NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

EE4491 Probability Theory & Applications

Tutorial No. 4 (Sem 1, AY2021-2022)

1. A CDF for a random variable *X* has the form

$$F_X(x) = \begin{cases} A\left[1 - e^{-(x-1)}\right], & x > 1 \\ 0, & x \le 1 \end{cases}$$

(a) Determine the value of *A*.
(b) Find $F_X(2)$.
(c) Compute Pr $(X > 2)$.
(d) Calculate the probability that *X* lies in the interval $1 < X \le 3$.

2. A random variable *Y* is related to the random variable *X* by $Y = 3X - 4$, where *X* has a PDF

$$f_X(x) = \alpha \exp(-2|x|)$$

(a) Determine the value of α.
(b) Find the probability that *Y* is negative.
(c) Compute the probability that *Y* is greater than *X*.

3. The random variable Θ is uniformly distributed over the interval $-\frac{\pi}{2} < \Theta < \frac{\pi}{2}$. Find the PDF of the random variable $Y = \sin\Theta$, and compute the probability that $Y > 1/2$.

4. A random variable *X* has a CDF of the form

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ x/2, & 0 < x \le 2 \\ 1, & x > 2 \end{cases}$$

(a) Find the mean value of *X*.
(b) Compute the variance of *X*.
(c) The third central moment of *X*.

5. A Gaussian random voltage *V* has a mean value of 10 and a variance of 25.
(a) What is the probability that an observed value is greater than zero?
(b) Compute the probability Pr $(0 < V \le 10)$.
(c) Calculate the probability that an observed value greater than twice the mean value?

Answer

(1) (a) $A = 1$; (b) $1 - e^{-1} = 0.6321$; (c) $e^{-1} = 0.3679$; (d) $1 - e^{-2} = 08647$
(2) (a) $\alpha = 1$; (b) $1 - 0.5e^{-\frac{8}{3}} = 0.9653$; (c) $0.5e^{-4} = 0.009158$
(3) $f_Y(y) = \frac{1}{\pi\sqrt{1-y^2}}$, $-1 < y < 1$; $\Pr(Y > 1/2) = 1/3$
(4) (a) 1; (b) 2nd moment 4/3, var = 1/3; (c) 0
(5) (a) $1 - Q(2) = 0.9772$; (b) $1 - Q(2) - Q(0) = 0.4772$; (c) $Q(2) = 0.0228$