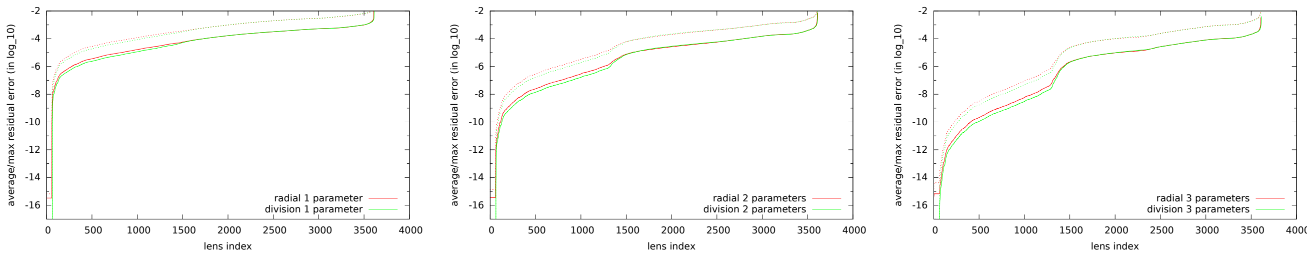


Fig. 2: Comparison of infinitely differentiable radial and division models with few parameters, used for correction. From left to right: one, two, and three parameters in the models. The plain curve represents the average residual error and the dashed curve the maximum error.

but could be performed on any distortion model obtained by blind correction. This method requires a highly textured planar pattern, which is obtained by printing a textured image and pasting it on a very flat object (a thick, heavy and rigid aluminium plate was used in the experiments). Another option is to use a high density grid pattern as in [18]. Two photos of the pattern were taken by a same camera. The distortion is estimated (up to a homography) as the diffeomorphism mapping the original digital pattern to its photograph. The algorithm is summarized below.

1) Take two slightly different photographs of a textured planar pattern (Fig. 4) with a camera whose settings are manually fixed (disable automatic mode);
2) apply the SIFT method [20] between the original digital pattern and two photographs respectively, to obtain matching pairs of points;
3) triangulate and interpolate the SIFT matches from the digital image to two photographs respectively;
4) use a loop validation to eliminate the outlier matches from the digital pattern to one of the two photographs;
5) use a vector filter to remove the few remaining outliers matches from the digital pattern to that photograph;
6) refine the precision of the SIFT matching by correcting each matching point in one image with the local homography estimated from its neighboring matching points;
7) triangulate and interpolate the refined inlier matches to get a dense reverse distortion field from the digital pattern to that photograph;
8) apply the reverse distortion field to any image produced by the same camera to correct the distortion.

For more details please refer to [11].

(a) (b) (c)

Fig. 4: (a) digital texture pattern. (b) and (c) two similar photographs of the flat pattern.

The matching pairs delivered by step 6 (about 8000 in our experiments) in the algorithm above do not contain gross outliers and are precise thanks to the local filtering. So we could directly try all the models to fit these matchings. The residual fitting error shows to what extent the models are faithful to a real camera distortion. Under the assumption that the textured pattern is flat, the mapping from the digital pattern to the photo can be modeled as $\mathbf{S}\mathcal{D}\mathbf{H}$, with $\mathbf{H}$ the homography from the digital pattern to the photo, $\mathcal{D}$ the non-linear lens distortion and $\mathbf{S}$ a diagonal matrix to model the slant of the CCD matrix:

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{24}$$

Since the polynomial and the rational function models can simulate well $\mathbf{H}$ and $\mathbf{S}$, we can use these models to simulate the distortion field without explicitly estimating the homography. Nevertheless, for the radially symmetric models, it is necessary to take into account $\mathbf{H}$ and $\mathbf{S}$ when approximating the distortion. Indeed $\mathbf{H}$ and $\mathbf{S}$ are generally not radially symmetric. Thus, we have 9 additional parameters to estimate, besides the parameters of radially symmetric distortion models and their distortion center. The radial+tangential model is also tried with $\mathbf{H}$ and $\mathbf{S}$ (with the distortion center fixed at the center of image). The polynomial model can again be solved linearly. For all the other models, an incremental LM minimization was used to estimate the distortion center, the distortion parameters, $\mathbf{H}$ and $\mathbf{S}$. The matrix $\mathbf{H}$ was initialized as the homography linearly estimated from the digital pattern to the photo and $s$ is initialized at 1. The other parameters were initialized in the same way as we did for the synthetic tests. We worked in the normalized image domain to avoid possible numerical problems.

Half the matching pairs were used to estimate the parameters for the different models, and the other half to evaluate the average fitting error. We tried two cameras: a Canon EOS 30D with EF-S 18–55mm lens and a Canon EOS 40D with EF 24–70mm lens. Both extreme focal lengths were tested: 18mm and 55mm for EF-S 18–55mm lens, 24mm and 70mm for EF 24–70mm lens. The results are recapitulated in Table IV. They show that by combining $\mathbf{H}$ and $\mathbf{S}$ to model the inclination between the camera and the pattern, all of the radially symmetric models give almost the same fitting error, which becomes stable with the increase of the model order. The similar estimation of $\mathbf{H}$ and $\mathbf{S}$ indicates that the minimization process is stable. The radial+tangential model is not always better than the radially symmetric models.