Coursework **Fall 2012**
Statistical Pattern Recognition (COMP136) **Roni Khardon**
Department of Computer Science, Tufts University

Assignment 2

This assignment is due by Tuesday October 2 in class.

1. Please read the solutions for exercises 1.7, 1.8, 1.9 (available on text web page). Then complete the details of the derivation in the solution of 1.8. In particular give the details of the derivatives showing how to get Eq (16) from Eq (15).

2. Develop an eigenvalue and eigenvector decomposition of the following matrix S.

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 6 & 2 \\ 0 & 2 & 6 \end{pmatrix}$$

   In particular,
   (1) develop and verify the form $S = V\Lambda V^T$ where V is the orthonormal matrix composed of eigenvectors,
   (2) develop and verify the form $S = \Sigma_i \lambda_i V_i V_i^T$,
   (3) show how $z = (123)^T$ can be expressed as a linear combination of eigenvectors $z = \sum w_i V_i$.

3. Consider a real-valued symmetric matrix S with eigen decomposition $S = V\Lambda V^T$. Now consider the optimization problem:

$$\text{argmax}_{\{x \mid x^T x \leq 1\}} \; x^T S x$$

   that is, we seek a vector x of norm at most 1 maximizing the quadratic form $x^T Sx$. What is the optimal solution x? derive your solution in general and illustrate it in the example of the previous question.

   Hint: since the columns of V form an orthonormal basis we can write $x = \sum a_k v_k = Va$ for some coefficients a_k. Use this fact to calculate the quadratic form and then analyze the result to identify the optimizing a.

4. Consider a multivariate random variable (of dimension 2) $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim \text{uniform}[1,2]^2$ and the random variable y define as $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1^2 \\ x_1 + x_2 \end{pmatrix}$.
   (1) Use the change of variables formulas given in class to calculate the distribution over y.
   (2) What is the range of values of y for which $Pr(y)$ is not zero.
   (3) Verify that $Pr(y)$ calculated in part (1) is normalized; that is, verify that $\int_y Pr(y)dy = 1$.

5. Solve problem 3.3 (page 174) in the textbook.

6. Consider a bi-variate normal variable X distributed $\mathcal{N}(0, I)$ and a univariate Y where $Y|X$ is distributed as $\mathcal{N}(\mu = 2x_1 + x_2 + 3, \sigma^2 = 4)$. Calculate an explicit form for $p(X|Y = 4)$ using our template for Bayes theorem for Gaussians. Are x_1, x_2 still independent after Y is observed?

7. Solve the first part of problem 3.21 (page 177). In particular, follow the textbook's directions and derive equation C.22 by using the eigen-decomposition of A, properties of determinants and trace w.r.t. such decompositions, and basic properties of the trace (linearity and C.9).