

stood, we may restrict ourselves to the study of the first. We shall therefore only consider affine transformations of the form

$$\begin{aligned} x' &= ax + by + cz \\ y' &= dx + ey + fz \\ z' &= gx + hy + kz \end{aligned} \quad \text{or} \quad \begin{aligned} x' &= ax + by \\ y' &= cx + dy \end{aligned}$$

with non vanishing determinants.

The results of section 3 (p. 25) for linear equations enable us to express the inverse transformation by the formulæ

$$\begin{aligned} x &= a'x' + b'y' + c'z' \\ y &= d'x' + e'y' + f'z' \\ z &= g'x' + h'y' + k'z' \end{aligned} \quad \text{or} \quad \begin{aligned} x &= a'x' + b'y' \\ y &= c'x' + d'y', \end{aligned}$$

in which  $a', b', \dots$  are certain expressions formed from the coefficients  $a, b, \dots$ . Because of the uniqueness of the solution, the original equations also follow from these latter. In particular, from  $x = y = z = 0$  it follows that  $x' = y' = z' = 0$ , and conversely.

The characteristic geometrical properties of affine transformations are stated in the following theorems

(1) *In space the image of a plane is a plane; and in the plane the image of a straight line is a straight line.*

For by section 1 (p. 9) we can write the equation of the plane (or the line) in the form

$$Ax + By + Cz + D = 0$$

$$(\text{or} \quad Ax + By + D = 0).$$

The numbers  $A, B, C$  (or  $A, B$ ) are not all zero. The co ordinates of the image points of the plane (or of the line) satisfy the equation

$$\begin{aligned} A(a'x' + b'y' + c'z') + B(d'x' + e'y' + f'z') \\ + C(g'x' + h'y' + k'z') + D = 0 \end{aligned}$$

$$(\text{or} \quad A(a'x' + b'y') + B(c'x' + d'y') + D = 0)$$

Hence the image points themselves lie on a plane (or a line), for the co efficient

$$\begin{aligned} A' &= a'A + d'B + g'O \\ B' &= b'A + e'B + h'O \\ C' &= c'A + f'B + k'O \end{aligned} \quad \left( \text{or} \quad \begin{aligned} A' &= a'A + c'B \\ B' &= b'A + d'B \end{aligned} \right)$$

of the co ordinates  $x', y', z'$  (or  $x', y'$ ) cannot all be zero, otherwise the equations

$$\begin{aligned} a'A + d'B + g'O &= 0 \\ b'A + e'B + h'O &= 0 \\ c'A + f'B + k'O &= 0 \end{aligned} \quad \left( \text{or} \quad \begin{aligned} a'A + c'B &= 0 \\ b'A + d'B &= 0 \end{aligned} \right)$$

would hold, and these we may regard as equations in the unknowns  $A, B, C$