

(8.) To find the volume of a parabolic spindle.

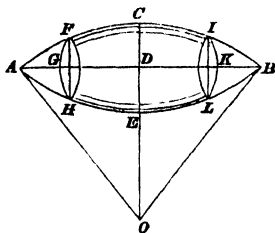
Put $CD = h$, $AB = 2c$, and

$x = AG$, and $y = FG$.

From the property of the parabola

$$AD^2 : AG \cdot GB :: CD : EG,$$

$$c^2 : x(2c - x) :: h : y,$$



$$\therefore y = \frac{h(2cx - x^2)}{c^2}, \therefore y^2 = \frac{h^2}{c^4} (4c^2x^2 - 4cx^3 + x^4),$$

$$\therefore V = \pi \int y^2 dx$$

$$= \pi \frac{h^2}{c^4} \int (4c^2x^2 - 4cx^3 + x^4) dx = \frac{\pi h^2}{c^4} \left(\frac{4c^2x^3}{3} - cx^4 + \frac{x^5}{5} \right),$$

the volume of AFH, since $C = 0$; when $x = c$, we have

$$\frac{\pi h^2}{c^4} \left(\frac{4c^5}{3} - c^5 + \frac{c^5}{5} \right) = \frac{8}{15} \pi h^2 c =$$

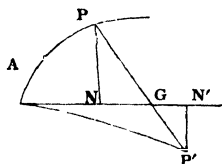
volume of the semi-spindle; and, as we have found the volume of the part AFH, if we subtract this from the whole semi-spindle, we shall have the frustum EHCF, the double of which will be volume of the whole frustum EHIL.

In the same manner as in the last example, we may find the surface of the parabolic spindle.

(9.) In a parabola find the area included between the curve, its evolute, and its radius of curvature.

$$\text{area parabola ANP} = \int y dx$$

$$= 2 \sqrt{a} \int \sqrt{x} dx = \frac{4}{3} \sqrt{a} x^{\frac{3}{2}},$$



$$\text{area evolute AN'P'} = \int \beta da = \frac{2}{3\sqrt{3a}} \int (a - 2a)^{\frac{3}{2}} da$$