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## Fast Exponentiation

→ normal solutions to find  $a^b \rightarrow O(b)$

$$2^{10} \Rightarrow \underbrace{2 \times 2 \times 2 \dots \times 2}_{10}$$

Loop  $\rightarrow [0 \rightarrow b-1] \quad \boxed{ans = 1}$   
 $\hookrightarrow ans = ans \times a ;$

Why learn this?  $\rightarrow$  to get fast sol<sup>n</sup>  
So that TLE  $\alpha$

2) Better sol<sup>n</sup>  $a^b \rightarrow O(\log b)$

$a^b \rightarrow$  if  $b \rightarrow$  even  
 $\hookrightarrow a^b \Rightarrow (a^{b/2})^2$   
 $\hookrightarrow$  if  $b \rightarrow$  odd  
 $\hookrightarrow a^b \Rightarrow (a^{b/2})^2 \times a$

for ex

$$2^{10} \Rightarrow \text{Even} \rightarrow (2^{10/2})^2 = (2^5)^2 = 2^{10}$$

$$2^{11} \Rightarrow \text{odd} \Rightarrow (2^5)^2 \times 2$$

$$2^5 \Rightarrow (2^4) * 2$$

↓

$$(2^2 * 2^2) * 2$$

↓↓

$$((2^1 * 2^1) * (2^1 * 2^1)) * 2$$

Divide  
&  
Conquer

```
class Solution {
public:
    int myPow(int a, int b) {
        int ans = 1;
        while(b > 0){
            if(b & 1){
                // odd
                ans = ans * a;
            }
            a = a * a;
            b >>= b >> 1; // b = b / 2;
        }
        return ans;
    }
};
```

Day Run

$$\text{one} = 1 \\ a = 5, b = 4$$

$$b = 4$$

$$b \Rightarrow \text{even} \Rightarrow a = a * a = 5 * 5 \Rightarrow \underline{\underline{25}} \\ a \uparrow$$

$$b = b >> 1 \quad // \quad b = b / 2$$

$$b = 2$$

$$b \Rightarrow \text{even} \Rightarrow a = a * a = 25 * 25 \Rightarrow \underline{\underline{625}} \\ a \uparrow$$

$$b = 1$$

$$b \Rightarrow \text{odd} \Rightarrow \text{one} = \text{one} * a \Rightarrow 1 * 625 \\ \Rightarrow \underline{\underline{625}} \quad (\text{one})$$

$$5^4 \rightarrow 5^2 \cdot 5^2 \\ \downarrow \quad \downarrow \\ 5 * 5 \quad 5 * 5$$

$$25 * 25 = \underline{\underline{625}}$$

Dot Product

$$\text{ans} = \cancel{2} \times \cancel{2}^{32}$$
$$a = \cancel{2} \times \cancel{4}_{16} \quad b = 5$$

$$b = 5$$

$$b = \text{odd} \Rightarrow \text{ans} = \text{ans} \times a \Rightarrow 1 \times 2 \Rightarrow \text{ans} = 2$$

$$a = a \times a \Rightarrow \underbrace{2 \times 2}_{2 \cdot 2} \Rightarrow a = 4$$

$$b = 2$$

$$b = \text{even} \Rightarrow a = a \times a \Rightarrow \underbrace{4 \times 4}_{(2^2 \cdot 2^2)} \Rightarrow a = 16$$

$$b = 1$$

$$b = \text{odd} \Rightarrow \text{ans} = \text{ans} \times a \Rightarrow \underbrace{2 \times 16}_{(2^2 \cdot 2^2) \cdot 2} \Rightarrow \text{ans} = 32$$

$$a = a \times a \Rightarrow 16 \times 16 \Rightarrow \underline{\underline{256}}$$

$$b = 0 \Rightarrow \text{given}$$

↓

$$\text{ans} \Rightarrow \underline{\underline{32}}$$

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$$2^5 \Rightarrow (2^2 \cdot 2^2) \cdot 2$$

$$\left( \underbrace{(2 \cdot 2)}_{\downarrow} \cdot (2 \cdot 2) \right) \cdot 2$$

