

denoting differentiation with respect to  $s$ . Using the relations  $x^2 = 1$ ,  $\dot{x}x = 0$ , we obtain the equations  $(y-x)x = 0$ ,  $(y-x)x = 1$ ,  $(y-x)x = 0$ . Hence we have  $y-x = \frac{[xx]}{[xx]x}$ .

5. Cf. Ex. 3 and also Ex. 5, p. 19.

7. From the definitions of  $\xi_1, \xi_2, \xi_3$  we have  $\xi_1 = x$ ,  $x^2 = 1$ ,  $\xi_2 = x/l$ ,  $\xi_3 = [\xi_1 \xi_2]$ ,  $\pm \sqrt{\xi_3^2} = 1/\tau$ . Obviously  $\xi_1 = k\xi_2$ . To determine  $\xi_2, \xi_3$ , we calculate their components with respect to a rectangular coordinate system  $O\xi_1, O\xi_2, O\xi_3$ . From the relations

$$\xi_2^2 = 1, \xi_3^2 = 1, \xi_1\xi_2 = \xi_2\xi_3 = \xi_3\xi_1 = 0$$

we obtain by differentiation

$$\xi_3\xi_1 = -\xi_1\xi_3 = 0, \xi_3\xi_2 = 0;$$

hence  $\xi_3$  is perpendicular both to  $\xi_1$  and to  $\xi_2$ , and therefore

$$\xi_3 = \pm \sqrt{(\xi_1^2)}\xi_2 = \pm \xi_2/\tau.$$

We define the sign of  $\tau$  so as to give  $\xi = -\xi_2/\tau$ . This implies that  $\tau$  is positive or negative according as the screw defined by the motion of the osculating plane in the direction of increasing  $s$  is right-handed or left-handed. To prove the second formula, note that

$$\xi_2\xi_1 = -\xi_1\xi_2 = -k, \xi_2\xi_3 = 0, \xi_2\xi_3 = -\xi_3\xi_2 = 1/\tau.$$

8. Use Ex. 6 and Ex. 3: (a)  $k\xi_2 - k^2\xi_1 + \frac{k}{\tau}\xi_3$ , (b)  $\frac{k}{k^2\tau}\xi_3 + \frac{\xi_2}{\tau}$

9.  $1/|\tau| = \sqrt{\xi_3^2} = 0$ , hence  $\xi_3$  is a constant vector  $\eta$ , say;  $\tau\eta = \xi_1\eta = \xi_2\eta = 0$ , so that  $x\eta = \text{const.}$ , where  $\eta$  is a fixed vector. That is, the curve lies in a fixed plane.

10. (b) If the curve is given by  $x = f(t)$ ,  $y = g(t)$ ,  $z = h(t)$ , the surface has the parametric equations

$$\begin{aligned}x &= f(t) + sf'(t) \\y &= g(t) + sg'(t) \\z &= h(t) + sh'(t),\end{aligned}$$

then express  $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}$  in terms of the derivatives with respect to  $t$  and  $s$ .

Appendix, § 1, p. 100

1. (a) As  $R$  is closed, there is a point  $B$  in  $R$  whose distance from  $A$  is less than that of any other point in  $R$ . Let  $n$  be the normal to  $AB$  at  $B$ . Then no point  $C$  in  $R$  lies on the same side of  $n$  as  $A$ , for otherwise not only  $B$  and  $C$ , but the whole segment  $BC$ , would belong to  $R$ , and on this segment there would be points nearer to  $A$  than  $B$  is. Hence the parallel to  $n$  through  $A$  cannot meet  $R$ .