Algorithm Foundations of Data Science and Engineering

Lecture 3: Sampling

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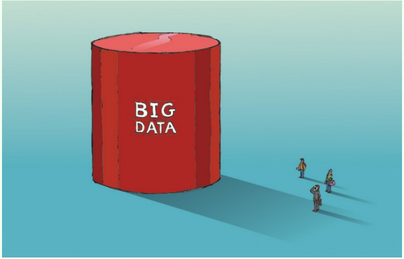
Outline

Motivation of Sampling

Sampling
- Simple Random Sampling
- Systematic Sampling
- Stratified Sampling
- Reservoir Sampling

Data volume in the world



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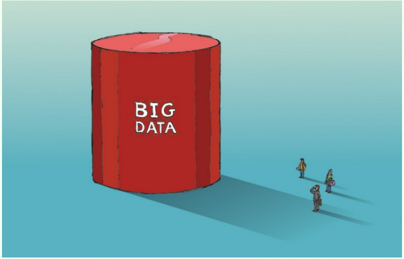

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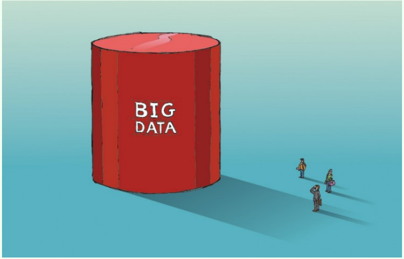

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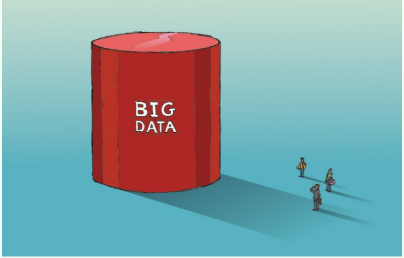

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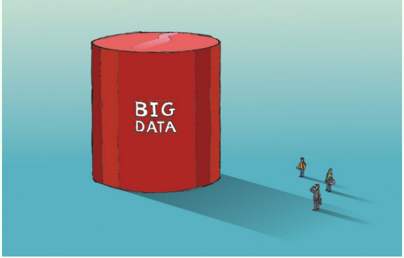

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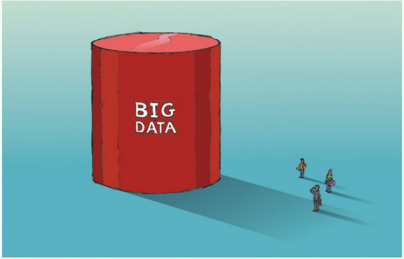

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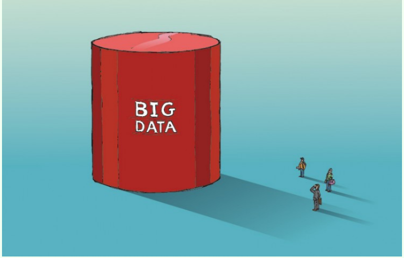

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- In February 2008, Yahoo! announced that its production search index was being generated by a 10,000-core Hadoop cluster.

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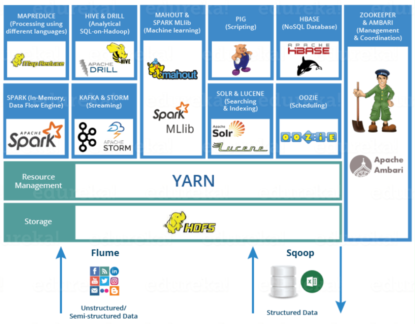

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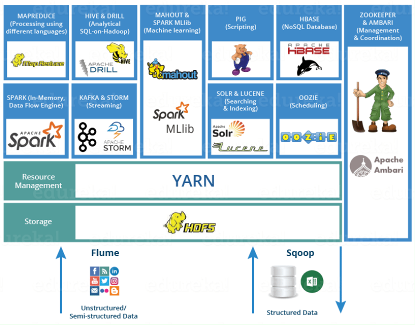

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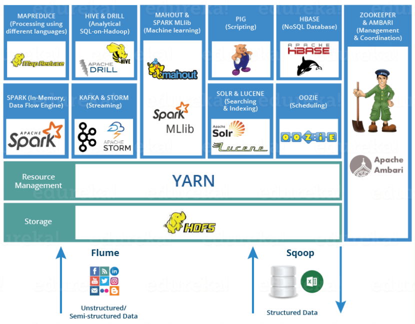

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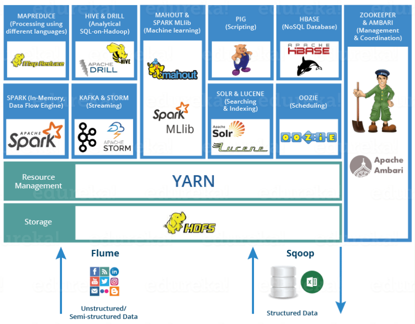

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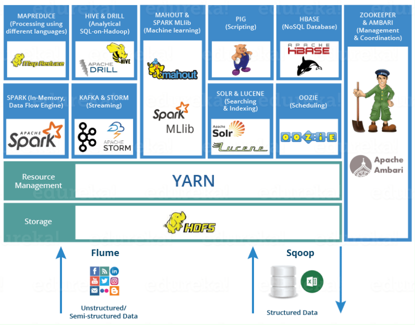

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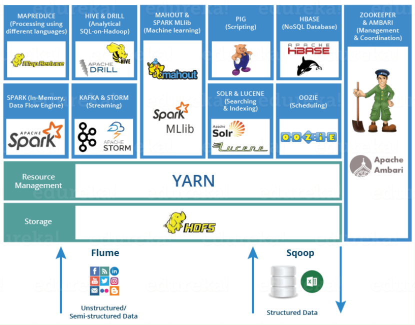

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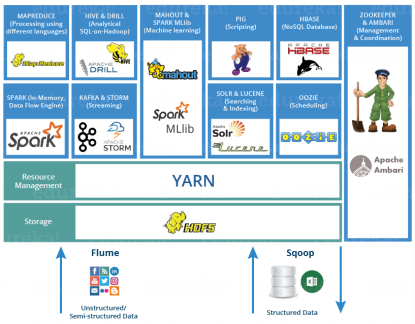

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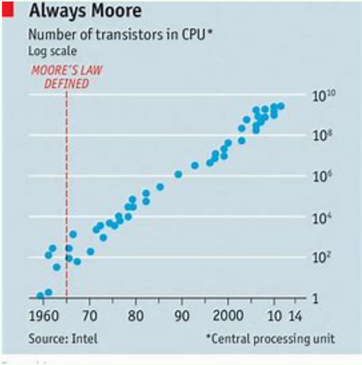

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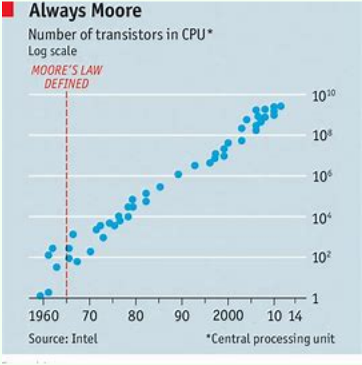

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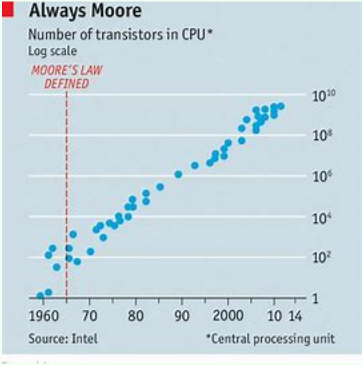

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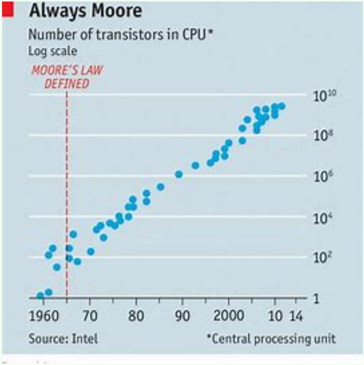

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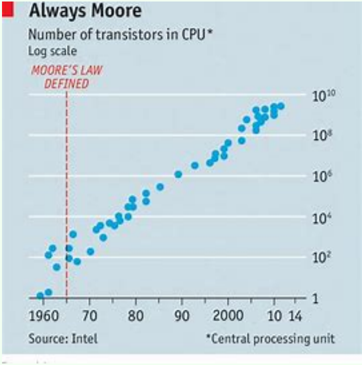

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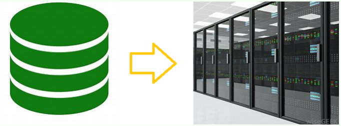
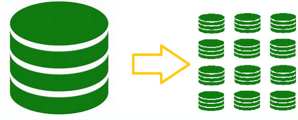
- The processing power of computers increases exponentially every couple of years has hit its limit, according to Jensen Huang, CEO of Nvidia.

Choices of big data processing





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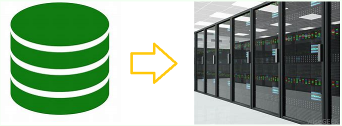

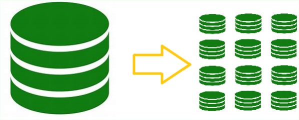

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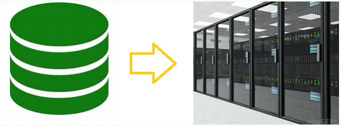

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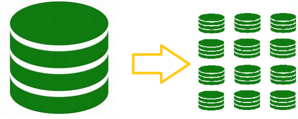

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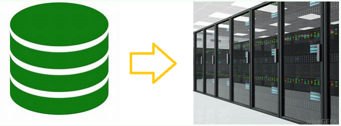

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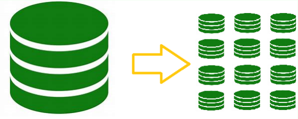

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- All are examples of probability sampling.

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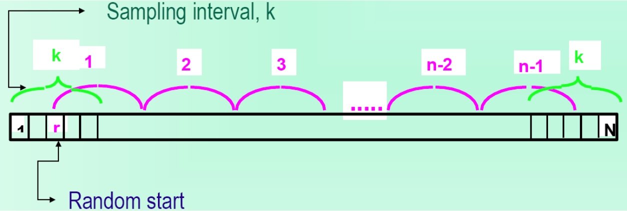

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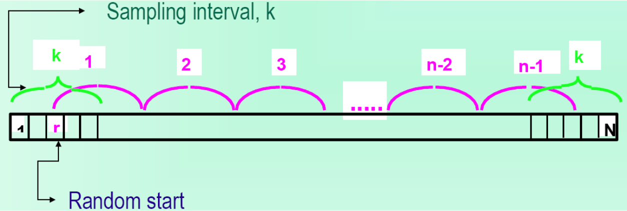

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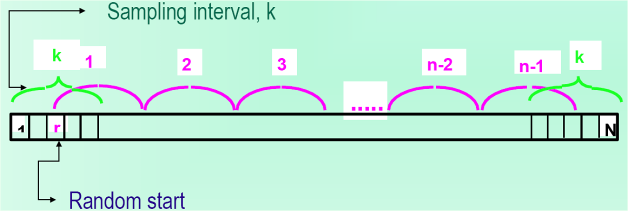

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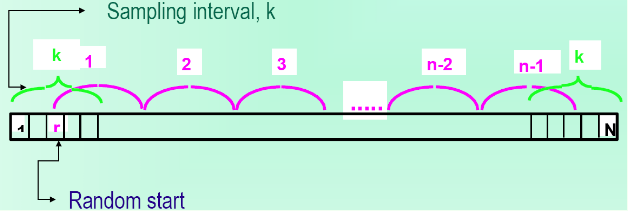

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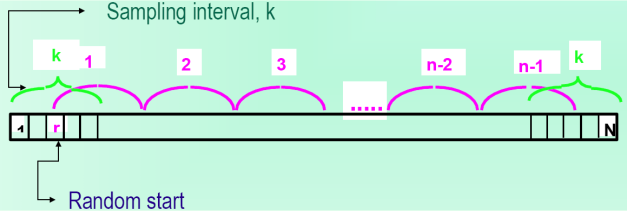

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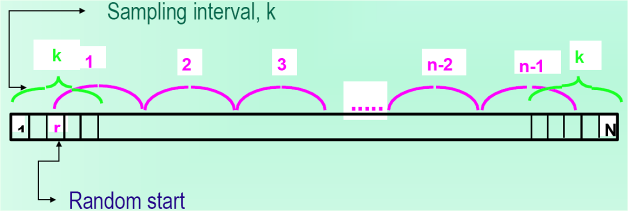

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- Instead of randomly choosing the n units in the sample, a skip pattern is run through a list (frame) of the N units to select the sample;
  - Decide on a sample size n and compute the skip, $k = \frac{N}{n}$;
  - Choose a random start r between 1 and k (inclusive);
  - Add "k" to selected random number to select the next unit repeatedly.

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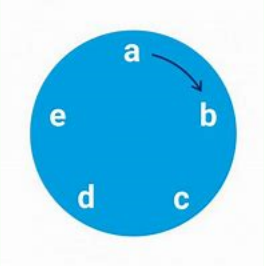
- This problem may be overcome by adopting a device, known as circular systematic sampling;

Circular systematic sampling



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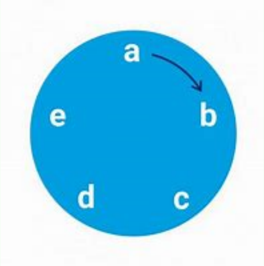

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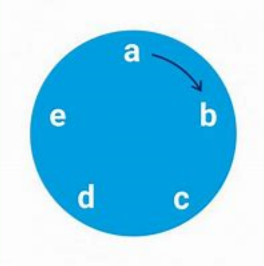

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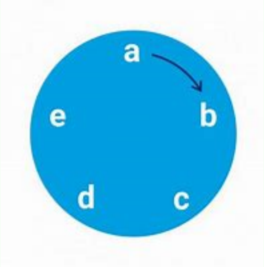

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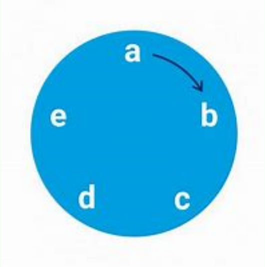

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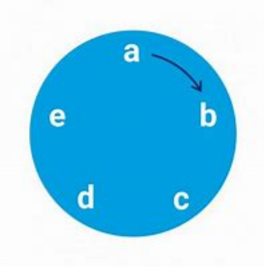

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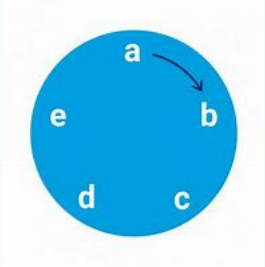

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Outline

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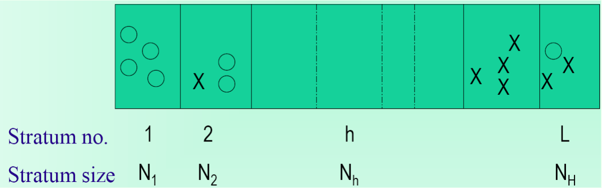

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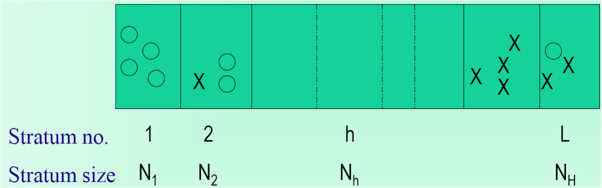

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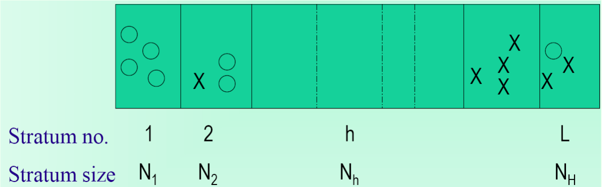

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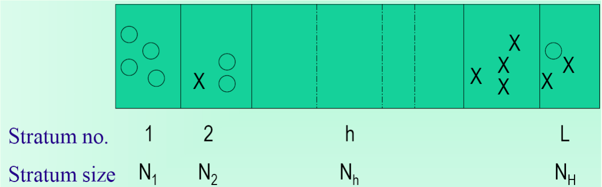

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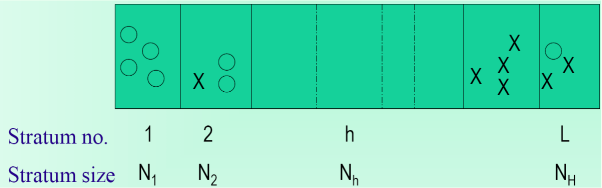

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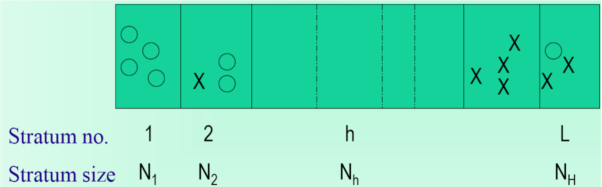

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So, let me make the problem harder. You do not know N (the size of the stream) in advance. How would you do it?

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algorithm `reservoir`(k, S)
/* take k random samples from the dataset S */

initialize an array *samples* of size k
for $i = 1$ to $n = |S|$
    $o =$ the i-th item
    if $i \leq k$ **then**
        $samples[i] = o$
    else
        generate a random integer from 1 to x
        if $x \leq k$ **then**
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The reservoir algorithm is very efficient: it spends $O(1)$ time per item. Next, we will show that the algorithm is correct, namely:

Reservoir Sampling

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Let $S = \{59, 100, 2, 30, 63, \cdots\}$, and $k = 3$.

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- Given the 5th item, again, the algorithm generates x randomly, but now from 1 to 5. Assume that $x = 2$ this time. Hence, the item is added to samples, and replaces the 2nd value there. Hence, samples becomes $(59, 63, 2)$.

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- The remaining items are processed in the same manner.

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Theorem

After $n \geq k$ items in S have been processed, each of those items is sampled with probability $\frac{k}{n}$.

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Basic step: for $n = k$ the statement is obviously correct.
Inductive step: assuming the correctness for $n = m$, next we show that the statement is also correct for $n = m + 1$.

- The $(m+1)-$th object o is sampled if and only if the random number x generated for o falls in the range from 1 to k. Hence, o is sampled with probability $\frac{s}{m+1}$.

□

Proof Cont'd

Theorem

- o' is sampled (after processing o) if and only if (i) it was sampled after processing the first m items, and (ii) the random number x generated for o is not equivalent to the index value of o in the array samples.

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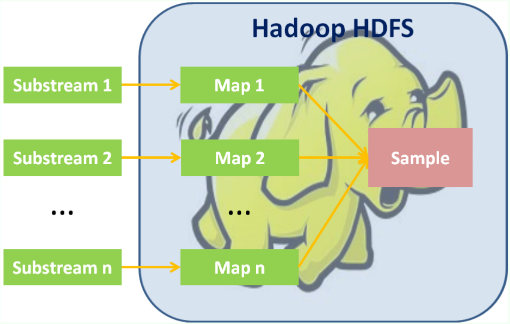
As the two events are independent, the probability that they happen simultaneously equals

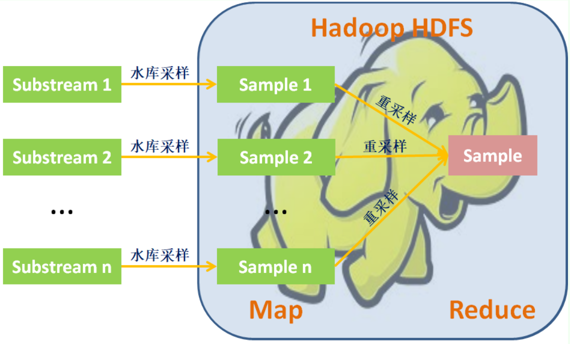
$$\frac{k}{m} \cdot \frac{m}{m+1} = \frac{k}{m+1}.$$

Application



If I want to break break up the problem on say 10 machines and solve it close to 10 times faster, how can I do that?

Distributed reservoir sampling



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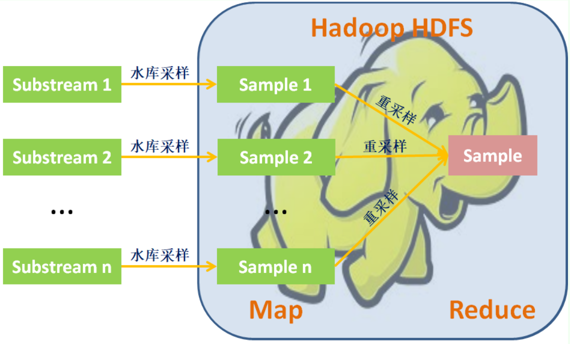

The answer is to have each of the 10 machines take roughly 1/10th of the input to process and generate their own reservoir sample from their subset of the data using the weighted variation above.

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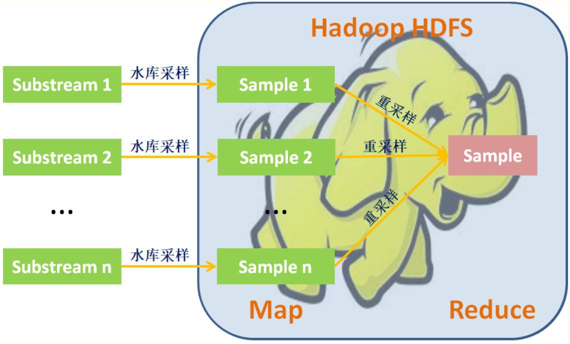

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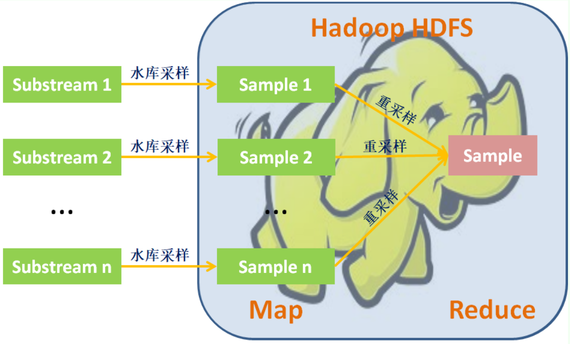

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Distributed reservoir sampling algorithm

Algorithm: Distributed reservoir sampling algorithm

Input : # Maps is n
Output: Sample H of size k

for *ith Map for* $1 \leq i \leq n$ **do**
    $F_i \leftarrow$ sample of k size in ith Map;
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Initialize reservoir H;
for $1 \leq j \leq k$ **do**
    $p \leftarrow random(0, 1)$;
    Determine m s.t., $\sum_{i=1}^{m-1} N_i < p \sum_{i=1}^{n} N_i \leq \sum_{i=1}^{m} N_i$;
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Take-home messages

- Motivation of sampling
- Sampling
  - Simple Random Sampling
  - Systematic Sampling
  - Stratified Sampling
  - Reservoir Sampling