PythonNotebook7_solution_2023

January 15, 2024

[1]:
```
# import modules
import numpy as np
import matplotlib.pyplot as plt
```

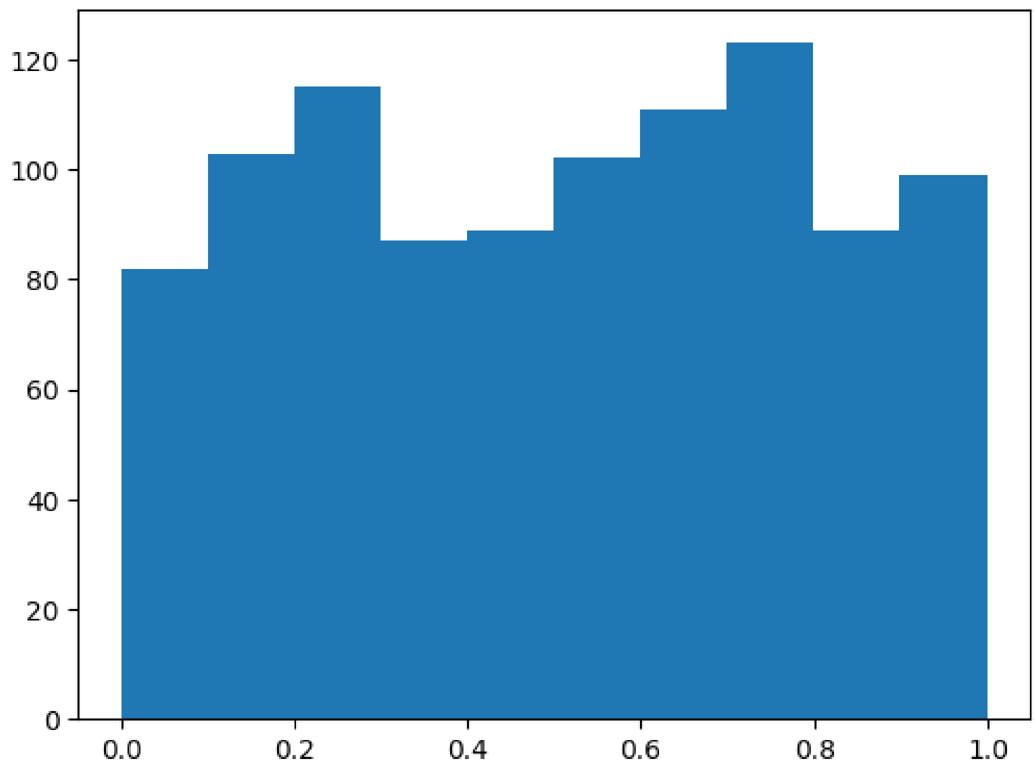
0.1 Exercise 7.1.1 Make a histogram of the uniform distribution

- create an array X of 1000 random numbers
- use `plt.hist(X)` to plot a histogram

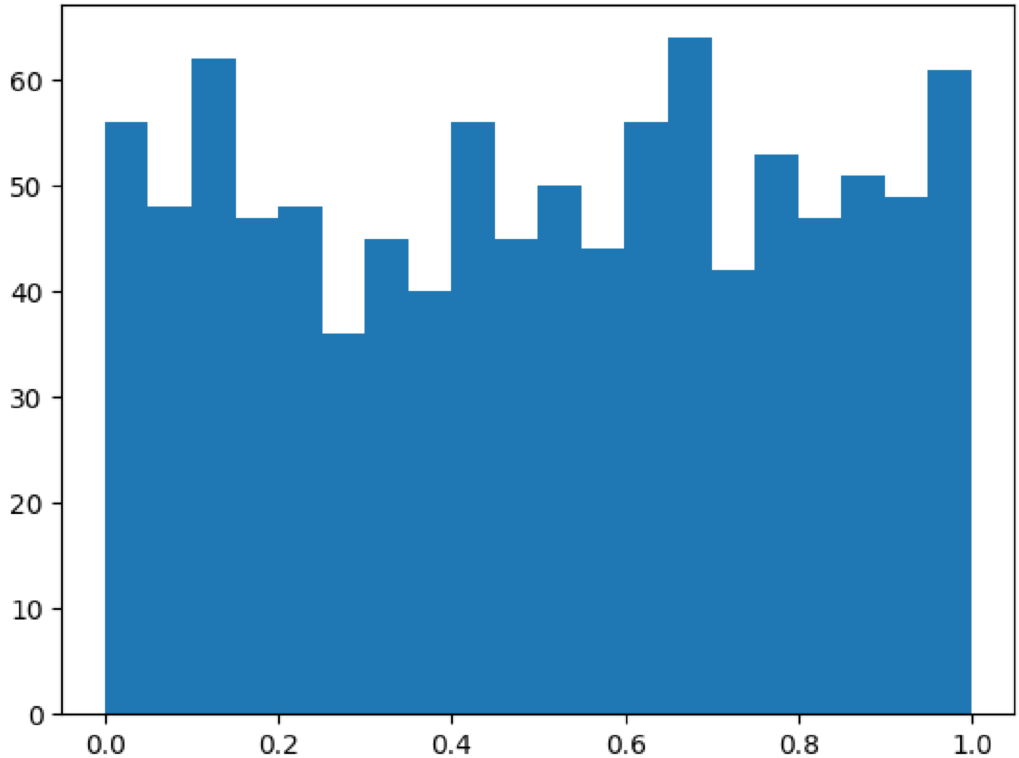
See the hist documentation

- make another histogram with 20 bins, using the `bins` argument.

[2]:
```
# solution
rng = np.random.default_rng()

x = rng.uniform(size=1000)
plt.hist(x)
plt.show()
```



[3]:
```
# solution
x = rng.uniform(size=1000)
plt.hist(x, bins=20)
plt.show()
```

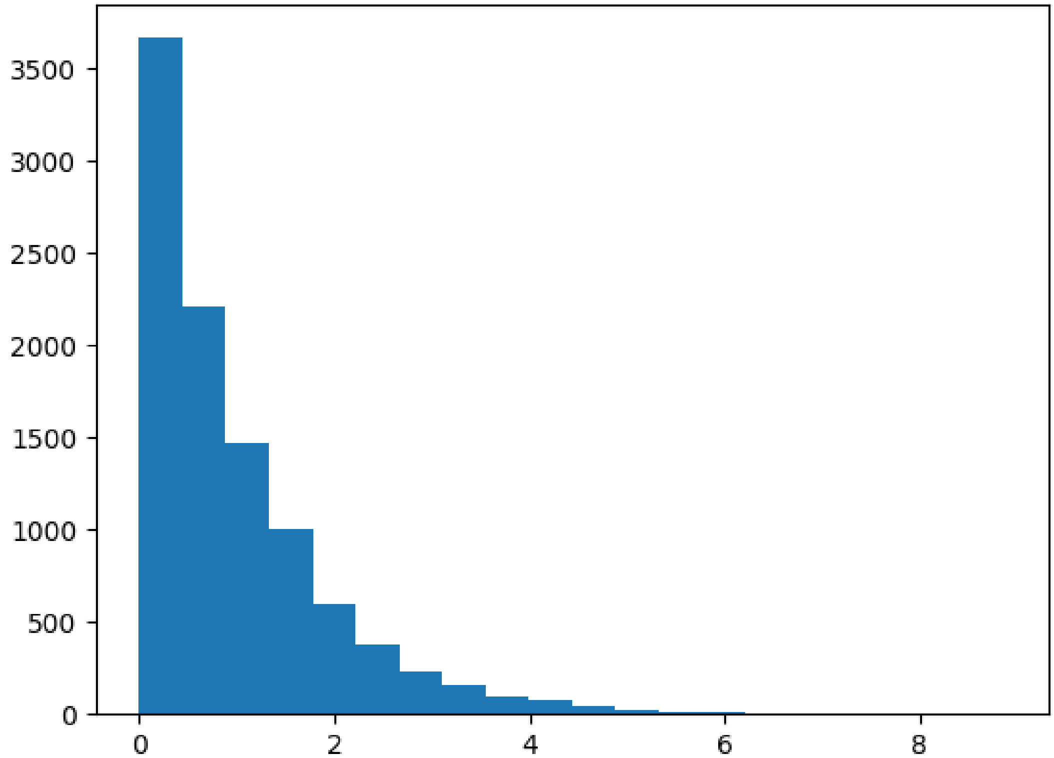


0.2 Exercise 7.1.2 Exponential and normal distributions

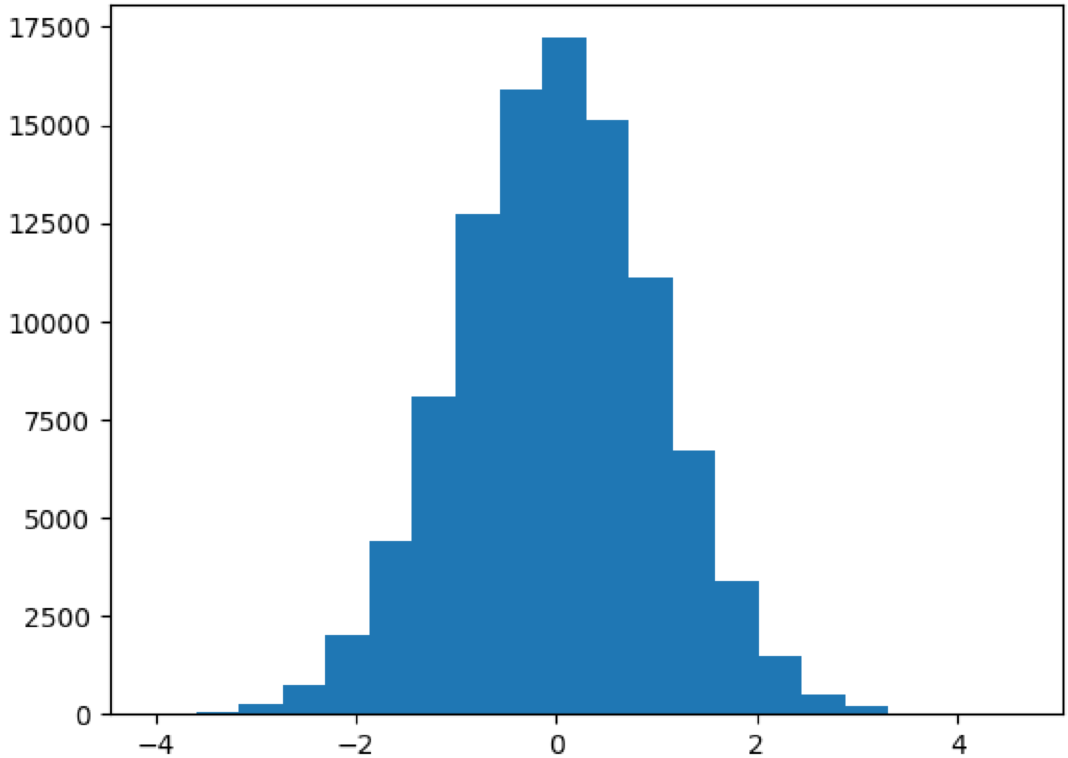
Now you can try other distributions. * Create similar histograms for the *normal* and *exponential* distribution.

Choose the number of random values and the number of bins for nice plots.

[4]:
```
# solution
x = rng.exponential(size=10000)
plt.hist(x, bins=20)
plt.show()
```



[5]:
```
# solution
x = rng.normal(size=100000)
plt.hist(x, bins=20)
plt.show()
```



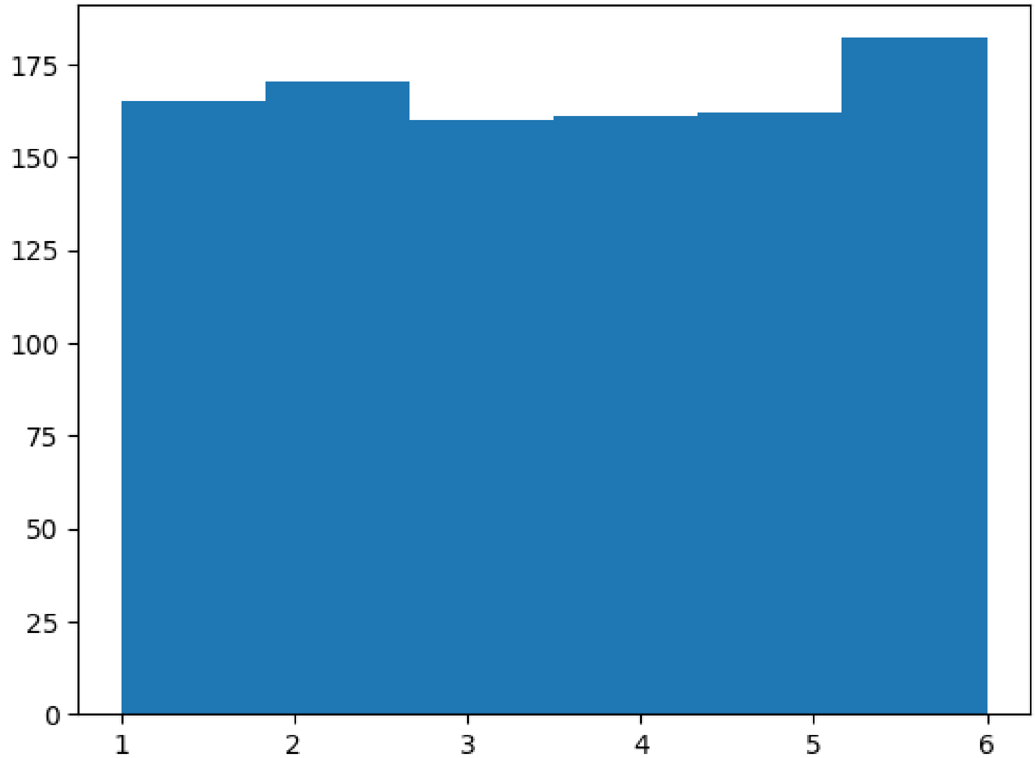
0.3 Exercise 7.1.3 Rolling multiple dice

With `rng.integers(low, high, size)` (documentation) you can generate integers, to simulate rolling dice.

- Use this function to make a histogram of the outcome of rolling a normal six-sided die. Hint: as with Python ranges, the low argument is inclusive, the high argument is exclusive.
- Now do the same for the sum of M dice, for M = 2, 3, … Hint: for N rolls of M dice, an easy way is to generate an N by M array of die results, and then summing this array over the M-direction. You need the `np.sum` function (documentation) and the axis ardument to select which direction to sum over. Hint 2: for the plot to look nice, you need to set the bins. The most robust is to ask for every integer to be its own bin, using bins=range(1,6*M)

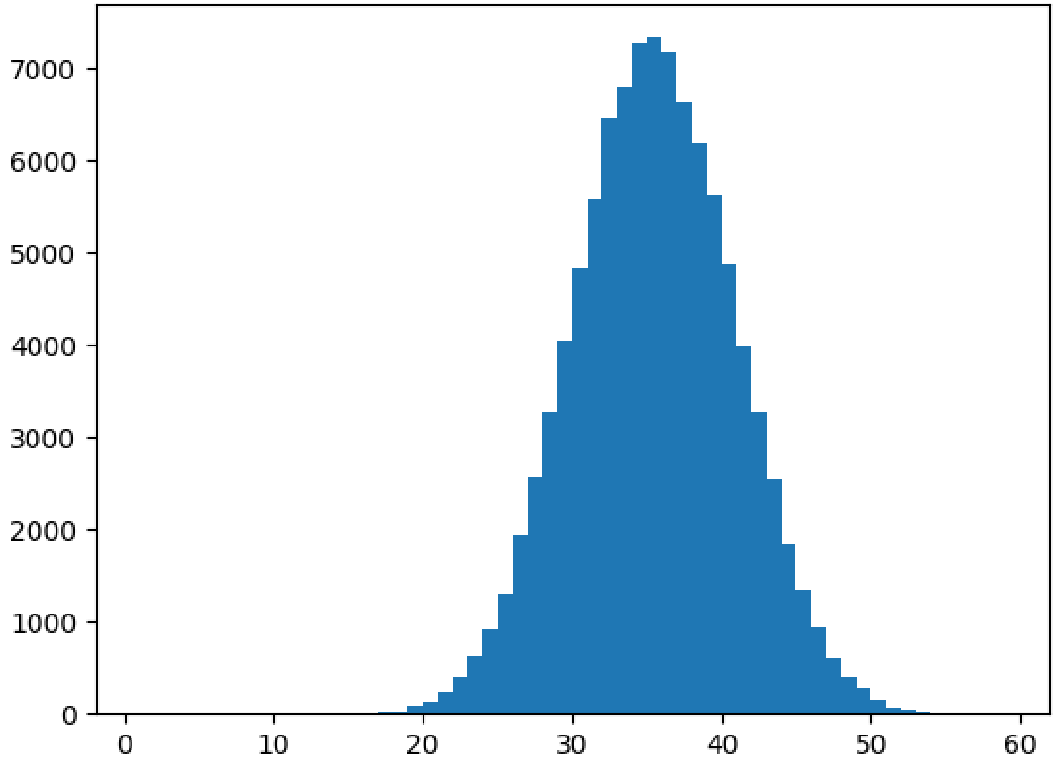
Notice that if you make M large, the distribution starts looking like the normal distribution you plotted above. This is known as the *Central Limit Theorem*.

[6]:
```
# solution
d = rng.integers(low=1, high=7, size=1000)
plt.hist(d, bins=6)
plt.show()
```



[7]:

```
# solution
M=10
d = rng.integers(low=1, high=7, size=(100000,M))
s = np.sum(d,axis=1)
plt.hist(s, bins=range(1,6*M))
plt.show()
```



[8]:

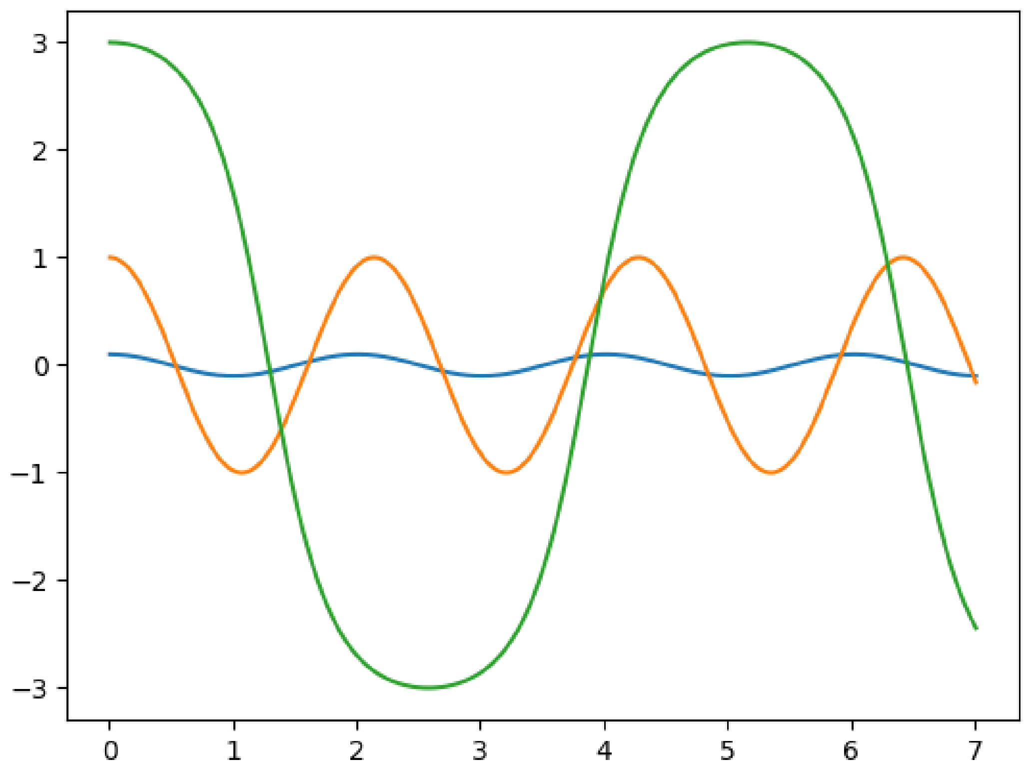
```python
# pendulum simulator - you don't need to change this

def pendulum(a, v=0, l=1, g=9.81, dt=0.01, t_end=7):
    """
    a:      initial angle (radians)
    v:      initial angular velocity (radians/s)
    l:      length of the pendulum (m)
    g:      gravitational acceleration (m/s**2)
    dt:     length of time step (s)
    t_end: end time of simulation (s)

    returns 3 arrays:
    T: times
    A: angle as function of time
    V: angular velocity as function of time
    """
    T = []
    A = []
    V = []
    t = 0
    while t < t_end:
        a += v * dt                # update angle with angular velocity
```

```
        v += -g/l * np.sin(a) * dt # update angular velocity with the force
        t += dt                    # update time

        T.append(t)                # collect output quantities
        A.append(a)
        V.append(v)
    return T, A, V
```

0.4 Exercise 7.2.1 Simulating a pendulum

- use the pendulum function above to simulate a pendulum with different initial angles. Plot the result. The angle is measured in radians, so half a turn is π radians. You should see that for small initial angles the result is sine-shaped, a well-known result in physics because the equations can be simplified when the angle is small. What happens for larger angles?

[9]:
```
# solution
T1,A1,V1 = pendulum(a=.1)
T2,A2,V2 = pendulum(a=1)
T3,A3,V3 = pendulum(a=3)

plt.plot(T1,A1)
plt.plot(T2,A2)
plt.plot(T3,A3)
plt.show()
```



0.5 Exercise 7.3.1 An iterative map

Doctor M. studies the following sequence:

$$z_0 = 0$$

$$z_n = z_{n-1}^2 + c$$

where z and c are complex numbers.

For some values of c, the sequence remains bounded, while for others the magnitude of z grows larger and larger. Write a function to test a value of c and determine the behavior of the sequence.

Input: Your function should take two parameters: c and a maximum number of iterations. If the value of $|z$ is ever larger than 2, it is known that $|z|$ will grow large, and you can stop iterating. **Output**: The function should return the number of iterations needed until $|z| > 2$, or 0 if $|z|$ remains bounded.

Hint 1: c will be a complex number, but since this is Python you don't need to worry much, normal math will just work. But remember `abs()` for $|z|$.

Hint 2: you don't need to save all the z_n. You can use a single z, and update it each iteration.

[10]:
```
# solution
def M(c, Nmax):
    z = 0
    n = 0
    while abs(z) < 2 and n < Nmax:
        z = z**2 + c
        n += 1

    if n == Nmax:
        return 0
    return n
```

0.6 Exercise 7.3.2 Plot M

Use Nmax = 100, and make a plot of M in the complex plane.

[11]:
```
# solution
plt.figure(figsize=(8,8))
X = np.linspace(-2.5,1.5, 100)
Y = np.linspace(-2,2, 100)
Nmax = 100
N = np.zeros((len(Y), len(X)))
for j in range(len(Y)):
    for i in range(len(X)):
        c = X[i] + 1j * Y[j]
        N[j][i] = M(c, Nmax)
```

```
plt.imshow(N)
```

[11]: <matplotlib.image.AxesImage at 0x245f48a6f40>