

$$\text{ex 4.20 } \sigma^2 = \frac{(b-a)^2}{12}$$

$$\int_a^b x^2 \frac{1}{b-a} dx = \left[\int_a^b x \frac{1}{b-a} dx \right]^2$$

$$= \frac{b^3 - a^3}{3(b-a)} - \left(\frac{b^2 - a^2}{2(b-a)} \right)^2 = \frac{b^3 - a^3}{3(b-a)} - \left(\frac{(b-a)(b+a)^2}{2^2(b-a)} \right)^2$$

$$= \frac{b^3 - a^3}{3(b-a)} - \frac{(b+a)^2}{4} = \frac{b^3}{3(b-a)} - \frac{a^3}{3(b-a)} - \frac{(b+a)^2}{4}$$

$$= \frac{4b^3}{12(a-b)} + \frac{4a^3}{12(a-b)} - \frac{3(b+a)^2(a-b)}{12(a-b)}$$

$$= \frac{-4b^3 + 4a^3 - 3(b^2 + 2ab + a^2)(a-b)}{12(a-b)} = \frac{4b^3 + 4a^3 - (3b^2 + 6ab + 3a^2)(a-b)}{12(a-b)}$$

$$= \frac{-4b^3 + 4a^3 + (-3ab^2 - 6a^2b - 3a^3 + 3b^3 + 6ab^2 + 3a^2b)}{12(a-b)}$$

$$= \frac{(-4b^3 + 3b^3) + (4a^3 - 3a^3) + (-3ab^2 + 6ab^2) + (-6a^2b + 3a^2b)}{12(a-b)}$$

$$= \frac{-b^3 + 3ab^2 - 3a^2b + a^3}{12(b-a)} = \frac{(-b+a)^3}{12(-b+a)} = \frac{(b-a)^2}{12}$$