# UMC 202
# PROBLEM SET 1

(1) Use the bisection method to find the solutions accurate to within $1e-4$ for $x^3 - 7x^2 + 14x - 6 = 0$ on $[0, 1]$.

(2) Use the bisection method to find the root of $x = exp(-x)$ with an accuracy of $10^{-4}$. How many iteration did you need?

(3) Use fixed point iteration method to determine a solution accurate to within $10^{-2}$ for $x^3 - 3x^2 - 3$ on $[1, 2]$.

(4) Determine an interval $[a, b]$ on which fixed point iteration will converge. Estimate the number of iterations necessary to obtain approximations accurate to within $10^{-5}$ and perform the calculations.

   (a) $x = \frac{2-exp(x)+x^2}{3}$,

   (b) $x = \frac{5}{x^2} + 2$,

   (c) $x = 5^{-x}$,

   (d) $x = 0.5(sin(x) + cos(x))$.

(5) Write down the code for computing a root of a given function $f(x) = 0$ using Newton Raphson's method.

(6) Let $f(x) = -x^3 - cosx$ and $p_0 = -1$. Use Newton's method to find $p_2$. Could $p_0 = 0$ be used?

(7) Use Newton's method to approximate to within $10^{-4}$, the value of $x$ that produces the point on the graph of $y = x^2$ that is closet to $(1, 0)$.

(8) Apply Newton's method to find the approximation of the root of $x = tanx$, starting with initial guess $x_0 = 4$ and $x_0 = 4.6$. Compare the results obtained from these two initial guesses. Does the method converge?

(9) Obtain an estimation (accurate till 4 decimal point) of the point of intersection of the curves $y = x^2 - 2$ and $y = \cos x$.

(10) Apply Newton's method to the function

$$f(x) = \begin{cases} x^{2/3}, & x \geq 0 \\ -x^{2/3}, & x < 0 \end{cases}$$

with the root $x^* = 0$. What is the behavior of the iterates? Do they converge, if yes, at what order?