1. Suppose we deal five cards from an ordinary 52-card deck.

   i What is the conditional probability that all five cards are spades, given that at least four of them are spades?

$$P\left(\text{All 5 are spades} \mid \text{At least 4 spades}\right) = \frac{P\left(\text{All 5 are spades}\right)}{P\left(\text{4 spades + 1 other card}\right) + P\left(\text{All 5 are spades}\right)}$$
$$= \frac{\frac{\binom{13}{5}}{\binom{52}{5}}}{\frac{\binom{13}{4}\binom{39}{1}}{\binom{52}{5}} + \frac{\binom{13}{5}}{\binom{52}{5}}}$$
$$= \frac{\binom{13}{5}}{\binom{13}{4}\binom{39}{1} + \binom{13}{5}}$$
$$\boxed{= \frac{3}{68}}$$

   ii What is the conditional probability that the hand contains no pairs, given that it contains no spades?

$P\left(\text{Containing no spades}\right) = \frac{\binom{39}{5}}{\binom{52}{5}}$. $P\left(\text{Containing no pairs and no spades}\right)$ means that for the first card chosen, any of 39 cards can be chosen, then for the second card, any of 36 cards can be chosen (excluding the other 3 cards that can make previous card a pair), etc.
i.e. $P\left(\text{Containing no pairs and no spades}\right) = \frac{39}{52} \cdot \frac{36}{51} \cdot \frac{33}{50} \cdot \frac{30}{49} \cdot \frac{27}{48} = \frac{8019}{66640}$
$\therefore P\left(\text{Containing no pairs } \textit{given} \text{ no spades}\right) = \frac{\frac{8019}{66640}}{\frac{\binom{39}{5}}{\binom{52}{5}}} \boxed{= \frac{2673}{4921}}$

2. (Q6 on p.84 From B&H)
A hat contains 100 coins, where 99 are fair but one is double-headed (always landing Heads). A coin is chosen uniformly at random. The coin is flipped 7 times, and it lands Heads all 7 times. Given this information, what is the probability that the chosen coin is double-headed?

From description: $P\left(\text{1-sided coin}\right) = \frac{99}{100}$, $P\left(\text{2-sided coin}\right) = \frac{1}{100}$, $P\left(\text{All heads} \mid \text{1-sided coin}\right) = \frac{1}{2^7}$ and $P\left(\text{All heads} \mid \text{2-sided coin}\right) = 1$. Therefore, by Bayes' Theorem,

$$P\left(\text{2-sided coin} \mid \text{All heads}\right) = \frac{P\left(\text{All heads} \mid \text{2-sided coin}\right) P\left(\text{2-sided coin}\right)}{P\left(\text{H} \mid \text{2-sided coin}\right) P\left(\text{2-sided coin}\right) + P\left(\text{H} \mid \text{1-sided coin}\right) P\left(\text{1-sided coin}\right)}$$
$$= \frac{1\frac{1}{100}}{1\frac{1}{100} + \frac{1}{2^7}\frac{99}{100}}$$
$$= \frac{\cancel{\frac{1}{100}}(1)}{\cancel{\frac{1}{100}}\left(1 + \frac{99}{2^7}\right)}$$
$$\boxed{= \frac{2^7}{2^7+99}}$$