Tomasz Prytuła: what is a quasi-isometry?

Context

Def: A map $\ell : (X, d_X) \longrightarrow (Y, d_Y)$ (metric spaces)

is an isometry if: 1) $\forall x_1, x_2 \in X \quad d_Y(\ell(x_1), \ell(x_2)) = d_X(x_1, x_2)$

2) $\forall y \in Y \; \exists x \text{ s.t. } d_Y(y, \ell(x)) = 0$ (onto)

in particular every isometry is a homeomorphism.

A quasi-isometry is a relaxation to consider more general things

Def $\ell : X \to Y$ is a quasi-isometry if

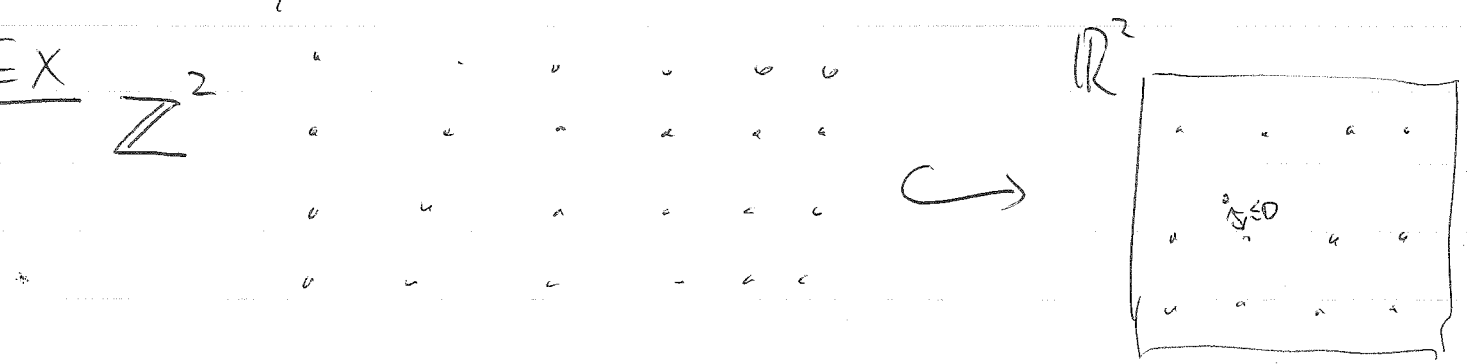
$\exists L > 0, \; D, C \geq 0$ s.t. ~~1) $\forall x_1, x_2 \in X$~~

1) $\forall x_1, x_2 \in X$ ~~$d_Y(\ell(x_1), \ell(x_2))$~~

$$\frac{1}{L} d_X(x_1, x_2) - C \leq d_Y(\ell(x_1), \ell(x_2)) \leq L \, d_X(x_1, x_2) + C$$

2) $\forall y \in Y \; \exists x \in X$ s.t. $d_Y(y, \ell(x)) \leq D$

Ex $\mathbb{Z}^2 \hookrightarrow \mathbb{R}^2$



If you look from far away they look similar.