Lab 1: Points and vectors

3D Computer Graphics

Exercise 1

Given are two vectors $\mathbf{u} = (2, -1, 1)$ and $\mathbf{v} = (1, 3, -2)$ and two points $P = (-2, -1, 5)$ and $Q = (1, -3, -4)$.

a) Compute $2\mathbf{u} + 3\mathbf{v}$.

b) Compute the length of $\mathbf{u}$.

c) Reverse $\mathbf{u}$.

d) Normalize $\mathbf{v}$.

e) Compute the dot product of $\mathbf{u}$ and $\mathbf{v}$.

f) Compute the cross product of $\mathbf{u}$ and $\mathbf{v}$.

g) Compute the vector $\mathbf{w}$ from point P to point Q.

Definition

Two vectors $\mathbf{u}$ and $\mathbf{v}$ are **perpendicular** if the angle between them is $90°$.

Definition

A **unit vector** is a vector with length $= 1$.

Exercise 2

a) What is the dot product of two perpendicular vectors $\mathbf{a}$ and $\mathbf{b}$?

b) What is the length of the cross product of two perpendicular unit vectors $\mathbf{u}$ and $\mathbf{v}$?

Exercise 3

Assume three points $P = (-2, -1, 5)$, $Q = (1, -3, -4)$ and $R = (-1, -4, 3)$. There is a unique plane in 3D space which contains these three points.

a) Find a unit vector $\mathbf{u}$ which is perpendicular to this plane.

b) Can you find an easy way to determine whether a vector $\mathbf{v}$ points to the same side of this plane as $\mathbf{u}$?

Exercise 4

Given a vector $\mathbf{u} = (u_x, u_y, u_z)$ and a positive real number c. Show that the length of the vector obtained by the scalar multiplication of $\mathbf{u}$ and c is equal to the length of $\mathbf{u}$ times c.

$$|c\mathbf{u}| = c|\mathbf{u}|$$

Exercise 5

Assume two vectors $\mathbf{u} = (1, 0, 0)$ and $\mathbf{v} = (0, 1, 0)$.

a) Draw these two vectors in a 3D coordinate system.

b) Use the property of the cross product to determine $\mathbf{u} \times \mathbf{v}$.

c) Verify your answer to b) by explicitly calculating the cross product of $\mathbf{u}$ and $\mathbf{v}$.

Exercise 6

Is the following statement true or false?

For any two 3D vectors $\mathbf{u}$ and $\mathbf{v}$ holds the property that

$$\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}.$$