The 3 questions below have been selected as the Assignment of the CONTINUOUS ASSESSMENT, with a total of 100 marks. You are required to solve the problems yourself. Note that plagiarism will result in serious consequences. Write your solutions neatly on A4-size paper sheets with dark ball pens. Follow the instructions to submit your solutions.

1. A communication channel consists of two inputs and three outputs. Each input-output path is described by a transition probability p_{ij}, standing for $\Pr(Y_j = y_j | X_i = x_i)$, $j = 1, 2, 3$ and $i = 1, 2$. Thus, p_{ij} is the probability of receiving output y_j given that the input is x_i. Accordingly, the channel transition is summarized by

$$i \begin{matrix} & j & \\ \begin{bmatrix} 0.92 & 0.05 & 0.03 \\ 0.02 & 0.10 & 0.88 \end{bmatrix} \end{matrix}$$

   If the input probabilities are $\Pr(X_1) = 0.6$ and $\Pr(X_2) = 0.4$,
   a) find the output probabilities $\Pr(Y_j)$, $j = 1, 2, 3$.
   b) determine $\Pr(X_i = x_i | Y_j = y_j)$, $i = 1, 2$, $j = 1, 2, 3$
   c) compute the probability of error $P_e = \Pr(x_1, \text{not } y_1) + \Pr(x_2, \text{not } y_3)$.

   Comment on the complexity of calculation in parts (a), (b) and (c) when i and j are large. Suggest a systematic approach to handle the computation.

   (30 marks)

2. In a probabilistic experiement, the sample space
   $$S = \{\text{Jan}, \text{Feb}, \text{Mar}, \cdots, \text{Nov}, \text{Dec}\}$$
   contains 12 elements. Let the sets
   $$A = \{x | x \text{ is a month with 31 days}\}$$
   and
   $$B = \{\text{Apr}, \text{May}, \text{Jun}, \text{Aug}, \text{Oct}\}$$
   Verify the deMorgan's laws
   $$\overline{A \cap B} = \bar{A} \cup \bar{B}$$
   $$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

   (30 marks)

3. Table 1 represents the inspection results of bearings from three different suppliers:

Table 1

	Supplier A	Supplier B	Supplier C
Satisfactory	3000	2500	1500
Oversized	880	100	120
Undersized	200	150	250

a) Find the probability that a defective item is undersized.
b) Find the probability that the defective item in part (a) is from supplier A.
c) Find the probability that a selected item is both defective and from supplier C.

(40 marks)