

EXERCISE 172

Solve by Cardan's method:

- | | |
|--------------------------|-----------------------------------|
| 1. $x^3 - 3x - 2 = 0.$ | 8. $x^3 - 72x - 280 = 0.$ |
| 2. $x^3 - 9x + 28 = 0.$ | 9. $x^3 - 6x + 9 = 0.$ |
| 3. $x^3 - 18x - 35 = 0.$ | 10. $x^3 + 9x - 26 = 0.$ |
| 4. $x^3 - 36x - 91 = 0.$ | 11. $x^3 - 9x - 28 = 0.$ |
| 5. $x^3 - 3x + 2 = 0.$ | 12. $x^3 - 72x + 280 = 0.$ |
| 6. $x^3 - 27x - 54 = 0.$ | 13. $x^3 - 6x^2 - 12x + 112 = 0.$ |
| 7. $x^3 + 9x + 26 = 0.$ | 14. $x^3 + 5x^2 + 8x + 6 = 0.$ |

SOLUTION OF BIQUADRATICS

594. A biquadratic is an equation of the fourth degree.

595. Descartes' solution of the biquadratic.

$$\text{Let } x^4 + px^2 + qx + r = 0. \quad (1)$$

$$\begin{aligned} \text{Assume } x^4 + px^2 + qx + r &= (x^2 + lx + m)(x^2 - lx + n) \\ &= x^4 + (-l^2 + m + n)x^2 - l(m - n)x + mn. \end{aligned} \quad (2)$$

Equating the coefficients, we have

$$-l^2 + m + n = p, \quad -l(m - n) = q, \quad mn = r. \quad (3)$$

Considering that $(m + n)^2 - (m - n)^2 - 4mn = 0$, it is easy to eliminate m and n . We obtain

$$l^6 + 2pl^4 + (p^2 - 4r)l^2 - q^2 = 0. \quad (4)$$

Equation (4) is a cubic in l^2 , which has always one real positive root. When l is known, the values of m and n are determined from equation (3). The solutions of the two equations

$$x^2 + lx + m = 0$$

and

$$x^2 - lx + n = 0$$

produce the required roots.