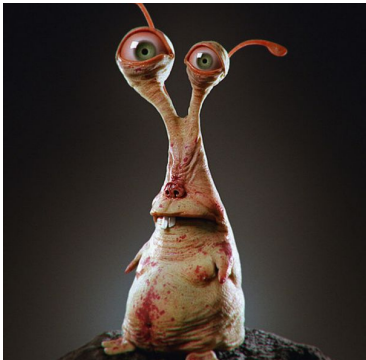


Ray tracing polygonal meshes

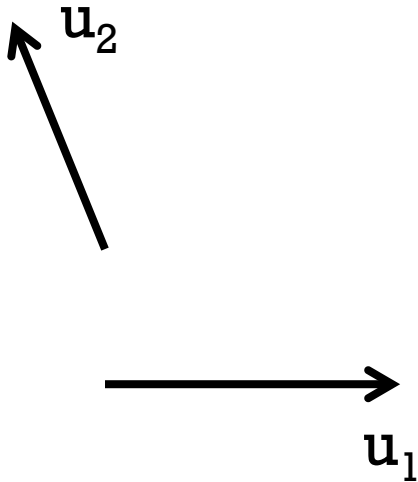
3D Computer Graphics (Lab 5)



Remember ...

$$\cos(\theta) = \frac{u_1 \cdot u_2}{|u_1| |u_2|}$$

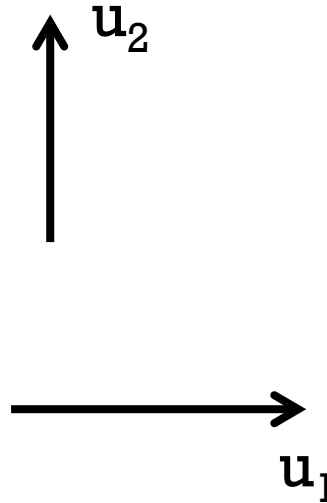
2



Angle θ between u_1
and $u_2 > 90^\circ$

$$\cos \theta < 0$$

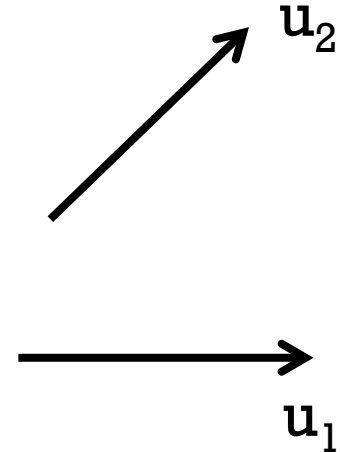
$$u_1 \cdot u_2 < 0$$



Angle θ between u_1
and $u_2 = 90^\circ$

$$\cos \theta = 0$$

$$u_1 \cdot u_2 = 0$$

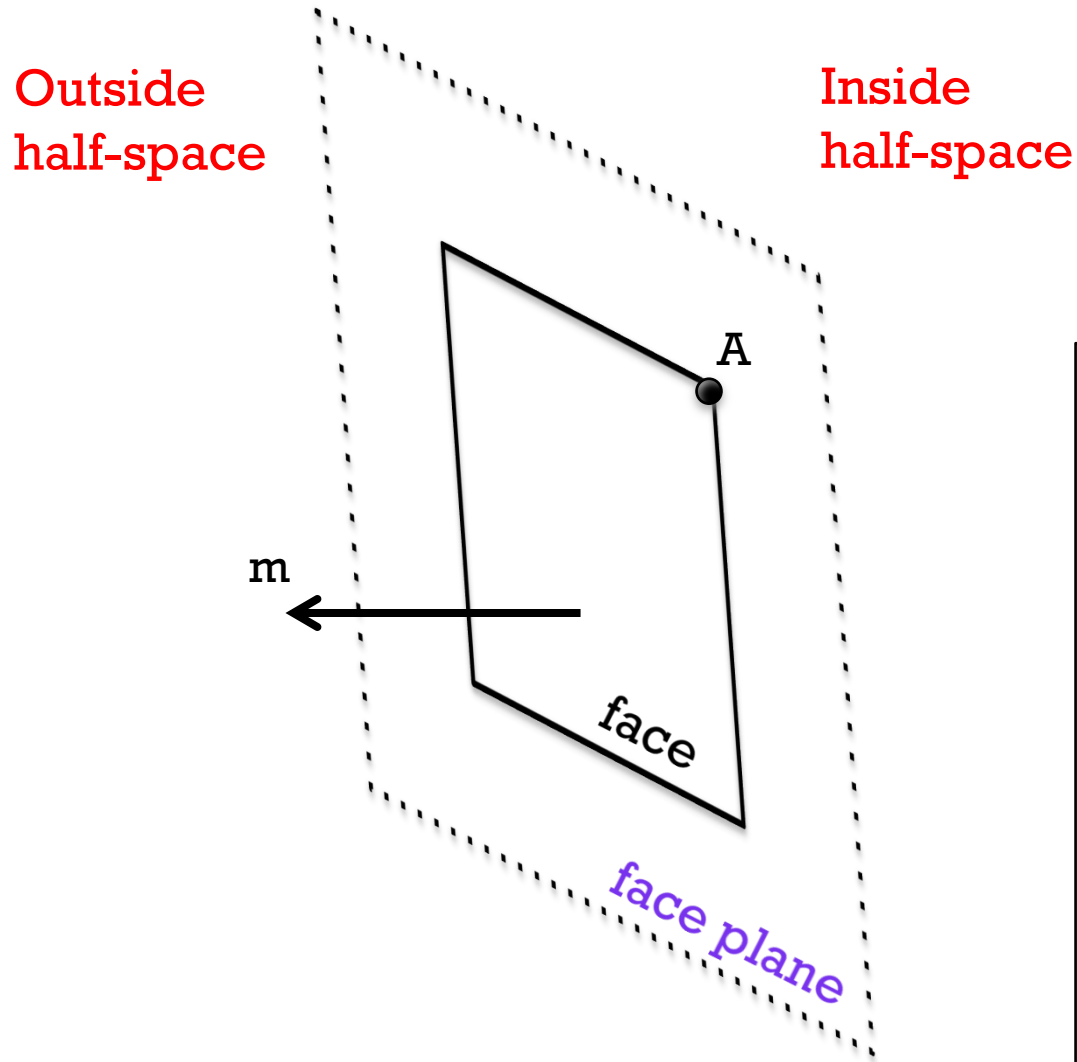


Angle θ between u_1
and $u_2 < 90^\circ$

$$\cos \theta > 0$$

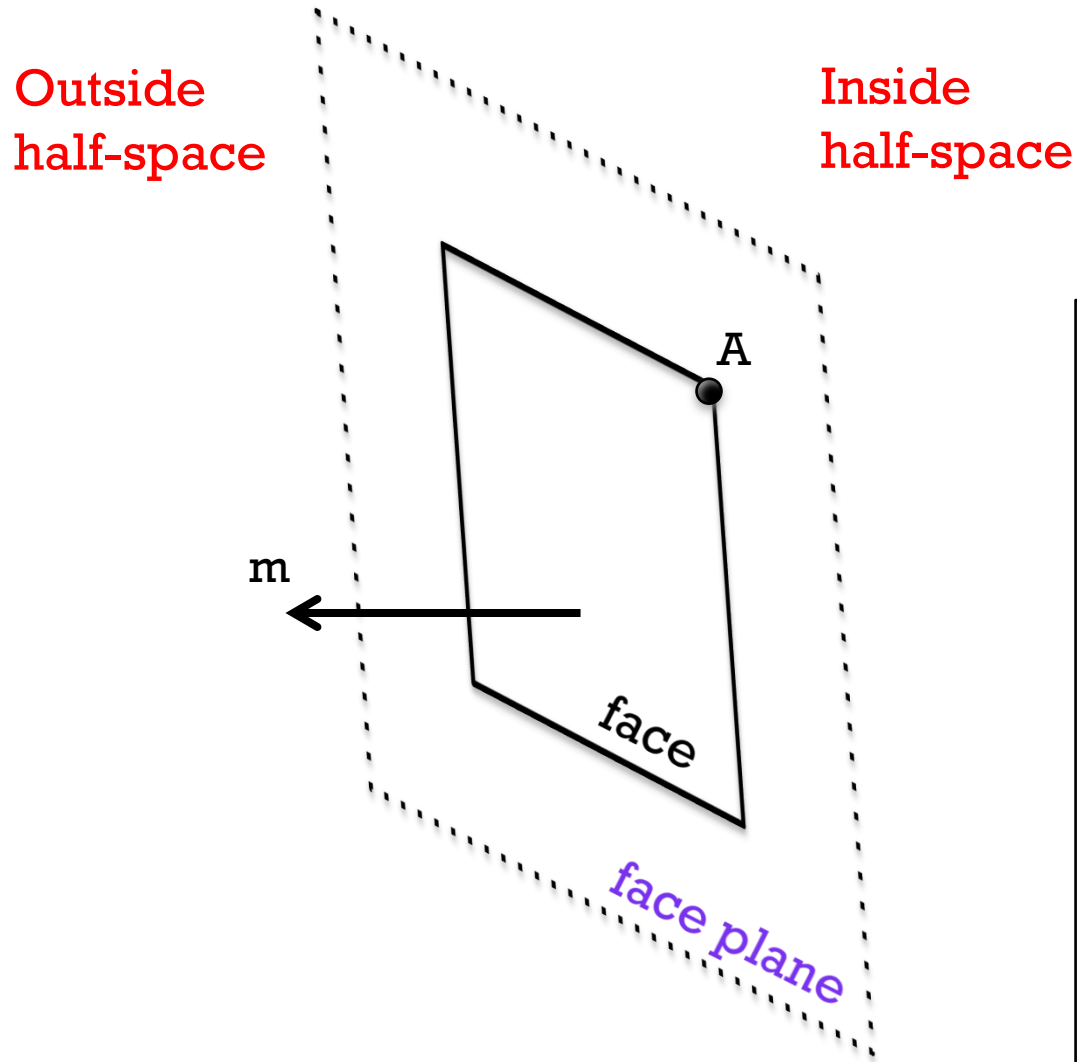
$$u_1 \cdot u_2 > 0$$

Face plane and half-spaces



- A = a vertex of the face
 m = normal vector of the face
- A **face plane** is the plane in which the face lies.
- The face plane divides the 3D space in two: the **outside** and **inside half-space**.
- m points in the direction of the outside half-space.

Face plane and half-spaces



- Note that we associated a normal vector with each vertex of a face in Lab 2.
- The normal vectors associated with these vertices may be different so how do we set m ?
- The purpose of m is to be able to differentiate between the inside and outside half-space, so any of the normal vectors of the face will do.



Position of a point with
respect to a plane

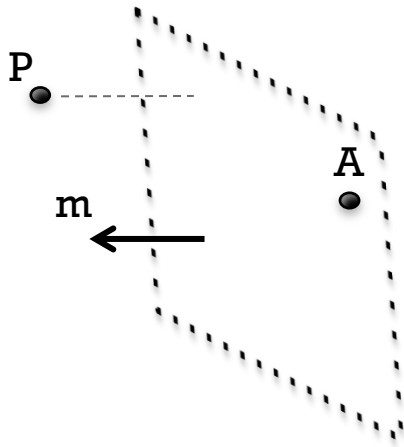
Position of a point w.r.t. a plane

6

How can we
determine the
difference?

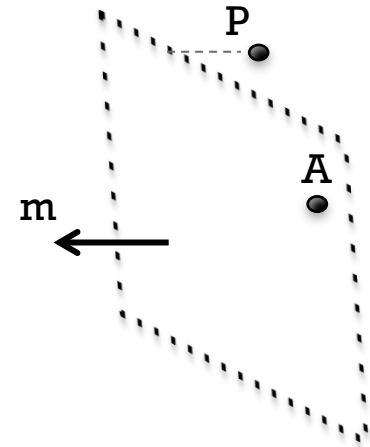
$$m.(A - P) < 0$$

P lies in outside half-space



$$m.(A - P) > 0$$

P lies in inside half-space

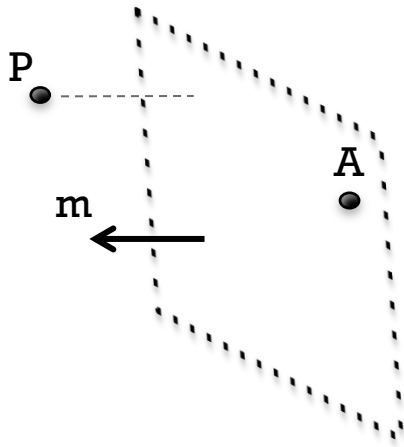


Position of a point w.r.t. a plane

7

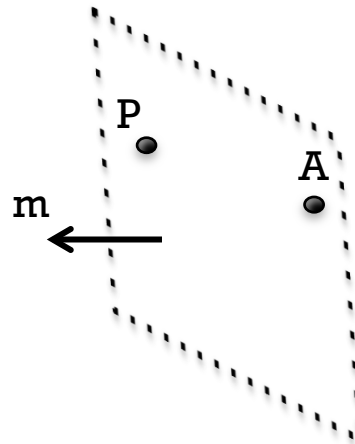
$$m.(A - P) < 0$$

P lies in outside half-space



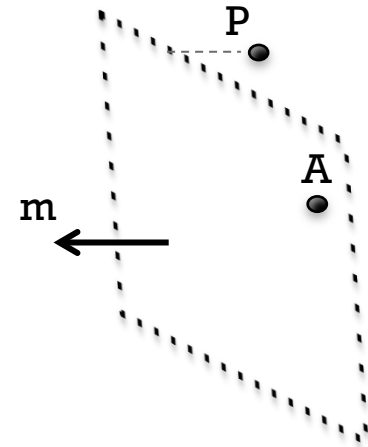
$$m.(A - P) = 0$$

P lies in face plane



$$m.(A - P) > 0$$

P lies in inside half-space

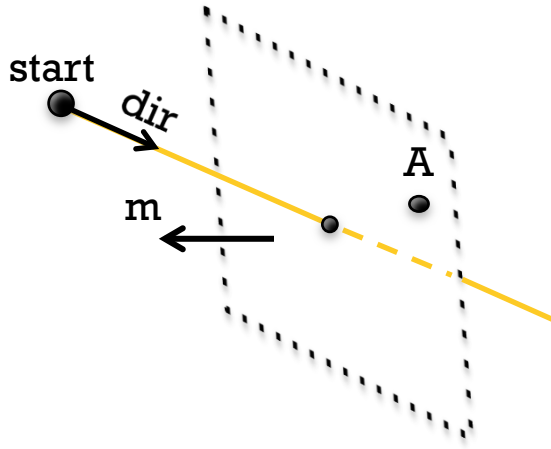




Position of a ray with
respect to a plane

$$m.dir < 0$$

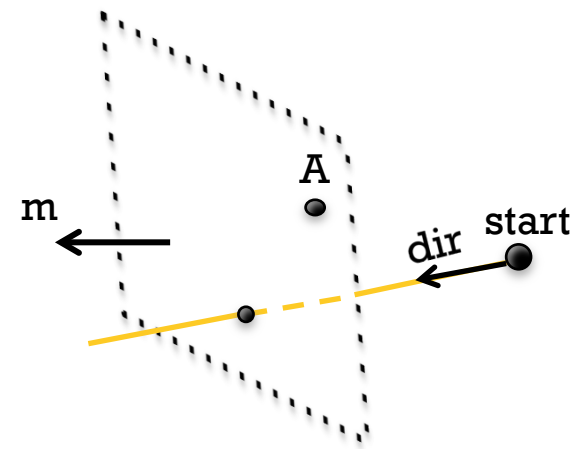
ray enters inside half-space



How can we
determine the
difference?

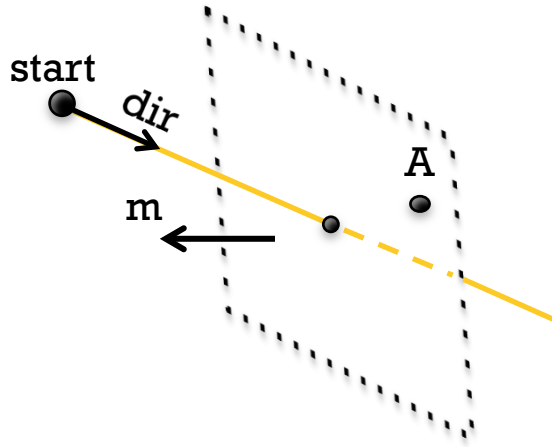
$$m.dir > 0$$

ray exits inside half-space



$$m.dir < 0$$

ray enters inside half-space



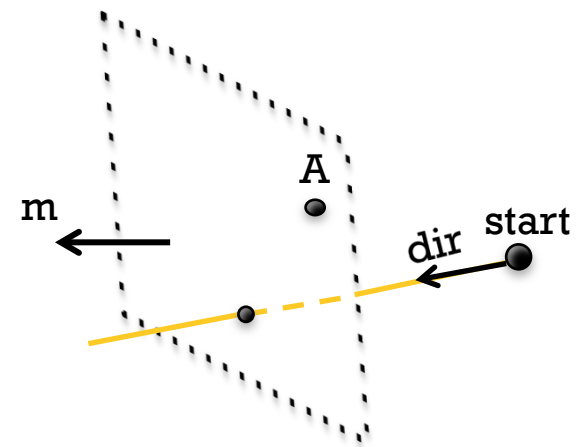
$$m.dir = 0$$

ray is parallel to face plane

Depending on the position of the start point, there are three sub scenarios.

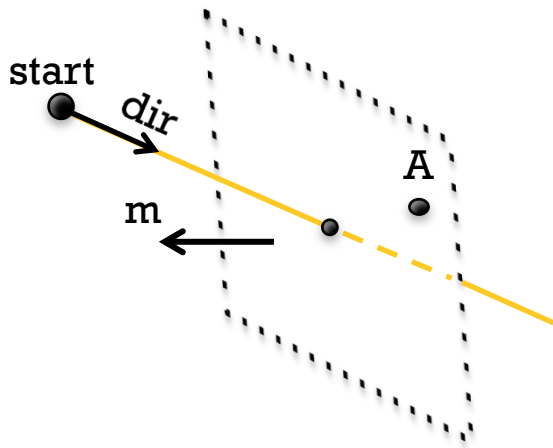
$$m.dir > 0$$

ray exits inside half-space



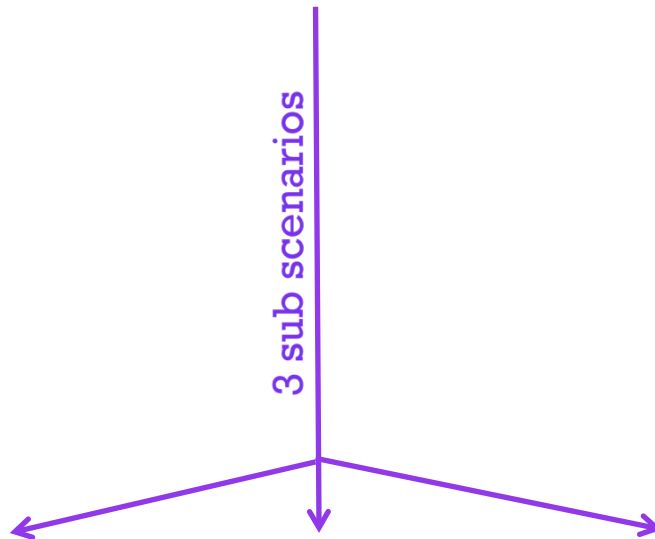
$$m.dir < 0$$

ray enters inside half-space



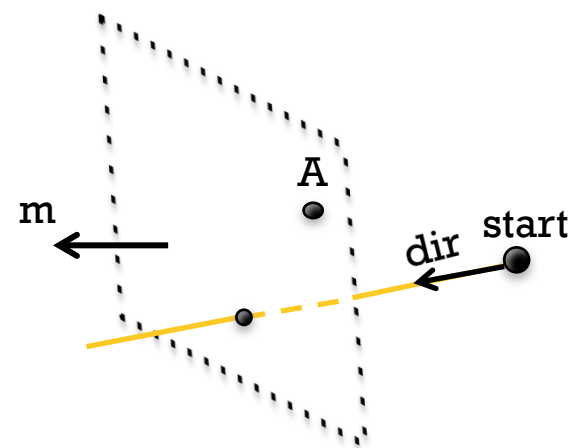
$$m.dir = 0$$

ray is parallel to face plane



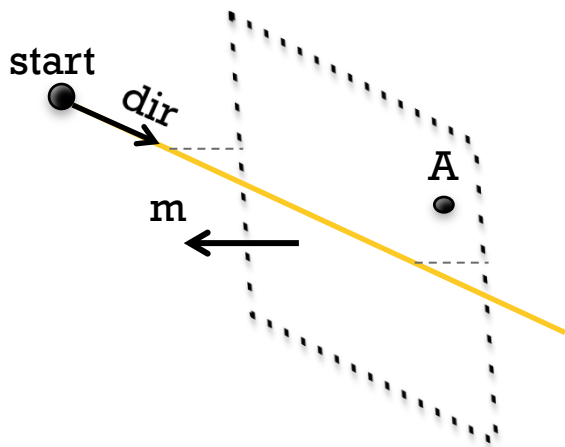
$$m.dir > 0$$

ray exits inside half-space



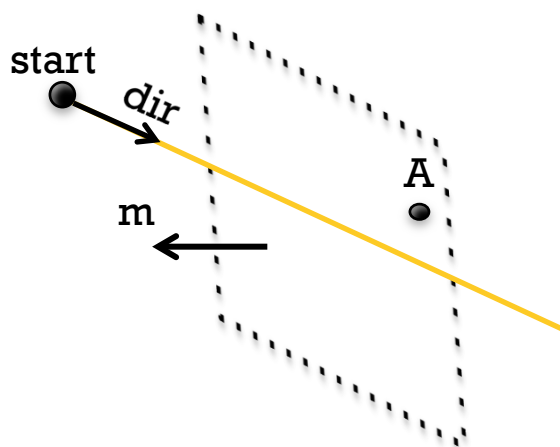
$$m.(A - start) < 0$$

ray lies in outside half-space



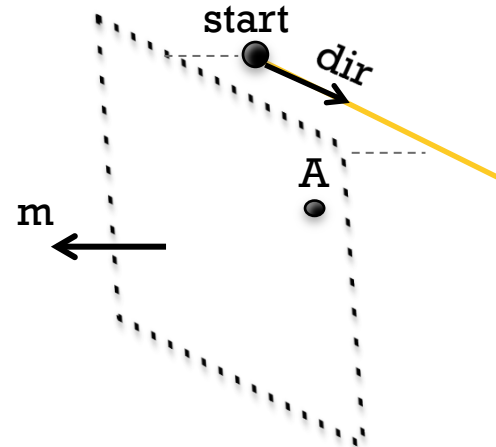
$$m.(A - start) = 0$$

ray lies in face plane



$$m.(A - start) > 0$$

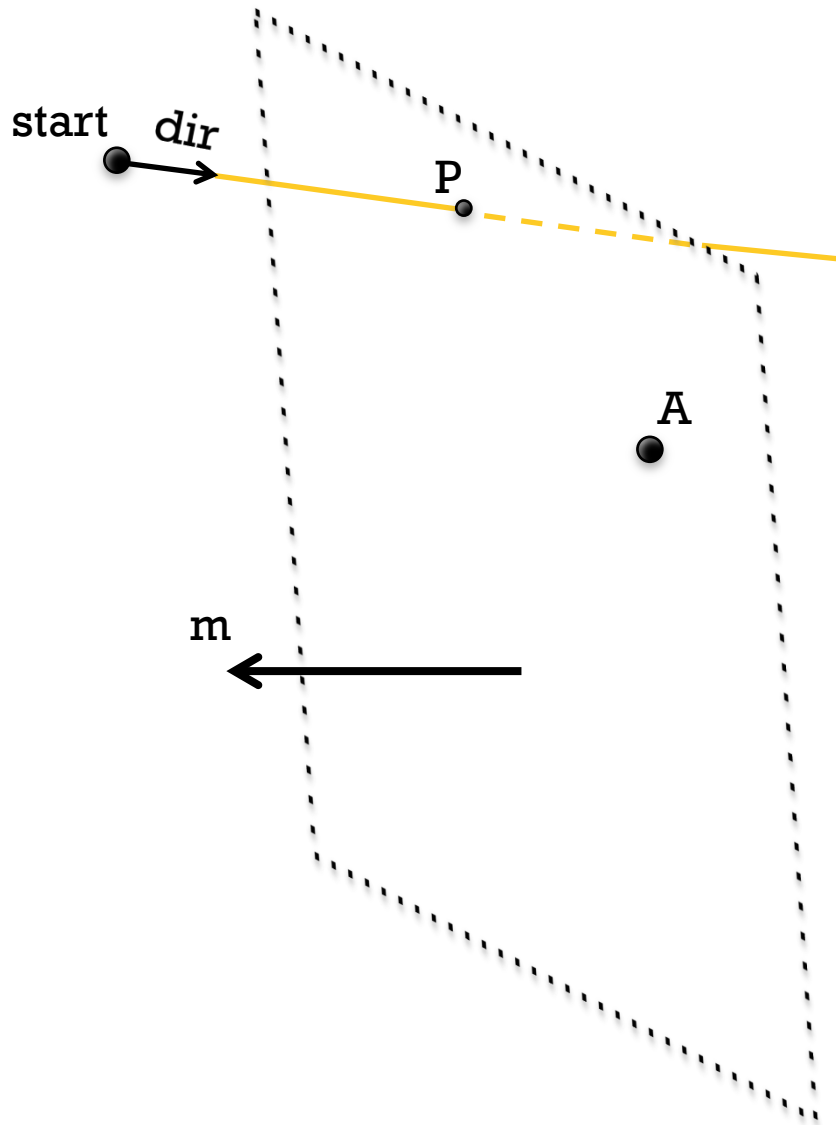
ray lies in inside half-space



The image features three solid-colored squares arranged horizontally. On the left is a red square, in the center is a larger yellow square, and on the right is a purple square. The yellow square is the largest and contains the text 'Intersection ray - plane' in a black, monospaced font.

Intersection ray - plane

Intersection ray - plane



We are searching for the point P which satisfies the following system

$$\begin{cases} P = start + t.dir & \boxed{\text{P is on ray}} \\ m.(A - P) = 0 & \boxed{\text{P is in plane}} \end{cases}$$

$$m.(A - (start + t.dir)) = 0$$

$$m.(A - start - t.dir) = 0$$

$$m.(A - start) - t(m.dir) = 0$$

$$m.(A - start) = t(m.dir)$$

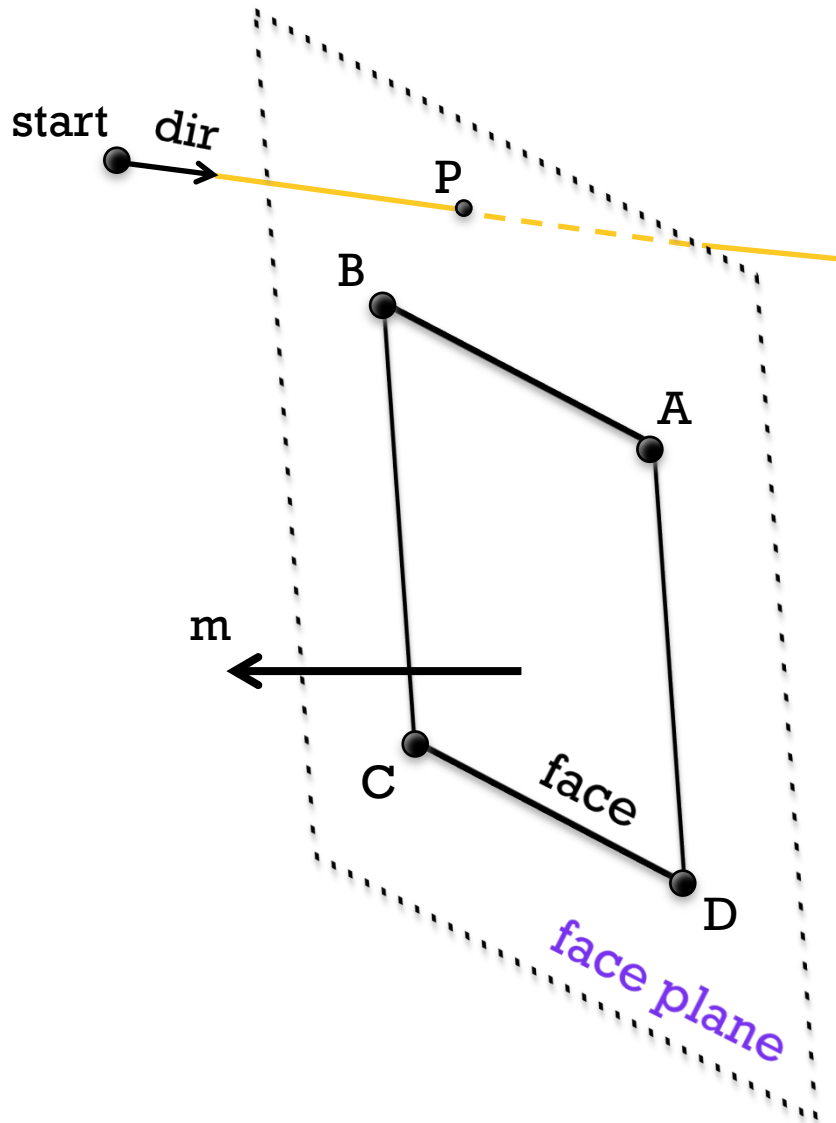
$$\boxed{t = t_{hit} = \frac{m.(A - start)}{m.dir}}$$

What if $m.dir = 0$?



Intersection
ray – polygonal mesh

Intersection ray – polygonal mesh



Pseudo code

```
for each face
  if m.dir is zero
    ignore face
  compute  $t_{hit}$ 
  if  $t_{hit} > 0$ 
    add hitInfo to list
```

Correct?

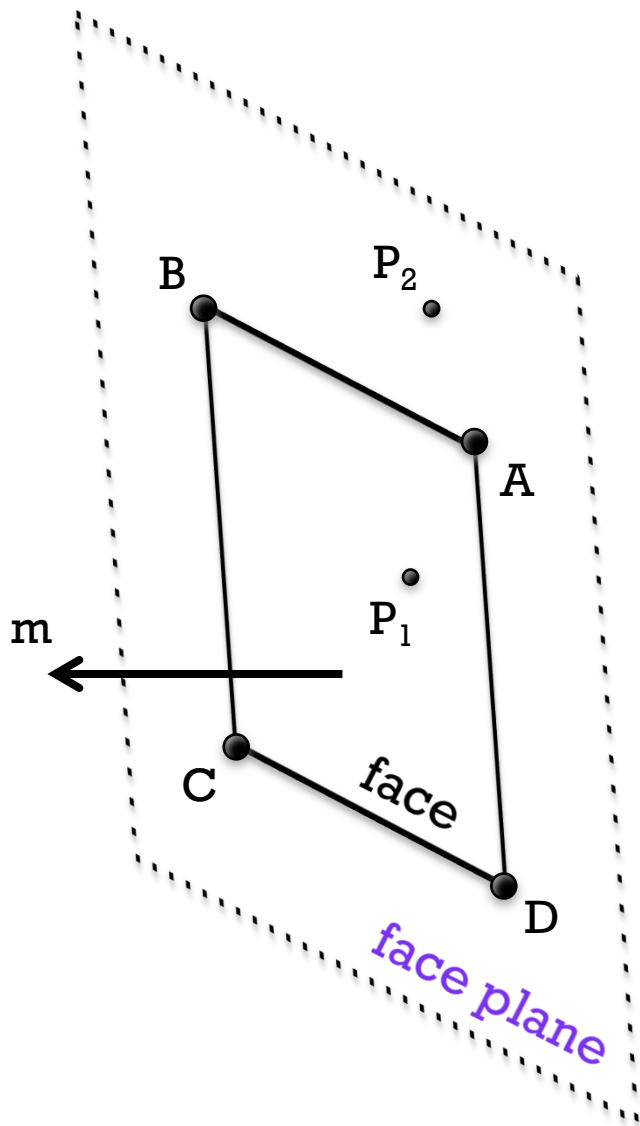
No, the hitPoint P does not necessarily lie in the face!

$$t = t_{hit} = \frac{m \cdot (A - start)}{m \cdot dir}$$

The image features three solid-colored squares arranged horizontally. On the left is a red square. In the center is a larger yellow square. On the right is a purple square. The text 'Point-in-polygon test' is centered within the yellow square.

Point-in-polygon test

Point-in-polygon test

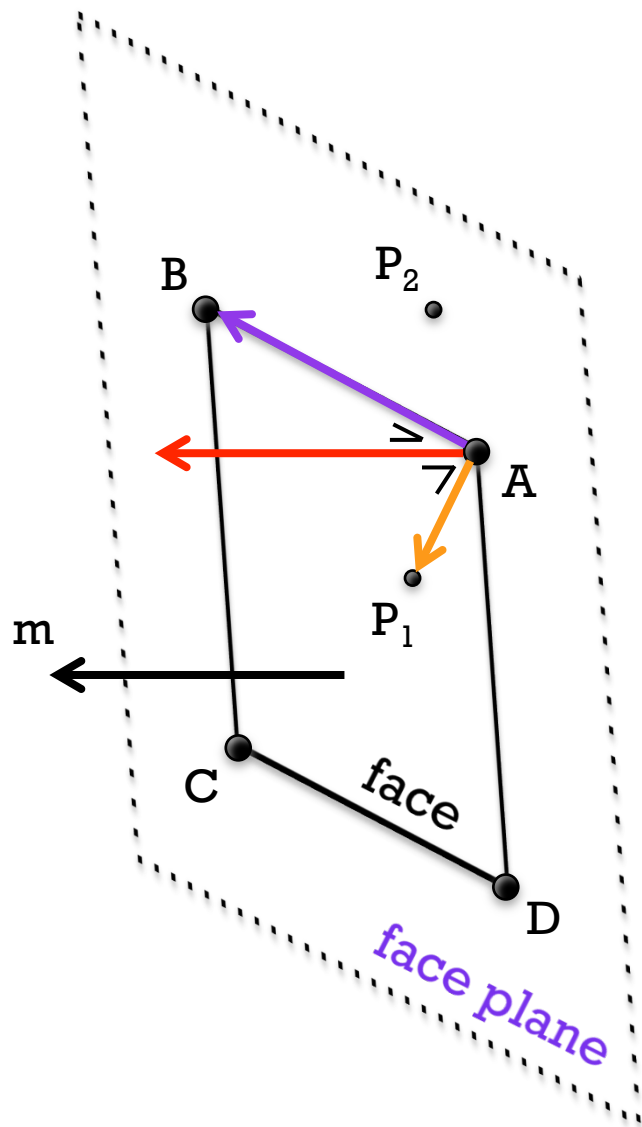


Idea ...

- Walk along the edges on the outside of the face so that you visit the vertices in the order in which they were specified in the face.
- If the point lies to the left of each edge, the point is inside the polygon.
- Else the point is outside the polygon.

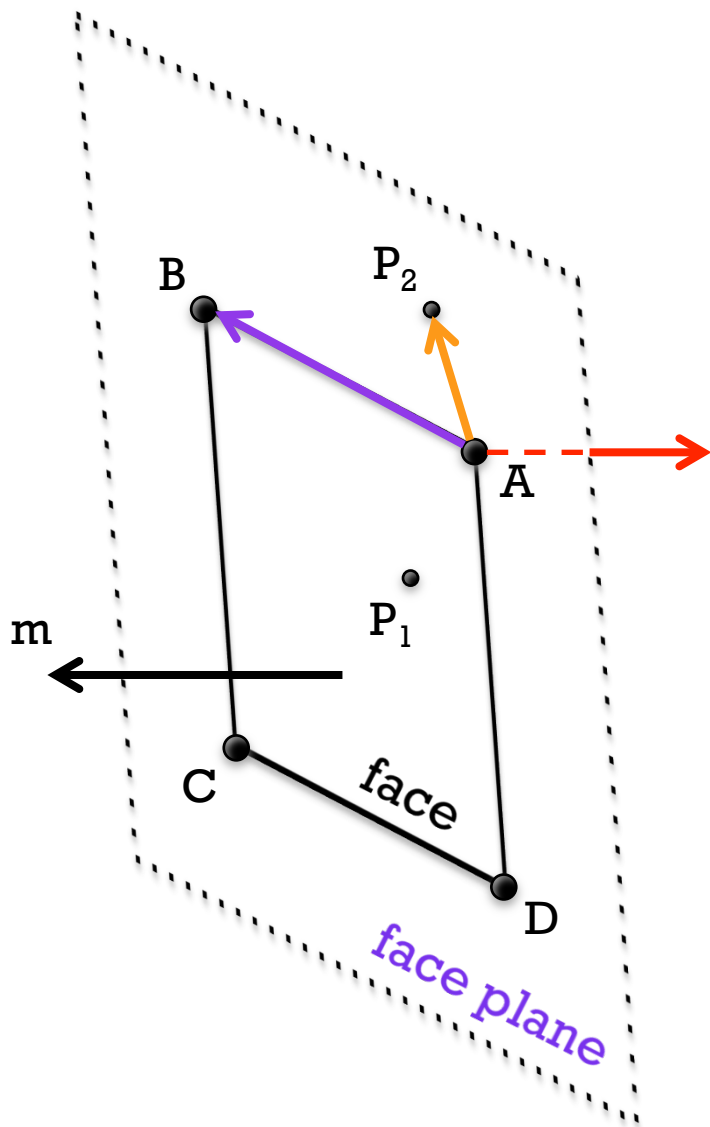
Note that this idea only works because of the convention we introduced in Lab 2 to order the vertices in a face counterclockwise as seen from outside the object.

Point-in-polygon test



- \mathbf{v}_1 = vector from A to B
- \mathbf{v}_2 = vector from A to P₁
- $\mathbf{v}_3 = \mathbf{v}_1 \times \mathbf{v}_2$
- \mathbf{v}_3 points to outside half-space

Point-in-polygon test



- v_1 = vector from A to B
- v_2 = vector from A to P_1
- $v_3 = v_1 \times v_2$
- v_3 points to outside half-space

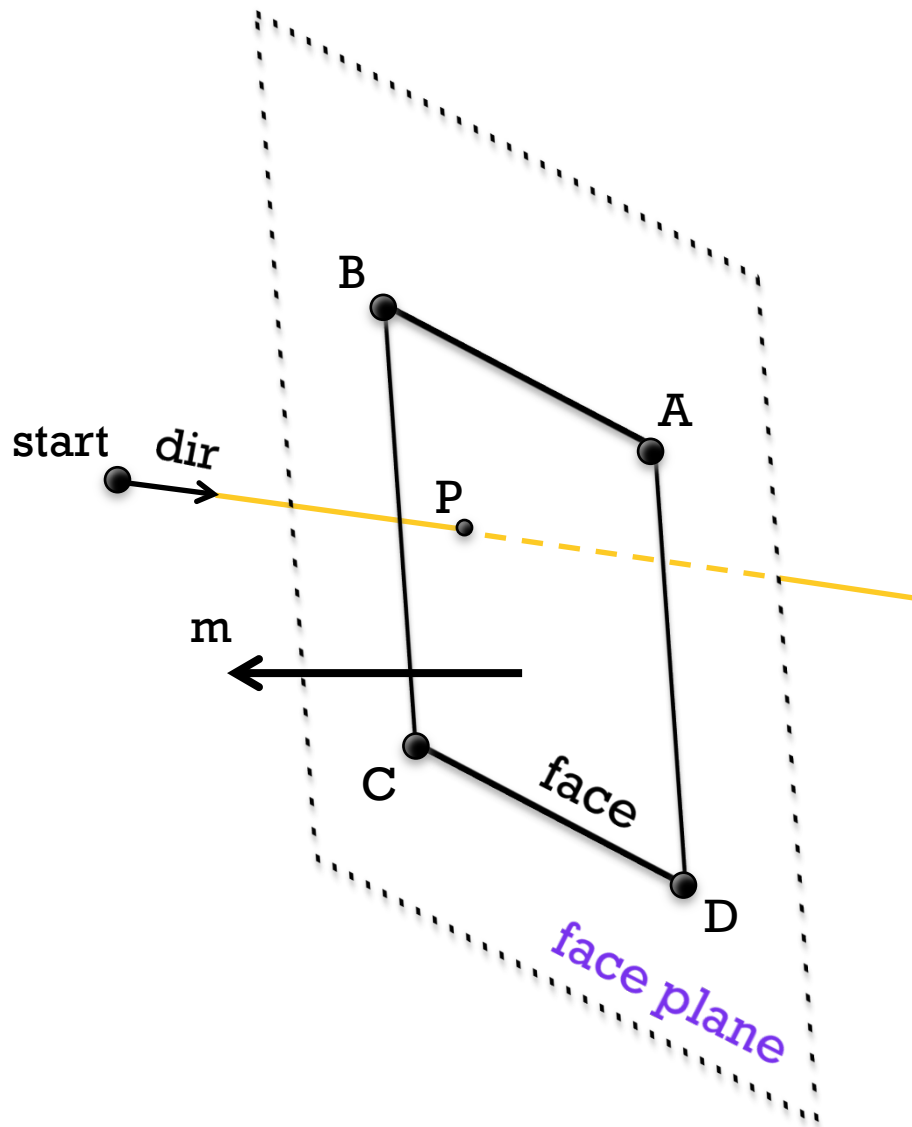
- v_1 = vector from A to B
- v_2 = vector from A to P_2
- $v_3 = v_1 \times v_2$
- v_3 points to inside half-space

A point P lies to the left of an edge if v_3 points to the outside half-space. This is the case if

$$v_3 \cdot m > 0$$

Intersection ray – polygonal mesh

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Pseudo code

```
for each face
  if m.dir is zero, ignore face
  compute  $t_{hit}$ 
  if  $t_{hit} < 0$ , ignore face
  for each edge
    if hitPoint not to the left of edge
      ignore face
  if hitPoint in face
    add hitInfo to list
return list
```

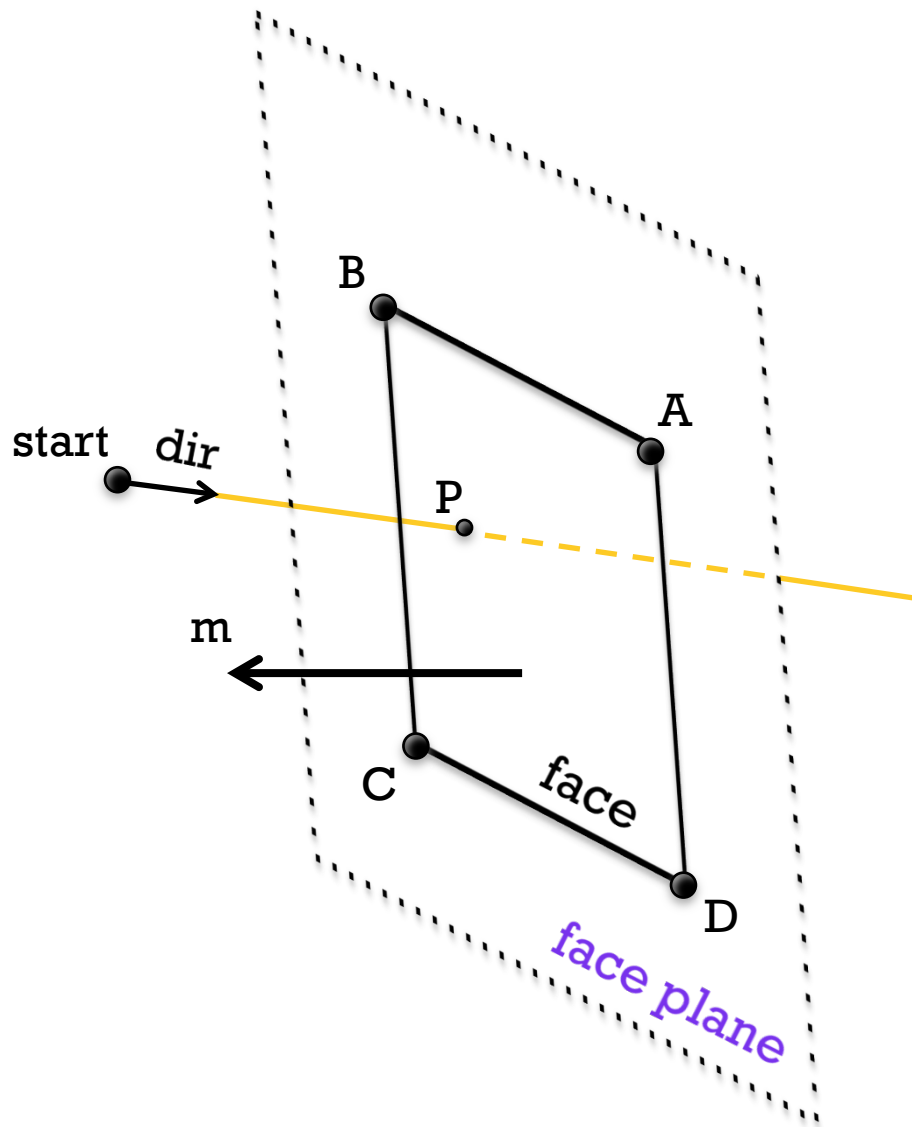
Correct?

No, the hit with the bestHit-Time should be at index 0!

How?

Intersection ray – polygonal mesh

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Pseudo code

In practise:
 $|m.dir| < 0.0000001$

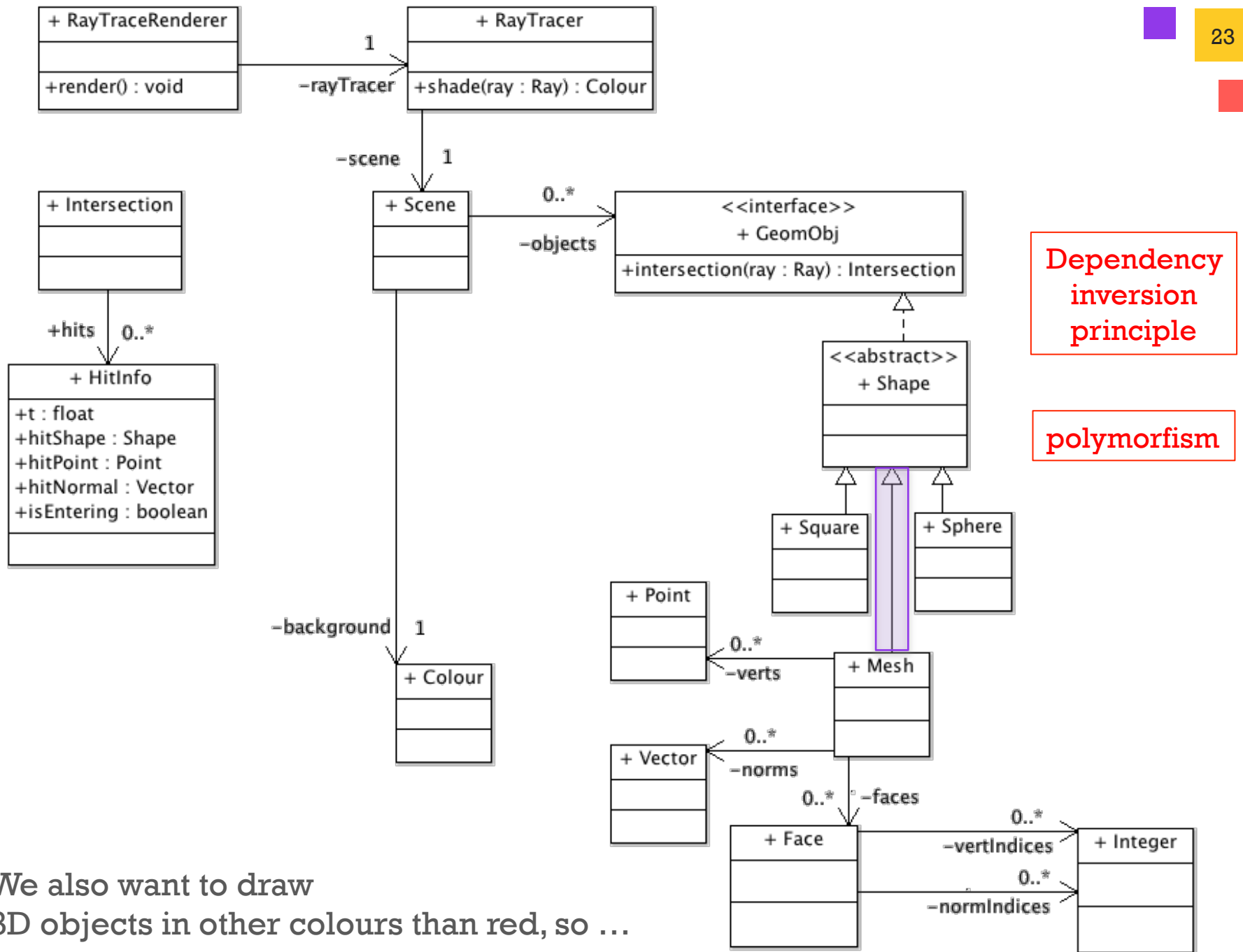
```
for each face
  if  $m.dir$  is zero, ignore face
  compute  $t_{hit}$ 
  if  $t_{hit} < 0$ , ignore face
  for each edge
    if hitPoint not to the left of edge
      ignore face
  if hitPoint in face
    add hitInfo to list
  set hit with bestHitTime at index 0
return list
```

Correct?

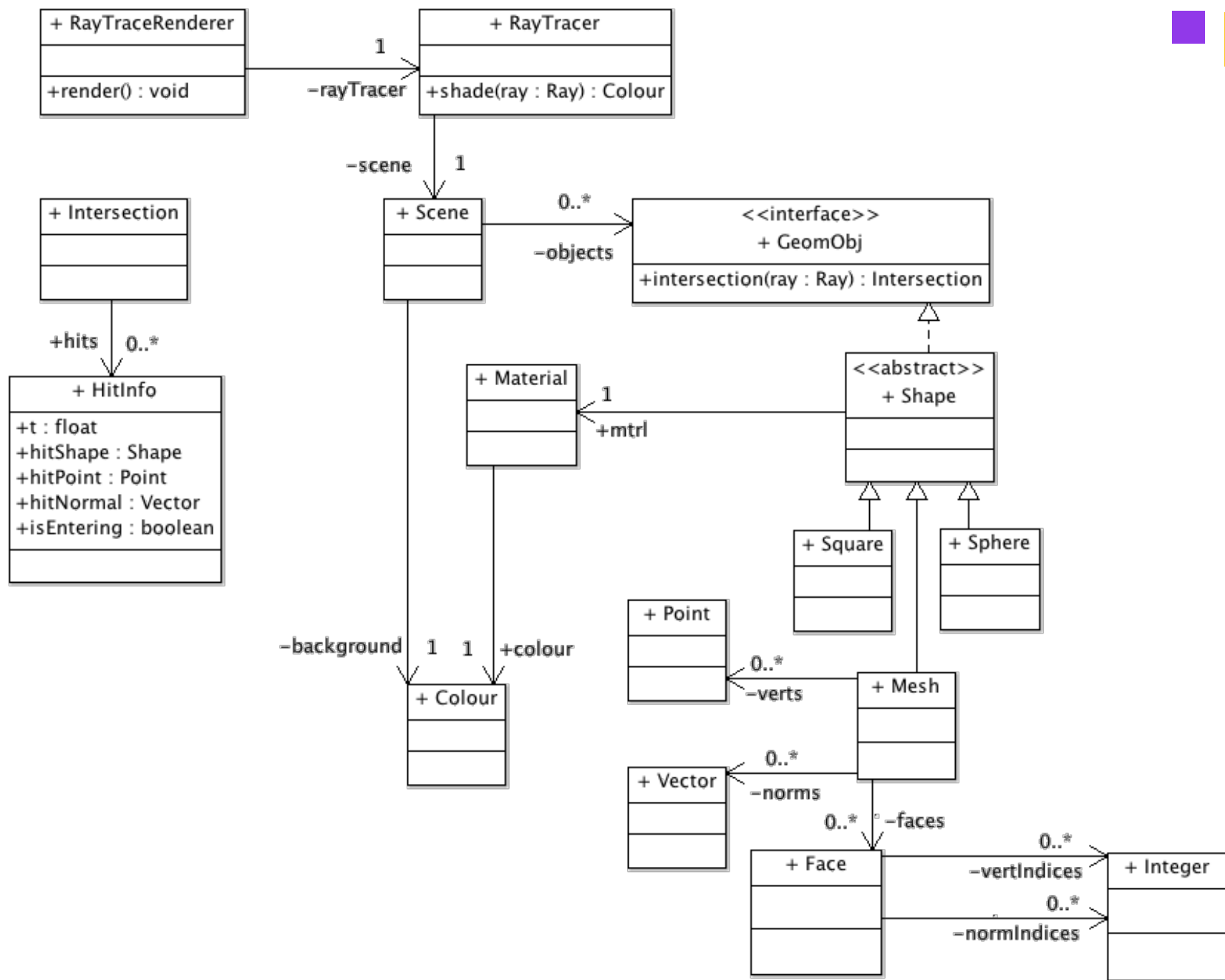
Yes!



(Simplified)
rendering framework



We also want to draw
3D objects in other colours than red, so ...





Questions?