1. (Setting up sample spaces) Assume two standard six-sided dice are rolled one after the other.

   i Restrict attention to the first roll, and list all possible outcomes of this random experiment.

   First roll can either be 1, 2, 3, 4, 5, or 6. $\left(S = \left\{x \in \mathbb{N} \mid 1 \leq x \leq 6\right\}\right)$
   We do not care about the outcomes of the second roll.

   ii Now consider both rolls, and list the outcomes of the most general sample space you can define.

   Each roll can either be 1, 2, 3, 4, 5, or 6. Therefore outcomes are: $\left(S = \left\{x, y \in \mathbb{N} \mid 1 \leq x, y \leq 6\right\}\right)$
   or
   {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
   (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
   (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
   (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
   (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
   (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

2. (Set theory) Consider two arbitrary events $A, B \in S$ and describe the following events using set operations.

   i Both events occur.

   $A \cap B$

   ii At least one event occurs.

   $A \cup B$

   iii Neither event occurs.

   $\neg(A \cup B)$

   iv Only event A but not event B occurs, i.e. $\left\{s \in S : s \in A \text{ and } s \notin B\right\}$.
   Note: this set operation is called *difference* and is denoted A – B (minus) or A\B (back-slash).

   $A \cap \neg B$

   v Exactly one event occurs.
   Note: this set operation is called *symmetric difference* and is denoted by $A \triangle B$. In logic it is also called *exclusive OR* (XOR).

   $(A \cap \neg B) \cup (\neg A \cap B)$

3. (Blitzstein: §1, Q42-43) For arbitrary events A, B use the probability axioms to show:
The definition of the difference ($-$) and symmetric difference ($\triangle$) set operations are given in problem 2.iv and 2.v.

i $P(A - B) = P(A) - P(A \cap B)$.

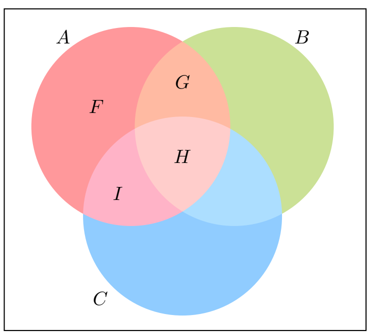
Proof. Recall: $A = (A \cap B) \cup (A \cap B^c) = (A \cap B) \cup (A - B)$, where $(A \cap B)$ and $(A - B)$ are disjoint.
$\therefore P(A) = \underbrace{P(A \cap B) + P(A - B)}_{\text{By the additivity axiom.}} \implies P(A - B) = P(A) - P(A \cap B)$ ■

ii $P(A \triangle B) = P(A) + P(B) - 2P(A \cap B)$.

Proof. Recall: $A - B$ and $B - A$ are disjoint.

$$\begin{aligned} P(A \triangle B) &= P((A - B) \cup (B - A)) \\ &= P(A - B) + P(B - A) && \text{By the additivity axiom.} \\ &= \big(P(A) - P(A \cap B)\big) + \big(P(B) - P(A \cap B)\big) && \text{By part i} \\ \therefore &= P(A) + P(B) - 2P(A \cap B) && \blacksquare \end{aligned}$$

4. Consider three arbitrary sets A, B, C represented by colored circles in the Venn diagram below.



i Describe the strictly pink area F using set operations on A, B, C.

$F = A \cap \neg(B \cup C)$

ii Describe the strictly brown area G using set operations on A, B, C.

$G = A \cap B \cap \neg C$