Depth-first search
COMS20017 (Algorithms and Data)

John Lapinskas, University of Bristol

Path-finding

One of the most basic problems in graph theory: Given a graph G and two vertices $x, y \in V(G)$, is there a path from x to y?

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E.g. can an enemy attack the base without breaking down a wall?

Often we want to know the **shortest** path from x to y — see next video!

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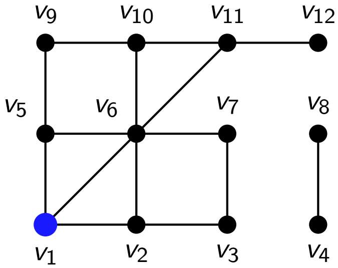

Input: v_1

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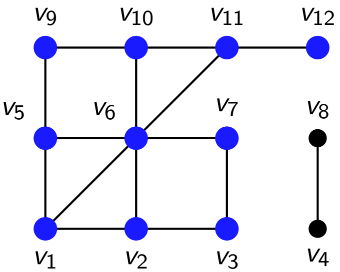

Input: v_1

Output: $[v_1, v_2, v_3, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}]$

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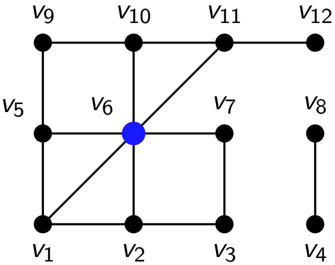

Input: v_6

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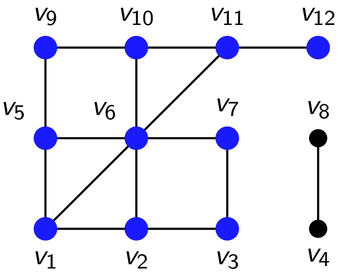

Input: v_6

Output: $[v_1, v_2, v_3, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}]$

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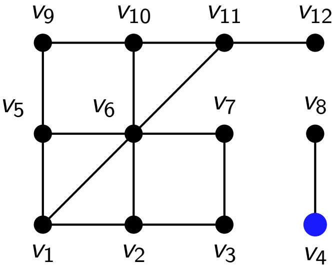

Input: v_4

Component-finding

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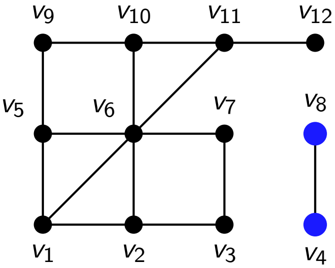

Input: v_4
Output: $[v_4, v_8]$

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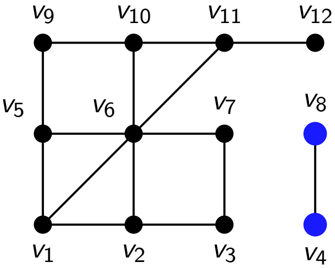

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In other words, we check whether there is a path from x to y for **all** y.
Turns out the worst-case running time is the same either way!

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Input: A graph G and a vertex $x \in V(G)$.
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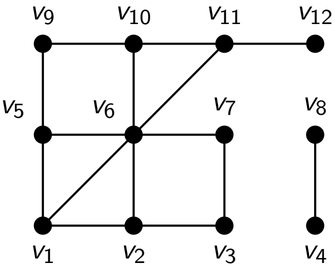
Idea: Think of the graph as like a **maze**: explore greedily until everything looks familiar, then backtrack.



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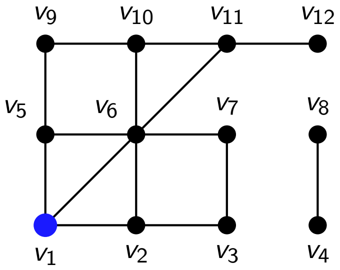

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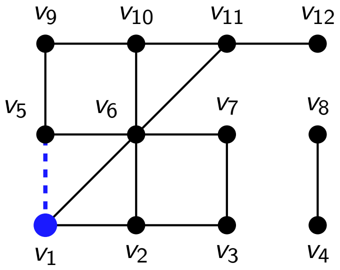

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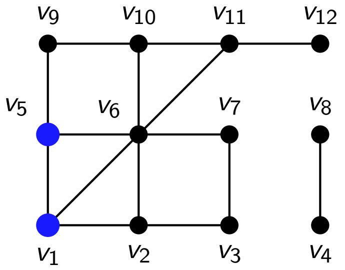

Input: G, v_1
Output: $[v_1, v_5$

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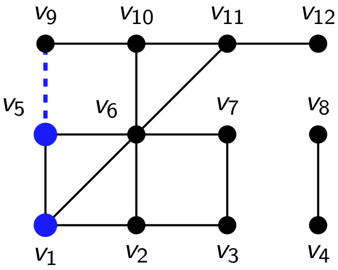

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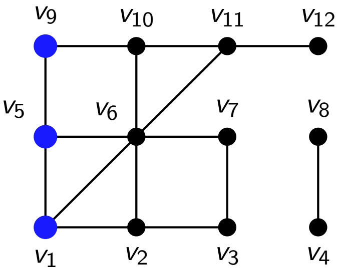

Input: G, v_1
Output: $[v_1, v_5, v_9$

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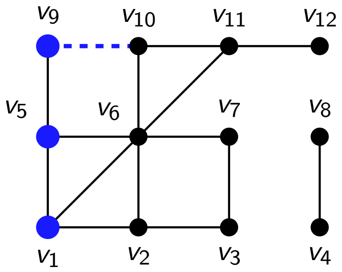

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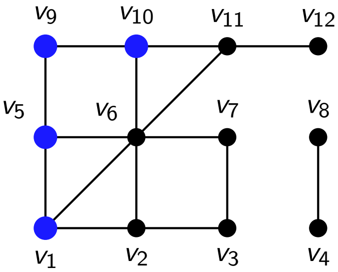

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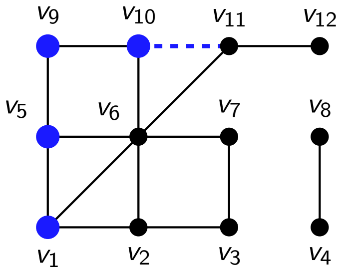

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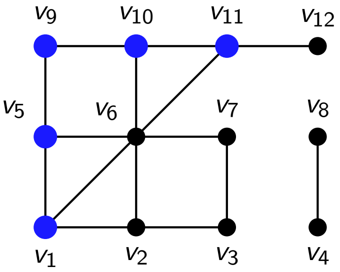

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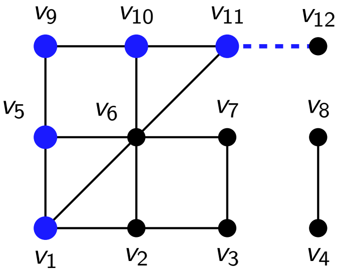

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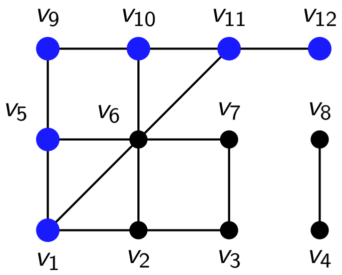

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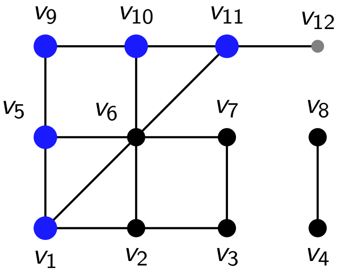

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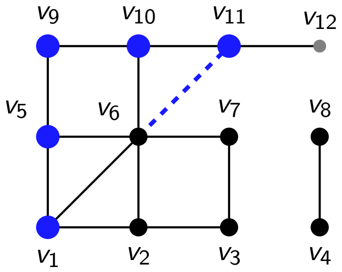

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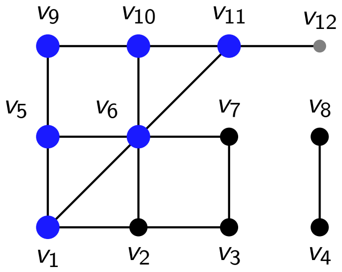

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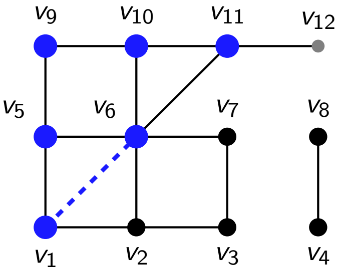

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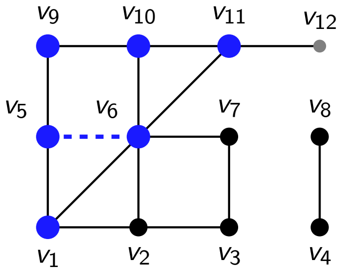

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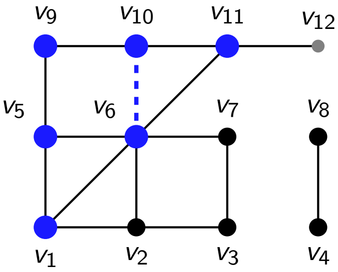

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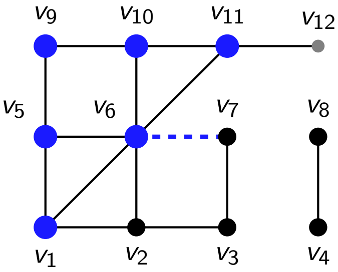

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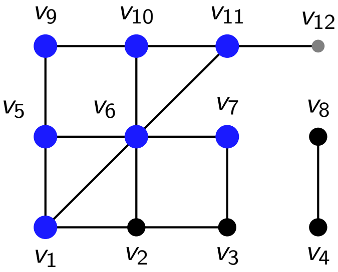

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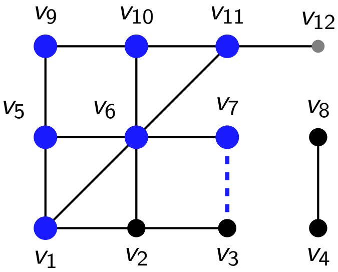

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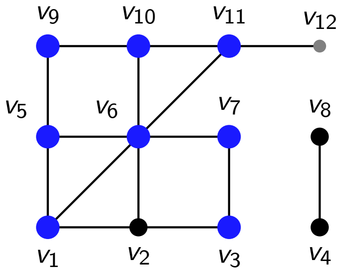

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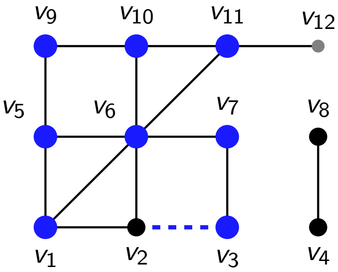

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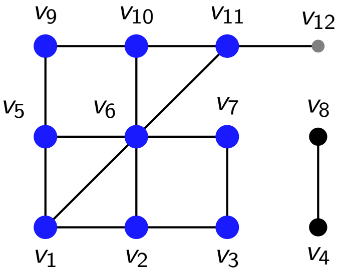

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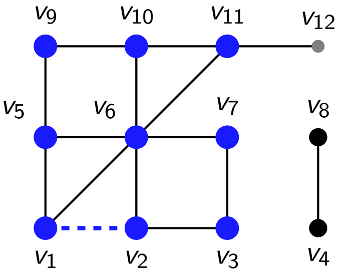

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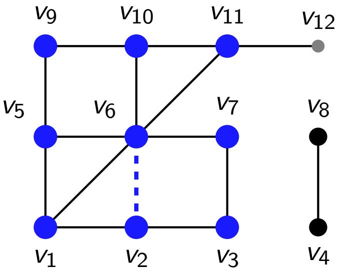

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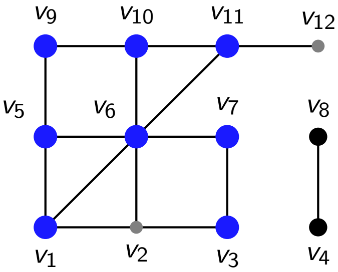

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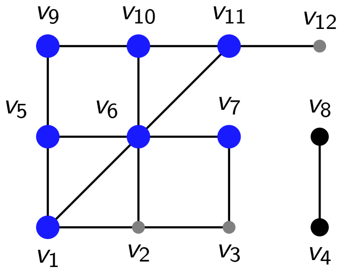

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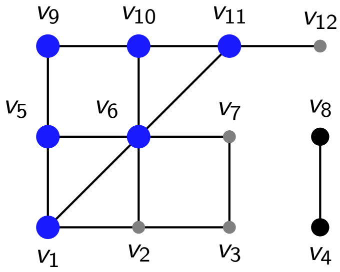

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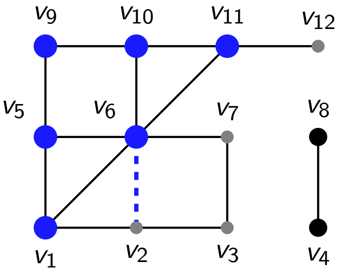

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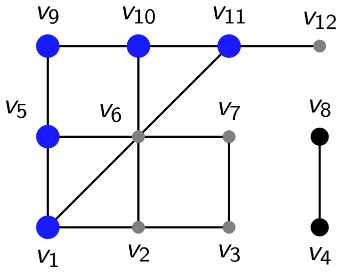

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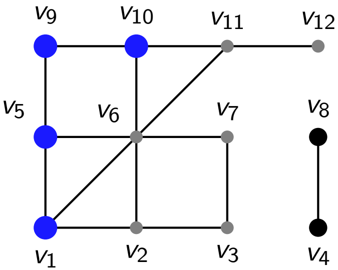
Output: A list of all vertices in the component of G containing x.

Idea: Think of the graph as like a **maze**: explore greedily until everything looks familiar, then backtrack.



Input: G, v_1

Output: $[v_1, v_5, v_9, v_{10}, v_{11}, v_{12}, v_6, v_7, v_3, v_2$

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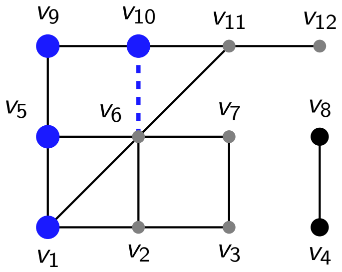

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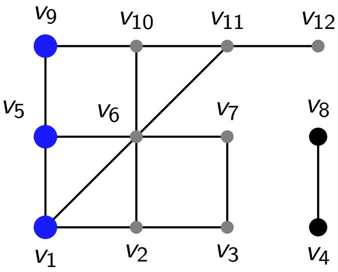

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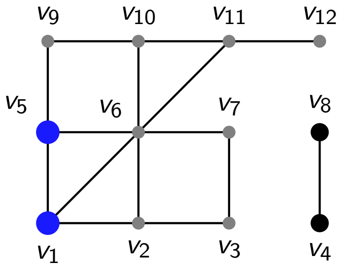

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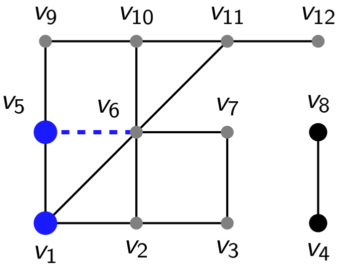

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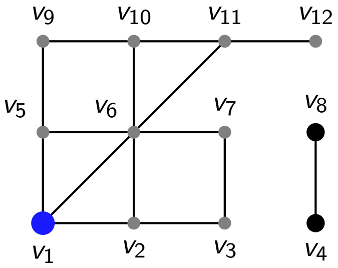

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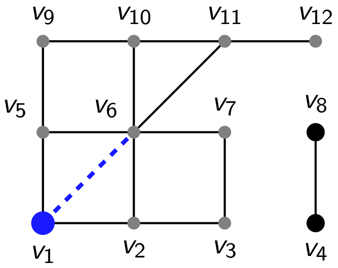

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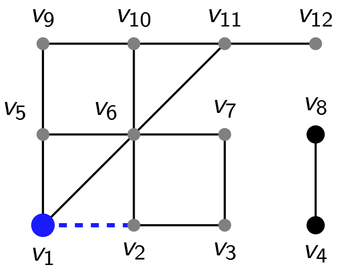

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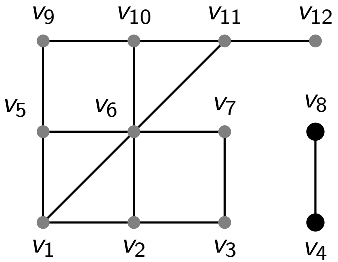

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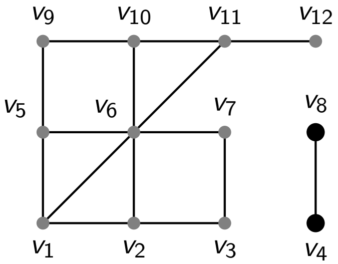

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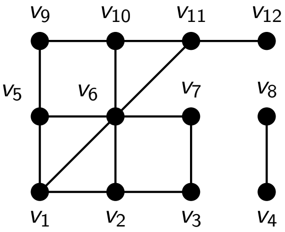
The slick way to implement this is to use recursion.

Pseudocode and example



Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
Output : List of vertices in v's component.

Number the vertices of G as $v_1, \dots, v_n$.
Let explored[i] $\leftarrow 0$ for all $i \in [n]$.
Procedure `helper(`v_i`)`
    if explored[i] $= 0$ **then**
        Set explored[i] $\leftarrow 1$.
        for v_j *adjacent to* v_i **do**
            if explored[j] $= 0$ **then**
                Call `helper(`v_j`)`.
Call `helper(`v`)`.
Return [v_i : `explored`[i] $= 1$] (in some order).

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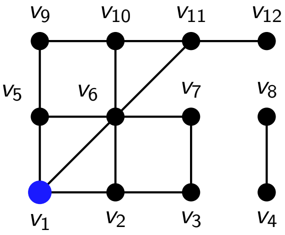

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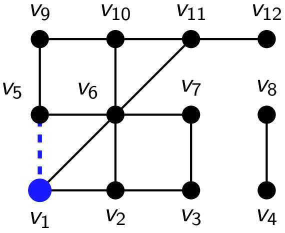

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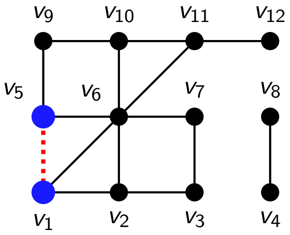

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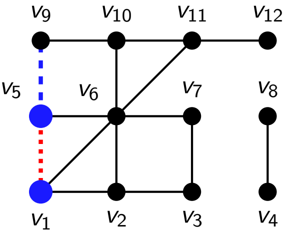

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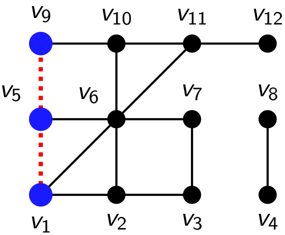

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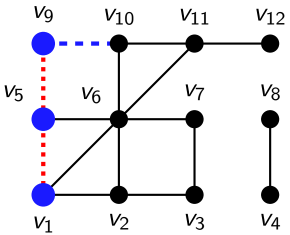

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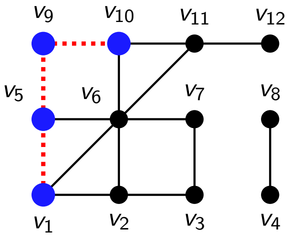

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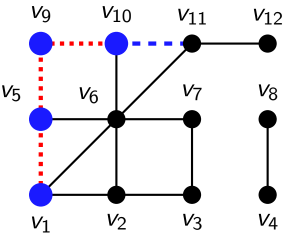

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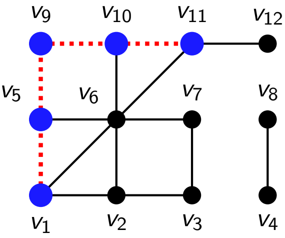

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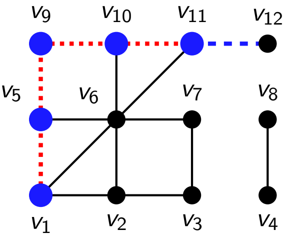

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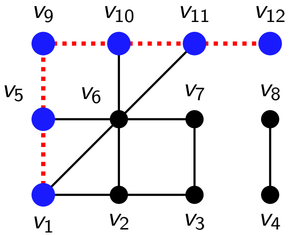

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Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
Output : List of vertices in v's component.

```
Number the vertices of G as v_1, ..., v_n.
Let explored[i] ← 0 for all i ∈ [n].
Procedure helper(v_i)
    if explored[i] = 0 then
        Set explored[i] ← 1.
        for v_j adjacent to v_i do
            if explored[j] = 0 then
                Call helper(v_j).
Call helper(v).
Return [v_i : explored[i] = 1] (in some order).
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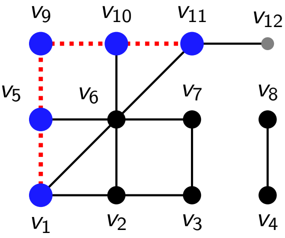
Pseudocode and example



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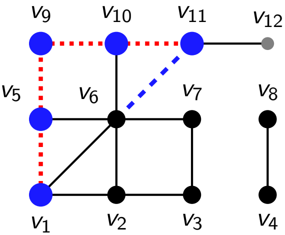

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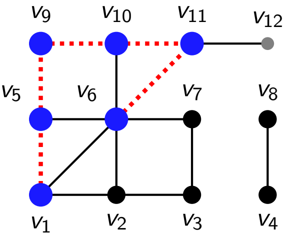

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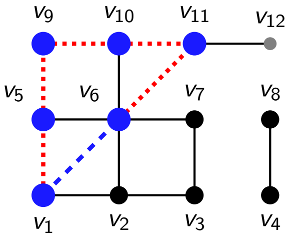

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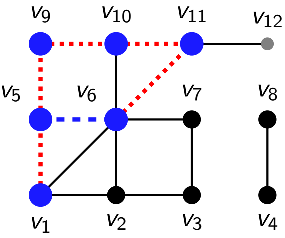

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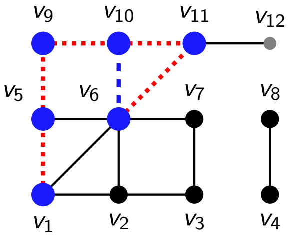

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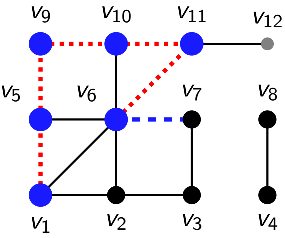

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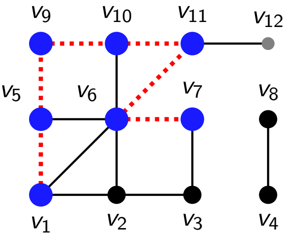
      if explored$[j] = 0$ **then**
        Call `helper`(v_j).
Call `helper`(v).
Return $[v_i : \texttt{explored}[i] = 1]$ (in some order).

Pseudocode and example



Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
Output : List of vertices in v's component.

Number the vertices of G as $v_1, \dots, v_n$.
Let explored[i] $\leftarrow 0$ for all $i \in [n]$.
Procedure `helper`(v_i)
    if explored[i] $= 0$ **then**
        Set explored[i] $\leftarrow 1$.
        for v_j *adjacent to* v_i **do**
            if explored[j] $= 0$ **then**
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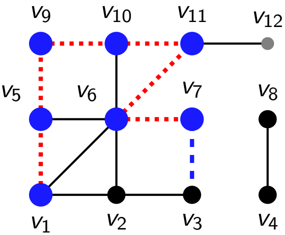
Return [v_i : `explored`[i] $= 1$] (in some order).



Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
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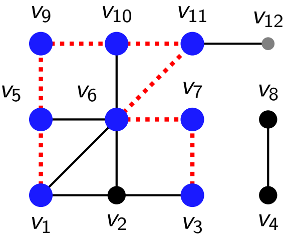
Pseudocode and example



Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
Output : List of vertices in v's component.

```
Number the vertices of G as v_1, ..., v_n.
Let explored[i] ← 0 for all i ∈ [n].
Procedure helper(v_i)
    if explored[i] = 0 then
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        for v_j adjacent to v_i do
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Call helper(v).
Return [v_i : explored[i] = 1] (in some order).
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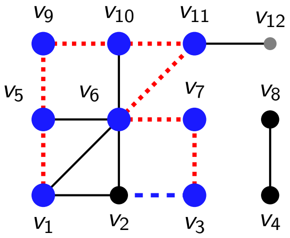
Pseudocode and example



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Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
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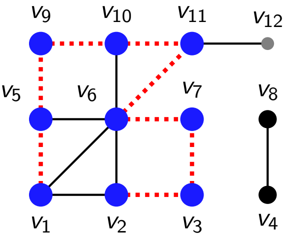

Input: v_1

Algorithm: DFS

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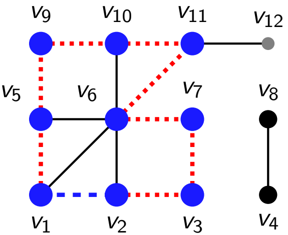

Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
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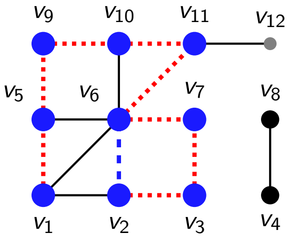

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Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
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Return [v_i : explored[i] = 1] (in some order).
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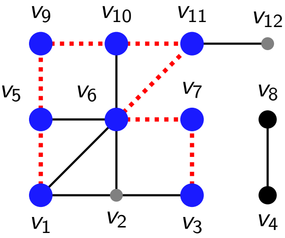
Pseudocode and example



Input: v_1

Algorithm: DFS

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Output : List of vertices in v's component.

Number the vertices of G as $v_1, \dots, v_n$.
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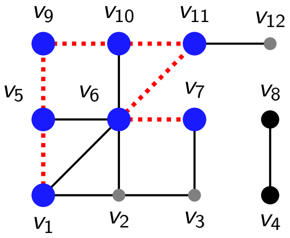
Pseudocode and example



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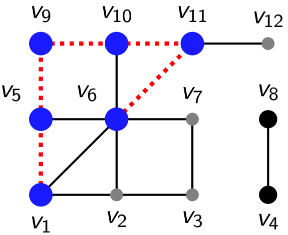

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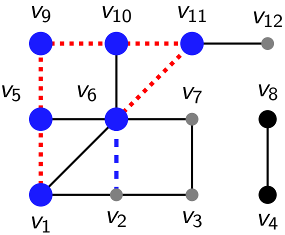
Pseudocode and example



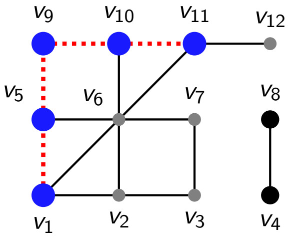
Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
Output : List of vertices in v's component.

Number the vertices of G as $v_1, \dots, v_n$.
Let explored[i] $\leftarrow 0$ for all $i \in [n]$.
Procedure `helper(`v_i`)`
    if explored[i] $= 0$ **then**
        Set explored[i] $\leftarrow 1$.
        for v_j *adjacent to* v_i **do**
            if explored[j] $= 0$ **then**
                Call `helper(`v_j`)`.

Call `helper(`v`)`.
Return [v_i : `explored`[i] $= 1$] (in some order).



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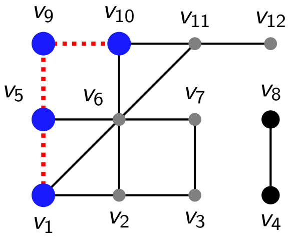

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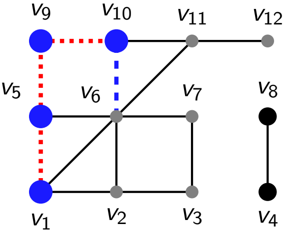

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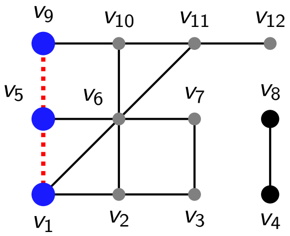
Pseudocode and example



Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
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Number the vertices of G as v_1, ..., v_n.
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    if explored[i] = 0 then
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        for v_j adjacent to v_i do
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Call helper(v).
Return [v_i : explored[i] = 1] (in some order).
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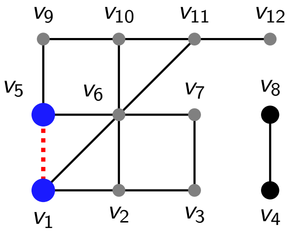
Pseudocode and example



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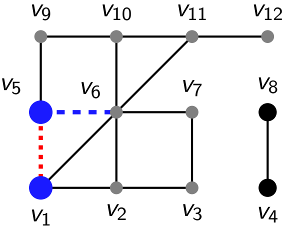
Pseudocode and example



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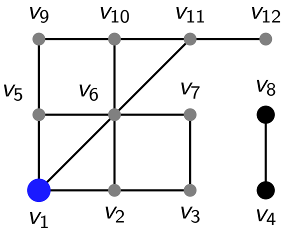

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Procedure `helper`(v_i)
    if explored$[i] = 0$ **then**
        Set explored$[i] \leftarrow 1$.
        for *v_j adjacent to v_i* **do**
            if explored$[j] = 0$ **then**
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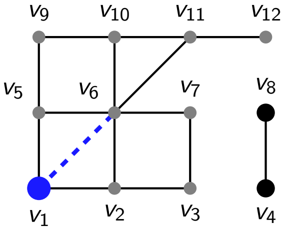
Call `helper`(v).
Return $[v_i :$ `explored`$[i] = 1]$ (in some order).

Pseudocode and example



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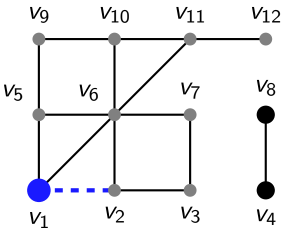

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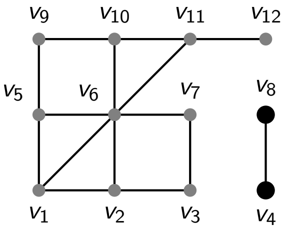


Input: v_1

Algorithm: DFS

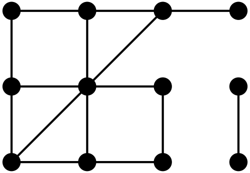
Input : Graph $G = (V, E)$, vertex $v \in V$.
Output : List of vertices in v's component.

```
1  Number the vertices of G as v_1, ..., v_n.
2  Let explored[i] ← 0 for all i ∈ [n].
3  Procedure helper(v_i)
4      if explored[i] = 0 then
5          Set explored[i] ← 1.
6          for v_j adjacent to v_i do
7              if explored[j] = 0 then
8                  Call helper(v_j).
9  Call helper(v).
10 Return [v_i : explored[i] = 1] (in some order).
```

We assume G is in adjacency list form.

Time analysis: In total there are $\sum_{v \in V} d(v) = O(|E|)$ calls to helper (each vertex only runs lines 5–7 once), and there is $O(1)$ time between calls. So the running time is $O(|V| + |E|)$.

Correctness I: Output is contained in v's component C



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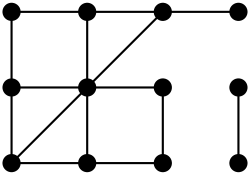
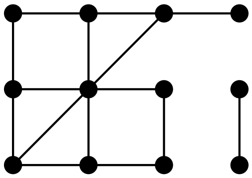


Invariant: "When `helper` is called, if `explored`$[i] = 1$ then $v_i \in V(C)$."

Correctness I: Output is contained in v's component C



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Proof by induction. Vacuously true for initial call and second call. ✓

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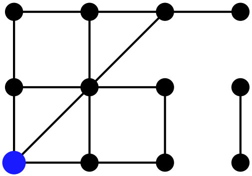

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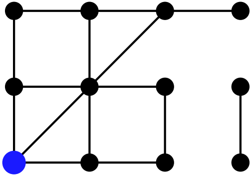
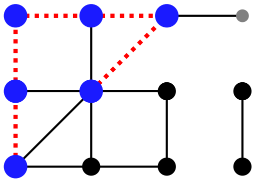


Invariant: "When `helper` is called, if `explored`$[i] = 1$ then $v_i \in V(C)$."

Proof by induction. Vacuously true for initial call and second call. ✓

Suppose it holds at the start of some call `helper`(v_j) from `helper`(v_i).
If v_j is already explored, we're done.

Correctness I: Output is contained in v's component C



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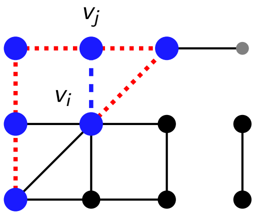

Invariant: "When `helper` is called, if `explored`$[i] = 1$ then $v_i \in V(C)$."

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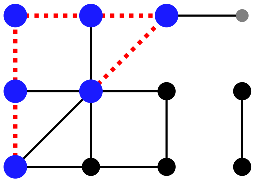
If v_j is already explored, we're done.

Correctness I: Output is contained in v's component C

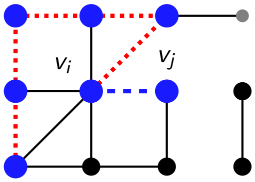


Invariant: "When `helper` is called, if `explored`$[i] = 1$ then $v_i \in V(C)$."

Proof by induction. Vacuously true for initial call and second call. ✓

Suppose it holds at the start of some call `helper`(v_j) from `helper`(v_i).
If v_j is already explored, we're done. If not, we must show $v_j \in V(C)$.

Correctness I: Output is contained in v's component C



Invariant: "When `helper` is called, if `explored`$[i] = 1$ then $v_i \in V(C)$."

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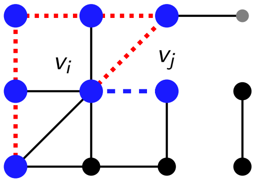

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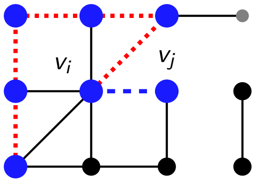

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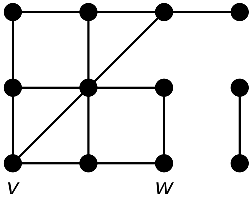
By induction there is a path P from v to v_i. Then Pv_iv_j is a walk from v to v_j, which contains a path, so $v_j \in V(C)$. □

Correctness II: Output contains v's component C



Let $w \in V(C)$. Then there is a path $P = x_1 \dots x_t$ from v to w.

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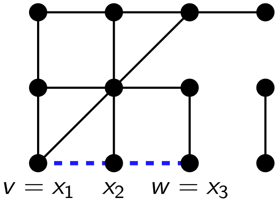

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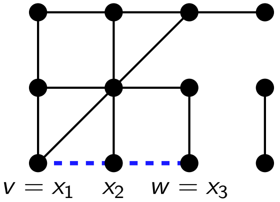

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Claim: Every vertex in P is explored.

Proof by induction: We prove $x_1, \dots, x_i$ are explored for all $i \leq t$.

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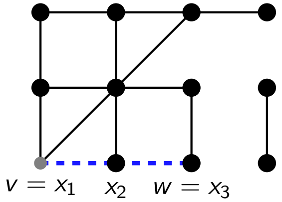

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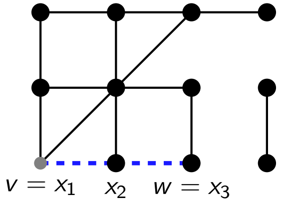

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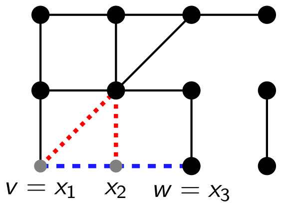

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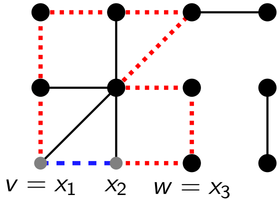

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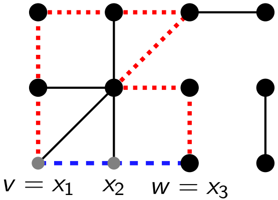

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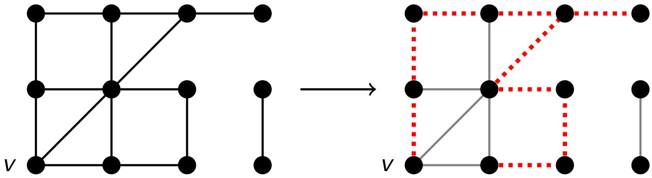
Consider the subgraph formed by the edges **traversed** in DFS:



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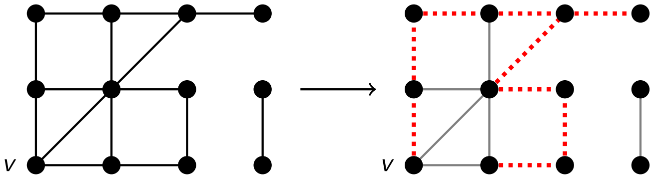

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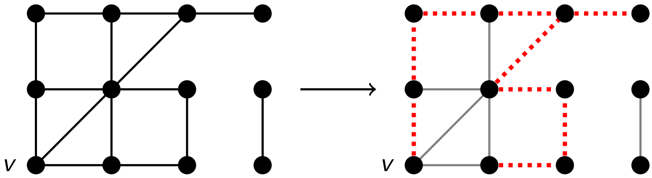

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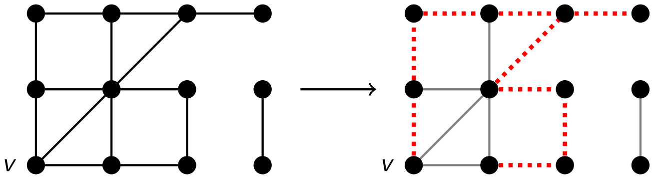

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Depth-first search works for directed graphs too, in exactly the same way. But paths **between** v and w are replaced by paths **from** v **to** w.