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Finite Element
Method

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Finite Element Method

Numerical Methods

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- The Finite Element Method (FEM) is a numerical technique for solving partial differential equations (PDEs) in complex domains.
- It is widely used in engineering and physics applications, and provides accurate and efficient solutions to a wide range of problems.
- In this presentation, we will discuss the steps to solve a PDE using FEM, and how to implement it using Python and the Matplotlib library.

What is FEM?

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- FEM involves dividing a complex domain into small, simple subdomains called elements.
- These elements are connected at shared points called nodes, forming a mesh.
- The solution to the PDE is approximated by piecewise polynomial functions within each element, and the solution over the entire domain is obtained by assembling the element solutions and solving the resulting system of equations.
- The accuracy of the FEM solution depends on the number and size of the elements in the mesh, as well as the order of the polynomial functions used to approximate the solution.

FEM is used in various applications, including:

- Structural Analysis
- Fluid Dynamics
- Electromagnetics
- Heat Transfer
- Acoustics

Steps to Solving a PDE using FEM

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The steps to solve a PDE using FEM are:

- 1 Discretize the domain into elements
- 2 Define the element shape functions
- 3 Assemble the stiffness matrix and load vector
- 4 Apply boundary conditions
- 5 Solve the resulting system of equations

Example PDE: Poisson's equation in 2D

$$-\nabla^2 u(x, y) = f(x, y)$$

Here, ∇^2 is the Laplacian operator, and $u(x, y)$ is the unknown function to be solved for. The equation states that the sum of the second partial derivatives of u with respect to x and y equals a given source term $f(x, y)$. For our specific problem, this equation takes the form:

Implementing the Finite Element Method in Python

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$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

where $u(x, y)$ is the temperature distribution and $f(x, y)$ is the heat source term.

To implement the Finite Element Method in Python, we can use the NumPy and Matplotlib libraries to perform numerical calculations and linear algebra operations. Here's an example of how we could solve the Poisson's equation using FEM in Python:

- 1 Define the mesh of nodes and elements
- 2 Assemble the global stiffness matrix and force vector
- 3 Apply boundary conditions to the system of equations
- 4 Solve the system of equations to obtain the nodal values of the solution

FEM is a powerful numerical technique for solving PDEs in complex domains. It involves dividing the domain into small subdomains called elements, and approximating the solution as piecewise polynomial functions within each element. The accuracy of the solution depends on the number and size of the elements in the mesh, as well as the