Robust Scheduling of Integrated Energy Systems with Decision-Dependent Uncertainties

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Abstract—Integrated energy system (IES) has been widely used in the field of energy supply due to the high efficiency of energy utilization. However, its economic operation is challenged by different uncertainties in the system. This paper establishes a multi-stage robust optimization model for IES with both decision-independent uncertainties (DIUs) and decision-dependent uncertainties (DDUs). Because some uncertainties in IES are affected by the decision, such as the price, they are modeled as DDUs. To solve the complicated scheduling problem with DDUs, an improved column-and-constraint generation (C&CG) algorithm is proposed. Finally, a 33 nodes system with 24 time slots is utilized to verify the efficacy of our design.

Keywords—Decision-dependent uncertainty, decisionindependent uncertainty, integrated energy system, energy storage, power market

I. INTRODUCTION

A. Motivation

With the development of society, how to use energy more efficiently and sustainably is an important issue at present. Among them, IES is a research hotspot. IES integrates a variety of energy carriers (such as gas, light, wind, heat, and electricity) and a variety of supply modes (such as pipelines, and power grids) to achieve renewable and repeatable energy use. IES has proven to be an effective solution for shaping an efficient, safe, and clean energy system of the future compared to traditional energy sources. The primary goal of researchers and companies in this field is to improve energy efficiency. In this context, the scheduling problem based on IES is quite necessary because the improved utilization rate is still at a low level by improving equipment and facilities[1][2]. Through scheduling and optimization, it can effectively improve the unreasonable behavior pattern of energy consumption of users and the market, and reduce the waste of energy. The research results in [3] also support our view. Therefore, our discussion of IES in the grid market can provide support for future energy utilization and research.

B. Related Works

When studying the scheduling problem of IES, various uncertainties should be considered to ensure the rationality of the model. The uncertainty-related work can be roughly divided into uncertainty based on scheduling and uncertainty based on the objective environment. In the first type, uncertainty is set at different optimization stages [4][5] or time nodes [6] to distinguish the demands of different stages. In the

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second category, the model takes some factors related to the natural environment, such as photovoltaic power generation[7] and energy storage unit capacity[4], as the basis for setting uncertainty. The above work provides great help for us to make better use of uncertainty to build models and get useful conclusions for practical work. However, there are two types of uncertainties to be considered, one is the decisionindependent uncertainties (DIU) involved in the above work, and the other is the DDU[8][9]. Since there are many uncertainties related to decision-making in practice, for example, the pricing strategy of the grid may fluctuate with the behavior of IES users[10][11]. So it is very necessary to study DDU in IES scheduling problems. Therefore, this paper adds the uncertainty related to IES users into the setting of the model and proposes a multi-stage robust optimization model with IES as the main body.

C. Contribution

The main contributions of this paper are as follows.

- 1) An energy scheduling model with both DIUs and DDUs is formulated for IES. Existing works mainly focus on DIUs, such as photovoltaic generation. However, many uncertainties are related to decisions in the IES operation, such as the price. Here, we consider the influence of energy consumption on the electricity purchase price. Additionally, this effect is a kind of market behavior rather than economic law, so it has strong uncertainty. which is a typical type of DDU.
- 2) A modified C&CG algorithm is proposed to solve the scheduling problem with DDU. The traditional C&CG algorithm has fast and stable convergence, but the solution range can only involve DIU, that is, there is no coupling relationship between uncertainty and other decision variables[12]. In this paper, we improve the C&CG algorithm by setting an extra link to pass DDU parameters so that it can solve the robust optimization model with DDU. Therefore, the improved C&CG algorithm effectively expands the range of problems that can be solved by C&CG.

D. Organization

The rest of this paper is organized as follows. In Section II, of this paper, a pre-analysis of the model building is made, which leads to the thermoelectric relationship. Section III focuses on the establishment of the model, starting with IES users and grid operators, a multi-stage robust optimization model is established. And in this part, the mathematical and practical meaning of DDU is clarified. Section IV is the establishment of a modified C&CG algorithm and case simulation study. Section V concludes the paper.

II. PREANALYSIS OF THE MODEL

Since the thermo-electric system is an important part of the IES, it is necessary to analyze the DHN in advance. The primary energy should be transformed by the DHN into heat and electricity through the source unit before being supplied to the user via the power grid and heating network. A series of underground pipe segments make up the DHN. Typically, a thermal insulator layer and a covering are present around each steel pipe section. The temperature dynamics along each line are expressed in Eq. (1). It can derive thermodynamic formulas from the law of conservation of energy through infinitesimal analysis[13].

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} + \frac{v}{mcR} (T - T^a) = \frac{\lambda^W}{\rho c} \frac{\partial^2 T}{\partial x^2}$$
 (1)

where v, c, m, λ^W , and ρ are the flow velocity, mass flow rate, specific heat capacity, thermal conductivity, and density of hot water, respectively; T^a is the ambient temperature outside the pipeline; R is the thermal resistance of the pipe segment; t and x are time and position variables.

Then, the basic pipeline relationship can be obtained by ignoring the relatively small quantities on the right-hand side of the equation. Then we use the applicable difference

schemes to simplify the problem.
$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} + \frac{v}{mcR} (T - T^a) = 0$$
 (2)

A. Establishment of Heat Network

The applicable difference scheme is an efficient and widely used numerical method for solving partial differential equations[10]. The fundamental concept is to approximate the discrete difference equation's solution with a finite set of unknown variables using the partial differential equation's

In this section, we construct a differential scheme, which is based on mesh partitioning, where region $\Gamma = \{(x, t) | 0 \le$ $x \le L, 0 \le t \le P$ is partitioned into a rectangular mesh by two sets of parallel lines described in Eq. (3)[14].

$$\begin{cases} x = x_i = ih, 0 \le i \le M \\ t = t_k = k\tau, 0 \le k \le N \end{cases}$$
 (3)

After building the t-x grid, we use the temperature of the four points around the grid node to represent the central region temperature, expressed by Eq. (4)

$$T_i^{i+1} = \Sigma_j \omega_j T_{x_j}^{t_j}, x_j \neq i, t_j \neq k+1$$
 (4)

Substituting Eq. (4) into the Eq. (1), the following Eq. (5) is obtained, and a difference scheme Eq. (6) with a second-

order convergence rate of
$$O(h^2 + \tau^2)$$
 is obtained:
$$\begin{cases}
\frac{\partial T}{\partial t}(x_{i-0.5}, t_{k+0.5}) + v \frac{\partial T}{\partial x}(x_{i-0.5}, t_{k+0.5}) \\
+ \frac{v}{mcR}[T(x_{i-0.5}, t_{k+0.5}) - T^a] = 0 \\
T(x_{i-0.5}, t_{k+0.5}) = 0.25(T_{i-1}^k + T_i^k + T_{i-1}^{k+1} + T_i^{k+1}) \\
+ O(h^2 + \tau^2) \\
\frac{\partial T}{\partial t}(x_{i-0.5}, t_{k+0.5}) = \frac{1}{2\tau}(-T_{i-1}^k - T_i^k + T_{i-1}^{k+1} + T_i^{k+1}) \\
+ O(\tau^2) \\
\frac{\partial T}{\partial t}(x_{i-0.5}, t_{k+0.5}) = \frac{1}{2h}(-T_{i-1}^k + T_i^k - T_{i-1}^{k+1} + T_i^{k+1}) \\
+ O(h^2)
\end{cases}$$

$$\begin{cases}
T_{i}^{k+1} = \frac{1+\alpha-\beta}{1+\alpha+\beta} T_{i-1}^{k} + \frac{\alpha-1-\beta}{1+\alpha+\beta} T_{i-1}^{k+1} + \frac{1-\alpha-\beta}{1+\alpha+\beta} T_{i}^{k} \\
+ \frac{4\beta}{1+\alpha+\beta} T^{a} (1 \le i \le M, 0 \le k \le N-1) \\
\alpha = \frac{v\tau}{h}, \beta = \frac{v\tau}{2cRm}
\end{cases} (6)$$

According to the simulation results in the[10], this model has stable convergence, and the error with the actual result is within a reasonable range and has good comprehensive calculation performance.

B. Facilities and Load Constraints

In the DHN, the weeping steam turbine combined heatand-power unit (CHP) unit is adopted because it has flexibility in regulating electric power and thermal power output[15], and is suitable for the behavior mode of real-time regulation of the grid market. Its feasible operating region is defined by several extreme operating points, represented as a polygon in two-dimensional space, see Eq. (7) and Fig. 1[16][17].

$$\begin{cases} P^{CHP} \ge \frac{P_a - P_b}{m_a - m_b} (m^s - m_a) + P_a \\ P^{CHP} \ge \frac{P_b - P_c}{m_b - m_c} (m^s - m_b) + P_b \\ P^{CHP} \ge \frac{P_d - P_e}{m_d - m_e} (m^s - m_d) + P_d \\ m_a \le m^s \le m_c \end{cases}$$
(7)

where m^s and P^{CHP} denote the extracted mass flow rate of steam and the electric power output of cogeneration unit.

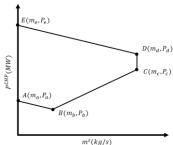


Fig. 1. Operation region of the extraction-turbine cogeneration unit

C. Heat-to-Electricity Conversion Relationship

In the pipeline network, the extracted steam is converted into hot water and injected into the DHN for heat transfer through the steam-water heat exchanger, whose heat exchange power satisfies:

$$Q = c^s m^s (T^{STM} - T^{PHS}) + r^s m^s \tag{8}$$

where c^s and r^s denote the specific heat capacity and the unit mass flow latent of steam; T^{STM} and T^{PHS} denote the temperature of the steam in the pipeline and the phase transition temperature of the steam to water.

Model typical conversion units such as electric boilers and heat pumps and the input electrical power P^{in} and output thermal power Q^{OUT} therein as linear equation constraints, by using fixed coefficients representing the efficiency in the thermoelectric conversion process. $Q^{OUT} = \eta^{p2h} P^{in}$

$$O^{OUT} = n^{p2h} P^{in} \tag{9}$$

where P^{in} and Q^{OUT} denote the input electric power and output thermal power respectively; η^{p2h} is the efficiency coefficient.

III. MODEL BUILDING AND REPRESENTATION OF DDU

In this section, we build a multi-stage robust optimization model, and we plan to study user and grid behavior during the day. Among them, we regard the users of the IES as the optimization objective of the master problem (MP), and the grid operator is involved in the subproblem (SP). In the two phases, there is a coupling relationship in terms of time and constraints: in terms of time, it is required to ensure the causal relationship between each time slot to ensure practical feasibility; in terms of constraints, grid operators and users of IESs share grid transactions environment, so there is a coupling relationship.

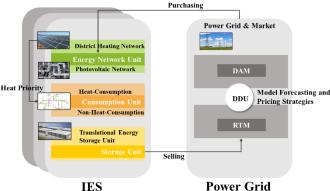


Fig. 2. Schematic diagram of the model

A. IES User Behavior Modeling

The structure of IES in this paper is shown in Fig. 2. In the part of the IES, the traditional energy supply-consumption model is expanded to a certain extent to form an energy network unit, a consumption unit, and a translational energy storage unit. Among them, the energy network unit mainly considers the DHN and the photovoltaic network (wind power lacks more common application scenarios compared with the photovoltaic network. Therefore, this paper uses a representative photovoltaic network); the consumption unit considers the user nodes based on the topological network relationship in the transmission grid and the DHN, where the load performance can be studied; compared with the conventional storage unit which limits user behavior to selling redundant PV power in real-time, the improved energy storage unit allows more flexible operation behavior: (1) Users can store electricity purchased from the grid or photovoltaic electricity into the unit. (2) As long as there is still power in the unit, users can take out the power at any time for use or sale. This change is more characteristic of a market economy and is now a research hotspot in the country and large enterprises.

1) Consumption Unit: Due to many changes in user behavior and grid pricing strategies within a day (there are peak and off-peak periods of electricity consumption), there can be more room for research and optimization. Therefore, the research period of this paper is within one day. In general, user load types can be divided into general heat load types and other types (e.g., lighting, electronics)[18][19]. And according to [16], it can be said that for an IES within a certain quarter, the total daily power consumption and the proportion of heat supply can be regarded as a certain constant. Therefore, at this stage, to minimize the power purchase expenditure of IES users, we established the following constraint model:

$$\min_{\mathbf{p}_{i,t}, p_{i,t}^b} b \Sigma_{i,t} p_{i,t} \tag{10}$$

$$s.t. \quad \mathbf{p}_{\min}^{H} \le p_{i.t}^{H} \le p_{max}^{H} \tag{11}$$

$$p_{\min}^0 \le p_{i,t}^0 \le p_{\max}^0 \tag{12}$$

$$p_{i,t} = p_{i,t}^0 + p_{i,t}^H (13)$$

$$\min_{\substack{p_{i,t}, p_{i,t}^{b} \\ p_{i,t}^{b} \\ p_{i,t}^{0} \\ p_{min}^{0} \leq p_{i,t}^{0} \leq p_{max}^{0}} \\
p_{min}^{0} \leq p_{i,t}^{0} \leq p_{max}^{0} \\
p_{i,t}^{0} = \sum_{t} p_{i,t}^{0} \\
h_{i}^{H} = \sum_{t} p_{i,t}^{H} \\
k_{t} \leq p_{i,t}^{0} = p_{i,t}^{0} \leq k_{t} \\
\end{cases} (10)$$
(11)

$$k_L \le p_{i,t} - p_{i,t-1} \le k_H \tag{15}$$

 $k_L \leq p_{i,t} - p_{i,t-1} \leq k_H \tag{15}$ where $b, p_{i,t}, p_{i,t}^b, h_i^T, h_i^H$ are the power purchase price, the total power consumption calculated by node i at in time slot t, the power purchased of node i in time slot t, the total load and heat load consumed by node i in a day. $p_{i,t}^H$, $p_{i,t}^O$ are the heating part and the rest part. Constraints (11)-(12) are upper and lower limit constraints, and constraint (15) is a climbing constraint.

2) Energy Network Unit: This section mainly considers the three energy carriers of thermal, electrical, and solar energy. The constraints brought by the thermal-electrical analysis are based on the DHN and CHP units mentioned above but have been partially corrected and improved[15]. First, we use CHP as the heating end of DHN, and there is a certain utilization rate between the output power P^{CHP} and the user's heat load $p_{i,t}^H$; secondly, we represent the temperature at a node by the temperature of the steam in the pipeline. In addition, the DIU for photovoltaic networks due to weather and component uncertainties should also be considered[20]. Add the following constraints to the model:

s.t.
$$(7) - (9)$$

 $p_{i,t}^H = \theta P_{i,t}^{CHP}$ (16)

$$g_{min} \le g_{it} \le g_{max} \tag{17}$$

s.t. (7) - (9) $p_{i,t}^H = \theta P_{i,t}^{CHP}$ (16) $g_{min} \le g_{i,t} \le g_{max}$ (17) where θ is conversion rate, $g_{i,t}$ is the photovoltaic power generation of the i node in time slot t.

3) Translational Energy Storage Unit: Compared with the conventional energy storage unit, the translational energy storage unit in this paper is not only a "user-grid" transfer station, but an intelligent processor[21]. It allows users to buy electricity from the grid and deposit it and allows users to delay selling. For this, the following constraints are introduced:

$$l_{i,t+1} = \alpha \left(p_{i,t}^b + g_{i,t} - p_{i,t} - \frac{n_{i,t}}{\beta} \right) + l_{i,t}$$
 (18)

$$p_{i,t}^{C} = \sum_{m \in u(i)} \left(p_{im,t} - r_{m,i} I_{m,i}^{2} \right) \tag{19}$$

$$p_{i,t}^{C} = \sum_{m \in u(i)} \left(p_{im,t} - r_{m,i} I_{m,i}^{2} \right)$$

$$\sum_{m \in u(i)} \left(p_{im,t} - r_{m,i} I_{m,i}^{2} \right) + p_{i,t} + n_{i,t} \le p_{i,t}^{b} + l_{i,t} + g_{i,t}$$
(20)

$$l_{i,0} = l_0 \tag{21}$$

where α , β , $n_{i,t}$, $l_{i,t}$ are the charge and discharge coefficients, the electricity sold by the i node to the grid at time slot t, and the remaining capacity of the storage container. $p_{im,t}$ and $r_{m,i}I_{m,i}^2$ are the network transmission constraints obtained according to the grid power flow equation. u(i) means the child nodes set of i. The positions of each node refer to the network. Constraint (19) calculates the line transmission loss for each node.

B. Grid Pricing Strategy

The structure of the power grid in this paper is shown in Fig. 2. In the power grid part, this paper establishes a realtime market (RTM) and a day-ahead market (DAM) based on actual research and simulates a more market-oriented transaction model with different electricity prices. Among them, the DAM observes the current traditional pricing model, which divides the time slots of a day into peak and off-peak periods, while the RTM includes power grid operators as decision-makers in model optimization[22]. According to the observation of the user's behavior, the corresponding pricing strategy is adjusted to update the DAM price as the RTM price. In this paper, DDU based on user behavior is introduced. In this part, the goal is to make the total price of electricity sold to the grid higher.

$$\min_{DDU} \max_{n_{i,t},l_{i,t}} \Sigma_{i,t} s_t n_{i,t}$$

$$s.t. \quad s_t = e + \gamma - \omega$$
(22)

$$s.t. \ s_t = e + \gamma - \omega \tag{23}$$

where s_t , e, γ are respectively the price of electricity sold by users (that is, the price of electricity purchased by the grid), policy support factors and DDU variables.

C. Introduction of DDU

Since the proposal of DDU is based on the prediction of user behavior patterns, we need to make behavior predictions for users from the perspective of grid operators in the absence of DDU. That is to solve the following problems:

$$\min_{\substack{p_{i,t}^{H}, p_{i,t}^{O}, p_{i,t}^{D}, l_{i,t} \\ s.t.}} \sum_{t} \Sigma_{i} b p_{i,t} - \Sigma_{t} \Sigma_{i} s_{t} n_{i,t}$$
(24)
$$\sum_{t} \sum_{t} \sum_{i} \sum_{t} \sum_{t}$$

$$Q_{i,t} = c^{s} m_{i,t}^{s} (T_{i,t} - T^{PHS}) + r^{s} m_{i,t}^{s}$$
(26)
$$s_{t} = e + \gamma$$
(27)

By solving the above optimization problem without DDU, we will get the expected power consumption of users $p_{i,t}^{E}$ in each period. In the problem we want to solve, we use the absolute deviation between the user's actual behavior and the prediction error as DDU. Its specific meaning is that when the user's electricity consumption behavior is in line with expectations, then maintaining a high price in the original peak period and a low price in the off-peak period can make greater profits; if the user's behavior deviates greatly from the predicted situation, even in the original If the electricity consumption in the peak period is greatly reduced, it is necessary to reduce the purchase electricity price to reduce the loss:

$$0 \le \omega_{i,t} \le \sum_{n=1}^{t-1} \left| p_{i,n}^E - p_{i,t} \right| \tag{28}$$

IV. OPTIMIZATION ALGORITHMS AND CASE STUDIES

A. Algorithm Design

By synthesizing the above models, we get the following final model:

$$\min_{\substack{p_{i,t}^H, p_{i,t}^O, p_{i,t}^b \\ s.t.}} \Sigma_i \Sigma_t b p_{i,t} - \max_{\omega} \min_{\substack{n_{i,t}, l_{i,t} \\ n_{i,t}, l_{i,t}}} \Sigma_i \Sigma_t s_t n_{i,t}$$
(29)

After conversion, a two-stage problem can be obtained. For SP, the solution can be further simplified by using the dual problem and Karush-Kuhn-Tucker (KKT) conditions.

This model is proposed based on multi-stage robust optimization, which can be solved by the Benders Decomposition method. But as mentioned in[12], this model can also be solved by the C&CG algorithm. By slacking the uncertain variables, solving the SP to determine the worst

case, and then putting the associated constraints and decision variables back to the MP, the so-called C&CG algorithm resolves the model. However, the C&CG in [12] is proposed based on DIU, so it needs to be improved based on the original algorithm. The improved algorithm adopted in this paper is to substitute DDU into SP after being solved in the MP stage and solve it in a similar way to DIU. The modified C&CG is given in Algorithm 1.

Algorithm 1. Modified C&CG

Step 0: Set LB= $-\infty$, UB= $+\infty$, k=0, O= \emptyset

Step 1: Solve the following master problem.

Step 1: Solve the following master problem.

MP:
$$\min_{\substack{p_{l}^{H}, p_{l}^{L}, p_{l}^{L}, p_{l}^{L} = 1 \\ p_{l}^{H}, p_{l}^{L}, p_{l}^{L}, p_{l}^{L}, p_{l}^{L}}} s_{l} \Sigma_{l} \Sigma_{l} b p_{l,t}^{b} + \eta$$
 $s.t. (24) - (27), (29) - (33)$

$$\sum_{m \in u(i)} (p_{im,t} - r_{m,i} l_{m,i}^{2}) + p_{i,t} + n_{i,t}^{l} \le p_{i,t}^{b} + l_{i,t}^{l} + g_{i,t} \quad \forall l \le k$$

$$l_{i,t+1}^{l} = \alpha \left(p_{i,t}^{b} + g_{i,t} - \sum_{m \in u(i)} (p_{im,t} - r_{m,i} l_{m,i}^{2}) - p_{i,t} - \frac{n_{i,t}^{l}}{\alpha} \right) + l_{i,t}^{l} \quad \forall l \le k$$

$$\eta \ge \Sigma_{l} \Sigma_{l} S_{l} n_{i,t}^{l}, \quad \forall l \in \mathbf{0}$$

$$g_{min} \le g_{i,t} \le g_{max}$$

Derive an optimal about ($p_{i,t}^{b*}, p_{i,t}^{0*}, p_{i,t}^{H*}$) and $\mathbf{L}\mathbf{B} = \min_{\substack{p_{i,t}^H, p_{i,t}^O, p_{i,t}^b}} \Sigma_i \Sigma_t b p_{i,t_{k+1}}^{b*} + \eta_{k+1}^*$

Step 2: Calculate the DDU with $(p_{i,t}^{0*}, p_{i,t}^{H*})$

Step 3: Call the oracle to solve SP and update

$$\begin{array}{c} \mathbf{P}_{i,t}^{H}, p_{i,t}^{O}, p_{i,t}^{L} \\ \mathbf{P}: Q(p_{i,t}^{H}, p_{i,t}^{O}, p_{i,t}^{H}) = \max_{\omega} \min_{n_{i,t}, l_{i,t}} -\Sigma_{i} \Sigma_{t} s_{t} n_{i,t} \end{array}$$

$$\begin{aligned} \mathbf{UB} = & \min \left\{ UB, \min_{\substack{p_{i,t}^H, p_{i,t}^O, p_{i,t}^D \\ p_{i,t}^D, p_{i,t}^O \\ p_{i,t}^D, p_{i,t}^D \\ p_{i,t}^O = p_{i,t}^D } \mathcal{E}_i \mathcal{E}_t b p_{i,t}^{b^*} + Q(p_{i,t_{k+1}}^{b^*}, p_{i,t_{k+1}}^O, p_{i,t_{k+1}}^D, p_{i,t_{k+1}}^O) \right. \\ & \mathbf{SP} : Q(p_{i,t}^H, p_{i,t}^O, p_{i,t}^b) = \max_{\substack{\alpha \\ m_{i,t}, l_{i,t} \\ m_{i,t}, l_{i,t}}} \min_{\substack{\alpha \\ m_{i,t}, l_{i,t} \\ m_{i,t} \leq g_{max}}} \mathcal{E}_i \mathcal{E}_t s_t n_{i,t} \\ & \sum_{\substack{m \in u(i)}} (p_{im,t} - r_{m,i} l_{m,i}^2) + p_{i,t} + n_{i,t} \leq p_{i,t}^b + l_{i,t} + g_{i,t} \\ & l_{i,t+1} = \alpha \left(p_{i,t}^b + g_{i,t} - \sum_{\substack{m \in u(i) \\ \eta \geq \Sigma_i \mathcal{E}_t s_t^*} n_{i,t}^l} (p_{im,t} - r_{m,i} l_{m,i}^2) - p_{i,t} - \frac{n_{i,t}}{\alpha} \right) + l_{i,t} \end{aligned}$$

Step 4: If **UB-LB** $\leq \varepsilon$, return $(p_{i,t_{k+1}}^{b^*}, p_{i,t_{k+1}}^{O^*}, p_{i,t_{k+1}}^{H^*}, p_{i,t_{k+1}}^{H^*})$ and terminate. Otherwise, do

(a) if $Q\left(p_{i,t_{k+1}}^{b^*}, p_{i,t_{k+1}}^{O^*}, p_{i,t_{k+1}}^{H^*}, p_{i,t_{k+1}}^{H^*}\right) \le +\infty$ add the following constraints to

MP. Update
$$k = k + 1$$
, $0 = 0 \cup \{k + 1\}$ and go to Step 2.

MP. Update
$$k = k + 1, 0 = 0 \cup \{k + 1\}$$
 and go to Step 2.
$$\sum_{m \in u(i)} (p_{im,t} - r_{m,i} l_{m,i}^2) + p_{i,t} + n_{i,t}^{k+1} \le p_{i,t}^b + l_{i,t}^{k+1} + g_{i,t}$$

$$l_{i,t+1}^{k+1} = \alpha \left(p_{i,t}^b + g_{i,t} - \sum_{m \in u(i)} (p_{im,t} - r_{m,i} l_{m,i}^2) - p_{i,t} - \frac{n_{i,t}^{k+1}}{\alpha} \right) + l_{i,t}^{k+1}$$

$$\eta \ge \Sigma_i \Sigma_t s_{t+1}^* n_{i,t}^{k+1}$$

(b) if $Q\left(p_{i,t_{k+1}}^{b^*}, p_{i,t_{k+1}}^{0^*}, p_{i,t_{k+1}}^{H^*}, p_{i,t_{k+1}}^{H^*}\right) \le +\infty$ add the following constraints to

MP. Update
$$k = k+1$$
 and go to Step 2.
$$\sum_{m \in u(i)} (p_{im,t} - r_{m,i} l_{m,i}^2) + p_{i,t} + n_{i,t}^{k+1} \le p_{i,t}^b + l_{i,t}^{k+1} + g_{i,t}$$
$$l_{i,t+1}^{k+1} = \alpha \left(p_{i,t}^b + g_{i,t} - \sum_{m \in u(i)} \left(p_{im,t} - r_{m,i} l_{m,i}^2 \right) - p_{i,t} - \frac{n_{i,t}^{k+1}}{\alpha} \right) + l_{i,t}^{k+1}$$

B. Case studies and Results Analysis

In this section, we choose a 33-node, 24-hour regional network for research. The tests are coded by Python 3.10.10 with toolbox seaborn 0.12.2, NumPy 1.24.2, matplotlib 3.7.0, and solved by gourbi 9.5.1 on a personal computer with Intel(R) Core (TM) i7-11800H CPU @2.30GHz and 16.0 GB memory. The results shown below are from the worst case.

(1) Basic Conclusion Analysis: First, it is worth noting that according to the simulation results in TABLE I., the improved C&CG retains the original fast and stable convergence, which proves the rationality of the change.

TABLE I. THE BASIC RESULTS OF THE SIMULATION

Iteration	LB	UB
1	175.432	174.287
2	173.470	173.470

TABLE II. shows the results for each of the three models. The function of raw data is to serve as a reference. It should be noted that the current electricity price purchased by users from the grid can be divided into off-peak hour and peak-hour prices. For the IES-based scheduling model provided in this paper, to prevent users from purchasing and selling electricity prices on a large scale in the off-peak period, e should be lower than the off-peak electricity price. e=0.1 is adopted in this paper. It can be observed that the IES model can effectively reduce the consumption of electricity by users. The reason for this is that users can save redundant power in storage units during low peak periods and use it during peak periods. Although the model without DDU has a lower user expenditure, the grid operator's total expenditure on purchasing power from the user will surge. This is not in line with reality, because to reduce losses, grid operators obviously cannot accept this model. While the DDU model is adopted, the analysis results show that when the user expenditure decreases, the power grid operator's power purchase expenditure increases very little, which is easier for the power grid to accept this result[23][24].

TABLE II. THE COST OF DIFFERENT MODELS

Models	Power Grid Expenditure	User Expenditure	Total Sum
Raw Data:	1.876	177.003	178.879
Model without DDU:	31.700	143.787	175.487
Model with DDU:	1.974	173.470	175.444

Furthermore, the expenditure of energy at different nodes can be understood as the cost of energy transfer between the user and the grid operator. Higher costs mean more energy needs to be transmitted and more line losses in the process. Therefore, if we sum up the expenditure of the user and the grid operator, the total sum can be understood as the resources or money consumed by the whole system in the process of energy transfer. Through summation, we find that the IES model with DDU has the smallest total expenditure. The result indicates that the model can effectively reduce the total amount of energy transmission. It shows that the IES model with DDU can increase local energy consumption, lower transmission line losses, and increase the sustainability and clean use of energy.

(2) DDU Curve Analysis: Fig. 4 shows the variation of the average DDU of 33 nodes. It can be seen from the figure that in the early stage, due to the lag in users' understanding and observation of DDU, DDU will rise rapidly. However, in the later stage, users will reduce DDU through reasonable behavioral decisions to improve their earnings. This behavior is in line with the law of the market. The results show that DDU can effectively interfere with the consumption behavior

pattern of users. The consumption behavior of users can operate according to the expectation and estimation of the grid operator. The above results are consistent with the law of market economy, which proves that DDU in this paper has important practical significance.

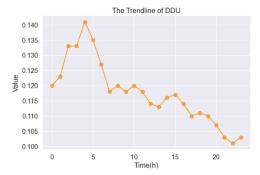


Fig. 3. Simulation results of the DDU curve

(2) Further Utilization of DDU: The state and grid operators can take advantage of DDU to effectively regulate the behavior of users. Peak-Cut, for instance, which reduces electricity consumption during peak hours while increasing it during off-peak hours, can effectively increase energy utilization rate while lowering users' electricity costs and easing load pressure on the power grid. So, the grid operator can achieve Peak-Cut by adjusting the DDU Settings. Assume a total power consumption of 9 KWH per node 24 hours a day, with an average of 0.375 KWH per time slot. Then the operator can achieve the purpose of Peak-Cut by reducing the variance, that is, improving the objective function in the DDU setting stage as follows.

$$\min_{\substack{p_{i,t}^{H}, p_{i,t}^{O}, p_{i,t}^{b} n_{i,t}, l_{i,t} \\ + \gamma \Sigma_{i} \Sigma_{t} \left(p_{i,t} - 0.375 \right)^{2}} (\Sigma_{t} \Sigma_{i} b p_{i,t}^{b} - \Sigma_{t} \Sigma_{i} s_{t} n_{i,t})$$
(30)

Under this setting, we get the user's electricity consumption behavior as shown in Fig. 5. It can be found that the model adding DDU achieves this purpose and makes the user's electricity consumption curve more stable.

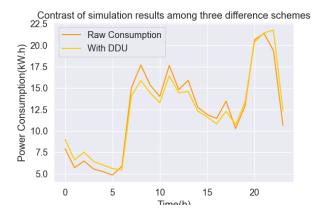


Fig. 4. Simulation Results of Improved User Behavior

V. CONCLUSION

This paper formulated a robust optimization model for IES with both DIUs and DDUs. Then, an improved C&CG algorithm is proposed to solve the complicated problem.

Numerical results on the 33-node system with 24 time slots show that it can reduce the electricity consumption cost and the load pressure on the power grid with the consideration of DDU. Future work will focus on incorporating more energy carriers into the model and improving the universality of the algorithm.

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