density, $\rho > \rho_{crit}$, it would collapse about itself. Conversely, the Universe would continue to expand indefinitely. The critical density is defined as:

$$\rho_{crit}(t) = \frac{3H(t)^2}{8\pi G}.$$

The equation 2.5 is divided by the Hubble constant H_o. It is algo defined the density parameter $\Omega_{i,o} = \rho_{i,o}/\rho_{crit}(t_o)$ with $i = m, r, \Lambda$. Then, it is obtained the next expression, where different contributions of the density to the Hubble parameter are observed, i.e., matter, radiation and vacuum density:

$$\frac{H^2(z)}{H_o^2} = \Omega_{m,o}\left(1+z\right)^3 + \Omega_{r,o}\left(1+z\right)^4 + \Omega_{\Lambda,o} + \left(1-\Omega_o\right)\left(1+z\right), \tag{2.6}$$

where $\Omega_o = \Omega_{m,o} + \Omega_{r,o} + \Omega_{\Lambda,o}$. The density parameter $\Omega_{m,o}$ accounts for the matter contribution to density, i.e., baryonic and dark matter components. $\Omega_{r,o}$ includes the contribution of the radition in the Universe density. And finally, $\Omega_{\Lambda,o}$ accounts for the contribution of vacuum to the total density. It has been introduced the relation between redshift and scale factor $1 + z = 1/a$ Padmanabhan (1996).

Every term of matter-energy density is a different function of the Universe expansion, although the vacuum energy does not depend on the redshift, this is, is constant through time.

Initially the Universe was dominated by the radiation, during this epoch ($z > 1100$) matter and radiation were coupled, i.e., the De Broglie electrons wavelenght were comparable to the wavelength of photons. Because of this, collisions between photons and electrons were very frequent causing that the mean free path of the photons be negligible and that the Universe would be opaque. During this coupling, radiation and matter had the same temperature and its behavior is explained as a black body. As can be seen in the figure 2.2, from $z_m = 3230$ matter becomes the major contribution to the Universe density. When $z_{rec} = 1100$ the temperature drop is big enough and the recombination rate turns higher than the ionization one. The last radiation dispersion due to matter still can be observed, and it is called cosmic radiation background (CMB). Because of the Universe expansion, its temperature has been dropping, and it is nowadays around $T = 2.7K$.

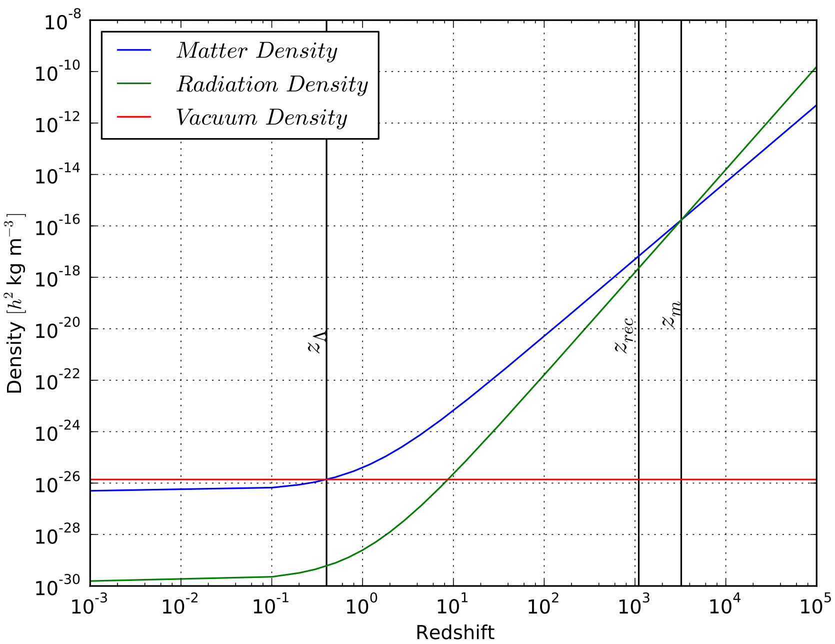

Figure 2.2: Dependence in redshift for Ω_Λ, Ω_m and Ω_r. The decoupling between matter and radiation is obtained when z_{rec}.

Nowadays, the dominant density component in the Universe is vacuum, ρ_Λ, though it is a constant since it does not depend on the scale factor, as already stated. In contrast, matter

Parameter	Symbol	Best fit
Hubble constant (km/Mpc-s)	H_0	67.74±0.46
Baryon density	$\Omega_b h^2$	0.02230±0.00014
Cold dark matter density	$\Omega_c h^2$	0.1188±0.0010
Dark energy density	Ω_Λ	0.6911±0.0062
Scalar spectral index	n_s	0.9667±0.0040
Sigma 8	σ_8	0.8159±0.0086

Table 2.1: Cosmological parameters from Planck results Ade et al. (2015).

depends on the scale factor as a^{-3} and radiation as a^{-4} causing both components diminish in time.

The cosmological constant could be associated to vacuum energy that causes an opposed behavior in the Universe dynamics compared to mass density, i.e., it accounts for the accelerated universe expansion.

There are several solutions to 2.5, for instance in the Einstein de Sitter Universe, there are no radiation or vacuum contributions to the density and the total density is $\Omega_o = 1.0$. In this particular case, the solution is

$$t = \frac{2}{3H_o}(1+z)^{-3/2}.$$

Therefore, depending on the chosen density values, the equation 2.5 has different solutions and one can expect several Universe models, i.e., depending on the parameters chosen, the Universe's evolution changes. The cosmological parameters measured from Planck, a space observatory that is operated by the European Space Agency, shown in table 2.1. When these values recovered by Planck are replaced in the ΛCDM model, it is concluded that our Universe is expanding at an increasing rate, i.e., an accelerated expansion is observed.

Other possible Universe models are for example, one obtained when matter density parameter is the only contribution to total universe density but it is bigger than 1. In such a case the Universe obtained is closed. Other one, it is one obtained when the Universe is dominated for the vacuum contribution. In this case, the Universe is always open. When all the contributions are present, the Universe can be open or closed depending on the total density parameter as shown in the figure 2.2.

2.4 Equation of state

As mentioned before, scale factor characterizes the Universe expansion, then finding relations between each density component of the Universe with this factor is an important task. A relation of proportionality between them is shown below. For further details Padmanabhan (1996) is a good reference.

Matter density: Assuming that all matter content in the Universe is an isolated system, the first law of thermodynamics is expressed as $dU = -pdV$, where relativistic terms are included in the internal energy term. Using the equipartition theorem, and deriving the internal energy with respect to the scale factor, is obtained:

$$T \propto a^{-2},$$

but from the equation of state $P = NkT$ and taking into account that the content of matter varies as $N = N_o a^{-3}$ Longair (2008)

$$P \propto a^{-5}. \tag{2.7}$$

Pressure exerted by matter decreases strongly with Universe's expansion, hence the dark energy pressure becomes more dominant as a increases. Density and temperature changes are slower.

Radiation density: Radiation energy density ξ expressed in terms of the photon density, $N(\nu)$, is:

$$\xi = \sum_{\nu} N(\nu) h\nu,$$

where $N(\nu)$ satifies the relation $N \propto (1+z)^3$. Furthermore, $\nu \propto (1+z)$ hence $\xi \propto a^{-4}$. When comparing the last result with Stefan-Boltzmann law it can be concluded that::

$$T \propto a^{-1}.$$

Finally, radiation pressure dependence on scale factor is found with the equation $P = \epsilon_{total}/3$, so:

$$P \propto a^{-4}. \tag{2.8}$$

Vacuum density: On the other hand, vacuum satisfies $\epsilon_{total} = \rho c^2$ where ρ is an effective density. Replacing this result in the first law of thermodynamics and deriving with respect to scale factor:

$$P = -\rho c^2 = -\frac{\Lambda c^4}{8\pi G}, \tag{2.9}$$

the vacuum density constancy is used for the deduction of the equation 2.9.

2.5 Perturbation evolution in the newtonian regime

There is no radiation comming toward us from a previous epoch to decoupling. Although, due to the last scattering between radiation and matter, highly homogeneous and isotropic distribution of matter is inferred from the patterns obtained from background cosmic radiation[1].

In the CMB radiation, small temperature perturbations are observed indicating the presence of small matter perturbations at this epoch. These are the initial seeds from where structures observed nowadays formed.

In this structure growth, density perturbations are increasing but it is not until they got a size of $\delta \sim 1$, where δ is a measure of the deviation of the current density respect to the background density, that this growth is dominated by gravity. The perturbations have grown enough to start talking about galaxy formation when their density gets around 1×10^6 compared to the background density, this happens for a epoch around $z \sim 100$ for halos with gas that have cooled enough.

But, it is still important to study the initial stages of the perturbations. This is, the perturbations do not deviate significantly from the background density. But their importance reside in that they are the seed of the larger structures, such as galaxies clusters and so on. Because of this, a linear regime treatment for perturbations when $\delta \ll 1$ are key in such a study Longair (2008).

2.5.1 Newtonian description

Inflation is an exponential expansion of the space-time in the initial stages of the Universe. In this stage quantum perturbations were magnified to cosmic size, forming inhomogeneities

[1] Image taken from as shown in Figure 2.3 `http://www.esa.int/spaceinimages/Images/2013/03/Planck_CMB`

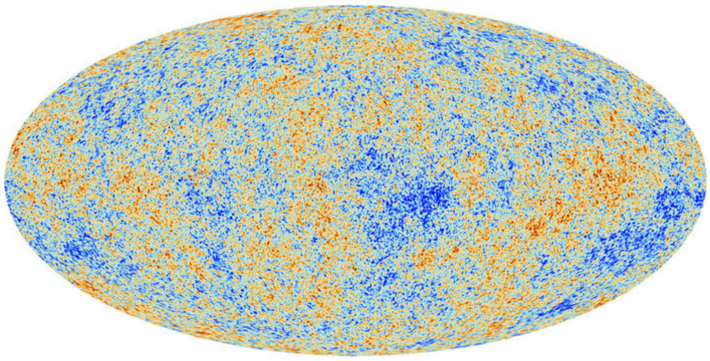

Figure 2.3: Cosmic background radiation image obtained by Planck satellite. Ade et al. (2015) It is a remnant from the first stages of the Universe. Before this radiation, the Universe was opaque, hence, we do not receive radiation from earlier stages.

of the density field, the seeds for the large scale structure of the Universe N.Gorobey et al. (2001). The initial density perturbations considered have a small characteristic length such that the relativistic effects can be neglected, i.e., perturbations do not affect the local metric.

In this Newtonian approximation the equations of gas dynamics for a fluid in a gravitational field can be considered. For a fluid in motion with a velocity distribution $\mathbf{u}$ and density ρ subject to a gravitational field ϕ that suffers changes in its pressure P, satisfies:

$$\begin{aligned}\frac{d\rho}{dt} &= -\rho\nabla_r \cdot \mathbf{u},\\ \frac{d\mathbf{u}}{dt} &= -\frac{\nabla_r P}{\rho} - \nabla_r\phi,\\ \nabla_r^2\phi &= 4\pi G\rho,\end{aligned} \tag{2.10}$$

where an Eulerian description is used, i.e., the partial derivatives in the expressions 2.10 describe the variations of the properties at a fixed point in space, r is the proper coordinate.

Since density perturbations are the ones that trigger potential wells, they are more of our interest than the density field itself. Then, it is useful to express the density as $\rho = \overline{\rho}(1+\delta)$, where $\overline{\rho}$ is the background density and δ is the overdensity (or perturbation) of interest Longair (2008).

Additionally, it is necessary to make clear another point. The velocity of the particles have two different contributions, the first one is because of the Universe expansion and the other one is the proper velocity of the particle, recessional and peculiar velocities respectively. Hence, the coordinate system could be changed from 2.10, an Euler description, to a Lagrangian one, i.e., moving with the Universe expansion. Let us see this in more detail, velocity in an Eulearian description is $\mathbf{u} = a\dot{\mathbf{x}} + \mathbf{x}\dot{a} = \mathbf{v} + \mathbf{x}\dot{a}$, where $\mathbf{v}$ is the peculiar velocity and $\mathbf{x}\dot{a}$ is the

Universe expansion velocity. Then, transforming in comoving coordinates, coordinates that move with the Universe expansion, and changing the density ρ to density perturbations δ, the equations 2.10 can be written as:

$$\begin{aligned}\frac{\partial\delta}{\partial t} &= -\frac{1}{a}\nabla\cdot[(1+\delta)\mathbf{v}],\\ \frac{\partial\mathbf{v}}{\partial t}+\frac{\dot{a}}{a}\mathbf{v}+\frac{1}{a}(\mathbf{v}\cdot\nabla)\mathbf{v} &= -\frac{\nabla\Phi}{a}-\frac{\nabla P}{a\overline{\rho}(1+\delta)},\\ \nabla^2\Phi &= 4\pi G\overline{\rho}a^2\delta, \end{aligned} \tag{2.11}$$

the first equation corresponds to the continuity equation, the second one is Euler's equation and the last one is Poissonian gravitational field equation with $\Phi=\phi+a\ddot{a}x^2/2$. Velocity components appear due to gravitational interactions and changes in pressure.

Additionally, equation of state relating the thermodynamic quantities pressure P, density ρ and entropy s for this cosmological fluid is:

$$P(\rho,s)=\left[\frac{h^2}{2\pi(\mu m_p)^{5/3}}e^{-5/3}\right]\rho^{5/3}\exp\left(\frac{2}{3}\frac{\mu m_p s}{k_B}\right). \tag{2.12}$$

Manipulating algebraically the continuity equation, Poisson equation and state equation, a wave equation for density perturbations could be obtained:

$$\frac{\partial^2\delta}{\partial t^2}+2\frac{\dot{a}}{a}\frac{\partial\delta}{\partial t}=4\pi G\overline{\rho}\delta+\frac{C_s^2}{a^2}\nabla^2\delta+\frac{2}{3}\frac{\overline{T}}{a^2}\nabla^2 s, \tag{2.13}$$

where $\overline{T}$ is the background temperature and C_s is the speed of sound. The Universe expansion is seen in the second term in the left hand side. Since for an expanding Universe the term $\dot{a}/a$ is positive, its effect opposes to the perturbation growth. This result was expected since the expansion is against collapse, leading to a a decrease of the growth of the perturbation.

The right hand side shows the causes for the evolution of the density perturbations. Entropy can be considered as the heat interchange between the perturbation and its surroundings, causing the expansion or growth of the perturbation. As expected, gravitational field is a source for perturbation growth.

A solution to the perturbation equation in terms of Fourier series is proposed as:

$$
\begin{aligned}
\delta(x,t) &= \sum_k \delta_k(t) e^{ik\cdot x}, \\
s(x,t) &= \sum_k s(t) e^{ik\cdot x},
\end{aligned}
$$

where **k** is the wave number and δ_k is a density mode that can be calculated using the discrete Fourier transform of the density field. Hence, every mode depends on all known values of the density perturbations.

An important aspect in the last expression is the independence of the functions $e^{ik\cdot x}$, allowing equation 2.13 to be expressed as:

$$
\frac{d^2\delta_k(t)}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\delta_k(t)}{dt} = \left[4\pi G\overline{\rho} - \frac{C_s^2 k^2}{a^2}\right]\delta_k(t) - \frac{2}{3}\frac{\overline{T}}{a^2}k^2 s_k(t), \tag{2.14}
$$

the solution of this equation provides expansion coefficients for the Fourier series, from where, the behavior of density perturbations, their growth or dissipation, is obtained Padmanabhan (1996).

2.5.2 Jeans Inestability

Before solving equation 2.14, it is important to develop some intuition about the physical phenomena. This can be achieved making some simplifications, for example, taking an isentropic static Universe ($\dot{a} = 0$) the expression becomes,

$$
\frac{d^2\delta_k(t)}{dt^2} + \omega^2\delta_k(t) = 0, \tag{2.15}
$$

with $\omega^2 = C_s^2 k^2/a^2 - 4\pi G\overline{\rho}$. Now, let us analyze the different solutions of this expression Longair (2008). When $C_s^2 k^2/a^2 > 4\pi G\overline{\rho}$, the frequency ω is positive. The solution obtained represents a sound wave, an oscillatory solution where gravity instabilities are balanced by radiation pressure. This particular solution is not of our interest, gravity is not strong enough to agglomerate matter. On the other hand, if $4\pi G\overline{\rho} > C_s^2 k^2/a^2$ the solution takes the form $\delta_k(t) \propto e^{\Gamma_k t}$, where $\Gamma_k = i\omega_k$ is the growth rate. Hence, the density mode can grow or dissipate depending on the growth rate, a negative rate causes dissipation but a positive one produces a gravitational collapse.

Therefore, density modes tend to collapse because of gravitational instabilities but in this process, pressure gradients appear due to atomic interactions causing dissipation. Nevertheless only the perturbations that collapse are the ones of our interest, they are the seeds of the large scale structure observed nowadays.

Furthermore, for a perturbation, a minimum length which stablishes if the perturbation is going to collapse is: $\lambda_J = 2\pi/k_j = C_s(\pi/G\rho)^{1/2}$. This length is called the Jeans' length and it is used to rewrite the growth rate in equation 2.15 . Now, with Jeans' length we could know the evolution of a perturbation, if $\lambda_{pert} >> \lambda_J$ is satisfied the perturbation collapses, where λ_{pert} is the length of the perturbation. A similar analysis can be performed with the Jean's mass defined as $M_J = \pi\bar{\rho}_{m,o}\lambda_J^3$. These expressions can be expressed as:

$$\lambda_J \approx 0.01(\Omega_{b,o}h^2)^{-1/2}Mpc,$$
$$M_J \approx 1.5 \times 10^5(\Omega_{b,o}h^2)^{-1/2}M_{\odot},$$

where $\Omega_{b,o}$ is the baryonic density parameter for the current time. The expresion is valid only for an epoch before decoupling. During this time, speed of sound was affected not only by matter but for radiation, the last one being the most important contribution to density as shown in figure 2.2. Since there is a change in the behavior of the density components with decoupling, and this way in the speed of sound, it appears a change in the Jean's length and Jean's mass. So, decoupling increases the gravitational collapse since the minimum characteristic length required for collapsing becomes smaller.

2.5.3 Evolution of perturbations

Until now dark matter has not been mentioned, a component that contrary to baryonic matter does not interact with radiation. But it is around 23% of the overall Universe matter-energy density content, making it responsible for the large scale mass distribution. First, let us see some observational evidence of dark matter.

1. One of the first observational evidence was found in the Coma cluster; a mass estimation of the cluster was done from the the virial theorem Zwicky (2009). Let us see this in more detail: the specific kinetic energy of the system is $T = v^2/2 \sim 3\sigma^2/2$ where σ is the galaxy velocity dispertion and the potential energy is $U = 3GM_{vir}/(5R_{vir})$. But since the virial theorem establishes that $T = 1/2|U|$, an estimate of the clusters mass can be found. But now, from the mass-luminosity ratio and the mean luminosity of the cluster, a second estimative of the mass can be found. There is a big discrepancy between the two values. A possible explanation is that not all of the cluster mass is visible, around $\sim 90\%$. And this contribution to the mass is what it is called dark matter.

2. The rotation curve of the galaxies can be other prove for dark matter existence. For example, the velocity measures performed with respect to the radius of Andromeda galaxy (or another spiral galaxies) is approximately the same independent of the radial distance of the stars to the center of the galaxy Jog (2002). From this, it could be affirmed that density is uniform along the galaxy contrary to expected for the observed number of star in function of the radius.

3. Another example is the Bullet cluster, composed by two two clusters that are colliding, an event not commonly observed. The gas of them reaches velocities around ~ 10 $millions\ of\ miles/h$ during the violent collision while they interact among them because of their charge Robertson et al. (2016). This interaction disminishes the gas velocity but this does not happen with dark matter cause it does not interact electrically. Using gravitational lensing a distortion map is obtained. Using the X rays detected and the distortion map, four different groups of matter are found, 2 bigger ones that correspond to the dark matter component and two smaller ones that correspond to luminous matter formed from the intercluster gas. These presents a strong evidence of dark matter existence.

Dark matter is studied as particles that interact among them and with baryonic matter through gravitational interaction. The interaction among dark matter particles leads to dark matter potential wells. But, dark matter initial seeds for potential wells are formed before decoupling, since they do not interact with radiation, they can gather forming early density perturbations. In this epoch, radiation and baryonic matter are modelled as a fluid that interacts gravitationally with dark matter, leading to no formation of baryonic seeds precisely because of the coupling between this two density components, i.e., radiation dissipates the baryonic seeds. Later a more detailed explanation will be provided.

In the linear regime an ansatz for perturbations is $\delta = \delta_o D(z)$, where δ_o is an initial seed from where a dark matter potential well formed and $D(z)$ is a growing function of an initial seed. Then, assuming isotropy and neglecting the velocity term since perturbations of our interest satisfy $\lambda_J << \lambda_{pert}$, equation 2.14 can be written as

$$\frac{d^2D(z)}{dz^2} + \left[\frac{H'(z)}{H(z)} - \frac{1}{1+z}\right]\frac{dD}{dz} = \frac{1}{H^2(z)}\left[\frac{4\pi G\overline{\rho}(z)}{(1+z)^2}\right]D(z).$$

In the last equation, different solutions are obtained by changing the density parameters. For example, an Einstein de Sitter Universe $\delta_k(z) = \delta_o(1+z)^{-1}$, an Universe dominated by

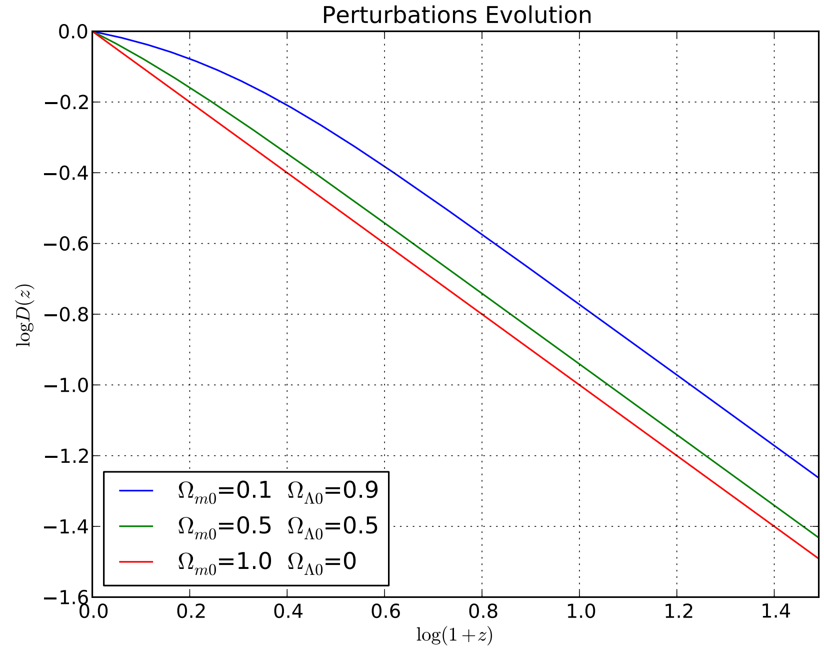

Figure 2.4: Perturbation evolution for a mass-vacuum model, the behavior for different contributions of each component. The redshift for the current time is $z = 0$.

radiation $\delta_k(z) = \delta_o(1+z)^{-1.22}$ and an Universe dominated by vacuum $\delta_k(z) = \delta_o(1+z)^{-0.58}$ Longair (2008). In figure 2.4 we show the evolution of perturbations for models with different matter and vacuum contributions. As expected, for bigger matter density perturbations (red curve), the growth is faster while for universes with bigger vacuum contribution (blue curve), perturbations grow slower because of the accelerated expansion induced by Λ. In the latter, a bigger initial mass is required to start the gravitational collapse.

2.6 Statistical properties of cosmological perturbations

To study the evolution of the Universe, the equation 2.13 can be used to know the density field and its evolution in time. But using the concept of density contrast $\delta = (\rho - \overline{\rho})/\overline{\rho}$, leads to a density perturbation field, which is a clearer way to analyse the evolution of the density field.

In linear regime, there are a infinite amount of perturbations described by different fourier modes. Since they evolve independently their amplitudes change with time and can be modelled using a transfer function $T(k)$ and a linear growth rate $D(t)$. To characterize such amount of density field values, statistical properties must be defined. This is, not considering individual positions or properties but instead moments defined from some distribution function. This idea is supported by the fact that there is no access to the primordial perturbations that originated the large scale structure observed nowadays. Hence, our Universe could be considered as a realization of a random process where a statistical treatment results as a natural way to study it.

Consider that the Universe, or certain region of it with volume V_u, can be divided into small cells, each of them with position x_i. These cells can be characterized statistically with a joint probability distribution and its moments that would describe the cosmic density perturbation field. Following this approach, the probability of having a mode between δ_k and $\delta_k + d\delta_k$ is,

$$\mathcal{P}(\delta_{\boldsymbol{\kappa}}) r_{\boldsymbol{\kappa}} dr_{\boldsymbol{\kappa}} d\phi_{\boldsymbol{\kappa}} = \exp\left[-\frac{r_{\boldsymbol{\kappa}}^2}{2V_u^{-1}P(\kappa)}\right] \frac{r_{\boldsymbol{\kappa}}}{V_u^{-1}} \frac{dr_{\boldsymbol{\kappa}}}{P(\kappa)} \frac{d\phi_{\boldsymbol{\kappa}}}{2\pi}, \tag{2.16}$$

where $r_{\boldsymbol{\kappa}}$ corresponds to perturbations amplitude, $\phi_{\boldsymbol{\kappa}}$ is the phase and varies between $0 < \phi_{\boldsymbol{\kappa}} < 2\pi$), i.e., $\delta_{\boldsymbol{\kappa}} = |\delta_{\boldsymbol{\kappa}}| \exp^{-i\phi_{\boldsymbol{\kappa}}} = r_{\boldsymbol{\kappa}} \exp^{-i\phi_{\boldsymbol{\kappa}}}$. The joint probability distribution function is useful because it allows the independence of the terms $\delta_{\boldsymbol{\kappa}}$, or in other words it is the product of every mode's probability:

$$\mathcal{P}_{\boldsymbol{\kappa}}(\delta_{\boldsymbol{\kappa}1}, ..., \delta_{\boldsymbol{\kappa}N}) = \prod_{\boldsymbol{\kappa}} \mathcal{P}_{\boldsymbol{\kappa}}(\delta_{\boldsymbol{\kappa}}).$$

This expression is not satisfied when the inverse fourier transform is done, since the probability density is not separable in the initial coordinate space. In the Fourier space, the term $P(\kappa)$ can be defined as the power spectrum and is related to the 2 point correlation function as:

$$P(k) = \frac{4\pi}{V_u} \int_0^{\infty} \xi(r) \frac{sin(kr)}{kr} r^2 dr = \langle |\delta(\boldsymbol{\kappa})|^2 \rangle, \tag{2.17}$$

from the latter expression can be seen that the isotropy of the Universe is taken into account, since during the power spectrum calculation, an average is done over all possible orientations of the vector $\boldsymbol{\kappa}$.

The correlation function, $\xi(r)$, describes the distribution of points, the excess probability of finding a particle at a distance r from another particle selected at random compared against a random distribution. The two point correlation is defined as:

$$\xi(r) = \langle \delta(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}) \rangle, \tag{2.18}$$

hence ξ only depends on the amplitude of r, it is due to the assumption of homogeneity and isotropy, i.e., depends on relative distances. Furthermore, from equations 2.17 and 2.18 it can be seen that the density field leads to a direct way to find the power spectrum using the Fourier transform.

In the correlation function, no clustering would imply that $\xi(r)$ is zero. A natural way to see this is through a conditional probability, given that there is a particle in a volume element dV_1 the probability there is other one in a volume element dV_2 at a distance r,

$$dP(2|1) = n[1 + \xi(x_{12})]dV_2,$$

if $\xi(x_{12}) > 0$ the probability of finding such pair of particles increases, i.e., there is clustering of structures. But if $\xi(x_{12}) < 0$ such probability diminishes leading to an anti-correlation. An anticorrelation appears when there is a negative relationship between two variables, higher values of one variable tend to be associated with lower values of the other one. In the case where $\xi(x_{12}) = 0$ there would be no clustering, the distribution of particles would be the one of a random catalogue. A random distribution where there are no patterns due to gravitational interaction.

Also, the function $\xi(r)$ can be expressed as a power law of the form

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\Gamma} \tag{2.19}$$

being valid in the range $100h^{-1}$ kpc to $10h^{-1}$ Mpc. The preferred scale $r_0 = 5h^{-1}$ Mpc is the one where the galaxy density is greater than twice of the background. The exponent value is $\Gamma = 1.8$. This fit overestimates the correlation function for distances bigger than $20h^{-1}$ Mpc.

So far, it has been shown two statistical measures, a fourier pair, in real space the correlation function and in fourier space the power spectrum. But other moments can be specified as previously mentioned, in general an l point correlation function can be defined through the next expression $\xi^l(\vec{x}_1, \vec{x}_2, \dots \vec{x}_l) \equiv \langle\delta_1\delta_2 \dots \delta_l)\rangle$, where the connected terms are the ones that contributed to the calculation. For example, the first moment of the distribution is $\langle\delta(x)\rangle = 0$, because of the definition of density perturbation field.

An important remark is that if initial density pertubations follow a gaussian distribution, all moments higher than two (2 point correlation function) are zero, i.e., the density perturbation field is completely described by the two first moments of the distribution.

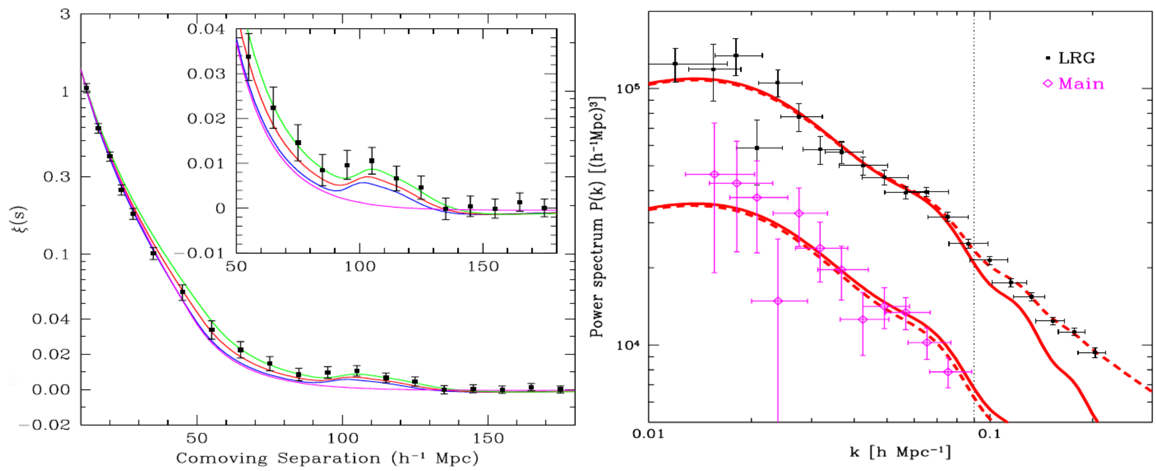

Figure 2.5: BAO peak in the correlation function in the left and the oscillations of BAO in the power spectrum in the right,Eisenstein et al. (2005). The lower curve is the main sloan digital sky survey (SDSS) sample and the upper one is the Luminous red galaxies (LGR) sample , Tegmark et al. (2006).

It is usually considered for the initial density field, that density contrasts follow a normal distribution centered in $\langle\delta\rangle = 0$ Bernardeau et al. (2001). This idea is supported by inflationary scenarios where a random gaussian perturbation field arises naturally from quantum perturbations during inflation, i.e., statistical behaviour lies on quantum perturbations. Since there are a large number of modes, the central limit theory would support this idea if the mode phases are independent. One of the advantages of this model is that the perturbation field remains gaussian during linear evolution.

Additionally it has been found that the initial power spectrum expected from inflation theories has the form $P(\kappa) = k^n$ Bernardeau et al. (2001). If $n = 1$ the power spectrum is called Harrison-Zeldovich which is commonly used.

With the initial power spectrum it can be done an inverse fourier transform and this way recreating the initial density field.

Furthermore, using inflationary models the shape of the linear power spectrum is well determined but there are not amplitude predictions, i.e., there is not a defined normalization of the power spectrum. One commonly way to do such thing is through the variance of the galaxy distribution when sampled with randomly placed spheres at radius R. The relation between the variance of the density field and the power spectrum is:

$$\sigma^2(R) = \frac{1}{2\pi^2}\int P(k)\hat{\omega}_R(k)^2 k^2 dk,$$

where $\hat{\omega}_R(k)$ is the Fourier transform of the spherical top hot model:

$$\hat{\omega}_R(k) = \frac{3}{(kR)^2}[\sin(kR) - kR\cos(kR)],$$

In this approach $\sigma(R)$ is taken ≈ 1 when $R = 8h^{-1}Mpc$ because of measures performed observationally Mo et al. (2010). Then, normalizing the power spectrum would imply to force $\sigma(R)$ to be one for the mentioned distance.

But several problems arise, 1) this normalization is not precisely valid for linear regime since $\sigma(R) \approx 1$ when $\delta(R) \ll 1$. 2) Baryonic matter is probably a bias tracer of the mass distribution.

It is necessary to consider that after recombination epoch, density perturbations started growing in size leading to a non lineal growth of the perturbations. This implies a change in the density field and likewise in the power spectrum.

2.7 Baryonic acoustic oscillations

Let us consider the epoch before recombination. The baryonic plasma (protons and electrons) was coupled with radiation via Thomson scattering, i.e., the electric component of radiation accelerate charged particles making small density perturbations to disperse. Nevertheless, considering that dark matter do not interact with radiation, small dark matter density contrasts or perturbations can form. Hence, the baryons are subject to two competing forces, radiation pressure and gravitation. Consider a particular dark matter density contrast that attract nearby baryons, they start clustering around the dark matter forming a bigger density contrast. But, due to the pressure caused by coupling, the outward force becomes bigger than gravity, making baryons to move outward as a sound wave (a baryonic plasma shell). **This oscillation of the baryonic plasma is known as baryonic acoustic oscillation.**

When decoupling occurs and temperature drops, the force responsible for the expansion of the shell disappear, this is, the pressure caused by the coupling between baryons and radiation, leading baryons in the last position they were located. The scale of the baryonic acoustic oscillation is usually called the sound horizon and it can be computed as:

$$s = \int_{z_{rec}}^{\infty} \frac{c_s dz}{H(z)}, \tag{2.20}$$

where c_s is the velocity of the propagation and $H(z)$ is the Hubble parameter, (Ruiz-Lapuente (2010)).

Therefore, there is a spherical shell formed around the dark matter density perturbation. Now, not only dark matter density contrast seed gravitational instability but the baryons in the shell as well.

The structures continue to grow reaching non linear growth and wipe out the imprint lead by the baryonic acoustic oscillations except for the bigger ones. The estimated size of the remaining BAO is 150 Mpc, causing that scale to be more likely to have galaxy formation activity. The reason this distribution is not observed at cosmological scales is because of the amount of imprints, too many galaxies following a BAO pattern, that lead to the smear out the preferred scale. Though, it is expected an enhancement in the two point correlation at scales of the BAO. As was seen before, there is a direct correspondence between the two point correlation and the power spectrum. Hence, a characteristic oscillation in the power spectrum caused by the imprint of BAO is naturally found.

Since BAO are primarily a linear phenomenon, they are preserved in the power spectrum despite of the temporal evolution. Then, BAO are used as an standard ruler, specifically for high redshift where other rulers tend to fail. This is commonly used for constraining dark matter models.

Although, the nonlinear collapse change the shape and position of BAO, broaden and shift the peak. This is clearly seen in the figure (2.7), the broadening of the peak initially shown as a very sharp causes a damping in the frequency in the power spectrum. It is expected that this effect that is going to be studied in this work, affects baryonic acoustic oscillations on scales around $\sim$ 10Mpc.

The diffusion damping (silk damping) also causes a reduction in size of density contrast by the diffusion of photons from hot regions to colder ones during the epoch of recombination.

2.7.1 Shift of the BAO

During the linear regime the BAO properties are not affected, i.e., the BAO shape does not change during this regime. But, once the non linear regime starts, it causes deviations in the BAO properties. But, let us see in more detail how these deviations affect the BAO properties. In order to do so, it must studied the velocity field. This gives account for the

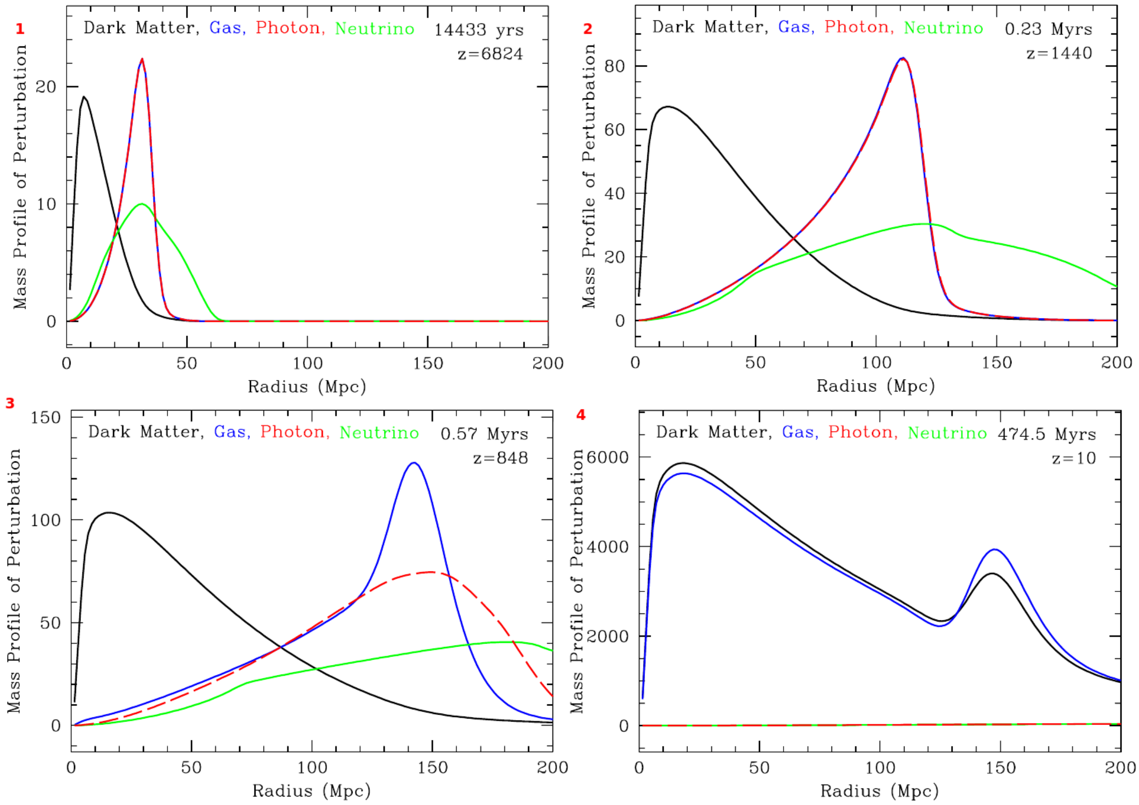

Figure 2.6: All the figures show the evolution of the radial mass profile of dark matter, baryons, photons and neutrinos. First one: Initial perturbations of the four species. Second one: the neutrinos do not interact and move away, the plasma of baryons and radiation overdensity expands because of radiation pressure, the dark matter continues to fall in the perturbation. Third one: The temperature drops enought to lead to decoupling, the baryons slows down until stopped, the radiation and neutrinos continue moving away. Fourth one: The dark matter and baryons eventually get the same distribution because of the gravitational interaction.

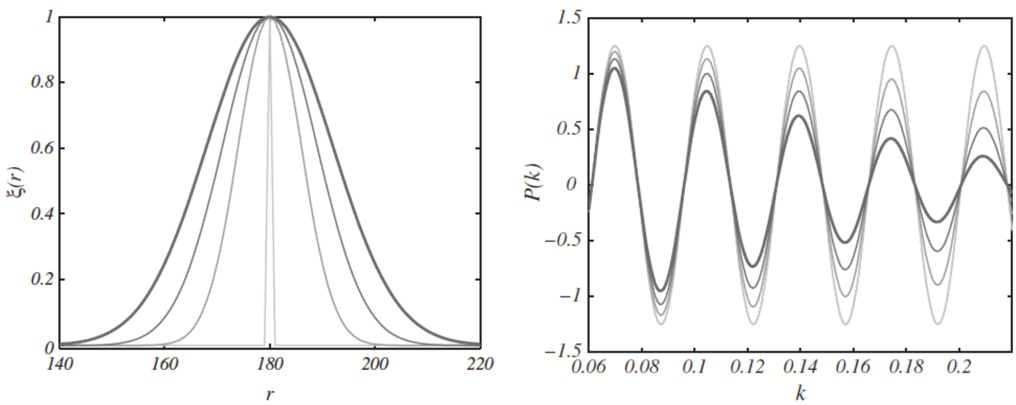

Figure 2.7: Left: the correlation function. Right: the power spectrum. When the width of the peak is increased the acoustic oscillation obtained in the power spectrum is damped.

changes in the mass distribution. Hence, we are going to focus in one model proposed by Smith et al. (2008) that models the shift suffered for the BAO signal during the non linear regime.

As mentioned, the correlation function allows to study the clustering of the mass distribution. But a specific mass distribution is affected by the existing velocity field at the same time. Hence, it is necessary to find a way to relate the correlation function with the velocity field. In this scenario, two particles are separated by a distance $\vec{r} = \vec{r_2} - \vec{r_1}$ and thus the pairwise infall velocity, $u_{12}(\vec{r}, \eta)$, is directed along the separation unit vector $\hat{r}$, where η is proportional to the conformal time. However, this distance diminishes with time due to clustering. In this direction, an important equation is the pair conservation equation:

$$\frac{\partial \ln[1+\xi(r,\eta)]}{\partial \eta} - u_{12}(r,\eta)\frac{\partial \ln[1+\xi(r,\eta)]}{\partial r} = \Theta(r,\eta), \tag{2.21}$$

where $\Theta(r,\eta) = \nabla \cdot [u_{12}(r,\eta)\hat{r}]$, is the divergence of the pairwise infall velocity. It allows to study the evolution of any feature, its movement, in the correlation function. Hence, the relation between the correlation function and the pairwise infall velocity allows to study the shift that BAO would suffer due to gravitational clustering. To solve the equation 2.21 is used the method of characteristics, the characteristics are the equations of motion of the pairs:

$$\frac{dr}{d\eta} = -u_{12}(r,\eta), \tag{2.22}$$

whose solutions allow to obtain an ordinary equation from 2.21,

$$\frac{d \ln[1+\xi(r,\eta)]}{d\eta} = \Theta(r,\eta). \tag{2.23}$$

Besides, the solution of the last equation 2.23 solution is given by

$$1+\xi(r,\eta) = (1+\xi_0[r_0(r,\eta)]) \times \exp\left[\int_0^{\eta} \Theta[r_{\eta'}(r,\eta),\eta']d\eta'\right], \tag{2.24}$$

where $r_0(r,\eta)$ is the initial separation that corresponds to r at time η and likewise for $r_{\eta'}$. To find this general solution, the shift in the correlation function due to gravitational clustering, it is necessary to know the divergence of the velocity field. A first approach that can tell us how the BAO peak evolves is through linear theory of velocities for $\Theta(r,\eta)$:

$$\Theta(r,\eta) = 2e^{2\eta}\xi_0(r), \tag{2.25}$$

where ξ_0 is the linear correlation function at $\eta_0 = 0$. In such case, the correlation function grows due to the infall velocities. It becomes the biggest contribution when η is large.

Now, using this expression and clearing the initial correlation function, is obtained the plot 2.8, the two curves for $z = 0$ (red) and $z = 1$ (blue), are represented with solid lines Smith et al. (2008). The solid green squares and solid red triangles, for $z = 0$ and $z = 1$ respectively, show the behavior of the correlation function in cosmological simulations. From this, it can be said that in non linear regime the BAO has been wash out hence it does not suffice the linear theory to produce such effect. This is in accordance with several studies that show a significant difference between numerical simulations and the predictions of the linear theory. So, a next step is to include in the equation 2.24 a divergence of the velocity that take into account the nonlinear effects.

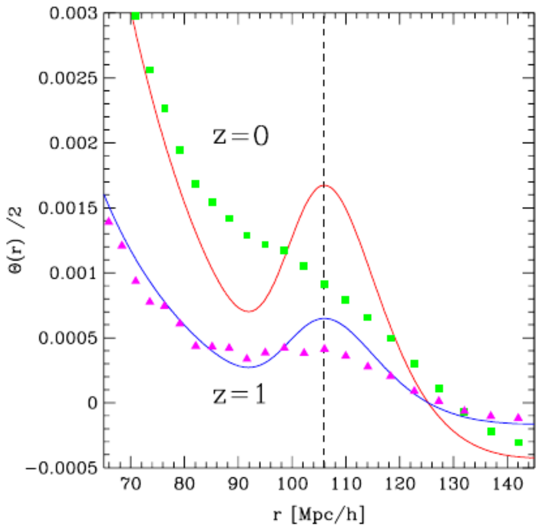

Figure 2.8: Using expression 2.25 for $z = 0$ and $z = 1$, the solid lines are obtained. From cosmological simulations the correlation function for $z = 0$ and $z = 1$ is shown, solid squares and solid triangles. Image taken from Smith et al. (2008).

For nonlinear regime, the perturbation theory (PT) can be used to find the pairwise infall velocity. This one has two nonlinear contributions and its divergence can be expressed as:

$$\Theta(r, \eta) = \Theta_2(r, \eta) + \Theta_3(r, \eta),$$

where $\Theta_2(r, \eta) = 2\nabla \cdot \langle \delta_1 \vec{u}_2 \rangle$ and $\Theta_3(r, \eta) = 2\nabla \cdot \langle \delta_1 \delta_2 \vec{u}_1 \rangle$. It can be shown that PT is used to find the last two terms as shown in Smith et al. (2008). Finally, the correlation function is expressed as:

$$1+\xi(r,\eta) = (1+b_i^2\xi_0[r_0(r,\eta)])\times \exp\left[\int_0^\eta d\eta'(b(\eta')\Theta_2[r_{\eta'}(r,\eta),\eta'] + b(\eta')^2\Theta_3[r_{\eta'}(r,\eta),\eta'])\right], \quad (2.26)$$

where b_i is the initial bias. This expression predicts the damping of the BAO that causes an apparent shifting to smaller scales. Hence, it can be concluded that indeed nonlinear regime induce changes on BAO properties such as its position. This idea is supported by Smith et al. (2008), Baldauf & Desjacques (2017), Anselmi et al. (2017), M. Ivanov (2016), Smith et al. (2007), Martín & Scoccimarro (2008), where it is shown that the position of the BAO peak is changed around 1% to 3%. Despite of the fact it is not a big difference, it induce a significant bias in the parameter of the dark energy equation. Then, a model for BAO must include the nonlinear contributions.

Although this model gives account for the shift due to the transport of matter by the velocity field, there is other term that can contribute to the shifting, the impact of tidal gravitational fields in the growth of structure as explained in Martín & Scoccimarro (2008).

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