1 Introduction

Various models of ordered random variables having different interpretations have been introduced and studied extensively in the literature. The most popular of these are *order statistics*, $X_{1:n} \leq \cdots \leq X_{n:n}$, obtained by arranging n random variables (rv's) $X_1, \cdots, X_n$ in non-decreasing order of magnitude. Order statistics appear in many areas of statistical theory and applications including quality control, robustness, outlier detection, and reliability analysis. If we assume that the underlying rv's $X_1, \cdots, X_n$ are independent and identically distributed (iid) and have probability density function (pdf) f, then the random vector $(X_{1:n}, \cdots, X_{n:n})$ has the joint pdf as

$$f_{X_{1:n},X_{2:n},\cdots,X_{n:n}}(x_1, x_2, \cdots, x_n) = \begin{cases} n! \prod_{i=1}^{n} f(x_i), & x_1 \leq \cdots \leq x_n, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

For more details on order statistics and their properties, interested readers may refer to the books by Arnold, Balakrishnan and Nagaraja (1992), Balakrishnan and Rao (1998a,b), Nevzorov (2001), and David and Nagaraja (2003).

Censored order statistics are generalizations of order statistics which arise naturally in life-testing experiments when the experimenter does not observe failure times of all units placed on the test. Various types of censoring schemes have been considered and discussed in the literature; see, for example, Cohen (1991) and Balakrishnan and Cohen (1991). One of them is progressive Type-II right censoring which arises as follows. Let n identical units with lifetimes $X_1, \cdots, X_n$ be placed on a life-test and at the time of the ith failure, one failed item and additional R_i remaining units are randomly withdrawn (or censored) from the experiment, $i = 1, \cdots, m$. Then, the $m = n - \sum_{i=1}^{m} R_i$ failure times so observed are referred to as *progressively Type-II right censored order statistics* with censoring scheme $(R_1, \cdots, R_m)$, and denoted by $X_{1:m:n}^{(R_1,\cdots,R_m)}, \cdots, X_{m:m:n}^{(R_1,\cdots,R_m)}$. If the underlying variables $X_1, \cdots, X_n$ are iid rv's with absolutely continuous cumulative distribution function (cdf) F and pdf f, then the joint pdf of all m progressively Type-II right censored order statistics is given by

$$\begin{aligned} &f_{X_{1:m:n}^{(R_1,\cdots,R_m)},\cdots,X_{m:m:n}^{(R_1,\cdots,R_m)}}(x_1, \cdots, x_m) \\ &\quad = \begin{cases} c \prod_{i=1}^{m} f(x_i)\{1 - F(x_i)\}^{R_i}, & x_1 \leq \cdots \leq x_m, \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{2}$$