

with the axes) By differentiation we see at once that these components are the derivatives of the function

$$\frac{O}{\sqrt{(\xi_1 - x_1)^2 + (\xi_2 - x_2)^2 + (\xi_3 - x_3)^2}}$$

with respect to the co ordinates x_1, x_2, x_3 respectively. The force vector apart from a constant factor is therefore the gradient of the function

$$\frac{1}{r} = \frac{1}{\sqrt{(\xi_1 - x_1)^2 + (\xi_2 - x_2)^2 + (\xi_3 - x_3)^2}}.$$

If a field of force is obtained from a scalar function by forming the gradient, this scalar function is often called the *potential function* of the field. We shall consider this concept from a more general point of view in the study of work and energy (Chapter V, p 350, and Chapter VI, pp 415, 468-81).

4. The Divergence and Curl of a Vector Field.

By differentiation we have assigned to every function or scalar a vector field, the gradient. Similarly, by differentiation we can assign to every vector field a certain scalar, known as the *divergence* of the vector field. Given a specific co-ordinate system, the x -system, we define the divergence of the vector u as the function

$$\text{div } u = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3},$$

i.e. the sum of the partial derivatives of the three components with respect to the corresponding co-ordinates. Suppose now that we change the co-ordinate system to the ξ -system. If the divergence is really to be a scalar function associated with the vector field and independent of the particular co-ordinate system, we must have

$$\text{div } u = \frac{\partial \omega_1}{\partial \xi_1} + \frac{\partial \omega_2}{\partial \xi_2} + \frac{\partial \omega_3}{\partial \xi_3},$$

where $\omega_1, \omega_2, \omega_3$ are the components of u in the ξ -system. In fact, the truth of the equation

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \frac{\partial \omega_1}{\partial \xi_1} + \frac{\partial \omega_2}{\partial \xi_2} + \frac{\partial \omega_3}{\partial \xi_3}$$

can be verified immediately by applying the chain rule and the transformation formulæ of p 84.