

Using V, with $u = x^2 + 1$, $v = x^2 - 1$,

$$\begin{aligned} dy &= \frac{(x^2 - 1) d(x^2 + 1) - (x^2 + 1) d(x^2 - 1)}{(x^2 - 1)^2} \\ &= \frac{(x^2 - 1) 2x dx - (x^2 + 1) 2x dx}{(x^2 - 1)^2} \\ &= -\frac{4x dx}{(x^2 - 1)^2}. \end{aligned}$$

Ex. 5. $y = \sqrt{x^2 - 1}$.

Using VI, with $u = x^2 - 1$,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (x^2 - 1)^{\frac{1}{2}} = \frac{1}{2} (x^2 - 1)^{-\frac{1}{2}} \frac{d}{dx} (x^2 - 1) \\ &= \frac{1}{2} (x^2 - 1)^{-\frac{1}{2}} (2x) = \frac{x}{\sqrt{x^2 - 1}}. \end{aligned}$$

Ex. 6. $x^2 + xy - y^2 = 1$.

We can consider y a function of x determined by the equation. Then

$$d(x^2) + d(xy) - d(y^2) = d(1) = 0,$$

that is,

$$2x dx + x dy + y dx - 2y dy = 0,$$

$$(2x + y) dx + (x - 2y) dy = 0.$$

Consequently,

$$\frac{dy}{dx} = \frac{2x + y}{2y - x}.$$

Ex. 7. $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$.

In this case

$$dx = dt - \frac{dt}{t^2}, \quad dy = dt + \frac{dt}{t^2}.$$

Consequently,

$$\frac{dy}{dx} = \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = \frac{t^2 + 1}{t^2 - 1}.$$

Ex. 8. Find an approximate value of $y = \left(\frac{1-x}{1+x}\right)^{\frac{1}{3}}$ when $x = 0.2$.