[CS166] Traffic Simulation Report

February 25, 2020

Traffic Simulation
Report

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1 Introduction

This report discusses the results of a discrete model of traffic simulation using cellular automata based on the work of Nagel & Schreckenberg (1992). The simulation then is extended to Two-Lanes model replicating the paper of Rickert, M., et al. (1996). We analyze the change in flow with respect to density for each model, then we compare the two models to disentangle whether a two-lane model can have a time-averaged flow greater than twice the single lane.

$$\overline{flow}\,(Double) \geq 2 \times \overline{flow}\,(Single)$$

Finally, we discuss to what extent the findings can be interpreted in the context of Buenos Aires (Argentina) given the existing the infrastructure and the model assumptions.

2 Model parameters

The model parameters are specified for traffic simulation:

- length: the number of positions in the road.
- density: the proportion of the cars with respect to length
- v_max: the maximum velocity of cars
- p_slow: the probability of randomly slowing down
- steps: the number of time steps to run the simulation.
- lanes: the number of lanes (either 1 or 2)
- warm-up: the time steps prior to counting the flow
- change: the probability of changing lanes (set to 1 throughout the simulations)

3 Assumptions

The traffic flow model is based on the following assumptions:

- Each cell is about the size of a car and cars travel along a road.
- All cars share the same velocity integer range (i.e., from 0 to 5).
- The model assumes periodic boundary conditions (closed system).
- All cars follow the same behavior (i.e., accelerating/decelerating with increment of one)
- All cars have to satisfy the same conditions for switching lanes in the Two-Lane model.

4 Methodologies & Results

The update of the system consists of the following steps:

- Acceleration: if the velocity v is lower than v_{max} and if the distance to the next car ahead is larger than $v + 1$, the speed is incremented by one.
- Decelerating: if a vehicle at position i sees the next vehicle at position $i + j$ (with $j \leq v$), it reduces the speed to $j - 1$ as $v \rightarrow j - 1$
- Randomization: if the velocity of each vehicle is greater than 0 then it would decremented by one with probability p.
- Car motion: each vehicle is moved with v positions.

4.1 Single-lane simulation

Based on the rules above, we generate a simulation for the Single-Lane model then visualize the average flow as a metric that measures the number of cars passing the end of the road at every time step. The analysis of the Monte Carlo simulation focuses on the change in flow as we increase density as well as the uncertainty of the simulation output (i.e., confidence interval)

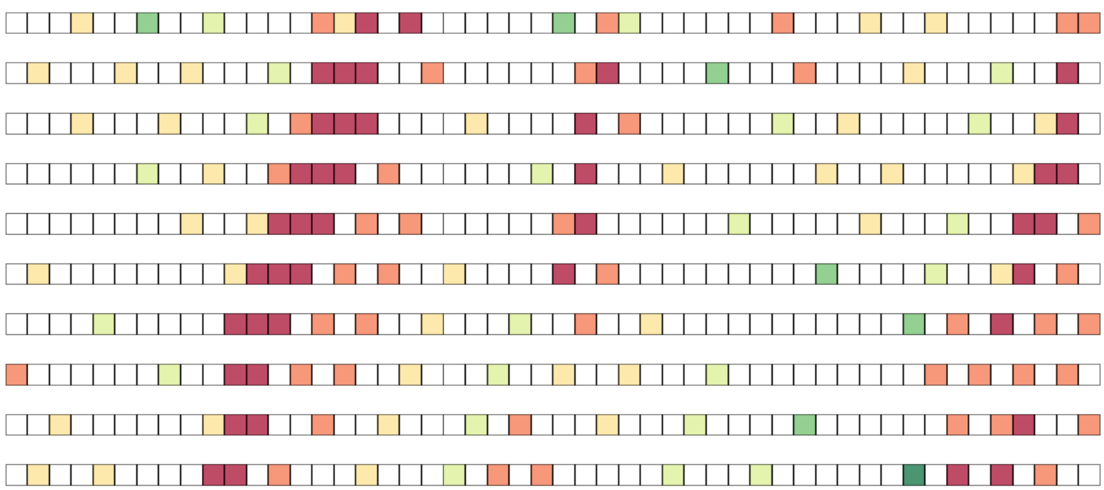


Figure 1. Output of the Single-Lane simulation after 10 time steps. Each colored cell correspond to a car and the colors represent the speeds (red: 0 to green:5). The density is set to 30 percent and 0.2 probability of slowing down. We notice the backward wave of stopping cars forming over time

Note: The simulation is ran for 50 warm-up steps then the flow is registered for 100 time-step. Furthermore, each density's flow is a result of 50 simulations averaged out.

The graph shows a steep increase in flow with narrow confidence intervals until density 9% where the flow reaches its maximum of 0.478 at 10%. at the 11% density, uncertainty increases significantly to reach its maximum (+-0.052). Afterwards, the flow gradually decreases until it reaches 0 at 100% density. Simultaneously, the confidence interval also decreases gradually throughout densities ranging from 11% to 100%.

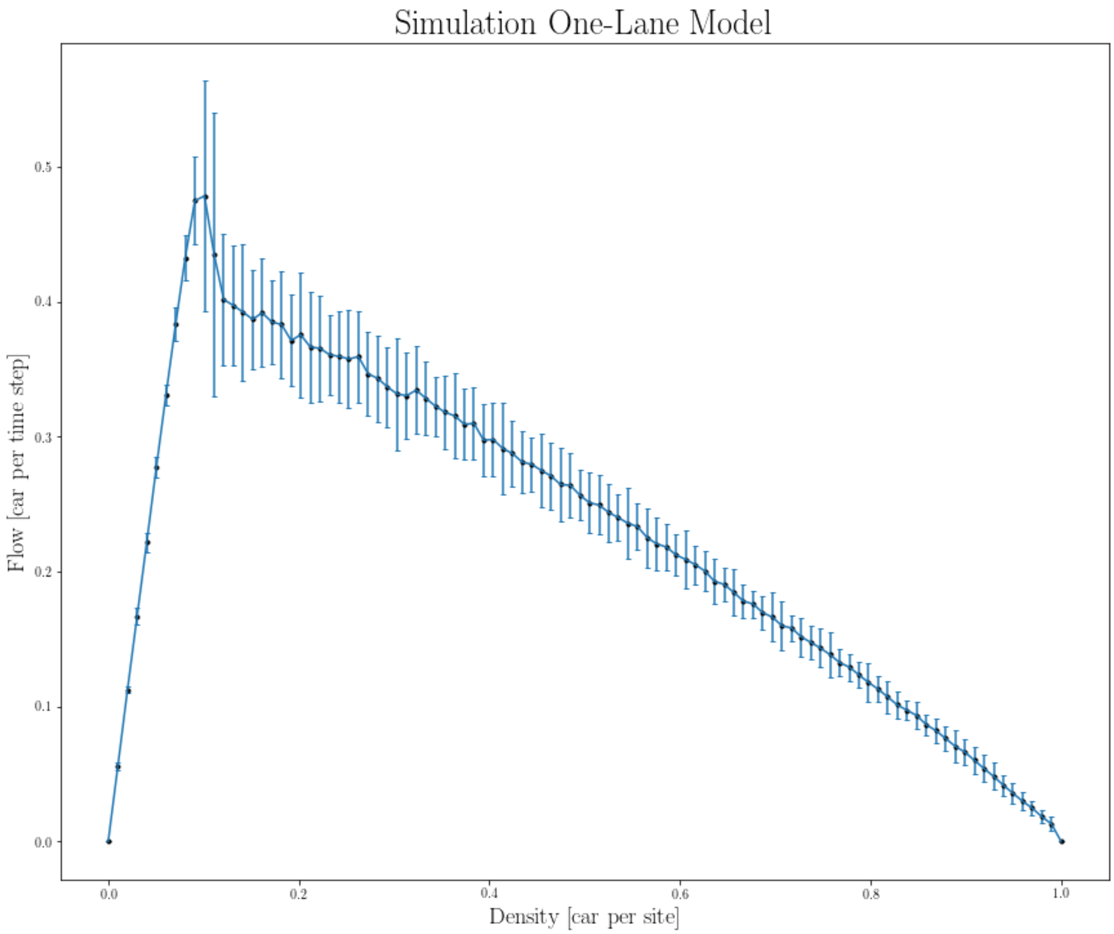


Figure 2. The average traffic flow for the Single-Lane model with densities ranging from 0 to 1 after 150 time steps and each density is averaged over 50 simulation.

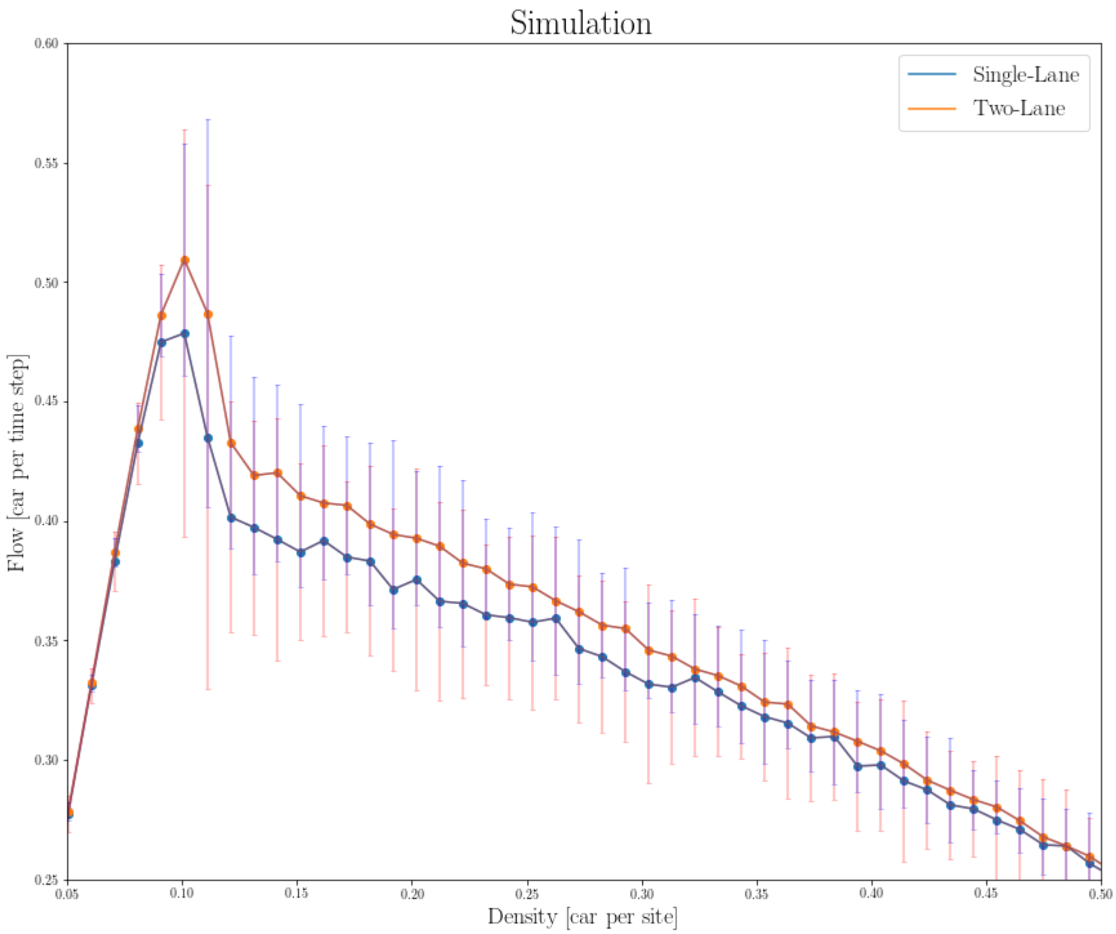
Interpretation At low densities (less than 10%), the flow of cars is barely interrupted by the random slow down and the output of each density's simulations lay around the same values (less uncertainty) but as soon as the density level reaches 9%, the output of the simulation starts varying significantly which suggests that the occurrences of the slow down impacts the traffic based on the initial configuration of the cars (since controlling for density and the probability of slow down, the random initialization created the difference). Moving up to higher densities, the road would be congested with cars leading to decreasing flow. In addition to the random slow down, cars are expected to form the backward wave more often which illustrate the forming of traffic jam as pictured in Fig 1.

4.2 Two-lane simulation

The traffic simulation is extended for the two-lane model, the main difference is that cars are now able to switch lanes when they following criteria are met:

1. If velocity of the car is greater than the space ahead.
2. The other lane has space equal or greater than $v_o + 1$
3. The space behind the car on the other lane is greater or equal to max speed $l_0 \geq v_max$

An example of the simulation output is illustrated in Fig 3. where we notice that each state is represented by two arrays filled with cars based on the density setting. Following the same methodology as the Single-Lane simulation, the Two-Lane model was ran for 50 times for every percentage density. The average flow is then divided by two so that we can compare it with the Single-Lane model output. The premise is that we want to check whether there's any advantage between having a Two-Lane model vs. Two separated Single-Lane model (see Fig 4.)



Interpretation: After controlling for the same density for each of the models (i.e., both models have the same proportion of cars), we notice that the flow of the Two-Lane model is greater than the Single-Lane model for densities starting from 0.09 to roughly 0.5. For the rest of densities, both models perform the same flow (i.e., same flow for either low densities, or really high densities)

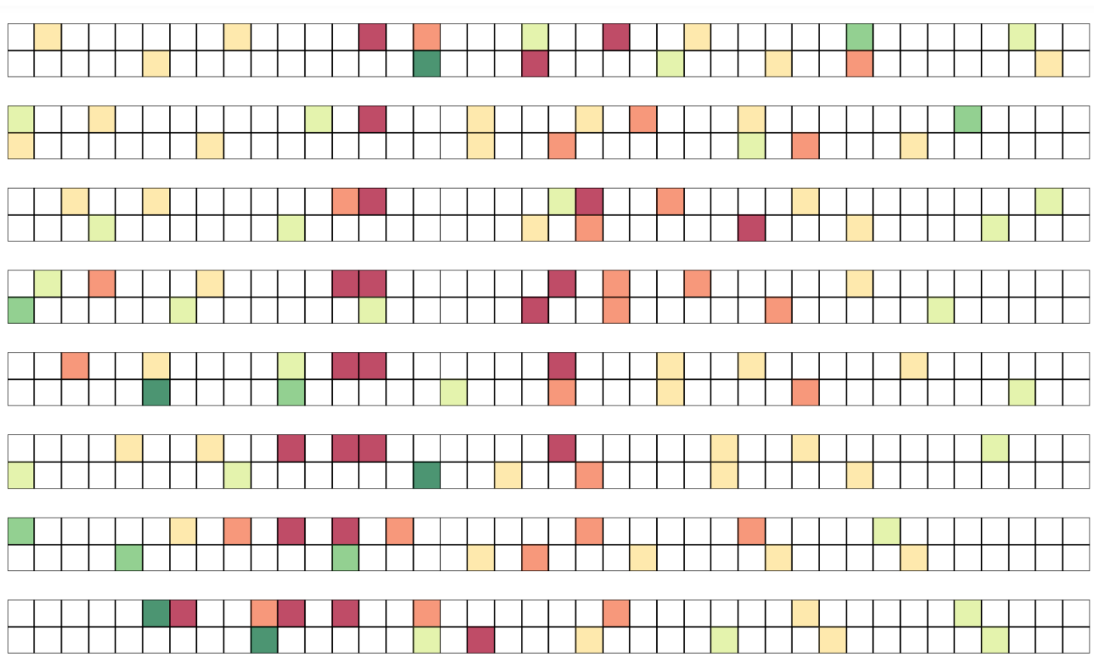


Figure 3. The Two-Lane simulation output after 7 time steps (Density = 40%, $p_s low = 0.5$

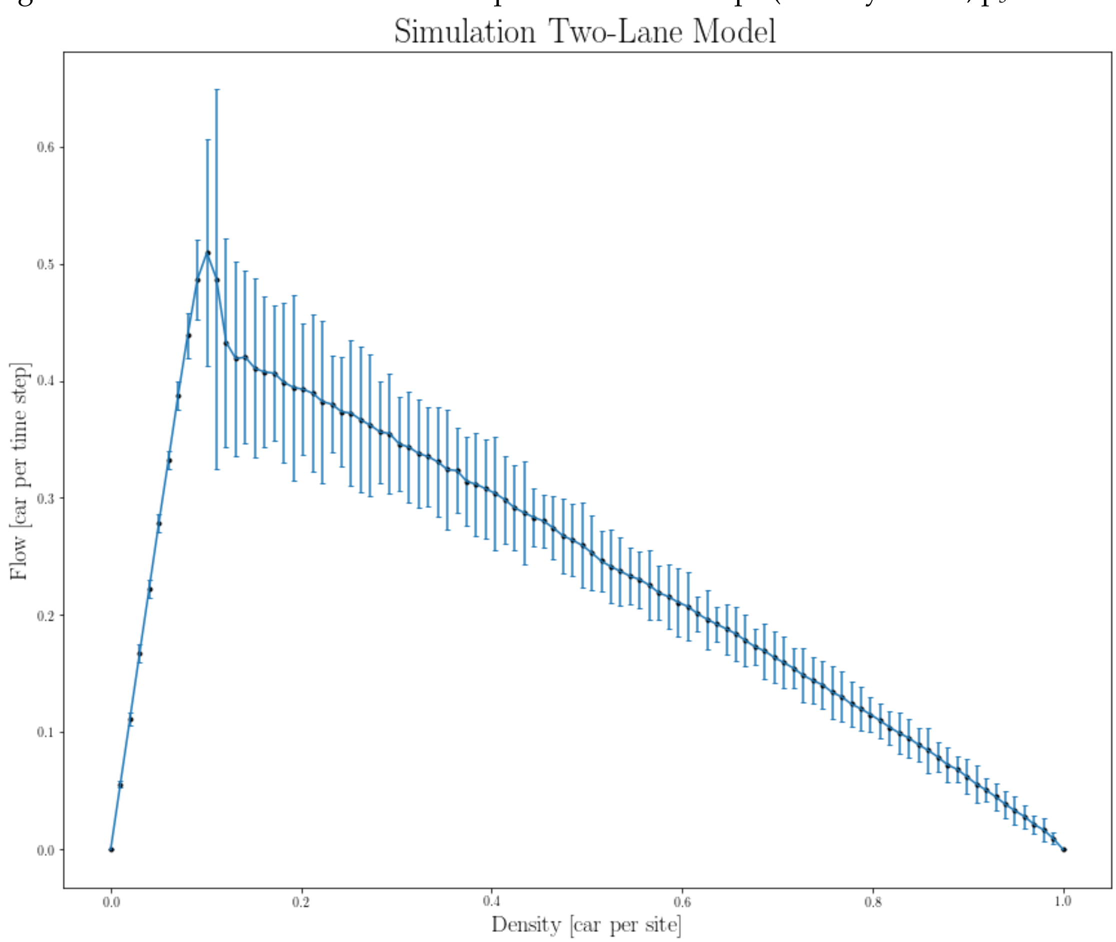


Figure 4. The average traffic flow for the Two-Lane model with densities ranging from 0 to 1 after 50 warm-up then 200 time steps and each density is averaged over 50 simulation.

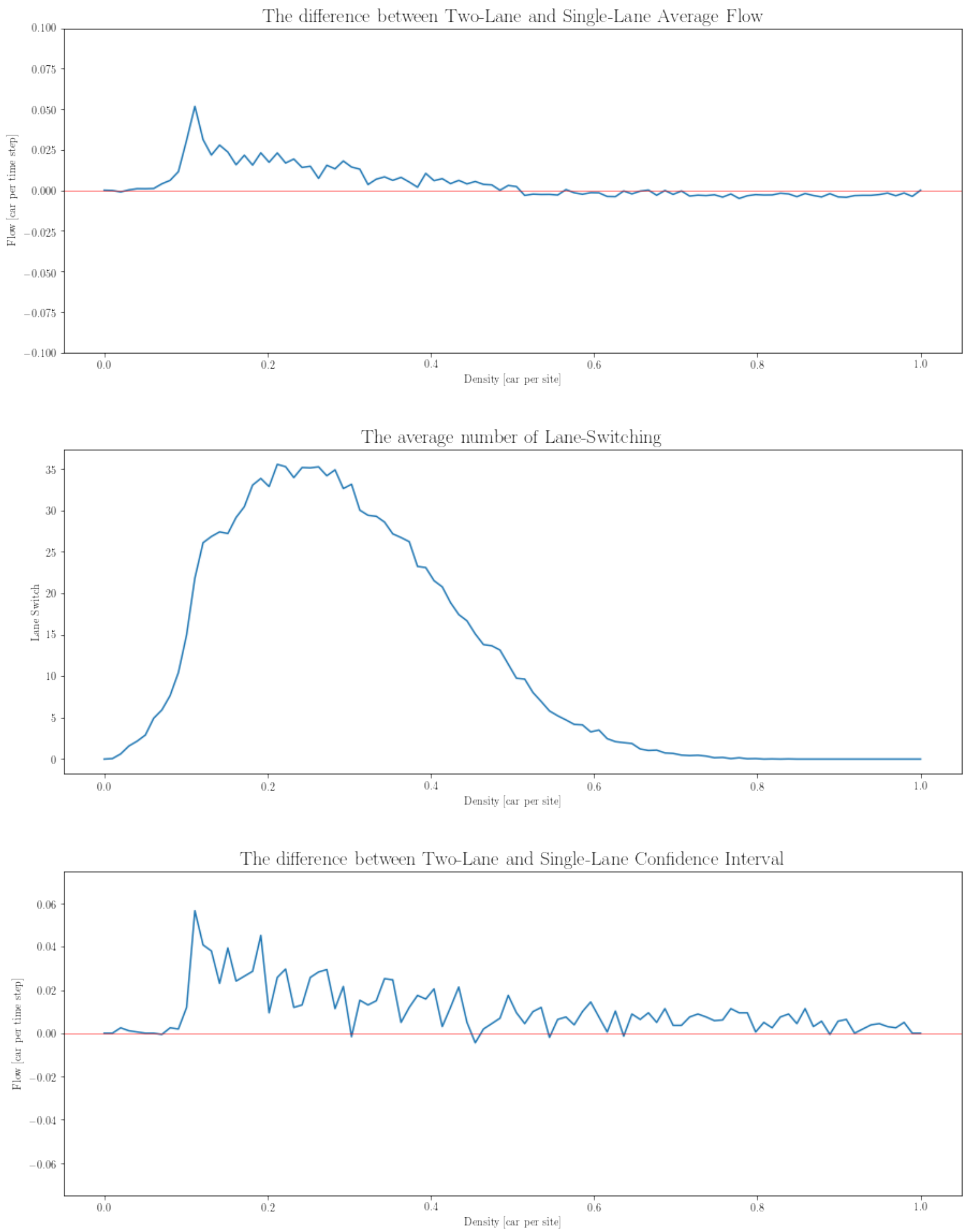


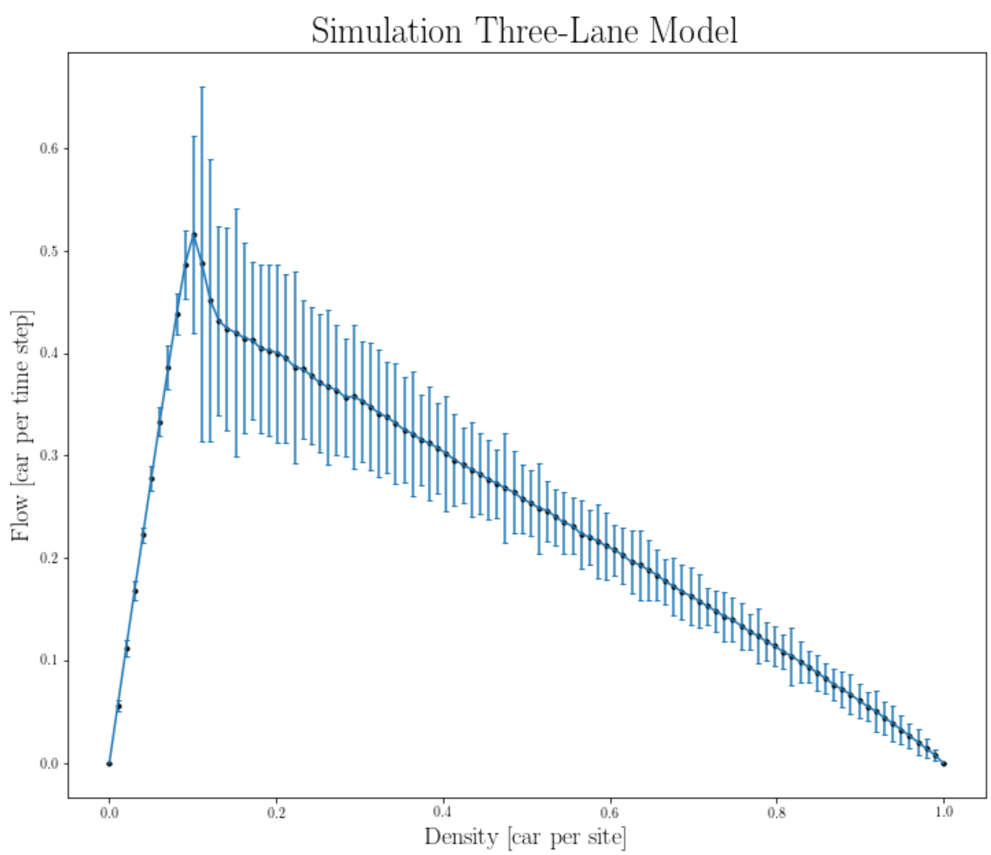
Figure 6. Visualizing differences: Figure I is illustrating the average flow difference between the Single-Lane and Two-Lane model. Figure II: Graph showing the average of lane switching occurrences at each density for the Two-Lane model. Figure III: illustrating the confidence interval difference between the Single-Lane and Two-Lane model

We notice that in low densities, both models have similar flow. Intuitively, the Two-Lane model performed fewer lane switching throughout low density simulations since there's small chance for congestion to form even with random decelerating. Similarly, in high densities, the conditions for lane switching are hardly met, hence, the Two-Lane model is just equivalent to a double separated Single-Lanes in the absence of switching. To confirm our intuition, we graph the average occurrences of lane-switching for each of the densities. We expect the graph to have a quick rise at density 9% then gradually deceasing until around 50% density (see Fig 6 Part II).

As discussed in the Single-Lane model, the confidence interval for the Two-Lane model follows the same behaviour as the Single-Lane model. However, after plotting the difference in the error bars between the two models, it yielded the graph in Fig 6 part III which shows that the Two-Lane model has higher variation. As an experiment, the simulation was ran without 0 probability of slow down and probability of lane switching of 1. The gap between the average flow for the two models was nearly zero. The finding supports the claim that around 9% to 50% densities, switching lanes mitigates the effect of congestion created by random slow down.

4.3 Three-lane simulation

The addition of a Three-Lane model was to to test the claim that an extra lane has a diminishing return and that the relation between the models is not linear (i.e., adding n lanes is not linearly related to the average flow).



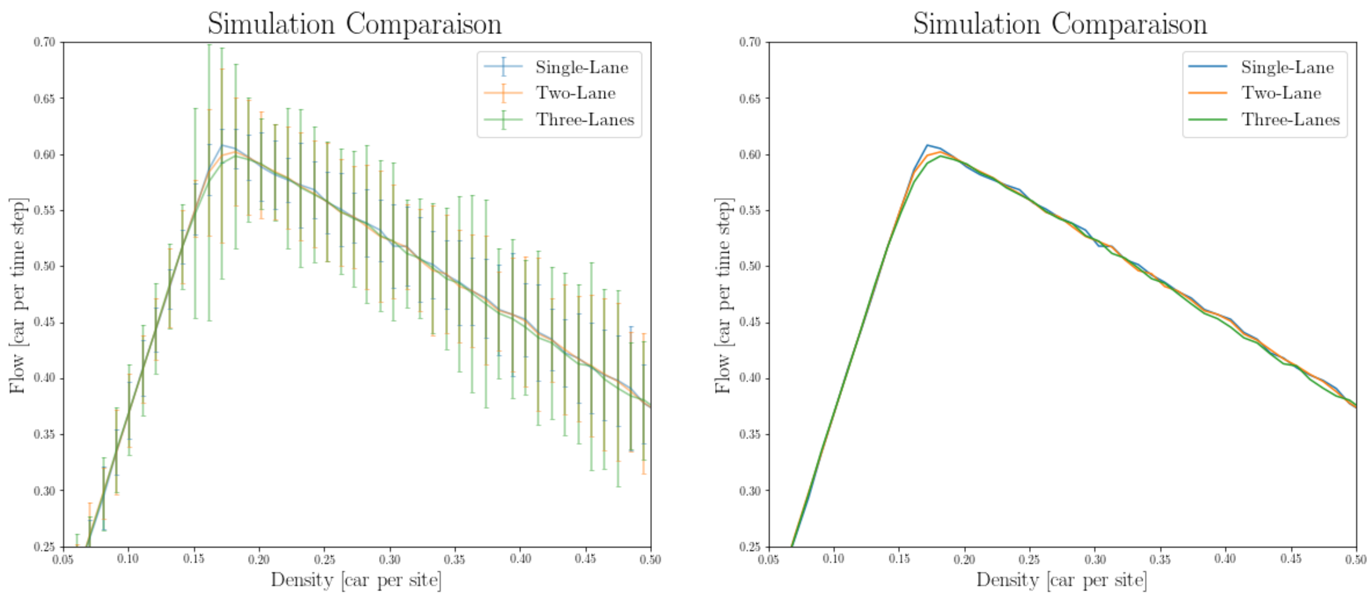


Figure 8. The average traffic flow for the all three models with densities ranging from 0 to 1 after 50 warm-up then 150 time steps and each density is averaged over 25 simulation.

Fig 8. is meant to test the claim that if the random slow down was set to be zero then -in the long run- adding multiple lanes is just equivalent to adding a separated single lane and the average flow for n lanes is just the same as n times the average flow of a single lane. In other words:

$$\overline{flow}\,(N \times lanes) = N \times \overline{flow}\,(Single)$$

Which is shown clearly after running the simulation for the three models.

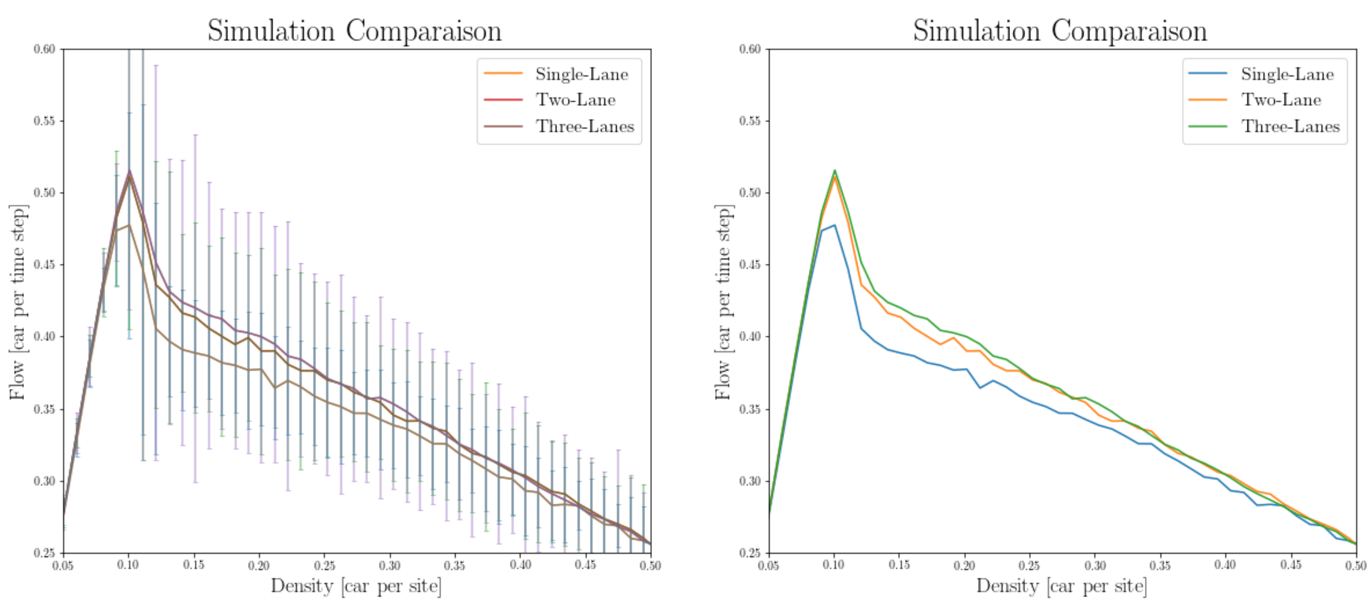


Figure 9. The average traffic flow for the all three models with densities ranging from 0 to 1 after 50 warm-up then 250 time steps + warm-up and each density is averaged over 50 simulation.

Fig 9. is meant to test the claim that adding an extra lane has a diminishing return (i.e., the improvement from Single to Two is much more bigger than the improvement from Two to Three). Numerically:

- At 0.1 density the max flow of One-Lane model 0.47740
- At 0.1 density the max flow of Two-Lane model 0.51085
- At 0.1 density the max flow of Three-Lane model 0.51556

5 Reflection

Controlling for density and given the assumptions stated, we found that the Two-lane model had a slight comparative advantage in flow for a specific range of car densities (between 9 and 40%). Quantitatively, both models reached a peak at 10% density but the Two-lane model had 0.04 higher flow (10% higher). However, that advantage starts vanishing as we increase the density (see Fig 6 part I).

In models with more than two lanes, the intuition from the findings suggest that adding more lanes would have diminishing returns (i.e. if the Two-lane model had at best 10% improvement, then I suspect that the Three-Lane model would perform a bit better than 10% but definitely less than 20%). That is to say, the relation between flow and number of lanes is not linear but more of a hyperbolic discounting shape.

As of 2017, the car fleet in Argentina amounted to nearly 10.7 million automobiles (Statista, 2017). Although the vast size of the country (2.7 sq km) covering roughly half of the South American continent. The highway network is not evenly distributed around the country for two reasons:

- The population is only 44 million (to give a perspective, Mexico has 20% less land but triple the population.
- The major Argentinian cities are concentrated in the northeast of the country, thus the necessity to build highways to the south is less urgent.

As a result, highways represent only 0.4% of the entire road network. The great majority of the network is composed of undivided or divided two-lane highways. Therefore, the simulation might not be widely applicable but still abides by the same rules as the highway structure (i.e., no intersection, no traffic lights).

6 Future work

Although the simulation modeled random slow down due to human error/miscalculation, other reasons for slowing down on a highway can have different effects (e.g., exits, merges, emergency stops due to accidents).

"Adding highway lanes to deal with traffic congestion is like loosening your belt to cure obesity." (Mumford. L., 1955).

This quote highlights the idea that managing demand is better than enhancing supply. In the context of the simulation results, it might be cost-effective to manage the car density in a highway segment to strike an optimal flow rather than adding a whole new lane which can be achievable by simple rules that individuals can follow (e.g., leaving enough distance with the car in front). Furthermore, the conditions for lane switching might differ depending on the drive (e.g., drivers can't accurately estimate the speed of the coming cars on the other lanes).
In a nutshell, the model -despite its simplicity since it's a coarse gaining of a complex phenomenon- still displays patterns that we see in real life (e.g., backward wave jam), hence, I suggest that we need to add more features based on their expected utility (i.e., we would rather have a simple model that display the major patterns rather than a complex model that over-fits the reality)

7 Appendix

1. HC/LO Application:

    - descriptive_stats (HC): throughout the report, the comparison between the Single-Lane and the Two-Lane model was based on the mean of multiple simulations per density. In addition, the confidence interval constructed for the flow v. density plots were interpreted to convey the degree of certainty about results.
    - interpret_results (LO): Each of the traffic models' results were interpreted based on the rules of the cellular automata and the intuition about how traffic behave in real life scenarios.
    - ca_analysis (LO): The traffic model was an example using discrete modeling method. Although it's a coarse-raining of real life traffic, we still see emergence of behaviors that we do witness in our roads.

8 References

- Nagel, K., Schreckenberg, M. (1992). A cellular automaton model for freeway traffic. Journal de Physique I, 2(12), 2221–2229. Retrieved from: https://course-resources.minerva.kgi.edu/uploaded_files/mke/YpqvNV/nagel-schreckenberg.pdf
- Rickert, M., et al. (1996). Two lane traffic simulations using cellular automata. Physica A: Statistical Mechanics and its Applications, 231(4), 534–550. Retrieved from https://course-resources.minerva.kgi.edu/uploaded_files/mke/00100888-7879/rickert-et-al.pdf