

and the integral required is therefore

$$\begin{aligned} \log \sqrt{x^2 + 1} + \frac{1}{x^2 + 1} - \frac{1}{4(x^2 + 1)^2}, \\ = \frac{4x^2 + 3}{4(x^2 + 1)^2} + \log \sqrt{x^2 + 1}. \end{aligned}$$

$$(22.) \int \frac{x^2 dx}{x^4 + 1},$$

$$(x^4 + 1) = \left(x^2 - 2x \cos \frac{\pi}{m} + 1 \right) \left(x^2 - 2x \cos \frac{3\pi}{m} + 1 \right) \dots$$

continued to the factor $\left(x^2 - 2x \cos \frac{m-1}{m} \pi + 1 \right)$ when m is an even number.

This gives

$$(x^4 + 1) = \left(x^2 - 2x \cos \frac{\pi}{4} + 1 \right) \left(x^2 - 2x \cos \frac{3\pi}{4} + 1 \right),$$

$$\text{or } (x^4 + 1) = (x^2 - x\sqrt{2} + 1)(x^2 + x\sqrt{2} + 1).$$

$$\text{Since, } \frac{\pi}{4} = 45^\circ \cos \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \cos \frac{3\pi}{4} = -\sin \frac{\pi}{4}.$$

Assume therefore

$$\frac{x^2}{x^4 + 1} = \frac{Ax + B}{x^2 - x\sqrt{2} + 1} + \frac{Cx + D}{x^2 + x\sqrt{2} + 1};$$

$$\therefore x^2 = (Ax + B)(x^2 + x\sqrt{2} + 1) + (Cx + D)(x^2 - x\sqrt{2} + 1).$$

$$\text{If } x^2 + x\sqrt{2} + 1 = 0, \quad x = \frac{\sqrt{-1} - 1}{\sqrt{2}} \quad (1.)$$

$$\text{If } x^2 - x\sqrt{2} + 1 = 0, \quad x = \frac{\sqrt{-1} + 1}{\sqrt{2}} \quad (2.)$$