

If the value of $\sqrt{9}$ had not been restricted (§ 255) to its positive value, 9 would be the root. On the other hand, no positive value of \sqrt{x} can satisfy the equation, since 5 plus a positive number cannot equal 2.

Consequently equation (1) has no root, although the student should bear in mind that this is the consequence of the arbitrary exclusion of negative values of roots.

280. If $x=9$ is not a root of the equation $5 + \sqrt{x} = 2$, it becomes necessary to ascertain why the solution produced this value.

In solving an equation, we usually proceed as if the given equation were true and we had to prove the correctness of each following one; or we prove that (3) is true if (1) is true. The real problem, however, is the opposite one, viz. to prove equation (1) is true if equation (3) is true. That is, we should start from equation (3) and prove successively (2) and (1).

But the members of (2) are the square roots of the members of (3), and if two quantities are equal, their square roots are not necessarily equal, as shown by the following illustration: $(-3)^2 = (+3)^2$, while -3 does not equal $+3$. Hence (2) does not need to be true, if (3) is true.

In general, squaring the two members of an equation introduces a new root, as can be seen from the following example:

Let $x = a$.

Squaring both members, $x^2 = a^2$.

The roots of the second equation are $+a$ and $-a$, while the first one has only one root, $+a$.

281. Equivalent equations are equations which have the same roots; as $x + 4 = \sqrt{x}$ and $x = \sqrt{x} - 4$.

282. A new root which is introduced by performing the same operations on both members of an equation is called an **extraneous root**.