

$$\text{Let } \frac{x^2}{(x^2 + 1)(x^2 + 4)} = \frac{A}{(x^2 + 1)} + \frac{B}{(x^2 + 4)}$$

$$x^2 = A(x^2 + 4) + B(x^2 + 1).$$

$$\text{Let } x = \sqrt{-1}, \text{ or } x^2 = -1,$$

$$\therefore -1 = 3A, \quad \therefore A = -\frac{1}{3},$$

$$\therefore x^2 + \frac{1}{3}(x^2 + 4) = B(x^2 + 1),$$

$$\therefore \frac{4(x^2 + 1)}{3} = B(x^2 + 1), \quad \therefore B = \frac{4}{3}$$

$$\int \frac{x^2 dx}{(x^2 + 1)(x^2 + 4)} = -\frac{1}{3} \int \left\{ \frac{dx}{(x^2 + 1)} - \int \frac{4 dx}{(x^2 + 4)} \right\}$$

$$= \frac{1}{3} \int \left\{ \frac{dx}{(1 + x^2)} - \frac{dx}{\left(1 + \frac{x^2}{4}\right)} \right\}$$

$$= \frac{1}{3} \left\{ 2 \tan^{-1} \frac{x}{2} - \tan^{-1} x \right\}.$$

$$\text{Or thus, } \int \frac{x^2 dx}{(x^2 + 1)(x^2 + 4)} = \frac{1}{3} \int \frac{3x^2 dx}{(x^2 + 1)(x^2 + 4)}$$

$$= \frac{1}{3} \int \frac{\{4x^2 + 4 - (x^2 + 4)\} dx}{(x^2 + 1)(x^2 + 4)}$$

$$= \frac{1}{3} \int \left(\frac{4 dx}{(x^2 + 4)} - \frac{dx}{x^2 + 1} \right)$$

$$= \frac{1}{3} \left(2 \tan^{-1} \frac{x}{2} - \tan^{-1} x \right)$$

$$(5.) \quad du = \frac{(3x^2 + x - 2) dx}{(x - 1)^3 (x^2 + 1)}.$$