

$$\begin{aligned}\therefore \text{semi-area} &= \frac{c(c^2 - a^2)}{4a} \sin^{-1} \frac{\sqrt{c^2 - a^2}}{\sqrt{c^2 - a^2}} \\ &= \frac{c(c^2 - a^2)}{4a} \frac{\pi}{2}\end{aligned}$$

$$\text{area circle} = \frac{a}{2} 2\pi b = \pi ab,$$

$$\begin{aligned}\text{area between epicycloid and circle} &= \frac{c(c^2 - a^2)}{4a} \pi - \pi ab \\ &= \frac{(a + 2b)4b(a + b)}{4a} \pi - \pi ab \\ &= \frac{\pi}{a} (a^2 b + 3ab^2 + 2b^3 - a^2 b) \\ &= \pi b^2 \left(3 + \frac{2b}{a} \right).\end{aligned}$$

(18.) Find the length of the curve where

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}},$$

$$\begin{aligned}s &= \int \sqrt{1 + \frac{dy^2}{dx^2}} = \int \sqrt{1 + \frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}}} \\ &= \int \sqrt{\frac{x^{\frac{2}{3}} + y^{\frac{2}{3}}}{x^{\frac{2}{3}}}} = \int \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}} = \frac{3}{2} a^{\frac{1}{3}} x^{\frac{2}{3}}\end{aligned}$$

Taking it between the limits $x = 0$, $x = a$,

$$s = \frac{3}{2} a.$$

The whole length of the curve $4 \times \frac{3}{2} a = 6a$.