UMC-202 Lab Test Fall 2023

Total 15 marks.

Consider the two point boundary value problem:

$$-ay''(x) + b(1 + x^2)y(x) = f(x), \qquad x \in [0, 1],$$

$$y(0) = y_0, \quad y(1) = y_1,$$

where a, b are positive constants, y_0, y_1 are real numbers $f(x)$ is continuous function.

On a paper do the following (write your name and roll number on the answer sheet):

1. For any positive integer N, let $h = 1/(N + 1)$ and $x_j = jh, j = 0, 1, \ldots, N + 1$. Use Centered difference formula (of $O(h^2)$) for the differential equation and derive the finite difference method to obtain a matrix system $A\alpha = L$, where $\alpha = [\alpha_j]$, $\alpha_j \approx y(x_j)$.

2. Write the Gauss-Jacobi and the Gauss-Seidel iterative methods to find approximate solution for the linear system.

On the computer do the following:

1. Write code for the above difference method and for the iterative methods. Use a tolerance *tol* to stop the iteration method when $|\alpha^m - \alpha^{(m-1)}| < tol$.

2. Given a fixed number of iterations, compare the approximate solutions α of the Gauss-Jacobi and the Gauss-Seidel iterative methods to that of true solution. Which of these two iterative methods give better solution.

3. When $a = b = 1$, $y_0 = y_1 = 0$ and $y(x) = 64x^2(1 - x)^2$, compare the approximate solution α_j and the true solution $y(x_j)$ (by plot or by computing the error $\max_{1 \le j \le N} |\alpha_j - y(x_j)|$)