

Hence by the definition of  $\lambda$  and  $\mu$

$$f_w + \lambda \phi_w + \mu \psi_w = 0.$$

Similarly, if we multiply the equations

$$\phi_v + \phi_z \frac{\partial z}{\partial y} + \phi_t \frac{\partial t}{\partial y} = 0$$

and

$$\psi_v + \psi_z \frac{\partial z}{\partial y} + \psi_t \frac{\partial t}{\partial y} = 0$$

by  $\lambda$  and  $\mu$  respectively and add them to the equation

$$f_v + f_z \frac{\partial z}{\partial y} + f_t \frac{\partial t}{\partial y} = 0,$$

we obtain the further equation

$$f_v + \lambda \phi_v + \mu \psi_v = 0$$

We thus arrive at the following result.

*If the point  $(\xi, \eta, \zeta, \tau)$  is an extreme point of  $f(x, y, z, t)$  subject to the subsidiary conditions*

$$\begin{aligned}\phi(x, y, z, t) &= 0, \\ \psi(x, y, z, t) &= 0,\end{aligned}$$

*and if at that point  $\frac{\partial(\phi, \psi)}{\partial(z, t)}$  is not zero, then two numbers  $\lambda$  and  $\mu$  exist such that at the point  $(\xi, \eta, \zeta, \tau)$  the equations*

$$\begin{aligned}f_w + \lambda \phi_w + \mu \psi_w &= 0, \\ f_v + \lambda \phi_v + \mu \psi_v &= 0, \\ f_z + \lambda \phi_z + \mu \psi_z &= 0, \\ f_t + \lambda \phi_t + \mu \psi_t &= 0,\end{aligned}$$

*and also the subsidiary conditions, are satisfied*

These last conditions are perfectly symmetrical. Every trace of emphasis on the two variables  $x$  and  $y$  has disappeared from them, and we should equally well have obtained them if, instead of assuming that  $\frac{\partial(\phi, \psi)}{\partial(z, t)} \neq 0$ , we had merely assumed that any one of the Jacobians  $\frac{\partial(\phi, \psi)}{\partial(x, y)}$ ,  $\frac{\partial(\phi, \psi)}{\partial(x, z)}$ ,  $\dots$ ,  $\frac{\partial(\phi, \psi)}{\partial(z, t)}$  did not