Quantised Random Embeddings for Dimensionality Reduction

V. Cambareri (joint work w. Prof. L. Jacques), ISP Group, ICTEAM/ELEN

Quantised Random Embeddings

$$\boldsymbol{y} = \mathsf{A}(\boldsymbol{x}) := \mathcal{Q}_\delta(\boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{\xi})$$

where:

- Dataset: $\boldsymbol{x} \in \mathcal{K} \subset \mathbb{R}^n \longrightarrow$ Signatures: $\boldsymbol{y} \in \mathsf{A}(\mathcal{K}) \subset \delta\mathbb{Z}^m$
- Uniform Scalar Quantiser: $\mathcal{Q}_\delta := \delta\lfloor \frac{\cdot}{\delta} \rceil$
- Gaussian (not necessary!) Random Projections: $\boldsymbol{\Phi} \sim \mathcal{N}^{m\times n}(0,1)$
- Uniform Dithering: $\boldsymbol{\xi} \sim \mathcal{U}^{m\times n}([-\frac{\delta}{2}, \frac{\delta}{2}])$

Quasi-Isometry Property

If $\boldsymbol{\Phi}$ is a ε-stable embedding (ℓ_2/ℓ_1 sense) w.r.t. $\mathcal{K}$,

$$(1-\varepsilon)\|\boldsymbol{x}'-\boldsymbol{x}''\|_2 - \Delta \le \frac{1}{m}\|\boldsymbol{y}'-\boldsymbol{y}''\|_1 \le (1+\varepsilon)\|\boldsymbol{x}'-\boldsymbol{x}''\|_2 + \Delta$$

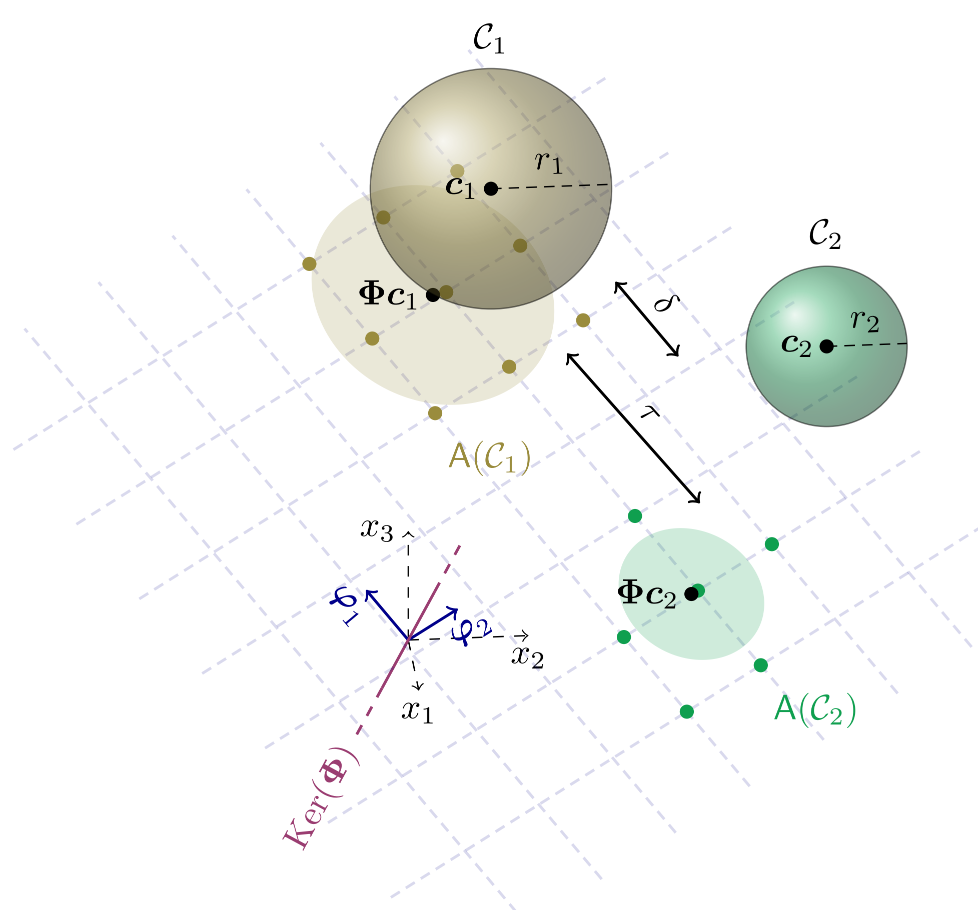
w.h.p. for $\Delta, \varepsilon > 0$ that decay with $m(\uparrow)$, Complexity($\mathcal{K}$) ($\downarrow$), δ ($\downarrow$).

"Rare Eclipses" and Class Separability

If $\boldsymbol{x}' \in \mathcal{C}' \subset \mathcal{K}, \boldsymbol{x}'' \in \mathcal{C}'' \subset \mathcal{K}, \mathcal{C}' \cap \mathcal{C}'' = \emptyset$ (convex sets), then

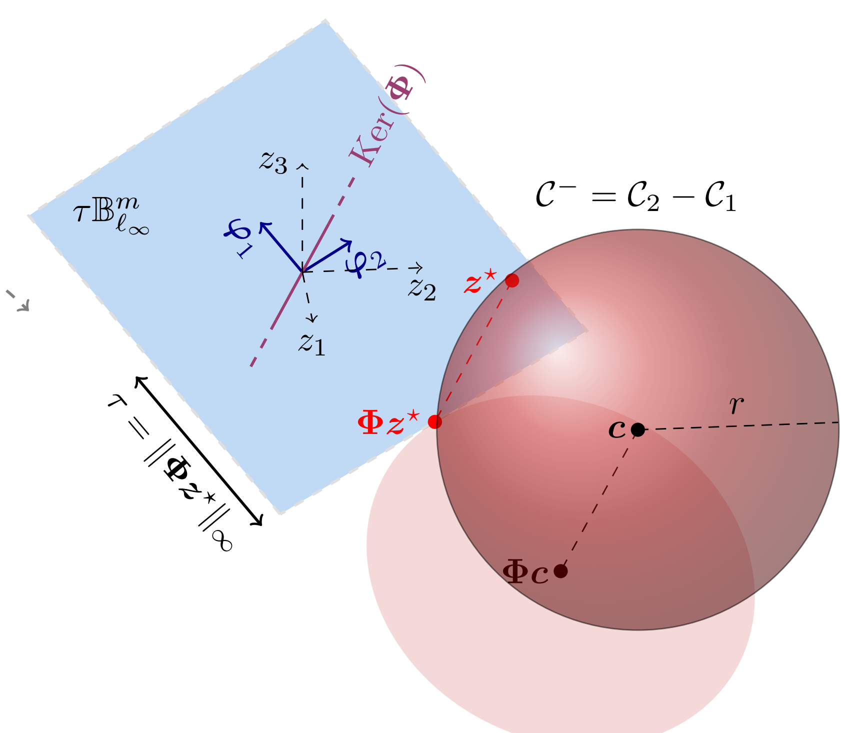
$$p_\delta := \mathbb{P}[\mathsf{A}(\mathcal{C}') \cap \mathsf{A}(\mathcal{C}') = \emptyset] \ge 1 - \eta$$

with η that decays with m ($\uparrow$), Complexity($\mathcal{K}$) ($\downarrow$), δ ($\downarrow$).



Avoid consistency
on difference set $\mathcal{C}^-$

(Inconsistent if $\tau > \delta$)



Code here