Word2vec Parameter Learning Explained

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Skip-Gram Model: $\vec{h} = \vec{x}^T \hat{W} = \hat{W}^T_{(k,\cdot)} := \vec{v}^T_{w_I}$ (24) $\quad p(w_{c,j} = w_{O,c} \mid w_I) = y_{c,j} = \dfrac{\exp(u_{c,j})}{\sum_{j'=1}^{V} \exp(u_{j'})}$ (25)

$u_{c,j} = \vec{v}'^T_{w_j} \cdot \vec{h} = u_j$ for $c = 1, \dots, C$ (26)

$$E = -\log p(w_{O,1}, \dots, w_{O,C} \mid w_I) = -\log \prod_{c=1}^{C} \frac{\exp(u_{c,j^*_c})}{\sum_{j'=1}^{V} \exp(u_{j'})} =$$

loss function
j^*_c ... index of the actual c-th output context word in the vocabulary

$$= -\log \frac{1}{C \cdot \sum_{j'=1}^{V} \exp(u_{j'})} - \log \prod_{c=1}^{C} \exp(u_{c,j^*}) = +C \log \sum_{j'=1}^{V} \exp(u_{j'}) - \underbrace{\log \prod_{c=1}^{C} \exp(u_{c,j^*_c})}_{= \sum \log \exp(\cdot)} =$$

from C-times $-\log \frac{\exp(\cdot)}{\Sigma \cdot}$

$$= C \cdot \log \sum_{j'=1}^{V} \exp(u_{j'}) - \sum_{c=1}^{C} u_{c,j^*_c} = -\sum_{c=1}^{C} u_{c,j^*_c} + C \log \sum_{j'=1}^{V} \exp(u_{j'}) \quad (29)$$

$$\frac{\partial E}{\partial u_{c,j}} = y_{c,j} - t_{c,j} := e_{c,j} \quad (30) \qquad EI_j = \sum_{c=1}^{C} e_{c,j} \quad (31)$$

$$\frac{\partial E}{\partial w'_{ij}} = \sum_{c=1}^{C} \frac{\partial E}{\partial u_{c,j}} \cdot \underbrace{\frac{\partial u_{c,j}}{\partial w'_{ij}}}_{= \frac{\partial}{\partial w'_{ij}} w'_{ij} \cdot h_i = h_i} = EI_j \cdot h_i \quad (32)$$

$$\frac{\partial E}{\partial u_{c,j}} = \frac{\partial}{\partial u_{c,j}} \left(-\sum_{c=1}^{C} u_{c,j^*_c} + C \log \sum_{j'=1}^{V} \exp(u_{j'}) \right) =$$

$$= \underbrace{-\sum_{c=1}^{C} \frac{\partial u_{c,j^*_c}}{\partial u_{c,j}}}_{} + C \frac{\partial}{\partial u_{c,j}} \log \sum_{j'=1}^{V} \exp(u_{j'}) =$$

$$= 1 \text{ if } c=j,\ 0 \text{ otherwise: } = -t_{c,j} \text{ (truth)} + \frac{C}{\sum_{j'=1}^{V} \exp(u_{j'})} \cdot \frac{\partial \sum_{j'=1}^{V} \exp(u_{j'})}{\partial u_{c,j}} =$$

$$= -t_{c,j} + \frac{C}{\sum_{j'=1}^{V} \cdot} \cdot \underbrace{\sum_{j'=1}^{V} \frac{\partial \exp(u_{j'})}{\partial u_{c,j}}}_{= j'=j:\ \exp(u_{c,j})} = -t_{c,j} + \underbrace{\frac{C \cdot \exp(u_{c,j})}{\sum_{j'=1}^{V} \exp(u_{j'})}}_{y_{c,j} \cdot \frac{1}{C}}$$

? (just for one context since $\frac{\partial}{\partial u_{c,j}}$) !

$$w'^{(new)}_{ij} = w'^{(old)}_{ij} - \eta \cdot EI_j \cdot h_i \quad (33) \qquad \text{"}\Delta w'_{ij}\text{"} = -\eta\, EI_j h_i = -\eta \frac{\partial E}{\partial w'_{ij}}$$

$$\vec{v}'^{(new)}_{w_j} = \vec{v}'^{(old)}_{w_j} - \eta \cdot EI_j \cdot \vec{h} \quad (34)$$

$\vec{v}^{(new)}_{w_I} = \vec{v}^{(old)}_{w_I} - \eta \cdot \vec{EH}^T$: update [redacted] input→hidden (35) $\quad EH_i = \sum_{j=1}^{V} EI_j \cdot w'_{ij}$ (36)

$$\frac{\partial E}{\partial h_i} = \sum_{j=1}^{V} \frac{\partial E}{\partial u_j} \frac{\partial u_j}{\partial h_i} = \sum_{j=1}^{V} e_j \cdot w'_{ij} = EH_i \quad (12)$$

$e_j \approx e_{c,j}$

$$\frac{\partial E}{\partial u_{c,j}} = e_{c,j}$$

$$\frac{\partial u_{c,j}}{\partial h_i} = \frac{\partial}{\partial h_i} (\vec{v}'_{w_j} \cdot h_i) = \frac{\partial}{\partial h_i} (w'_{ij} \cdot h_i) = w'_{ij}$$