

NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE4491 Probability Theory & Applications

Tutorial No. 6 (Sem 1, AY2021-2022)

1. A calculator can produce three-digit random numbers that are uniformly distributed between 0.000 and 0.999.
- Find the mean, second moment and variance of the random numbers.
 - How large should the sample size be in order to obtain a sample mean whose standard deviation is no greater than 0.01?
 - Suppose the following random sequence is generated: 0.102, 0.085, 0.345, 0.993, 0.299, 0.837, 0.065, 0.180, 0.326, 0.124. Estimate the population mean and population variance.

$$[\text{Hint: } \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}]$$

2. Company X produces a line of bipolar transistors that have an average current gain of 120 with a standard deviation of 10. Company Y produces a similar line of transistors with the same average current gain but with a standard deviation of 5. 20 transistors are purchased from each company and they are mixed together.
- If 5 transistors are randomly selected with replacement, find the variance of the sample mean.
 - If 5 transistors are randomly selected without replacement, find the variance of the sample mean.
 - How large a sample size should use, with replacement and without replacement, in order to obtain a standard deviation of the sample mean equal 2.
3. A zero-mean random time function is sampled so as to obtain independent sample values. How many sample values are required to obtain an unbiased estimate of the variance of the time function with a standard deviation that is two percent of the true variance? [Hint: For a Gaussian random variable, $\mu_4 = 3\sigma_X^2$.]
4. Data are collected for a random variable Y as a function of another variable X .

i	1	2	3	4	5	6	7	8
X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

- Plot the scatter diagram for the data
- Find the linear regression equation that best fits the data.

Answer

- (1) (a) 0.4995, 0.3328, 0.0833; (b) $n = 834$; (c) 0.3356, 0.1050
(2) (a) 12.5; (b) 11.218; (c) 16, 12
(3) 5003
(4) (b) $y = 0.545 + 0.636x$