Linear Systems

ECE3150 Test III November 18, 2019

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Signature:

Instructions:

1) This exam is closed book, closed notes, and closed neighbor. You may have one $8.5'' \times 11''$ note sheet with notes written on one side only. **Turn in your notesheet with your test.**

2) There are 8 pages to this exam including this cover sheet. You have 50 minutes to work the exam. Start when the instructor tells you to start.

3) Work the problems on the exam in the space provided. If you need additional space, *use the back side of the previous page.*

4) If you believe a problem cannot be solved, for full credit state exactly *why* it cannot be solved.

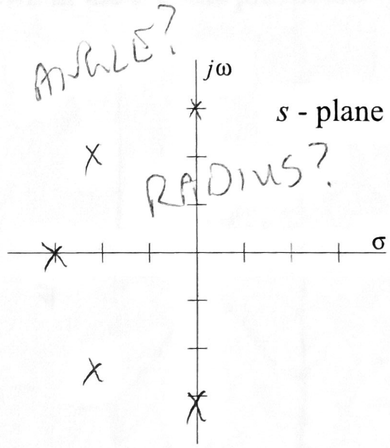
5) If you believe a problem has ambiguous notation, ask the instructor for clarification.

Question #	Max Points	Points
1	25	12
2	25	20
3	25	10
4	25	24
Total:	100	66

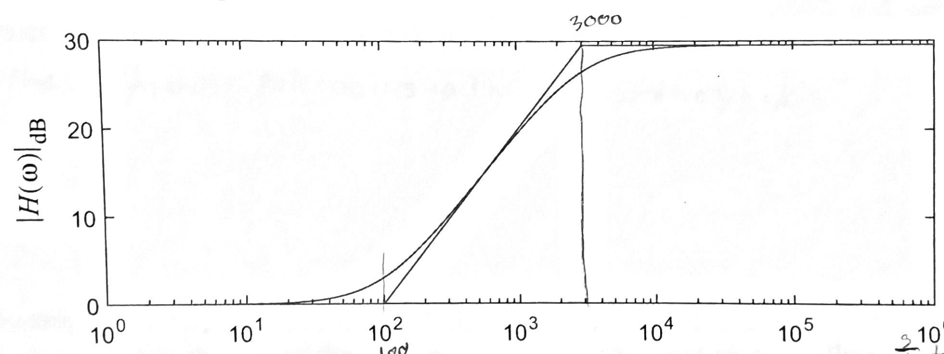
12

1) (25 pts)

a) A 5th-order Butterworth *lowpass* filter has cutoff frequency $\omega_c = 2000$ rad/sec. Draw the pole-zero constellation for this filter on the s-plane shown. Clearly label the axis tic marks so the plot scale is defined. Also, clearly label the position of each point in the pole-zero constellation so it is clearly defined.



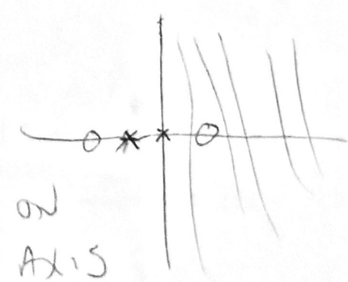
b) The magnitude $|H(\omega)|_{dB}$ and bode plot asymptotes for a filter are shown below. If $|H(0)|_{dB} = 0$, write an expression for the filter transfer function $H(s)$.



$H(s) = \dfrac{\frac{s}{100}+1}{\frac{s}{3000}+1}$ CLOSE

c) The transfer function for a system is given by $H(s) = \dfrac{4(s-2)(s+2)}{s(s+1)}$. Is this system (circle one) (1) stable, (2) marginally stable, or (3) unstable? Clearly explain your answer.

the rightmost pole lies on the ~~y~~ axis.
AT $S=0$
+ NO OTHER ON
$j\omega$ AXIS

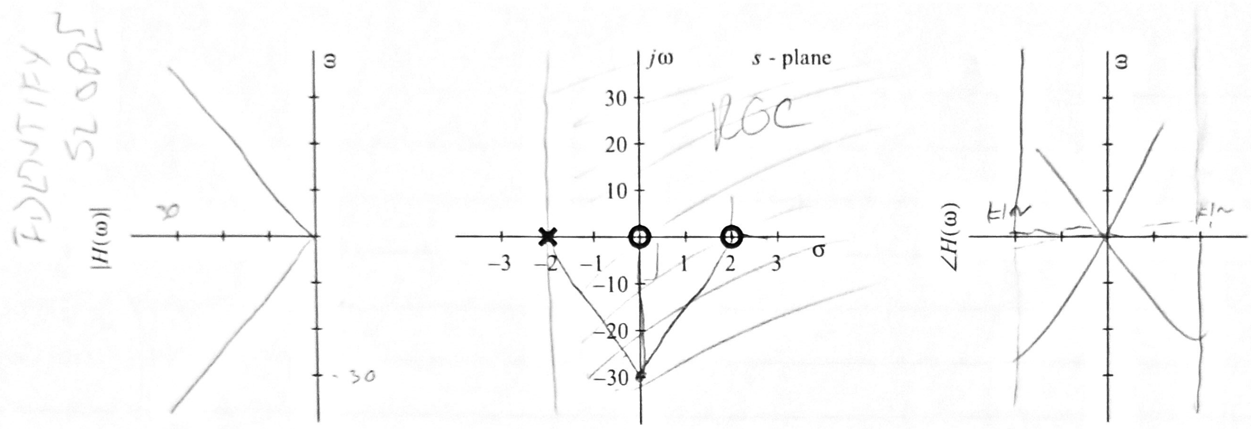


d) The transfer function for a system is given by $H(s) = \dfrac{4(s-2)(s+2)}{s(s+1)}$. Find $h(0^+)$ and $h(\infty)$. If either do not exist, then simply state that and explain why.

H(s)

20

2) (25 pts) The pole-zero constellation for *transfer function* $H(s)$ is shown in the s-plane plot below.



a) Draw the *Region of Convergence* for this transfer function directly on the s-plane plot.

b) Is this system *stable*? Clearly explain why or why not.

Yes, the rightmost pole occurs to the left of the y-axis.

c) Draw the *magnitude response* $|H(\omega)|$ over the frequency range $[-30, 30]$ rad/sec directly on the diagram above and to the *left* of the s-plane plot. Draw the plot as carefully and accurately as you can and clearly label any critical points of the plot.

d) Draw the *phase response* $\angle H(\omega)$ over the frequency range $[-30, 30]$ rad/sec directly on the diagram above and to the *right* of the s-plane plot. Draw the plot as carefully and accurately as you can and clearly label any critical points of the plot.

e) Write a *complete expression* for the transfer function $H(s)$ assuming that $|H(10)| = 20$.

$H(s) = \frac{ks(s-2)}{s+2}$

$H(10) = \left| \frac{k(j10)(j10-2)}{j10+2} \right| = \frac{k\,80}{12} = \frac{k\,20}{3} = 20$

$k = 3$

$$\boxed{H(s) = \frac{3(s)(s-2)}{s+2}}$$

10

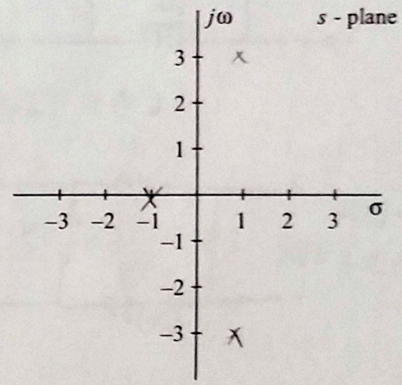
$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\frac{1 \pm \sqrt{1-184}}{4}$

$\frac{1 \pm j\sqrt{183}}{4}$ = Zeros

3) (25 pts) The transfer function for a Linear, Time-Invariant system is shown at the right.

$$H(s) = \frac{2s^2 - s + 23}{(s^2 - 2s + 10)(s+1)}$$

a) ~~Plot the *pole-zero constellation* on the s-plane plot shown at the right~~



b) Is this system *stable*? Clearly explain why or why not.

Unstable, poles occur to the right of the y-axis.

c) Find the *impulse response* $h(t)$.

* Carefully organize your solution

* Circle your final answer.

* For full credit, show all your work.

$\frac{2 \pm \sqrt{4-4(10)}}{2} = \frac{1 \pm j\sqrt{36}}{2}$

$s = 1 \pm j3$

$H(s) = \frac{2s^2 - s + 23}{(s^2-2s+10)(s+1)} = \frac{2s - s + 23}{((s-2s+4)+6)\, s+1} = \frac{2s - s + 23}{\cancel{((s-2)^2+6)}(s+1)}$

$\frac{2s^2 - s + 23}{} = \frac{A}{s+1} + \frac{Bs + C}{s^2 - 2s + 10}$

$= As^2 - 2As + 10A + Bs^2 + Cs + Bs + C$

$A = \frac{2s^2 - s + 23}{s^2 - 2s + 10}\Big|_{\substack{s+1=0 \\ s=-1}} = \frac{2+1+23}{1+2+10} = \frac{26}{13} = 2$

$2s^2 = As^2 + Bs^2$ $\quad -s = -2As + Cs + Bs \quad 23 = 10A + C$

$Bs + C \Big| = \frac{2s^2 - s + 23}{s+1}\Big|$

$s = 1 + j3$

$s^2 - 2s + 10 = 0$

$s = 1 \pm j3$

$\frac{2(1+j3)^2 - 1 + j3 + 23}{1 + j3 + 1}$

$H(s) = 2 \frac{1}{(s+1)} +$

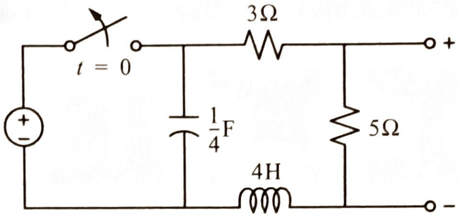
2

$h(t) = 2e^{-t} \frac{t}{2} +$

$= \cancel{e^{t} t} +$

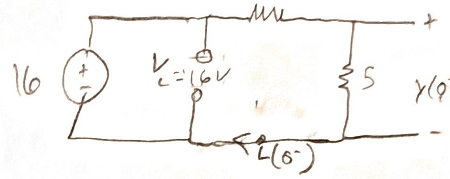
24

4) (25 pts) The circuit shown at the right contains a switch that changes state at $t = 0$. Solve for the *output voltage* $y(t)$ over the *entire* time range from $[-\infty, \infty]$.



* Carefully organize your solution
* *Circle* the *steady-state* part of your final answer.
* *Underline* the *transient* part of your final answer.
* For full credit, show all your work.

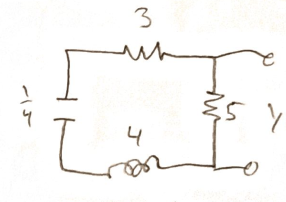
Case I: $t(0^-)$



$i(0^-) = \frac{16}{8} = 2A$

$y(0^-) = 2(5) = 10$

Case II: $t \geq 0$

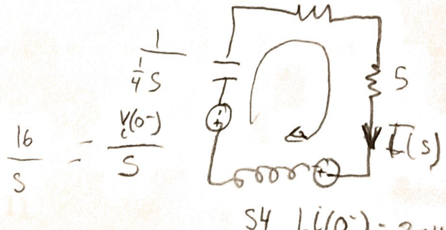


$\mathcal{L}$

$V_L = L\frac{di}{dt}$

$V_L = L(I(s)s - i(0))$

$= LI(s)s - Li(0^-)$



$s4 \quad Li(0^-) : 2 \cdot 4$

KVL: $4\frac{1}{s}I(s) + 3I(s) + 5I(s) - 2\cdot 4 + 4sI(s) + \frac{16}{s} = 0$

$4sI(s) + 8I(s) + \frac{4}{s}I(s) + \frac{16}{s} - 8 = 0$

$I(s)\left[4s + 8 + \frac{4}{s}\right] = \frac{16}{s} - 8$

$I(s) = \frac{\frac{16}{s} + 8}{4s + 8 + \frac{4}{s}} \left(\frac{s}{s}\right)$

$I(s) = \frac{16 + 8s}{4s^2 + 8s + 4} = \frac{4 + 2s}{s^2 + 2s + 1}$

$= \frac{4}{(s+1)^2} + \frac{2s}{(s+1)^2} = 4\frac{1}{(s+1)^2} + \frac{2(s+1)}{(s+1)^2} - \frac{2}{(s+1)^2}$

$\mathcal{L}^{-1}$

$i(t) = 4e^{-t}\,t\,u(t)$

$+ 2e^{-t}u(t)$

$- 2e^{-t}\,t\,u(t)$

$i(t) = 2e^{-t}\,t\,u(t) + 2e^{-t}u(t)$

TRANSIENT