

respectively, then the relations between the components u_1, u_2, u_3 with respect to the x -system and the components $\omega_1(\xi_1, \xi_2, \xi_3), \omega_2(\xi_1, \xi_2, \xi_3), \omega_3(\xi_1, \xi_2, \xi_3)$ with respect to the new ξ -system are given by the equations of transformation

$$\omega_1 = a_1 u_1 + \beta_1 u_2 + \gamma_1 u_3$$

$$\omega_2 = a_2 u_1 + \beta_2 u_2 + \gamma_2 u_3$$

$$\omega_3 = a_3 u_1 + \beta_3 u_2 + \gamma_3 u_3$$

and

$$u_1 = a_1 \omega_1 + a_2 \omega_2 + a_3 \omega_3$$

$$u_2 = \beta_1 \omega_1 + \beta_2 \omega_2 + \beta_3 \omega_3$$

$$u_3 = \gamma_1 \omega_1 + \gamma_2 \omega_2 + \gamma_3 \omega_3$$

respectively. (Cf Chap I, p 6) The components $\omega_1, \omega_2, \omega_3$ in the new system thus arise from the introduction of the new variables and the simultaneous transformation of the functions representing the components in the old system.

When in physical applications each point of a portion of space has assigned to it a definite value of a function $u = f(x_1, x_2, x_3)$, such as the density at the point, and we wish to emphasize that the property is not a component of a vector, but on the contrary is a property which retains the same value although the co-ordinate system is altered, we say that the function is a *scalar function* or *scalar*, or, if we wish to emphasize the association between the values of the function and the points of the portion of space, we speak of a *scalar field*. Thus for every vector field \mathbf{u} the quantity $|\mathbf{u}|^2 = u_1^2 + u_2^2 + u_3^2$ is a scalar, for it represents the square of the length of the vector and therefore retains the same value independently of the co-ordinate system to which the components of the vector are referred.

In the examples above the vector field \mathbf{u} is given us to begin with, and its components with respect to any system of rectangular co-ordinates are therefore determined. If, conversely, in a definite co-ordinate system, say an x -system, there are given three functions $u_1(x_1, x_2, x_3), u_2(x_1, x_2, x_3), u_3(x_1, x_2, x_3)$, these three functions define a vector field with respect to that system, the components of the field being given by the three functions. To obtain the expressions for the components $\omega_1, \omega_2, \omega_3$ in any other system we have only to apply the equations of transformation deduced above.