Lemma B.2. *Let $\pi\colon \mathcal{Y} \to \mathcal{W}$ be a morphism of ringed topoi. Let $\mathcal{B}$ be a sheaf of $\mathcal{O}_{\mathcal{W}}$-algebras. Let $\mathcal{W}'$ and $\mathcal{Y}'$ be the ringed topoi $(\mathcal{W}, \mathcal{B})$ and $(\mathcal{Y}, \pi^*\mathcal{B})$, respectively. There is an induced 2-commutative diagram of ringed topoi:*

$$\begin{tikzcd} \mathcal{Y}' \arrow[r, "\pi'"] \arrow[d, "j'"'] & \mathcal{W}' \arrow[d, "j"] \\ \mathcal{Y} \arrow[r, "\pi"] & \mathcal{W}. \end{tikzcd}$$

If π and j are tor-independent and $\mathcal{N} \in \mathsf{D}(\mathcal{W}')$, then there is a natural isomorphism:

$$\mathsf{L}\pi^*\mathsf{R}j_*\mathcal{N} \simeq \mathsf{R}j'_*\mathsf{L}\pi'^*\mathcal{N}.$$

In particular, if $\mathcal{Q} \in \langle \mathsf{R}j_*\mathsf{D}^-_{\mathrm{pc}}(\mathcal{W}')\rangle$, then $\mathsf{L}\pi^*\mathcal{Q} \in \langle \mathsf{R}j'_*\mathsf{D}^-_{\mathrm{pc}}(\mathcal{Y}')\rangle$.

Proof. Now j_*, j'_* are exact and $\pi^{-1}j_* = j'_*\pi'^{-1}$. By tor-independence of π and j:

$$\begin{aligned} \mathsf{L}\pi^*\mathsf{R}j_*\mathcal{N} = \mathcal{O}_{\mathcal{Y}} \otimes^{\mathsf{L}}_{\pi^{-1}\mathcal{O}_{\mathcal{W}}} \pi^{-1}j_*\mathcal{N} &\simeq (\mathcal{O}_{\mathcal{W}} \otimes^{\mathsf{L}}_{\pi^{-1}\mathcal{O}_{\mathcal{Y}}} \pi^{-1}j_*\mathcal{O}_{\mathcal{Y}'}) \otimes^{\mathsf{L}}_{\pi^{-1}j_*\mathcal{O}_{\mathcal{Y}'}} \pi^{-1}j_*\mathcal{N} \\ &\simeq j'_*\mathcal{O}_{\mathcal{Y}'} \otimes^{\mathsf{L}}_{\pi^{-1}j_*\mathcal{O}_{\mathcal{Y}'}} \pi^{-1}j_*\mathcal{N} \\ &\simeq j'_*\mathcal{O}_{\mathcal{Y}'} \otimes^{\mathsf{L}}_{j'_*\pi'^{-1}\mathcal{O}_{\mathcal{W}'}} j'_*\pi'^{-1}\mathcal{N} \\ &\simeq j'_*(\mathcal{O}_{\mathcal{Y}'} \otimes^{\mathsf{L}}_{\pi'^{-1}\mathcal{O}_{\mathcal{W}'}} \pi'^{-1}\mathcal{N}) = \mathsf{R}j'_*\mathsf{L}\pi'^*\mathcal{N}. \end{aligned}$$

The latter claim follows from a similar argument to that in Lemma B.1. □

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