Linear Systems

ECE3150 Test II October 18, 2019

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Signature: [signature]

Instructions:

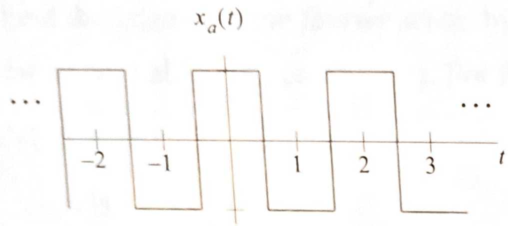
1) This exam is closed book, closed notes, and closed neighbor. You may bring in and use one note sheet, an $8.5'' \times 11''$ sheet of paper, with notes written on *one* side only. **Turn in your notesheet with your test.**
2) There are 7 pages to this exam including this cover sheet. You have 50 minutes to work the exam. Start when the instructor tells you to start.
3) Work the problems on the exam in the space provided. If you need additional space, *use the back side of the previous page.*
4) If you believe a problem cannot be solved, for full credit state exactly *why* it cannot be solved.
5) If you believe a problem has ambiguous notation, ask the instructor for clarification.

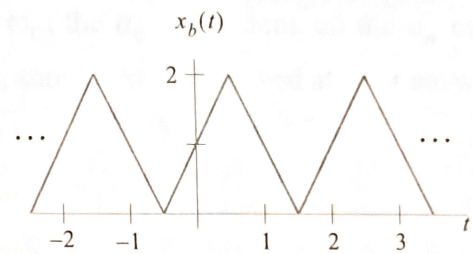
Question #	Max Points	Points
1	25	15
2	25	5
3	25	21
4	25	22
Totals:	100	63

15

1) (25 pts)

a) For each signal shown below, check the appropriate boxes that describe the properties of the complex exponential fourier series coefficients X_n.





$X_0 = 0$	Purely imaginary	Purely Real	Complex	
☒	☐	☒	☐	$x_a(t)$
☐	☒	☐	☐	$x_b(t)$

b) The input to an LTI system is $x(t) = 5\cos(4\pi t)$ and the system transfer function is $H(f) = j3f$. What is the output $y(t)$? For full credit, show all your work.

$\frac{X(f)}{Y(f)} = j3f$

$Y(f) = \frac{X(f)}{j3f} = \frac{5(\delta(f-2)+\delta(f+2))}{2j3f}$

5

$x(t) = 5\cos(4\pi t) = 5\cos(2\cdot 2\pi t)$

$X(f) = 5\cdot\frac{1}{2}(\delta(f-2)+\delta(f+2))$

$y(t) = ?$

c) For each LTI system defined below, check the box that *best* describes the filter type. Note that two systems are described by their impulse response and one by its transfer function.

Lowpass	Highpass	Bandpass	
☐	☐	☒	$h(t) = \text{sinc}(50t)\cos(4000\pi t)$
☐	☒	☐	$h(t) = \delta(t) - \text{sinc}(50t)$
☒	☐	☐	$H(f) = \frac{1}{10 + j6\pi f}$

d) A lowpass filter with impulse response $h(t)$ has a cutoff frequency of 100Hz. What is the cutoff frequency of the lowpass filter with impulse response $h(2t)$? Hint: the factor of 2 in the argument *compands* the impulse response. Clearly explain your answer.

~~50Hz~~, as the signal doubles in the time domain, it halves in the frequency domain.

$\sin(10\pi t) = \frac{e^{j10\pi t} + e^{-j10\pi t}}{2}$

5

GCD of $4\pi, 6\pi = 2\pi = \omega_0$

2) (25 pts) Given the periodic signal

$\frac{1}{2}(\sin(4\pi t - 6\pi t) + \sin(4\pi t + 6\pi t)) =$

$f = \frac{2\pi}{2\pi} = 1$

$$x(t) = 2 + \sin(4\pi t)\cos(6\pi t)$$

$\frac{1}{2}(\sin(10\pi t) - \sin(2\pi t))$ $T_0 = \frac{1}{f} = 1$

0 for all integer

a) Find the *trigonometric fourier series* by finding ω_0, the a_0 coefficient, all the a_n coefficients, and all the b_n coefficients. For full credit, show how you arrived at your answer.

$a_0 = 1\int_0^1 (2 + \frac{1}{2}\sin(10\pi t) - \sin(2\pi t))dt$

$= \int 2dt + \frac{1}{2}\int_0^1 \sin(10\pi t)dt - \frac{1}{2}\int_0^1 \sin(2\pi t)dt$

$= 2t|_0^1 + \frac{1}{2}[-\frac{\cos(10\pi t)}{10\pi}]|_0^1 - \frac{1}{2}[-\frac{\cos(2\pi t)}{10\pi}]|_0^1$

$= 2 + \frac{1}{2}\frac{1}{10\pi} + \frac{1}{2}\frac{1}{10\pi} + \frac{1}{2}\frac{1}{10\pi} - \frac{1}{2}\frac{1}{10\pi}$

$\boxed{a_0 = 2}$

$a_n = 2\int_0^1 (2 + \frac{1}{2}\sin(10\pi t) - \sin(2\pi t)) \cdot \cos(n\omega_0 t)$

$= 2\int_0^1 2\cos(n\omega_0 t) + \frac{1}{2}\sin(10\pi t)\cos(n\omega_0 t) - \frac{1}{2}\sin(2\pi t)\cos(n\omega_0 t)$

$= 2[2\frac{\sin(n\omega_0 t)}{n\omega_0}]|_0^1 + \frac{1}{4}\sin$

$\omega_0 = ?$

$a_n = ?$

$b_n = ?$

b) Find the *complex exponential fourier series* by finding ω_0 and all the X_n coefficients. For full credit, show how you arrived at your answer.

$X_n = 1\int (2 + \frac{1}{2}\sin(10\pi t) - \frac{1}{2}\sin(2\pi t))e^{-jn2\pi t}dt$

$= \int_0^1 2e^{-jn2\pi t} + \int \frac{1}{4}(e^{j10\pi t} - e^{-j10\pi t})e^{-jn2\pi t}dt - \int_0^1 \frac{1}{4}(e^{j2\pi t} - e^{-j2\pi t})e^{-jn2\pi t}dt$

$\frac{2e^{-jn2\pi t}}{-jn2\pi}|_0^1 + \int_0^1 \frac{1}{4}(e$

c) What is the average power in $x(t)$? For full credit, show how you arrived at your answer. Hint: use Parseval's theorem

$= \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt$

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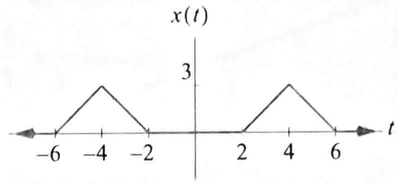
$\Lambda\left(\frac{t}{\tau}\right) = \tau \, \mathrm{Sinc}^2(f\tau)$

3) (25 pts)The time-domain signal $x(t)$ is shown at the right



a) Find the Fourier Transform $X(f)$. *Simplify your answer* so that it can be easily plotted, i.e. so that there are *no* complex exponentials in the answer.

$X(t) = \underbrace{3\Lambda(t+4)}_{2} + \underbrace{3\Lambda(t-4)}_{2}$

$X(f) = \mathcal{F}\{\Lambda(t+4)\} + \mathcal{F}\{\Lambda(t-4)\}$

$= e^{-j2\pi f\cdot(-4)}\,\mathcal{F}\{A(t)\} + e^{-j2\pi f(4)}\,\mathcal{F}\{\Lambda(t)\}$

$= \mathcal{F}\{\Lambda(t)\}\left[e^{-j8\pi f_0} + e^{j8\pi f_0}\right]$

$= \mathrm{Sinc}^2(f)\cdot 2\left[\frac{e^{-j8\pi f} + e^{j8\pi f}}{2}\right]$

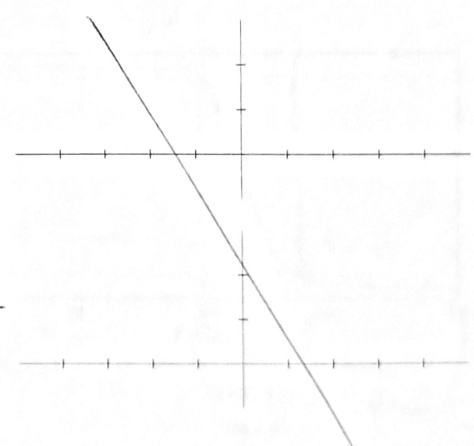
$\boxed{= \mathrm{Sinc}^2(f)\cdot 2\cos[8\pi f]}$

CLOSE

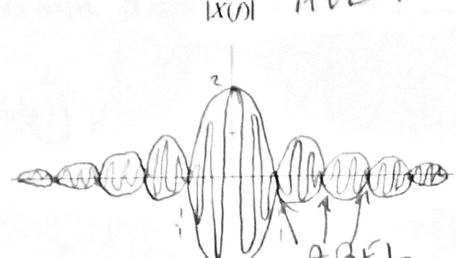
$W = 8\pi$

$f = 4$

$T_0 = \frac{1}{4}$



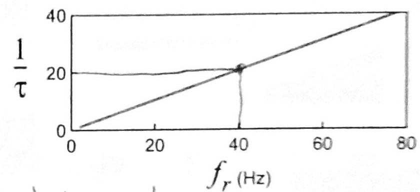
b) Sketch the Fourier Transform *magnitude* $|X(f)|$. If it is composed of the product of two signals, try drawing the magnitude of each one individually and then combine them as a product. Be sure to make the plot as accurate as possible and fully label all critical values in the plot.



22

$40Hz \Rightarrow 20 = \frac{1}{\tau} = \tau = \frac{1}{20} = .05$

4) (25 pts) A set of filters are to be designed using the *window method*. The window function, $w(t)$, to be used is $\Pi(t)$. The curve shown at the right shows the rolloff frequency f_r to the reciprocol time width $1/\tau$ of $w(t)$ using thresholds $\delta_p = 0.1$ and $\delta_s = 0.1$.



~~Hz $\Rightarrow$ time $= \frac{1}{40Hz} = .025$~~

$W = 40Hz$

a) Write an expression for the *impulse response*, $h_L(t)$, of a lowpass filter with a passband of 100Hz and with rolloff of 40Hz. Be sure to give the impulse response for a filter that has *unity passband gain* and is *causal*. Be sure to clearly provide *numeric values* for any variables used.

$f_c = \frac{\Delta F}{2} + f_p = \frac{f_s - f_p}{2} + f_p = 120$

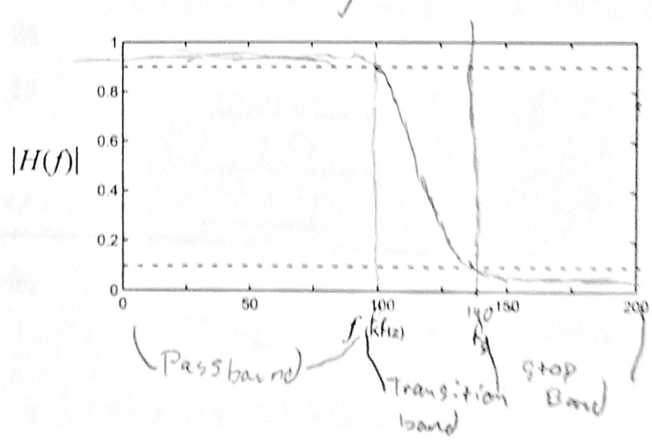
$h_L(t) = 2f_c \, \text{sinc}[2f_c(t-\tau)] \left[\Pi\left(\frac{t-\tau}{2\tau}\right)\right]$

$= 2f_c \, \text{sinc}[2f_c(t-.05)] \Pi\left[\frac{t-.05}{.1}\right]$

$= 240 \, \text{sinc}[240(t-.05)] \Pi\left[\frac{t-.05}{.1}\right]$

0/12

b) Draw a reasonable sketch of the lowpass filter transfer function magnitude $|H(f)|$. Your representation of the curve does not need to be exact, but the *rolloff* and *passband* frequencies should be consistent with the parameters of the design.



c) Write an expression for the *impulse response*, $h_B(t)$, of a bandpass filter with a 5000Hz center freuency, with a 200Hz bandwidth and with a 40Hz highside rolloff. Be sure to give the impulse response for a filter that has *unity passband gain* and is *causal*. Be sure to clearly state the *numeric value* for any variables used. It is *not* necessary to sketch $|H(f)|$.

$H_B(t) = 2(5000) \, \text{sinc}[2f_c(100t)] \, \Pi\left(\frac{t-5000}{.01}\right) 2\cos(2\pi(5000)(t))$

$= 200$

$f_c = ?$

Trig F. Series

$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$

$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$ even $a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(n\omega_0 t)\,dt$

odd $b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin(n\omega_0 t)\,dt$

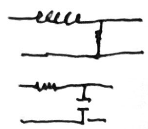
$\omega = 2\pi f$

$T_0 = \frac{1}{f}$

Ideal Lowpass:

$H(f) = \Pi\left(\frac{f}{2f_c}\right)$

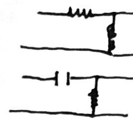
$h(t) = 2f_c \,\mathrm{sinc}(2f_c t)$

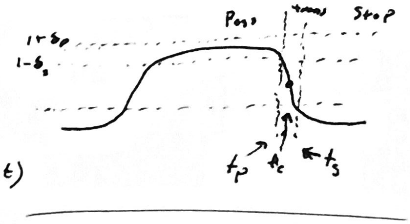


Ideal Highpass:

$H(f) = 1 - \Pi\left(\frac{f}{2f_c}\right)$

$h(t) = \delta(t) - 2f_c \,\mathrm{sinc}(2f_c t)$





Complex exp F. Series

$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$

$X_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}\,dt$

$e^{j\theta} = \cos\theta + j\sin\theta$

Symmetry:

$x(t)$ is real → coeff have even mag, odd phase

$X_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}\,dt$, $X_n = X_{-n}^* \to |X_n| = |X_{-n}| \to \underline{/X_n} = -\underline{/X_{-n}}$

$x(t)$ is real & even, X_n even & real

$X_n = \frac{1}{T_0}\int_{T_0} x(t)\cos(n\omega_0 t)\,dt$

$x(t)$ real & odd, X_n has even mag and purely imaginary

$X_n = -j\frac{1}{T_0}\int_{T_0} x(t)\sin(\omega_0 t n)\,dt$

Band Pass:

$H(f) = \Pi\left(\frac{f-f_0}{B}\right) + \Pi\left(\frac{f+f_0}{B}\right)$

$h(t) = B\,\mathrm{sinc}(Bt) \cdot 2\cos(2\pi f_0 t)$

Band reject:

$H(f) = 1 - BP$

$h(t) = \delta(t) - BP$

Window method:

Uses a function to time limit the ideal filter.

for lowpass, using Π window:

$h_i(t)$: above

$w = \Pi\left(\frac{t}{2\tau}\right)$

multiply and use time delay

$h(t) = h_i(t-\tau)\,w(t-\tau) =$

$= 2f_c\,\mathrm{sinc}[2f_c(t-\tau)]\left[\Pi\left(\frac{t-\tau}{2\tau}\right)\right]$

$h_i(t) \to$ Bandwidth

$w(t) \to$ rolloff

$\tau \to$ to make filter causal

for bandpass

make Lowpass w/ timedelay $h_L(t)$

make a signal to shift to center

$m(t) = 2\cos(2\pi f_0 t)$

make $h(t)$ by mult. & time delay

$h(t) = h_L(t-\tau)\,m(t-\tau)$

$\tau \to$ large enough for impulse response

Complex → Trig	Trig → complex
$a_0 = X_0$	$X_0 = a_0$
$a_n = X_n + X_{-n}$	$X_n = \frac{1}{2}(a_n - jb_n)$
$b_n = j(X_n - X_{-n})$	$X_{-n} = \frac{1}{2}(a_n + jb_n)$

Parseval's Theorem Avg power

$\sum_{n=-\infty}^{\infty} |X_n|^2 = \frac{1}{T_0}\int_{T_0} |x(t)|^2\,dt$

1st Order Lowpass rise time

TF: $H(f) = \frac{1}{\left(\frac{j2\pi f}{\alpha}\right)+1} = \frac{\alpha}{j2\pi f + \alpha}$

Impulse resp.: $h(t) = \alpha e^{-\alpha t}u(t)$

Step resp: $h_s(t) = \int h(t) = \int_{-\infty}^{t} h(\lambda)\,d\lambda$

$\Rightarrow [1 - e^{-\alpha t}]\,u(t)$

Diff:

$H(f) = \frac{j2\pi fRC}{1+j2\pi fRC}$, for $f \ll \frac{1}{2\pi RC}$

$H(f) \approx j2\pi fRC$

$|H(f)| = 2\pi f$

$\underline{/H(f)} = \frac{\pi}{2}$; $f>0$

$-\frac{\pi}{2}$; $f<0$

integrator:

$H(f) = \frac{1}{1+j2\pi fRC}$

if $f_{min} \gg \frac{1}{2\pi RC}$

$H(f) \approx \frac{1}{j2\pi f}$

$|H(f)| = \frac{1}{2\pi|f|}$ $\underline{/H(f)} = \frac{\pi}{2}$ $f<0$

$-\frac{\pi}{2}$ $f>0$

FT Symmetry

$x(t)$ real, $X(f)$ even mag, odd phase

$x(t)$ real, even, then $X(f)$ even, real

$x(t)$ real, odd, then $X(f)$ even, imaginary

TD	FD	TD	FD
∞	∞	Period	delta
∞	∞	delta	Period

Rayleigh's Energy Theorem

$x(t)$ is energy (goes to 0, $-\infty \le t \le \infty$)

$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df$

Time $E_{t_0} = \int_{t_0}^{t} |x(t)|^2\,dt$

Freq $E_{f_0} = \int_{f_0}^{f} |X(f)|^2\,df$

Transfer function:

$\mathcal{F}\{h(t)\} = H(f)$; generally complex

~~Discrete~~ :

Cap. $i(t) = C\frac{dV}{dt}$

Ind $V(t) = L\frac{di}{dt}$