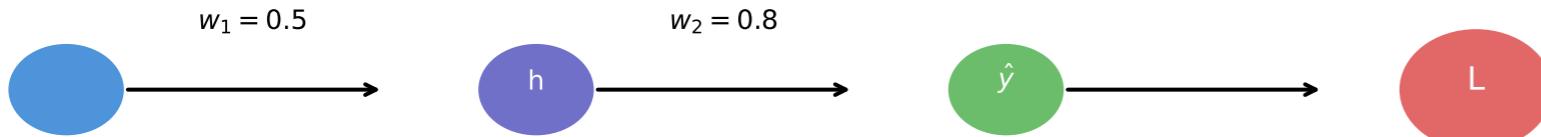


## Worked Example: Backpropagation

**Backward Pass:**

$$x = 2$$



$$w_1 = 0.5$$

$$w_2 = 0.8$$

$$\begin{aligned} h &= \sigma(w_1 \cdot x) \\ &= \sigma(1) = 0.73 \end{aligned}$$

$$\begin{aligned} \hat{y} &= \sigma(w_2 \cdot h) \\ &= \sigma(0.58) = 0.64 \end{aligned}$$

$$\begin{aligned} L &= (\hat{y} - y)^2 \\ y &= 1, L = 0.13 \end{aligned}$$

$$\frac{\partial L}{\partial \hat{y}} = 2(\hat{y} - y) = -0.72$$

$$\frac{\partial \hat{y}}{\partial w_2} = h \cdot \sigma'(\cdot) = 0.17$$

$$\frac{\partial L}{\partial w_2} = -0.72 \times 0.17 = -0.12$$

$$\frac{\partial L}{\partial h} = w_2 \cdot \sigma'(\cdot) \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_1} = x \cdot \sigma'(\cdot) \cdot \frac{\partial L}{\partial h}$$

**Weight Updates:**

$$w_2^{new} = w_2 - \eta \cdot \frac{\partial L}{\partial w_2} = 0.8 - 0.1 \times (-0.12) = 0.812$$

$$w_1^{new} = w_1 - \eta \cdot \frac{\partial L}{\partial w_1} \text{ (similar calculation)}$$

$$\eta = 0.1 \text{ (learning rate)}$$