NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE4491 Probability Theory & Applications

Tutorial No. 5 (Sem 1, AY2021-2022)

1. Two random variables *X* and *Y* have a joint PDF given by

$$f_{XY}(x,y) = \begin{cases} kxy, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Find the value of k.
   (b) Determine the joint CDF $F_{XY}(x,y)$.
   (c) Compute the probability of the event $X \le 1/2$ and $Y > 1/2$
   (d) Derive the marginal PDF $f_X(x)$.

2. Two random variables *X* and *Y* have zero mean and variances of 16 and 36. Their correlation coefficient is 0.5.
   (a) Determine the variance of $Z = X + Y$.
   (b) Find the variance of $W = X - Y$.
   (c) Repeat (a) and (b) if the correlation coefficient is -0.5.

3. A random variable *X* has a PDF of

$$f_X(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

   and an independent random variable *Y* is uniformly distributed between 0 and 1.0.
   (a) Derive the PDF of the random variable $Z = X + Y$.
   (b) Find the probability that $0 < Z \le 1$.

4. A Bernoulli random variable *X* has two possible outcomes, 1 and 0, with probabilities p and $1 - p$, respectively.
   (a) Determine the characteristic function of X.
   (b) The mean value and the mean-square value of X.
   (c) The third central moment of X.

Answer

(1) (a) $k = 4$; (b) $x^2y^2,\ 0 \le x \le 1,\ 0 \le y \le 1$; (c) $3/16$;
(d) $f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$

(2) (a) 76; (b) 28; (c) $28, 76$

(3) (a) $f_Z(z) = \begin{cases} z^2 & 0 < z \le 1 \\ 1-(z-1)^2 & 1 < z \le 2 \\ 0 & \text{otherwise} \end{cases}$ (b) $1/3$

(4) (a) $\varphi_X(\omega) = 1 - p + pe^{j\omega}$; (b) p, p; (c) $p(1-p)(1-2p)$