

Fig. 4. Damping per period for the forward and backward propagating waves as a function of the width of the flow profile, l^* . We use the same notation as in Fig. 3. In this plot $v_i/v_{Ai} = 0.02$.

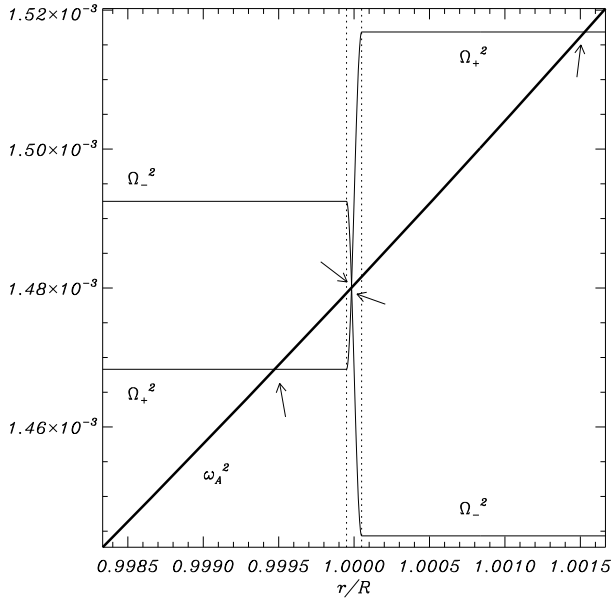


Fig. 5. Forward (Ω_+), backward (Ω_-) and local Alfvén frequency (see thick line) as a function of the radial position. The arrows indicate the location of the resonances. In this plot $v_i/v_{Ai} = 0.02$, $l/R = 0.1$ and $l^*/R = 0.0001$, $L = 100R$ and $\rho_i/\rho_e = 3$. The vertical dotted lines mark the limits of the inhomogeneity of the flow.

layer, the forward wave has two additional resonances, one at $R - l/2 < r_A < R - l^*/2$ and the second at $R + l^*/2 < r_A < R + l/2$. The backward wave still has a single resonance situated, as before, inside the velocity layer. Although the situation is more complicated, we can still understand the role of the resonances. The derivatives of Ω^2 with respect to r at the resonance inside the velocity layer, located around $r = R$ (for both the forward and backward waves), become very large (in absolute value) for small l^* , thus dominating over the derivative of ω_A^2 (see Eq. (10)). This means that the factor $|\Delta|_{r_A}$ is large and tending to infinity (for $l^* \ll l$), therefore γ will tend to zero, i.e., they will

not produce any damping. However, the other two resonances of the forward wave still behave as ordinary resonances since the derivative of the flow is zero (they are located outside the velocity layer where the flow is constant) and in this situation the total damping of the mode will be finite due to the combined contribution of the two resonances.

3.2. Linear approximation of frequency and damping time in the TTB approximation

Visual inspection of Fig. 3 shows that the damping per period varies smoothly and appears, to very good approximation, to be a linear function of v_i/v_{Ai} . This has motivated us to derive a linear approximation of the frequency ω_{kf} and the damping rate γ as function of v_i/v_{Ai} . Actually, it turns out that

$$x = \frac{kv_i}{\omega_k}, \quad (14)$$

is a more convenient variable for obtaining the linear approximation. Since

$$x = \frac{1}{\sqrt{2}} \left(\frac{\rho_i + \rho_e}{\rho_i} \right)^{1/2} \frac{v_i}{v_{Ai}}, \quad (15)$$

it follows that $v_i/v_{Ai} \ll 1$ is equivalent to $x \ll 1$.

The first order approximation to ω_{cm} (given by Eq. (2)) is

$$\omega_{cm} = \omega_k, \quad (16)$$

so that the linear approximation to ω_{k+} (for the forward wave) is

$$\omega_{kf+} = \omega_k \left(1 + x \frac{\rho_i}{\rho_i + \rho_e} \right). \quad (17)$$

In order to derive a linear approximation to γ we need some intermediate results to be used in Eq. (8). The linear approximation to quantity $\rho_i(\Omega_i^2 - \omega_{A,i}^2)^2$ is

$$\rho_i(\Omega_i^2 - \omega_{A,i}^2)^2 = \frac{\rho_i^2 \rho_e^2}{(\rho_i + \rho_e)^2} (\omega_{A,e}^2 - \omega_{A,i}^2)^2 \left\{ 1 - 4 \frac{\omega_k^2}{\omega_{A,e}^2 - \omega_{A,i}^2} x \right\}, \quad (18)$$

and the linear approximation to $\rho(r_A)\Delta$ can be written as

$$\rho(r_A)\Delta = \rho(r_A)\omega_k^2 \left\{ \frac{x}{l_v} - \frac{(1+y)}{l_\rho} \right\}. \quad (19)$$

Here l_ρ and l_v are the length scales of variation of density ρ and velocity v_i , respectively. They are defined as

$$\frac{1}{l_\rho} = \frac{1}{\rho(r_A)} \left| \frac{d\rho}{dr} \right|_{r_A}, \quad (20)$$

$$\frac{1}{l_v} = \frac{1}{v(r_A)} \left| \frac{dv}{dr} \right|_{r_A}. \quad (21)$$

Note that l_ρ and l_v are not equal to the width of the non-uniform layer of density l nor to the width of the non-uniform layer of velocity l^* . E.g, for the sinusoidal profile $v(r_A) = v_i/2$ (since we assume that $r_A = R$) and $\rho(r_A) = (\rho_i + \rho_e)/2$, the resultant characteristic spatial scales are

$$l_\rho = \frac{l}{\pi} \frac{\rho_i + \rho_e}{\rho_i - \rho_e}, \quad l_v = \frac{l^*}{\pi}. \quad (22)$$

The quantity y in Eq. (19) is defined as

$$y = x \frac{\rho_i - \rho_e}{\rho_i + \rho_e}. \quad (23)$$