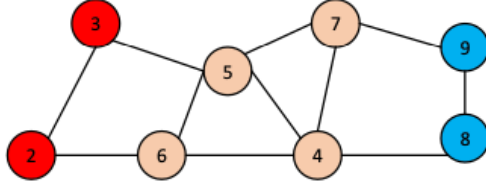


1. 利用模块度的定义 (公式 13.1) 计算下图中划分的模块度。

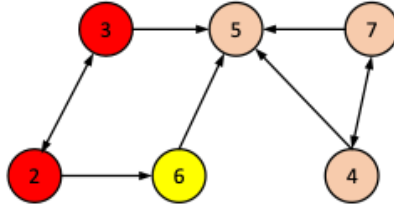


$$m = 11 \quad k_2 = k_3 = k_8 = k_9 = 2 \quad k_4 = k_5 = 4 \quad k_6 = k_7 = 3$$

$$\begin{aligned} Q &= \frac{1}{2m} \left[\left(0 - \frac{k_2^2}{2m}\right) + 2\left(1 - \frac{k_2 k_3}{2m}\right) + \left(0 - \frac{k_3^2}{2m}\right) + \left(0 - \frac{k_4^2}{2m}\right) + 2\left(1 - \frac{k_4 k_5}{2m}\right) + 2\left(1 - \frac{k_4 k_6}{2m}\right) + 2\left(1 - \frac{k_4 k_7}{2m}\right) \right. \\ &\quad \left. + \left(0 - \frac{k_5^2}{2m}\right) + 2\left(1 - \frac{k_5 k_6}{2m}\right) + 2\left(1 - \frac{k_5 k_7}{2m}\right) + \left(0 - \frac{k_6^2}{2m}\right) + \left(0 - \frac{k_7^2}{2m}\right) + \left(0 - \frac{k_8^2}{2m}\right) + 2\left(1 - \frac{k_8 k_9}{2m}\right) + \left(0 - \frac{k_9^2}{2m}\right) \right] \\ &= \frac{1}{22} \left[\left(0 - \frac{4}{22}\right) + 2\left(1 - \frac{4}{22}\right) + \left(0 - \frac{4}{22}\right) + \left(0 - \frac{16}{22}\right) + 2\left(1 - \frac{16}{22}\right) + 2\left(1 - \frac{12}{22}\right) + 2\left(1 - \frac{12}{22}\right) + \left(0 - \frac{16}{22}\right) \right. \\ &\quad \left. + 2\left(1 - \frac{12}{22}\right) + 2\left(1 - \frac{12}{22}\right) + \left(0 - \frac{9}{22}\right) + \left(0 - \frac{9}{22}\right) + \left(0 - \frac{4}{22}\right) + 2\left(1 - \frac{4}{22}\right) + \left(0 - \frac{4}{22}\right) \right] \\ &= \frac{20}{121} \end{aligned}$$

因此, 模块度为 $\frac{20}{121}$

2. 计算下面有向带权图中划分的模块度。



$$m = 9, k_2^{in} = k_3^{in} = k_4^{in} = k_6^{in} = k_7^{in} = 1, k_5^{in} = 4$$

$$\begin{aligned} k_2^{out} &= k_3^{out} = k_4^{out} = k_7^{out} = 2, k_5^{out} = 0, k_6^{out} = 1 \\ Q &= \frac{1}{m} \left[\left(0 - \frac{k_2^{out} k_2^{in}}{m}\right) + \left(1 - \frac{k_2^{out} k_3^{in}}{m}\right) + \left(1 - \frac{k_3^{out} k_2^{in}}{m}\right) + \left(0 - \frac{k_3^{out} k_3^{in}}{m}\right) + \left(0 - \frac{k_4^{out} k_4^{in}}{m}\right) \right. \\ &\quad \left. + \left(1 - \frac{k_4^{out} k_5^{in}}{m}\right) + \left(1 - \frac{k_4^{out} k_7^{in}}{m}\right) + \left(0 - \frac{k_5^{out} k_5^{in}}{m}\right) + \left(0 - \frac{k_5^{out} k_4^{in}}{m}\right) + \left(0 - \frac{k_5^{out} k_7^{in}}{m}\right) \right. \\ &\quad \left. + \left(0 - \frac{k_7^{out} k_7^{in}}{m}\right) + \left(1 - \frac{k_7^{out} k_4^{in}}{m}\right) + \left(1 - \frac{k_7^{out} k_5^{in}}{m}\right) + \left(0 - \frac{k_6^{out} k_6^{in}}{m}\right) \right] \\ &= \frac{7}{27} \end{aligned}$$

因此, 模块度为 $\frac{7}{27}$