

$j_1 \in \{2, \dots, \ell + 2\}$ with $\mu_1 \in (\widehat{Q_{\mathcal{B}}(Z)})^{T_{j_1}}_{I_{j_1-1}}$. Similar, to $\mu_2 := (\lambda_1, \lambda_2) \in \widehat{Q_{\mathcal{B}}(Z)}$, there exists $j_2 \in \{2, \dots, \ell + 2\}$ with $\mu_2 \in (\widehat{Q_{\mathcal{B}}(Z)})^{T_{j_2}}_{I_{j_2-1}}$.

Suppose $j_2 < j_1$. Since $\widehat{Q_{\mathcal{B}}(Z)}^{T_{j_2}}$ is open, by (2.5) there exist open sets $V_k \subseteq \mathbb{R}^{H_k}$ with $\lambda_k \in V_k$, $k = 1, 2$, such that $V_1 \times V_2 \uplus V_1 \uplus V_2 \subseteq \widehat{Q_{\mathcal{B}}(Z)}^{T_{j_2}} \subseteq \widehat{Q_{\mathcal{B}}(Z)}^{T_{j_1-1}}$ which contradicts our choice of j_1 .

Now suppose $j_1 = j_2$, i.e. $\mu_1, \mu_2 \in (\widehat{Q_{\mathcal{B}}(Z)})^{T_{j_1}}_{I_{j_1-1}}$. By Lemma 3.4 the space $(\widehat{Q_{\mathcal{B}}(Z)})^{T_{j_1}}_{I_{j_1-1}}$ is Hausdorff in its relative topology, which is impossible because we have $\mu_1 \in W_2 \cap W_1$ for any choice of open sets $W_k \subseteq \widehat{Q_{\mathcal{B}}(Z)}$ with $\mu_k \in W_k$. Consequently, we have $j_1 < j_2$, and iterating this procedure leads to a sequence $2 \leq j_1 < \dots < j_{k_0} \leq \ell + 2$ which gives $\ell \geq k_0 - 1$, and completes the proof in the case $k_0 = n$. On the other hand, if $k_0 < n$ note that by Lemma 3.4 we have

$$\psi\left(\biguplus_{k=1}^{k_0} (\widehat{Q_{\mathcal{B}}(Z)})^{T_{j_k}}_{I_{j_k-1}}\right) = \biguplus_{k=1}^{k_0} T_{j_k-1} \subseteq \biguplus_{j=1}^{\ell+1} T_j$$

where each T_{j_k} corresponds to a μ_k , i.e. to an infinite-dimensional irreducible representation. Since $\widehat{Q_{\mathcal{B}}(Z)}$ also has one-dimensional irreducible representations by Theorem 2.1 and (2.4), we have $k_0 + 1 \leq \ell + 1$, i.e. the length of $Q_{\mathcal{B}}(Z)$ is at least k_0 . \square

Of course, a combination of Lemma 3.1, Lemma 3.2, and Lemma 3.5 completes the proof of Theorem 1.1.

It remains to prove Proposition 1.2. Recall that the composition series (1.1) is called *stratified* if for any irreducible representation π of \mathcal{B} with $\pi(\mathcal{J}_k) \neq \{0\}$ we have $\pi(\mathcal{J}_{k+1}) \neq \pi(\mathcal{J}_k)$ [13, Definition 0.3]. Moreover, a solving, stratified composition series coincides with the maximal radical series by [13, Proposition 0.4]. If we abbreviate $\ell := \text{length } Q_{\mathcal{B}}(Z)$, then the series

$$(3.1) \quad Q_{\mathcal{B}}(Z) \supseteq \mathcal{J}_{\ell+1} \supseteq \mathcal{J}_{\ell} \supseteq \dots \supseteq \mathcal{J}_1 = \{0\}$$

with \mathcal{J}_k given by (2.3) is solving of minimal length by Theorem 1.1 and the proof of Lemma 3.2. Using the description (2.4) of the irreducible representations of $Q_{\mathcal{B}}(Z)$ and the compatibility conditions (2.1) and (2.2), it is straightforward to check that the series (3.1) is stratified. Thus, it is the maximal radical series for $Q_{\mathcal{B}}(Z)$, hence, an application of Lemma 3.1 completes the proof of Proposition 1.2.

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