

On output sampling based sliding mode control for discrete time systems

Srinath Govindaswamy, Thierry Floquet, and Sarah K. Spurgeon

Abstract—This paper considers the problem of designing an output feedback sliding mode controller for discrete time systems. A minimal set of current and past outputs are identified to determine an extended output signal. This augmented system permits the design of a sliding manifold based upon output information only, which renders the sliding mode dynamics stable. It is shown that any transmission zeros of the augmented system will also be among the transmission zeros of the original plant. The class of discrete time systems which can be stabilised by output feedback sliding mode control is broadened by the proposed method. The theoretical results are constructive and are demonstrated via a motivational example which could not previously be solved by static output feedback.

I. INTRODUCTION

In continuous time, a sliding mode is generated by means of discontinuities in the control signals about a surface in the state space [27]. It is required that the discontinuity surface, usually called the sliding surface, is attained from any initial condition in a finite time interval. For an appropriately selected controller, the motion on the surface, or *sliding mode* is completely insensitive to any matched uncertainty in the system [27], [6]. In a discrete control implementation, the control signal is held constant during the sample period and hence it is not possible, in general, to attain a sliding mode which requires the control to switch at infinite frequency. As a result, the invariance properties of continuous time sliding-mode control can be lost. The obvious solution of sampling at high frequency, which will closely approximate continuous time, may not be possible for given hardware specifications. This has led to interest in the idea of discrete time sliding-mode control (DSMC). For the case of uncertain discrete systems, it is not possible to ensure the states remain on a surface within the state space and for this reason much of the early DSMC literature focused on establishing a discrete time counterpart to the (continuous time) reachability condition [21], [11], [3]. A comprehensive overview of these early developments is given in [18]. One distinctive feature is that DSMC does not necessarily require the use of a discontinuous control strategy [24]. The results presented in [24], [13] show that an appropriate choice of sliding surface, used with the equivalent control required to ensure sliding, can guarantee a bounded motion about the

surface in the presence of bounded matched uncertainty and that the use of a relay/switch in the control law is detrimental to performance.

Early contributions in sliding mode control were developed in a framework in which all the system states are available. This may not be realistic for practical engineering problems and has motivated the need for output feedback controllers. A number of algorithms have been developed for robust stabilization of uncertain systems which are based on sliding surfaces and output feedback control schemes [28], [5]. In [28], a geometric condition is developed to guarantee the existence of the sliding surface and the stability of the reduced order sliding motion. Edwards and Spurgeon derived an algorithm [5], [6] which is convenient for practical use. Based on the work in [28], some dynamic feedback sliding mode controllers have been proposed [15], [22]. In all the above output feedback sliding mode control schemes, it is an *a priori* requirement that the system under consideration is minimum phase and relative degree one and that a particular sub-system must be output feedback stabilisable [6].

Compared with continuous time sliding-mode strategies, the design problem in discrete time is much less mature. Other than early work in [23], much of the literature assumes all states are available [13], [10], [26]. Discrete sliding mode control schemes which have restricted themselves to output measurements alone have often been observer based schemes with or without disturbance estimation [17], [25]. Recent exceptions have been the work in [12] which considers both static and dynamic output feedback problems, and the discrete time versions of certain higher-order sliding-mode control schemes [1], [2].

For continuous time systems, it was shown in [9] that the relative degree condition associated with the solution of the existence problem can be weakened if a classical sliding mode observer is combined with sliding mode exact differentiators to generate additional independent output signals from the available measurements. In this paper, it will be shown that by using the output signal at the current time instant together with a limited amount of information from previous sample instants, the class of systems for which an output feedback based sliding mode controller can be developed is significantly broadened.

The paper is structured as follows. Section 2 presents the problem motivation. The existence problem is considered in Section 3 and a solution to the reachability problem is given in Section 4. A motivational example, from the class of discrete systems which could not previously be stabilised by output feedback sliding mode control, is presented to

Srinath Govindaswamy and Sarah Spurgeon are with the Instrumentation, Control & Embedded Systems Research Group, Department of Electronics, University of Kent, Canterbury CT2 7NT, UK. S.K.Spurgeon@kent.ac.uk

T. Floquet is with LAGIS UMR CNRS 8146, École Centrale de Lille, BP 48, Cité Scientifique, 59651 Villeneuve d'Ascq and with Équipe Projet ALIEN, INRIA Lille-Nord Europe, France. thierry.floquet@ec-lille.fr