2/6 The Trigonometric Functions (3.5)

Derivatives of sine and cosine functions

The derivative of the sine function is $\frac{d}{dx}\left(\sin(x)\right) = \lim\limits_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} =$ $\lim\limits_{h \to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h} = \lim\limits_{h \to 0} \sin(x) \cdot \frac{\cos(h) - 1}{h} + \lim\limits_{h \to 0} \cos(x) \cdot \frac{\sin(h)}{h}$.

We can estimate $\lim\limits_{h \to 0} \frac{\cos(h) - 1}{h} \approx \frac{\cos(.001) - 1}{.001} \approx -.0005$ or about 0. We can also estimate $\lim\limits_{h \to 0} \frac{\sin(h)}{h} \approx \frac{\sin(.001)}{.001} \approx .9999998$ or about 1. Then $\frac{d}{dx}\left(\sin(x)\right) \approx$ $\sin(x) \cdot 0 + \cos(x) \cdot 1 = \cos(x)$. This formula is only valid when we measure *x* in radians!

Similarly, we can show that $\frac{d}{dx}[\cos(x)] = -\sin(x)$.

FIND $\frac{d^2}{dx^2}[\cos(x)]$.

Derivative of tangent.

FIND $\frac{d}{dx}[\tan(x)]$. Since $\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\frac{d}{dx}[\tan(x)] = \frac{d}{dx}[\frac{\sin(x)}{\cos(x)}]$. Hint: Use the Quotient Rule.

Derivatives of products and quotients of trigonometric functions

SOLVE 3.5.9.

FIND $\frac{d}{dx}\left(\frac{1 - \sin x}{1 - \cos x}\right)$.

Derivatives of trigonometric functions using the Chain Rule

SOVLE 3.5.10.

SOLVE 3.5.24.

SOLVE 3.5.37.

SOLVE 3.5.44. (a) $\frac{d}{dt}[5 + 4.9\cos(\frac{\pi}{6}t)] = 4.9(\frac{\pi}{6})(-\sin[\frac{\pi}{6}t]) = -\frac{4.9\pi}{6}\sin(\frac{\pi}{6}t)$.
The derivative represents the rate of change of the depth of the water in feet/hour.
(b) $\frac{dy}{dt} = 0 = -\frac{4.9\pi}{6}\sin(\frac{\pi}{6}t)$. Solving for *t*, we get $\sin(\frac{\pi}{6}t) = 0$, so $\frac{\pi}{6}t = k\pi$ (for any integer *k*). Therefore, $t = 6k$ or $t = 6,\ 12,\ 18,\ 24,\ \ldots$ hours. In other words, at 6 AM, 12 AM, 6 PM and 12 PM daily, the depth of the water is not changing at that moment in time. Therefore, the water level has either just finished rising to a maximum height (high tide) or fallen to a minimum height (low tide).