UMC 202
PROBLEM SET 2

(1) $f(x) = x - 2 + \log(x)$ has a root near $x = 1.5$. Use the Newton Raphson formula to obtain the better estimate.

(2) Use Newton's method, secant method and Regula Falsi method for finding the approximations of the two zeros, one in $[-1, 0]$ and other in $[0, 1]$ to within 10^{-3} accuracy of $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$. Use the end points of the interval as initial guesses for the secant method and the midpoint for Newton's method.

(3) Use Newton's method to find solutions accurate to within 10^{-5} to the following problems

(a) $x^3 - 2x^2 - 5 = 0$ on the interval $[1, 4]$.

(b) $x^2 - 2\,xe^{-x} + e^{-2x} = 0$ on the interval $[0, 1]$.

(c) $x^3 - 3x^2(2^{-x}) + 3\,x(4^{-x}) - 8^{-x} = 0$ on the interval $[0, 1]$.

(4) Repeat the above problem using the modified Newton's method with

$$g(x) = x - \frac{f(x)\,f'(x)}{[f'(x)]^2 - f(x)f''(x)}.$$

Is there an improvement in speed or accuracy over the above problem.

(5) Use appropriate Lagrange interpolating polynomials of degree one, two and three to approximate each of the following:

(a) $f(8.4)$ if $f(8.1) = 16.94410, f(8.3) = 17.56492, f(8.6) = 18.50515,$ $f(8.7) = 18.82091.$

(b) $f(0.25)$ if $f(0.1) = 0.62049958, f(0.2) = -0.28398668, f(0.3) = 0.00660095,$ $f(0.4) = 0.24842440.$

(6) Construct the Lagrange interpolating polynomials for the following functions, and find a bound for the absolute error on the interval $[x_0, x_n]$.

(a) $f(x) = e^{2x}cos(3\,x),\;\; x_0 = 0,\, x_1 = 0.3,\, x_2 = 0.6,\, n = 2,$

(b) $f(x) = sin(ln\,x),\; x_0 = 2.0,\, x_1 = 2.4,\, x_2 = 2.6,\, n = 2,$

(c) $f(x) = ln\,x,\; x_0 = 1,\, x_1 = 1.1,\, x_2 = 1.3,\, x_3 = 1.4,\, n = 3.$