

Logarithm change of base rule

Suppose we want to find the value of the expression $\log_2(64.5)$. Since 64.5 is not a rational power of 2, it is difficult to evaluate this without a calculator.

However, most calculators only directly calculate logarithms in base-10 (\log) and base-e (\ln). So in order to find the value of $\log_2(64.5)$, we must *change the base* of the logarithm first.

The change of base rule

We can change the base of any logarithm by using the following rule:

$$\log_b(a) = \log_x(a) / \log_x(b)$$

Notes:

- When using this property, you can choose to change the logarithm to *any* base x .
- The arguments of the logarithms must be positive and the bases of the logarithms must be positive and not equal to 1.

Example: Evaluating $\log_2(64.5)$

If the goal is to find the value of a logarithm with base 2, change the base to 10 or e since these logarithms can be calculated on most calculators.

So let's change the base of $\log_2(64.5)$ from 2 to 10.

To do this, we apply the change of base rule with $b = 2$, $a = 64.5$ and $x = 10$.

$$\begin{aligned}\log_2(64.5) &= \log_{10}(64.5) / \log_{10}(2) && // \text{Change of base rule} \\ &= \log(64.5) / \log(2) && // \text{Since } \log_{10}(x) = \log(x)\end{aligned}$$

We can now find the value using the calculator.

$$\log_2(64.5) \approx 6.0$$