

is no longer an "affine" transformation, but a *transformation to general curvilinear co-ordinates*

We again assume that when (x, y) ranges over a region R of the xy -plane the corresponding point (ξ, η) ranges over a region B of the $\xi\eta$ -plane, and also that for each point of B the corresponding (x, y) in R can be uniquely determined, in other words, that the transformation is one-to-one. The inverse transformation we again denote by $x = g(\xi, \eta)$, $y = h(\xi, \eta)$.

By the *co-ordinates of a point P* in a region R we can mean any number-pair which serves to specify the position of the point P in R uniquely. Rectangular co-ordinates are the simplest case of co-ordinates which extend over the whole plane. Another typical case is the system of polar co-ordinates in the xy -plane, introduced by the equations

$$\begin{aligned}\xi &= r = \sqrt{x^2 + y^2}, \\ \eta &= \theta = \arctan(y/x) \quad (0 \leq \theta < 2\pi).\end{aligned}$$

When we are given a system of functions $\xi = \phi(x, y)$, $\eta = \psi(x, y)$ as above, we can in general assign to each point $P(x, y)$ the corresponding values (ξ, η) as new co-ordinates. For each pair of values (ξ, η) belonging to the region B uniquely determines the pair (x, y) , and thus uniquely determines the position of the point P in R , this entitles us to call ξ, η the co-ordinates of the point P . The "co-ordinate lines" $\xi = \text{const}$ and $\eta = \text{const}$, are then represented in the xy -plane by two families of curves, which are defined implicitly by the equations $\phi(x, y) = \text{const}$, and $\psi(x, y) = \text{const}$, respectively. These co-ordinate curves cover the region R with a co-ordinate net (usually curved), for which reason the co-ordinates (ξ, η) are also called *curvilinear co-ordinates* in R .

We shall once again point out how closely these two interpretations of our system of equations are interrelated. The curves in the $\xi\eta$ -plane which in the mapping correspond to straight lines parallel to the axes in the xy -plane can be directly regarded as the co-ordinate curves for the curvilinear co-ordinates $x = g(\xi, \eta)$, $y = h(\xi, \eta)$ in the $\xi\eta$ -plane; conversely, the co-ordinate curves of the curvilinear co-ordinate system $\xi = \phi(x, y)$, $\eta = \psi(x, y)$ in the xy -plane in the mapping are the images of the straight lines parallel to the axes in the $\xi\eta$ -plane. Even in the interpretation of (ξ, η) as curvilinear co-ordinates in the xy -plane