

Using the formula just proved,

$$d \cos u = d \sin \left( \frac{\pi}{2} - u \right) = \cos \left( \frac{\pi}{2} - u \right) d \left( \frac{\pi}{2} - u \right) = -\sin u \, du.$$

**38. Proof of IX, X, XI, and XII.** — Differentiating both sides of the equation

$$\tan u = \frac{\sin u}{\cos u}$$

and using the formulas just proved for the differentials of  $\sin u$  and  $\cos u$ ,

$$\begin{aligned} d \tan u &= \frac{\cos u \, d \sin u - \sin u \, d \cos u}{\cos^2 u} = \frac{\cos^2 u \, du + \sin^2 u \, du}{\cos^2 u} \\ &= \sec^2 u \, du. \end{aligned}$$

By differentiating both sides of the equations

$$\cot u = \frac{\cos u}{\sin u}, \quad \sec u = \frac{1}{\cos u}, \quad \csc u = \frac{1}{\sin u},$$

and using the formulas for the differentials of  $\sin u$  and  $\cos u$ , we obtain the differentials of  $\cot u$ ,  $\sec u$  and  $\csc u$ .

*Example 1.*  $y = \sin^2 (x^2 + 3)$ .

Since

$$\sin^2 (x^2 + 3) = [\sin (x^2 + 3)]^2,$$

we use the formula for  $u^2$  and so get

$$\begin{aligned} dy &= 2 \sin (x^2 + 3) \, d \sin (x^2 + 3) \\ &= 2 \sin (x^2 + 3) \cos (x^2 + 3) \, d (x^2 + 3) \\ &= 4 x \sin (x^2 + 3) \cos (x^2 + 3) \, dx. \end{aligned}$$

*Ex. 2.*  $y = \sec 2x \tan 2x$ .

$$\begin{aligned} \frac{dy}{dx} &= \sec 2x \frac{d}{dx} \tan 2x + \tan 2x \frac{d}{dx} \sec 2x \\ &= \sec 2x \sec^2 2x (2) + \tan 2x \sec 2x \tan 2x (2) \\ &= 2 \sec 2x (\sec^2 2x + \tan^2 2x). \end{aligned}$$

## EXERCISES

In the following exercises show that the derivatives and differentials have the values given:

$$\checkmark 1. \quad y = 2 \sin 3x + 3 \cos 2x, \quad \frac{dy}{dx} = 6 (\cos 3x - \sin 2x).$$