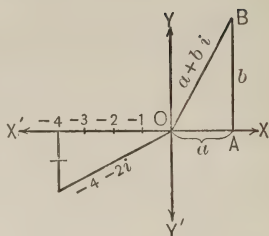


**438. Complex numbers.** If  $a$  and  $b$  are real numbers, the complex number  $a + bi$  may be represented by  $OB$ , the sum of  $a$  and  $bi$ . *I.e.* Draw  $OA = a$ , and  $AB$  equal and parallel to  $bi$ .  $OB$  represents  $a + bi$ .

*E.g.*  $OE$  represents  $-4 - 2i$ .



**439.** The **absolute value** or **modulus** of any number (*i.e.* real, pure imaginary, and complex) is the length of the line which represents the number. It is always taken as positive.

The absolute value of  $a + bi = OB$ , or  $+\sqrt{a^2 + b^2}$ .

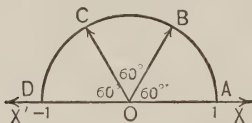
The absolute value of  $-4 - 2i = \sqrt{4^2 + 2^2} = 2\sqrt{5}$ .

**440.** The **amplitude** of  $OB$  is the angle  $XOB$ , *i.e.* the angle between  $OX$  and  $OB$ , measured from  $OX$  counter-clockwise.

**Ex. 1.** Determine the algebraic meaning of the rotation of a line through an angle of  $60^\circ$ .

Let  $OA = OB = OC = OD = 1$ ,

and  $\angle AOB = \angle BOC = \angle COD = 60^\circ$ .



If  $x$  is the number which, applied as a factor, produces the required rotation,

then  $OB = x$ ,  $OC = x^2$ ,  $OD = x^3$ .

*I.e.*  $x^3 = -1$ ,

or  $x = \sqrt[3]{-1}$ .

The rotation through an angle of  $60^\circ$  represents therefore a multiplication by  $\sqrt[3]{-1}$ , and line  $OB$  represents  $\sqrt[3]{-1}$ . A simple geometrical deduction shows that  $OB$  or  $\sqrt[3]{-1} = \frac{1}{2} + \frac{1}{2}\sqrt{3} \cdot i$ .