

Ex. 1. Prove that 5 is a root of the equation

$$x^3 - 3x^2 - 18x + 40 = 0.$$

$$\begin{array}{r} \text{Dividing by } x - 5, \quad 1 - 3 - 18 + 40 \quad \underline{5} \\ + 5 + 10 - 40 \\ \hline 1 + 2 - 8; \quad 0 \end{array}$$

Since there is no remainder, 5 is a root of the given equation.

507. If $f(x)$ is divided by $x - a$, the remainder of the division is equal to $f(a)$. (Remainder Theorem, Chapter XVI.)

Using the notation of § 505, we have

$$(x - a)Q(x) + R = f(x).$$

$$\text{Substituting } x = a, \quad R = f(a).$$

Ex. 2. If $f(x) = x^4 - 2x^3 - 9x^2 + 2$, find $f(4)$.

Dividing $f(x)$ by $x - 4$,

$$\begin{array}{r} 1 - 2 - 9 + 0 + 2 \quad \underline{4} \\ + 4 + 8 - 4 - 16 \\ \hline 1 + 2 - 1 - 4; \quad -14 \end{array}$$

$$\text{Hence} \quad f(4) = -14.$$

508. The preceding *method of substitution* may also be demonstrated as follows:

$$\begin{array}{r} x^4 - 2x^3 - 9x^2 + 0 \cdot x + 2 \quad \underline{4} \\ + 4x^3 + 8x^2 - 4x - 16 \\ \hline x^4 + 2x^3 - x^2 - 4x; \quad -14 \end{array}$$

I.e. if $x = 4$,

$$x^4 = 4x^3, \text{ which added to the next term gives } +2x^3.$$

$$2x^3 = 8x^2, \text{ which added to the next term gives } -x^2.$$

$$-x^2 = -4x, \text{ which added to the next term gives } -4x.$$

$$-4x = -16, \text{ which added to the next term gives } -14.$$

$$\text{Therefore} \quad f(4) = -14.$$