

(ii) The Referee calculates $c_R = [\lambda_{AR} < \lambda_{RB}] \in \{0, 1\}$ and sends it to Bob (1 bit).

(iii) After reception of j_A , Bob calculates $\beta = \sin[\sin(\phi_B - j_A \frac{\pi}{4})]$ and $\phi'_B = (\phi_B - j_A \frac{\pi}{4} \bmod \pi) \in [0, \pi]$. He determines the index $j_B = [\frac{4}{\pi} \phi'_B] \in \{0, 1, 2, 3\}$ of the sector where his angle ϕ'_B lies, and the bit $c_B = [\lambda_{RB} < \phi'_B - j_B \frac{\pi}{4}] \in \{0, 1\}$. Depending on j_B , c_A , c_R , c_B , ϕ'_B and λ_{RB} , he outputs $b = \beta$ with probability $\wp_{c_A c_R c_B}^{j_B}(\phi'_B - \lambda_{RB})$, and $b = -\beta$ with probability $1 - \wp_{c_A c_R c_B}^{j_B}(\phi'_B - \lambda_{RB})$, for the functions $\wp_{c_A c_R c_B}^{j_B}$ defined in Table I.

As explicitly shown in the Supplemental Material [36], the above protocol gives the desired probability $P(a=b|\phi_A, \phi_B) = \sin(\frac{\phi_A - \phi_B}{2})^2$ for all possible equatorial measurements, using 4 bits of communication. It can then be extended in the following way to all measurement directions on the Bloch sphere, using a similar technique as in [10]:

Protocol 2.—For measurement directions $\mathbf{x} = (\sin\theta_A \cos\phi_A, \sin\theta_A \sin\phi_A, \cos\theta_A)$ and $\mathbf{y} = (\sin\theta_B \cos\phi_B, \sin\theta_B \sin\phi_B, \cos\theta_B)$, the three parties

(i) run Protocol 1 with input angles ϕ_A and ϕ_B ; Alice and Bob obtain intermediate outputs a_0 and b_0 .

(ii) run Protocol 1 a second time (using a new set of independent variables λ_{AR} , λ_{RB}), now with input angles $a_0\theta_A$ and $-b_0\theta_B$; Alice and Bob output the outcomes a and b of this second run of Protocol 1.

This second protocol now simulates the desired correlation $E(\mathbf{x}, \mathbf{y}) = -\mathbf{x} \cdot \mathbf{y}$ for all possible projective measurements by Alice and Bob, with 8 bits of communication; for more details on the calculations, see Supplemental Material. Note that Protocols 1 and 2 do not simulate the correct marginals. In order to randomize the marginals, Alice can—at the very end of the protocol—generate a random bit and send it to Bob; depending on the value of this bit, they will both flip their outcomes or not. All in all, the entanglement swapping correlations can thus be simulated with 9 bits of communication.

Discussion.—We thus have proved that remarkably, the entanglement swapping process can be simulated with bounded communication, even in a bilocal scenario where Alice and Bob are (as in the quantum case) completely uncorrelated before the protocol is run, and therefore do not have any prior shared randomness. Our protocol provides an upper bound on the nonlocality of entanglement swapping in terms of its communication cost. It is an open question whether fewer bits of communication are actually sufficient: it might indeed be possible to simulate equatorial measurements more efficiently than with Protocol 1, or to find a more direct protocol that does not treat separately the azimuth and zenith angles of the measurement settings, more in the spirit of the Toner-Bacon simulation protocol for singlet correlations [13].

Next, it is natural to consider the simulation of multi-stage entanglement swapping, which is essential for long

distance quantum communication. Now, N referees (R_1, R_2, \dots, R_N) are placed on a line between Alice and Bob. Two neighboring referees share a singlet state, while R_1 and R_N share singlet states with A and B , respectively; each referee performs a joint measurement, leaving at the end the particles of Alice and Bob entangled. Whereas the quantum protocol has a straightforward and nice iterative character, we were not able to find a simulation protocol with a finite amount of communication in a $(N+1)$ -locality scenario [29]. Consider for instance the case with one additional referee R_2 . Analogously to our Protocol 1, assume that Alice and R_1 share the random variable λ_{AR_1} , R_1 and R_2 share $\lambda_{R_1 R_2}$, and R_2 and Bob share $\lambda_{R_2 B}$, all uniformly and independently distributed on some interval $[0, \frac{\pi}{m}]$. After some finite communication, Bob could for instance [as in our first attempt, before Eq. (3)] output $b = a = 1$ if and only if $[\phi_A < \lambda_{AR_1}] = [\lambda_{AR_1} < \lambda_{R_1 R_2}] = [\lambda_{R_1 R_2} < \lambda_{R_2 B}] = [\lambda_{R_2 B} < \phi_B]$. This would result in the probability $P(a=b|\phi_A, \phi_B \in [0, \frac{\pi}{m}]) = \frac{m^3}{6\pi} |\phi_A - \phi_B|^3$, which scales cubically with $\phi_A - \phi_B$, and is therefore too small when ϕ_A is close to ϕ_B . It is unclear how to change the cubic scaling with finite communication. The following questions remain open: can multistage entanglement swapping be simulated with finite communication? Or can one prove, that above a certain value of N , an infinite amount of communication is necessary?

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