TP 4: Nonlinear Transformation

You will implement logistic regression to predict whether microchips from a fabrication plant passes quality assurance (QA). During QA, each microchip goes through various tests to ensure it is functioning correctly.
Suppose you are the product manager of the factory and you have the test results for some microchips on two different tests. From these two tests, you would like to determine whether the microchips should be accepted or rejected. To help you make the decision, you have a dataset of test results on past microchips, from which you can build a logistic regression model.

1- Before starting to implement any learning algorithm, visualize your data.

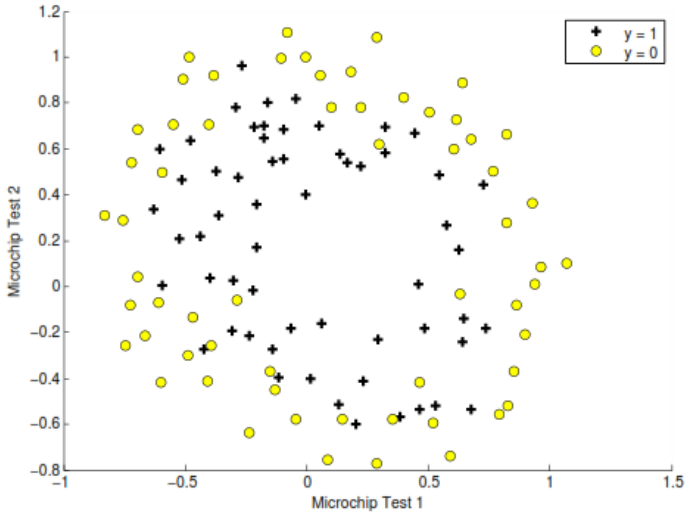

Figure 1: Plot of training data.

2- What do you notice from the graph?
3- Can logistic regression perform well in this dataset?
4- Implement the sigmoid activation function:

$$h_w(x) = \frac{1}{1 + e^{-w^T x}}$$

5- Implement the cost function for the logistic regression:

$$L_S(h_w) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)}\log\left(h_w\left(x^{(i)}\right)\right) - (1 - y^{(i)})\log(1 - h_w(x^{(i)}))\right]$$

Where m is the number of training data.

6- Implement the gradient of the cost function:

$$\frac{\partial}{\partial \mathrm{w_j}} L_S(h_w) = \frac{1}{m}\sum_{i=1}^{m}\left(h_w\left(x^{(i)}\right) - y^{(i)}\right)x_j^{(i)}$$

Where $j = \{1, \dots, d\}$.

7- Use the logistic regression algorithm then compute the training and testing errors.
8- Now, let's create more features from each data point. You should map the features into all polynomial terms of x_1 and x_2 for $Q = \{1, \dots, 6\}$.

$$\text{mapFeature}(x) = \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ x_1^2 \\ x_1 x_2 \\ x_2^2 \\ x_1^3 \\ \vdots \\ x_1 x_2^5 \\ x_2^6 \end{bmatrix}$$

9- What is the number of the features that you obtain for each case of Q (you will have 6 cases).
10- Run the logistic regression algorithm for each case of Q. Visualize the data and their separation in two dimensional degree.
11- Report for each case the training and the testing errors. What can you notice?
12- What is the best model?