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~~102~~ 温兆和 10205501432 数据科学算法作业8

1. 解: (1) $\nabla f(x,y) = \begin{pmatrix} 50x \\ 2y \end{pmatrix}$ 故 $c=\begin{pmatrix} 0.6 \\ 4 \end{pmatrix}$ 处梯度向量为 $\begin{pmatrix} 30 \\ 8 \end{pmatrix}$

(2) 将 $\begin{pmatrix} 30 \\ 8 \end{pmatrix}$ 标准化后为 $\frac{\sqrt{241}}{482}\begin{pmatrix} 30 \\ 8 \end{pmatrix}$

(3) $c^{(1)} = c - \varepsilon \nabla f(c) = \begin{pmatrix} 0.6 \\ 4 \end{pmatrix} - 0.5\begin{pmatrix} 30 \\ 8 \end{pmatrix} = \begin{pmatrix} -14.4 \\ 0 \end{pmatrix}$

(4) $c^{(1)} = c - \varepsilon (1 \;\; 0)^T = \begin{pmatrix} 0.6 \\ 4 \end{pmatrix} - 0.5\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 4 \end{pmatrix}$

4. 解: $\cancel{p_{uj}^{(t+1)} = p_{uj}^{(t)} + \varepsilon \sum e_{ui} q_{ji}^{(t)} + \frac{\lambda \partial \|P\|_2^2}{\partial f}}$

$$P^{(t+1)} = P^{(t)} + \varepsilon\left(E^{(t)} Q^{(t)} + \frac{\lambda \partial \|P\|_2^2}{\partial P}\right)$$

$$Q^{(t+1)} = Q^{(t)} + \varepsilon\left(E^{(t)T} P^{(t)} + \frac{\lambda \partial \|Q\|_2^2}{\partial Q}\right)$$

5. 解: (1) 记 $J(R: \alpha P_1 + (1-\alpha) P_2, \; \alpha Q_1 + (1-\alpha) Q_2)$

$$= \frac{1}{2} \sum_{(u,i)\in K} \left(r_{ui} - (\alpha p_{1u}^T + (1-\alpha) p_{2u}^T)(\alpha q_{1i} + (1-\alpha) q_{2i})\right)^2$$

$$= \frac{1}{2} \sum_{(u,i)\in K} \Big(r_{ui} - \underbrace{\left(\alpha^2 p_{1u}^T q_{1i} + \alpha(1-\alpha)(p_{1u}^T q_{2i} + p_{2u}^T q_{1i}) + (1-\alpha)^2 p_{2u}^T q_{2i}\right)}_{\text{记为 } A}\Big)^2$$

$$= \frac{1}{2} \sum_{(u,i)\in K} (r_{ui}^2 - 2A r_{ui} + A^2).$$

$$\alpha J(R: P_1, Q_1) + (1-\alpha) J(R: P_2, Q_2)$$

$$= \frac{\alpha}{2} \sum_{(u,i)\in K} (r_{ui} - p_{1u}^T q_{1i})^2 + \frac{1-\alpha}{2} \sum_{(u,i)\in K} (r_{ui} - p_{2u}^T q_{2i})^2$$

$$= \frac{1}{2} \sum_{(u,i)\in K} r_{ui}^2 - \frac{\alpha}{2} \sum_{(u,i)\in K} 2 r_{ui} p_{1u}^T q_{1i} - \frac{1-\alpha}{2} \sum_{(u,i)\in K} 2 r_{ui} p_{2u}^T q_{2i}$$

$$+ \frac{\alpha}{2} \sum_{(u,i)\in K} (p_{1u}^T q_{1i})^2 + \frac{1-\alpha}{2} \sum_{(u,i)\in K} (p_{2u}^T q_{2i})^2.$$

而 $\frac{1}{2} \sum_{(u,i)\in K} (r_{ui}^2 - 2A r_{ui} + A^2)$

$$= \frac{1}{2} \sum_{(u,i)\in K} r_{ui}^2 - \frac{\alpha^2}{2} \sum_{(u,i)\in K} 2 r_{ui} p_{1u}^T q_{1i} - \frac{(1-\alpha)^2}{2} \sum_{(u,i)\in K} 2 r_{ui} p_{2u}^T q_{2i} - \frac{\alpha(1-\alpha)}{2} \sum_{(u,i)\in K} 2 r_{ui} (p_{1u}^T q_{2i} + p_{2u}^T q_{1i})$$

$$+ \frac{\alpha^4}{2} \sum_{(u,i)\in K} (p_{1u}^T q_{1i})^2 + \frac{\alpha^2(1-\alpha)^2}{2} \sum_{(u,i)\in K} (p_{1u}^T q_{2i} + p_{2u}^T q_{1i})^2 + \frac{(1-\alpha)^4}{2} \sum_{(u,i)\in K} (p_{2u}^T q_{2i})^2$$

$$+ \frac{\alpha^3(1-\alpha)}{2} \sum_{(u,i)\in K}$$

只需举反例即可.

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当 $R=\begin{pmatrix}2&7&0\\9&7&1\\3&9&0\end{pmatrix}$ $P_1=\begin{pmatrix}9&6\\0&0\\8&7\end{pmatrix}$ $P_2=\begin{pmatrix}4&3\\5&1\\3&2\end{pmatrix}$ $Q_1=\begin{pmatrix}1&2&0\\2&5&2\end{pmatrix}$ $Q_2=\begin{pmatrix}0&8&5\\6&4&5\end{pmatrix}$

$\alpha=0.9$ 时:

$J(R: \alpha P_1+(1-\alpha)P_2, \alpha Q_1+(1-\alpha)Q_2)-[\alpha J(R:P_1,Q_1)+(1-\alpha)J(R:P_2,Q_2)]$

$=57.4>0$

故: $J(R:P,Q)$ 不是关于 P, Q 的凸函数.

(2) $\frac{\partial J}{\partial p_{uj}}=-2e_{ui}q_{ji}=-2(r_{ui}-p_u^Tq_i)q_{ji}$

$\frac{\partial J}{\partial q_{ji}}=-2e_{ui}p_{uj}=-2(r_{ui}-p_u^Tq_i)p_{uj}$

(3) $p_{uj}^{(t+1)}=p_{uj}^{(t)}+\varepsilon\sum_{(u,i)\in R}(r_{ui}-p_u^{(t)T}q_i^{(t)})q_{ji}^{(t)}$

$q_{ji}^{(t+1)}=q_{ji}^{(t)}+\varepsilon\sum_{(u,i)\in R}(r_{ui}-p_u^{(t)T}q_i^{(t)})p_{uj}^{(t)}$