

1. (Evans 2.3.1) Suppose that we roll two fair six-sided dice, and let Y be the sum of the two numbers showing. Compute $P(Y = y)$ for every real number y .

The sample space S would be $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$ with 36 elements. Therefore, $Y \in \{1, 2, \dots, 12\}$. $P(Y = y)$ for every real number y :

y	2	3	4	5	6	7	8	9	10	11	12
$P(Y = y)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

2. Suppose that a bowl contains 100 chips: 30 are labelled 1, 20 are labelled 2, and 50 are labelled 3. The chips are thoroughly mixed, a chip is drawn, and let X be the number on the chip.

- i Compute $P(X = x)$ for every real number x .

$X \in \{1, 2, 3\}$ since those are the numbers for the chips.

$P(X = x)$ for every real number x :

x	1	2	3
$P(X = x)$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{5}{10}$

- ii Suppose the first chip is replaced, a second chip is drawn, and let Y be the number on the second chip. Compute $P(Y = y)$ for every real number y .

Since the first chip is replaced, Y has the same probabilities as X . i.e. $P(Y = y)$ for every real number y :

y	1	2	3
$P(Y = y)$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{5}{10}$

- iii Let $W = X + Y$ and compute $P(W = w)$ for every real number w .

$W \in \{2, 3, 4, 5, 6\}$

$$P(W = w) = \begin{cases} 2 & = (1, 1) & = (0.3 \cdot 0.3) = 0.09 \\ 3 & = (1, 2) \text{ or } (2, 1) & = (0.3 \cdot 0.2) + (0.2 \cdot 0.3) = 0.12 \\ 4 & = (1, 3) \text{ or } (3, 1) \text{ or } (2, 2) & = (0.3 \cdot 0.5) + (0.5 \cdot 0.3) + (0.2 \cdot 0.2) = 0.34 \\ 5 & = (2, 3) \text{ or } (3, 2) & = (0.2 \cdot 0.5) + (0.5 \cdot 0.2) = 0.2 \\ 6 & = (3, 3) & = (0.5 \cdot 0.5) = 0.25 \end{cases}$$

$$\text{i.e. } P(W = w) = \begin{cases} 0.09 & , w = 2 \\ 0.12 & , w = 3 \\ 0.34 & , w = 4 \\ 0.2 & , w = 5 \\ 0.25 & , w = 6 \end{cases}$$

3. Suppose that a bowl contains 10 chips, each uniquely numbered 0 through 9. The chips are thoroughly mixed, one is drawn and let X_1 be the number on it. This chip is then replaced in the bowl, and a second chip is drawn, with number X_2 . Compute $P(W = w)$ for every real number w when $W = X_1 + 10X_2$.

Any number from 0–99 can be made since chips are replaced,

i.e. $X_1 \in \{0, 1, \dots, 9\}, X_2 \in \{0, 1, \dots, 9\}, W \in \{00, 01, \dots, 99\}$.

Therefore the probability of any w is the probability of getting the corresponding x_1 and x_2 value.

$P(W = w)$, where $w = x_2x_1$, $= P(X_2 = x_2) \cap P(X_1 = x_2) = \frac{1}{10} \cdot \frac{1}{10} = \boxed{\frac{1}{100}}$

4. (Blitzstein: §3, Q1) People are arriving at a party one at a time. While waiting for more people to arrive they entertain themselves by comparing their birthdays. Let X be the number of people needed to obtain a birthday match, i.e. before the X th person arrives there are no two people with the same birthday, but when person X arrives there is a match. Find the distribution of X .