- 1. (Setting up sample spaces) Assume two standard six-sided dice are rolled one after the other.
 - i Restrict attention to the first roll, and list all possible outcomes of this random experiment.

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First roll can either be 1, 2, 3, 4, 5, or 6. \left(S = \left\{x \in \mathbb{N} \mid 1 \leq x \leq 6\right\}\right) We do not care about the outcomes of the second roll.
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ii Now consider both rolls, and list the outcomes of the most general sample space you can define.

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Each roll can either be 1, 2, 3, 4, 5, or 6. Therefore outcomes are: \left(S = \left\{x, y \in \mathbb{N} \mid 1 \leq x, y \leq 6\right\}\right) or \left\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \right\}
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- 2. (Set theory) Consider two arbitrary events $A, B \in S$ and describe the following events using set operations.
 - i Both events occur.

$$A\cap B$$

ii At least one event occurs.

$$A \cup B$$

iii Neither event occurs.

$$\neg (A \cup B)$$

iv Only event A but not event B occurs, i.e. $\{s \in S : s \in A \text{ and } s \notin B\}$. Note: this set operation is called *difference* and is denoted A - B (minus) or $A \setminus B$ (back-slash).

$$A \cap \neg B$$

v Exactly one event occurs.

Note: this set operation is called *symmetric difference* and is denoted by $A\triangle B$. In logic it is also called *exclusive OR* (XOR).

$$(A \cap \neg B) \cup (\neg A \cap B)$$

3. (Blitzstein: §1, Q42-43) For arbitrary events A, B use the probability axioms to show:

The definition of the difference (*△*) and symmetric difference (*△*) set operations are given in problem 2.iv and 2.v.

i
$$P(A - B) = P(A) - P(A \cap B)$$
.

Proof. Recall:
$$A = (A \cap B) \cup (A \cap B^c) = (A \cap B) \cup (A - B)$$
, where $(A \cap B)$ and $(A - B)$ are disjoint. $\therefore P(A) = \underbrace{P(A \cap B) + P(A - B)}_{\text{By the additivity axiom.}} \implies P(A - B) = P(A) - P(A \cap B) \blacksquare$

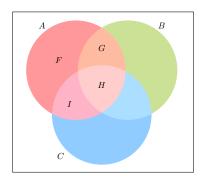
ii
$$P(A\triangle B) = P(A) + P(B) - 2P(A \cap B)$$
.

Proof. Recall:
$$A - B$$
 and $B - A$ are disjoint.
$$P(A \triangle B) = P((A - B) \cup (B - A))$$

$$= P(A - B) + P(B - A)$$

$$= (P(A) - P(A \cap B)) + (P(B) - P(A \cap B))$$
By the additivity axiom.
$$= (P(A) + P(B) - 2P(A \cap B))$$
∴ = $P(A) + P(B) - 2P(A \cap B)$

4. Consider three arbitrary sets A, B, C represented by colored circles in the Venn diagram below.



i Describe the strictly pink area F using set operations on A, B, C.

$$F = A \cap \neg (B \cup C)$$

ii Describe the strictly brown area G using set operations on A, B, C.

$$G = A \cap B \cap \neg C$$