Exam 1

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Friday, March 5, 2021

Place your answers into this markdown document, knit it, and hand in the result as a PDF. There are 9 questions, each worth 10 points. Additionally, you get 10 points for handing in a PDF of a knit markdown document.

You may use R, the internet, and any reference material, but do not work together and do not get help (except from Dr. Clair).

Honor Pledge

The work I have submitted represents my own effort. While working on this exam, I did not communicate in any form with individuals other than the instructor.

Signed: Robert V. Campbell

Problem 1

Suppose you roll an ordinary six sided die 20 times and add up the total. What is the probability the sum is 55 or less?

```
x<-replicate(10000,sum(sample(1:6,20,replace=TRUE)))
mean(x<56)
```

```
## [1] 0.0298
```

Problem 2

McDonald's happy meals currently contain a package of four Pokemon cards. There are 25 different cards, all equally likely.

a. If you buy one happy meal (4 cards), what is the probability you get a Pikachu card?

```
x<-replicate(10000,sum(sample(1:25, 4, replace=TRUE) == 1))
mean(x>0)
```

```
## [1] 0.1482
```

b. If you buy 10 happy meals, what is the probability you get a Pikachu card?

```
x<-replicate(10000,sum(sample(1:25, 40, replace=TRUE) == 1))
mean(x>0)
```

```
## [1] 0.804
```

c. How many happy meals do you need to buy to have a 99% chance of getting a Pikachu?

```
x<-replicate(10000,sum(sample(1:25, 116, replace=TRUE) == 1))
mean(x>0)
```

```
## [1] 0.9919
```

```
116/4
```

```
## [1] 29
```

Problem 3

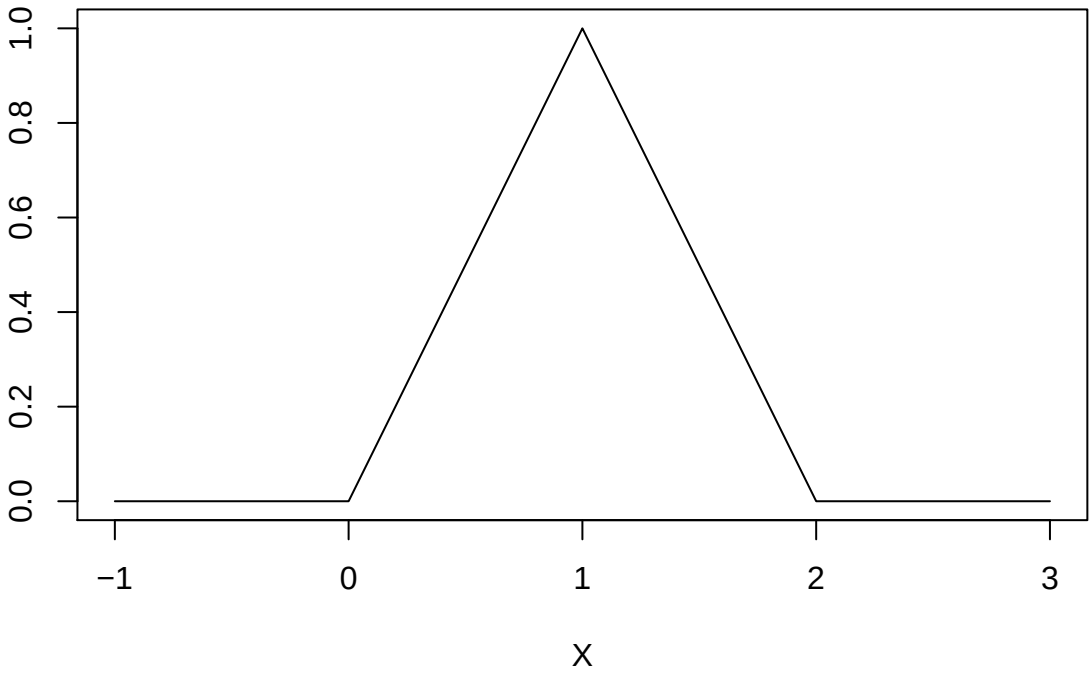
Roll two dice and consider the events

- A: both dice are even
- B: the sum of the dice is 8

a. What is $P(A)$? $P(A) = \frac{9}{36} = \frac{1}{4}$
b. What is $P(A|B)$? $P(A|B) = \frac{P(A\&B)}{P(B)} = \frac{3/36}{5/36} = \frac{3}{5}$
c. Are A and B independent events? No.

Problem 4

The random variable X has the pdf shown here:



a. What is $P(X < 1)$? $P(X < 1) = \frac{1}{2}$
b. What is $P(X > 1.5)$? $P(X > 1.5) = \frac{1}{8}$
c. What is $E[X]$? $E[X] = 1$

Problem 5

A high voltage spark can be used to create tiny aerosol particles of gold. These particles have diameters which vary according to a normal distribution with mean 1.35nm and standard deviation 0.24nm.

a. What proportion of these particles have diameter less than 1nm?

```
x<-rnorm(10000,1.35,0.24)
mean(x<1)
```

```
## [1] 0.0707
```

b. What proportion of these particles have diameter larger than 2nm?

```
mean(x>2)
```

```
## [1] 0.0025
```

Problem 6

Back in 2006, the website MySpace leaked a large number of its user's passwords. The passwords had this distribution:

- Numbers only: 1.3%
- Letters only: 9.5%
- Alphanumeric (both letters and numbers): 81%
- Includes symbols: 8.2%

a. What is the probability that a MySpace user had no Symbols in their password? $1 - .082 = 0.918$

b. If you chose 10 MySpace users at random, what is the probability that all of them had Alphanumeric passwords?

```
x<- replicate(10000,sum(sample(c('N','L','AN','S'), 10, replace=TRUE,prob=c(.013,.095,.81,.082)) == 'AN
mean(x==10)
```

```
## [1] 0.1248
```

Problem 7

Let X and Y be normal random variables with mean 10 and sd 5.

Find the expected distance from the point (X, Y) to the origin $(0, 0)$.

```
sqrt((10^2)+(10^2))
```

```
## [1] 14.14214
```

Problem 8

The `ChestSizes` data from the `HistData` library has measurements of Scottish Militiamen. There are two variables: `chest`, which gives the chest size in inches, and `count` which gives the number of soldiers with that chest size.

a. Calculate the total number of soldiers by summing the `count` variable.

```
sum(HistData::ChestSizes$count)
```

```
## [1] 5738
```

b. Calculate the mean chest size by summing (chest size)*(count)/(total number of soldiers).

```
sum((HistData::ChestSizes$count)*(HistData::ChestSizes$chest))/sum(HistData::ChestSizes$count)
```

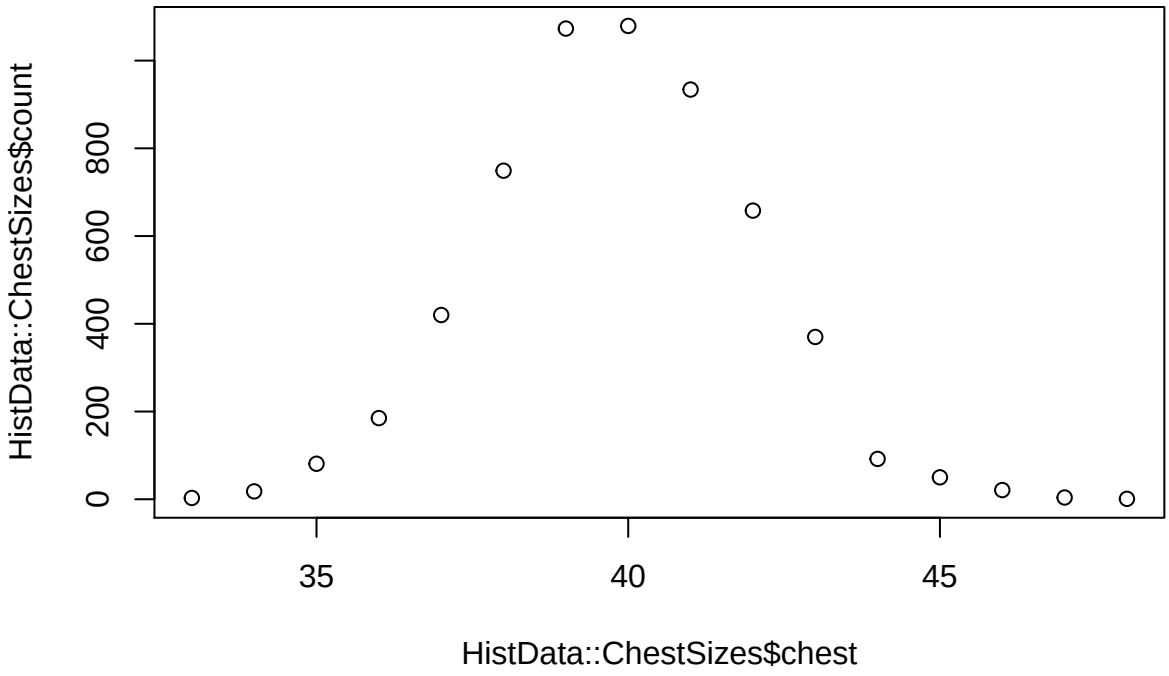
```
## [1] 39.83182
```

Problem 9

Continue with `HistData::ChestSizes`.

a. Plot the distribution of chest sizes with `chest` on the x-axis and `count` on the y-axis.

```
plot(HistData::ChestSizes$chest,HistData::ChestSizes$count)
```



b. What sort of random variable could model this distribution? Normal RV

c. From your plot, estimate (by eye) the mean and standard deviation of chest sizes. Mean: 40, sd: 2