Observation 1. *Let W be a set of propositional interpretations and $U \subseteq$ At be a set of atoms. Then, for any subjective literal L with $Atoms(L) = \{a\}$:*

i) if $a \in U$, then $W \models L$ iff $W_{|U} \models L$,

ii) if $a \notin U$, then $W \models L$ iff $W_{|\overline{U}} \models L$,

Proposition 1. *Let Π be a program that accepts an epistemic splitting set $U \subseteq At$ and let W be a set of propositional interpretations. Let $W_b = W_{|U}$ and $W_t = W_{|\overline{U}}$. Then, we get*

i) $B_U(\Pi)^W = B_U(\Pi)^{W_b}$,

ii) $T_U(\Pi)^W = E_U(\Pi, W_b)^{W_t}$, and

iii) $\Pi^W = B_U(\Pi)^{W_b} \cup E_U(\Pi, W_b)^{W_t}$.

Proof. First, since every rule $r \in B_U(\Pi)$ satisfies $Atoms(Body^{sub}(r)) \subseteq U$, from Observation 1, it follows that $B_U(\Pi)^W = B_U(\Pi)^{W_b}$. Furthemore, for any program Γ, it is easy to check that $\Gamma^W = (\Gamma_U^{W_b})^{W_t}$, that is, applying the reduct w.r.t W is the same than applying it w.r.t. to its projection in U and afterwards to the remaining part. Thus, we get $T_U(\Pi)^W = (T_U(\Pi)_U^{W_b})^{W_t} = E_U(\Pi, W_b)^{W_t}$. Finally, we have that $\Pi^W = (B_U(\Pi) \cup T_U(\Pi))^W = B_U(\Pi)^W \cup T_U(\Pi)^W$ and, thus, the result holds. □

Theorem 4 (Main theorem)**.** *Semantics G91 satisfies epistemic splitting.*

Proof. Let W be some set of propositional interpretations and let $W_b = W_{|U}$ and $W_t = W_{|\overline{U}}$. By definition, W is a world view of Π if and only if $W = \mathrm{SM}[\Pi^W]$. Furthermore, since U is a modal splitting set of Π, it is easy to check that U is also a regular splitting set of the regular programa Π^W. Hence, from Corollary 1, we get that W is a world view of Π iff $W = \mathrm{SM}[\Pi^W] =$

$$\left\{ I_b \cup I_t \;\middle|\; I_b \in \mathrm{SM}[\hat{b}_U(\Pi^W)] \text{ and } I_t \in \mathrm{SM}[\hat{e}_U(\Pi^W, I_b)] \right\}$$

for some arbitrary splitting $\langle \hat{b}_U(\Pi^W), \hat{t}_U(\Pi^W) \rangle$. Note that all rules belonging to $B_U(\Pi)$ have all atoms from U. Hence, we take $\hat{b}_U(\Pi^W) \stackrel{\text{def}}{=} B_U(\Pi)^W = B_U(\Pi)^{W_b}$ (Proposition 1). Similarly, we also take $\hat{t}_U(\Pi^W) \stackrel{\text{def}}{=} T_U(\Pi)^W = E_U(\Pi, W_b)^{W_t}$. Then, we get

$$\hat{e}_U(\Pi^W, I_b) \;=\; \hat{e}_U(\hat{t}_U(\Pi^W), I_b) \;=\; \hat{e}_U(E_U(\Pi, W_b)^{W_t}, I_b)$$

Notice also that no atom occurring in $E_U(\Pi, W_b)^{W_t}$ belongs to U, which implies that $\hat{e}_U(E_U(\Pi, W_b)^{W_t}, I_b) = E_U(\Pi, W_b)^{W_t}$. Replacing above, we have that W is a world view of Π iff W is equal to

$$\{I_b \cup I_t \mid I_b \in \mathrm{SM}[B_U(\Pi)^{W_b}], I_t \in \mathrm{SM}[E_U(\Pi, W_b)^{W_t}]\}$$

iff

$$W = \{I_b \cup I_t \mid I_b \in W'_b \text{ and } I_t \in W'_t\}$$

with $W'_b = \mathrm{SM}[B_U(\Pi)^{W_b}]$ and $W'_t = \mathrm{SM}[E_U(\Pi, W_b)^{W_t}]$ iff $W = W'_b \sqcup W'_t$. Hence, it only remains to be shown that both $W_b = W'_b$ and $W_t = W'_t$ hold. Note that $I \in W_b = W_{|U}$ iff $I = I' \cap U$ for some $I' \in W$ iff $I = (I_b \cup I_t) \cap U$ for some $I_b \in W'_b$ and $I_t \in W'_t$ iff $I = (I_b \cap U) \cup (I_t \cap U)$ for some $I_b \in W'_b$ and $I_t \in W'_t$ iff $I = I_b$ for some $I_b \in W'_b$. The fact $W_t = W'_t$ follows in an analogous way. □

A similar proof can be developed to show that (Truszczyński, 2011), that generalises[4] (Gelfond, 1991) from subjective literals to subjective formulas, also satisfies epistemic splitting.

To illustrate the behaviour of other semantics with respect to splitting, we will use several examples. Let us take the program Π_6 consisting of $\{(10), (11)\}$ and the rule:

$$c \vee d \;\leftarrow\; \texttt{not}\, \mathbf{K}\, a \tag{16}$$

The set $U = \{a, b\}$ splits the program into the bottom, (10)-(11) and the top (16). The bottom has a unique world view $W_b = [\{a\}, \{b\}]$ so $\mathbf{K}\, a$ does not hold and the top is simplified as $E_U(\Pi_6, W_b)$ containing the unique rule:

$$c \vee d \leftarrow \texttt{not}\; \bot \tag{17}$$

This program has a unique world view $W_t = [\{c\}, \{d\}]$ that, combined with W_b yields $[\{a, c\}, \{b, c\}, \{a, d\}, \{b, d\}]$ as the unique solution for Π_6, for any semantics satisfying epistemic splitting (and so, also for G91). Let us elaborate the example a little bit further. Suppose we add now the constraint:

$$\bot \leftarrow c \tag{18}$$

The top must also include this rule and has now a unique stable model $W_t = [\{d\}]$, so the world view for the complete program would be $[\{a, d\}, \{b, d\}]$. Finally, let us forbid the inclusion of atom d too:

$$\bot \leftarrow d \tag{19}$$

so we consider $\Pi_7 = \{(10), (11), (16), (18), (19)\}$. This last constraint leaves the simplified top program $E_U(\Pi_6, W_b) = \{(17), (18), (19)\}$ without stable models, so epistemic splitting would yield that program Π_7 has no world view at all. This is the result we obtain, indeed, in (Gelfond, 1991, 2011)[5] and in (Truszczyński, 2011). Surprisingly, recent approaches like (Kahl et al., 2015; Fariñas del Cerro, Herzig, and Su, 2015; Shen and Eiter, 2017; Son et al., 2017) yield world view $[\{a\}]$, violating the epistemic splitting property. For instance, in the case of (Kahl et al., 2015), the reduct of Π_7 with respect to $[\{a\}]$ is the program

$$\begin{aligned} a &\;\leftarrow\; \texttt{not}\; b \\ b &\;\leftarrow\; \texttt{not}\; a \\ c \vee d &\;\leftarrow\; \texttt{not}\; a \\ \bot &\;\leftarrow\; c \\ \bot &\;\leftarrow\; d \end{aligned}$$

[4] In fact, Truszczyński (2011) defines several semantics but, among them, we refer here to the epistemic stable model semantics.

[5] These two semantics actually produce empty world views, but as we said before, we disregard them, as they just point out that the program has no solution.