Linear Systems

ECE3150 Test I September 27, 2019

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Signature: [signature]

Instructions:

1) This exam is closed book, closed notes, and closed neighbor. You may have one $8.5'' \times 11''$ note sheet with notes written on one side only. **Turn in your notesheet with your test.**
2) There are 6 pages to this exam including this cover sheet. You have 50 minutes to work the exam. Start when the instructor tells you to start.
3) Work the problems on the exam in the space provided. If you need additional space, *use the back side of the previous page*.
4) If you believe a problem cannot be solved, for full credit state exactly *why* it cannot be solved.
5) If you believe a problem has ambiguous notation, ask the instructor for clarification.

Question #	Max Points	Points
1	25	18
2	25	13
3	25	22
4	25	24
Total:	100	77

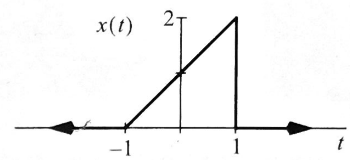
+5

18

1) (25 pts)

a) What is the *step response* of a system that performs the *antiderivative* operation?

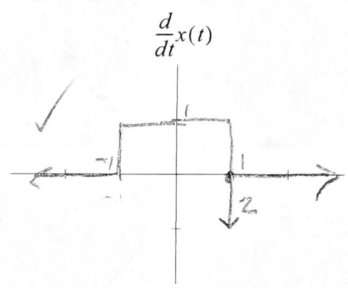
b) Find an expression for the signal shown in terms of the *singularity functions* $u(t)$ and $r(t)$.



$y(t) = r(t+1) - r(t-1) - 2u(t-1)$

c) Find an expression for the *derivative* of the signal shown in part (b) above and plot it.

$\frac{dy}{dt} = u(t+1) - u(t-1) - 2\delta(t-1)$



d) Check *true* or *false* as appropriate.

true☑ false☐ $[1-e^{-t}]u(t)$ is a *power* signal

true☑ false☐ $\int_{-4}^{0} \cos(\pi t)\delta(t+2)dt = 1$

$t_0 = -2$

$\int_{-4}^{0} \cdots = \cos(-2\pi) = 1$

e) For each *LTI system* with impulse response shown, check the box if the property *holds*.

causal☐ stable☑ $h(t) = e^t \Pi\left(\frac{t-2}{4}\right)$ starts at $t=0$, so stable

causal☑ stable☑ $h(t) = u(2-t)$

f) For each dynamical system defined below, check the box if the property *holds*.

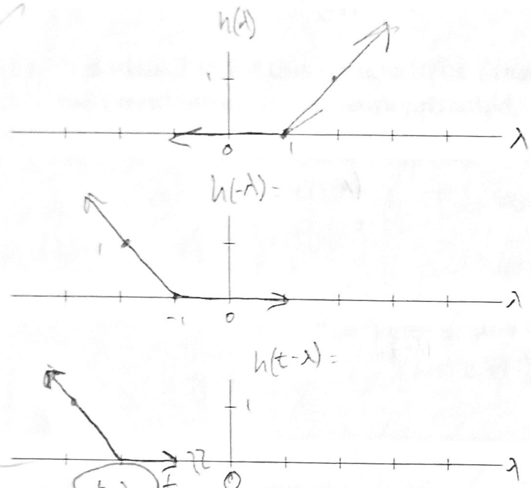
linear☑ time-invariant☐ $y(t)\cos(t) = \frac{d}{dt}x(t)$

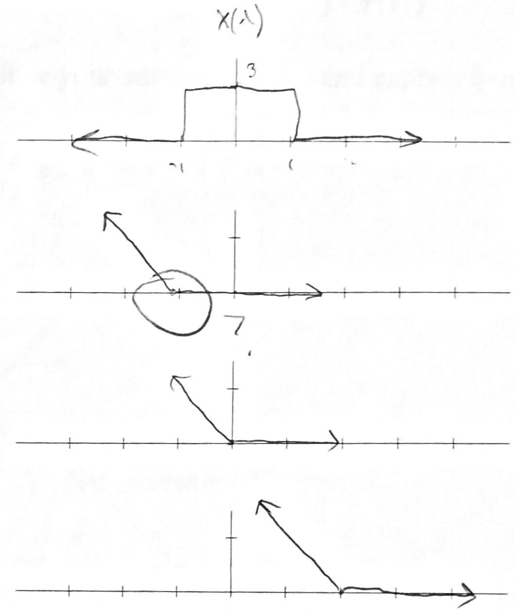
linear☑ time-invariant☑ $x(t) = \frac{d}{dt}y(t) + \underbrace{y(t)x(t)}_{\text{not linear}}$

13

2) (25 pts) A continuous-time, LTI system has input $x(t) = 3\Pi(t/2)$ and impulse response $h(t) = r(t-1)$.

a) Find the *output*, $y(t)$. Clearly show how you arrived at your solution.





0 for $\lambda \le -1$

$t < -1$

$t-1$ $\quad -1 < t \quad t-1 < -1$

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda = \begin{cases} 0 \\ 3(-\lambda+t) & -1 \le t < 1 \\ 0 & 1 < t \end{cases}$$

$$y(t) = \cancel{\int_{-\infty}^{-1} 0} + 3\int_{-1}^{1} t-\lambda \, d\lambda + \int$$

$$y(t) = 3\left[t\lambda - \frac{\lambda^2}{2}\right]_{-1}^{1}$$

$$3t + 3t - \left[\frac{1}{2} + \frac{1}{2}\right]$$

$$y(t) = 6t - 1 \text{ for}$$

b) Carefully sketch the graph of $y(t)$. Be sure to *label all critical points* of the graph.

22

3) (25 pts) A system $y(t) = H\{x(t)\}$ is known to have the following input/output signal pairs

$$\text{(a) } r(t+1) = H\{u(t)\}\ ,\quad \text{(b) } (t+1)^2 u(t+1) = H\{r(t)\}\ ,\quad \text{(c) } r(t) = H\{u(t-1)\}$$

$$\text{(d) } 2r(t+1) = H\{2u(t)\}\ ,\quad \text{(e) } (t+1)(t+2)u(t+1) = H\{u(t) + r(t)\}.$$

a) Is it plausible for this system to be *Linear*? State your answer (yes/no) and explain it using the input/output signal pairs provided.

c & d demonstrate scaling property YOU MEAN (a) + (d)

additive (a, b, c) you mean (a), (b), + (e)

$H\{u(t)+r(t)\} = H\{u(t)\} + H\{r(t)\}$

$\cancel{(t+1)}(t+2)u(t+1) = r(t+1) + (t+1)^2 u(t+1)$

$\cancel{(t+1)} + (t+1)^2 u(t+1)$

$(t+2)u(t+1) = (1+)(t+1)u(t+1)$ $\neq$ $\neq$

Does not satisfy additive property ∴ Not Linear

b) Is it plausible for this system to be *Time-Invariant*? State your answer (yes/no) and explain it using the input/output signal pairs provided. $x_1(t) = x_2(t-t_0) \Rightarrow y_1(t) = y_2(t-t_0)$

a and c you mean (a) + (c)

$x_1(t) = u(t)$ $y_1(t) = r(t+1)$

$x_2(t) = u(t-1)$ $y_2(t) = r(t)$

$t_0 = +1$

$x_2(t+1) = u(t) = x_1(t)$; $y_2(t+1) = r(t+1) = y_1(t)$

Plausibly time invariant.

c) Is it plausible for this system to be *Causal*? State your answer (yes/no) and explain it using the input/output signal pairs provided.

(a)

No, the output precedes the input

d) Is it plausible for this system to be *Stable*? State your answer (yes/no) and explain it using the input/output signal pairs provided.

no.

(a) has a bounded input $(u(t))$, but the output is not $(r(t+1))$.

24

4) (25 pts)

a) Create a block diagram that implements the following differential equation. Your diagram may only use the *summer*, *integrator*, and *gain* devices. Each prime denotes derivative.

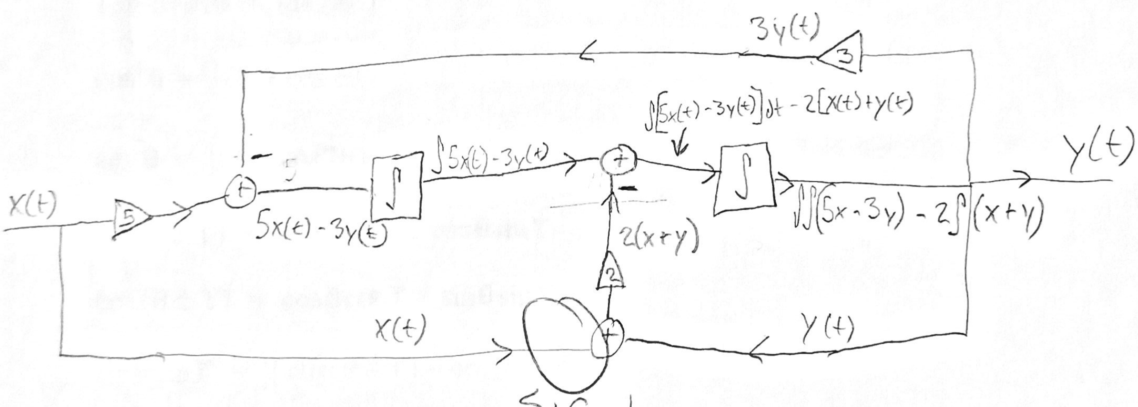
$$3y(t) - 2y'(t) + y''(t) = 5x(t) - 2x'(t)$$

$$3y(t) - 2\frac{dy}{dt} + \frac{d^2y}{dt^2} = 5x - 2\frac{dx}{dt}$$

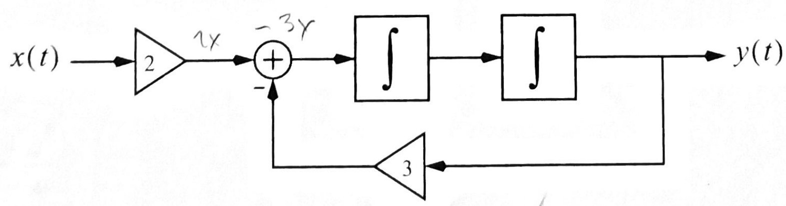
$$\frac{d^2y}{dt^2} = 5x - 3y - 2\frac{dx}{dt} + 2\frac{dy}{dt}$$

$$\iint \frac{d^2y}{dt^2} = \iint \frac{d^0}{dt^0}[5x - 3y] - 2\int \frac{d}{dt}[x+y]$$

$$y = \iint[5x - 3y] - 2\int[x+y]$$



b) Find the system equation for the system defined by the block diagram shown



$$y(t) = \iint (2x(t) - 3y(t))$$