UMC 202
PROBLEM SET 7

(1) Use Taylor's series method of order 2 to approximate the solution for each of the following initial value problems

(a) $y' = \frac{y}{x} - \frac{y^2}{x^2}$, $x \in [1, 2]$, $y(1) = 1$ with $h = 0.1$,

(b) $y' = sin\, x + e^{-x}$, $x \in [0, 1]$, $y(0) = 0$ with $h = 0.5$,

(c) $y' = \frac{(y^2+y)}{x}$, $x \in [1, 3]$, $y(1) = -2$, with $h = 0.5$,

(d) $y' = -x\, y + \frac{4x}{y}$, $x \in [0, 1]$, $y(0) = 1$, with $h = 0.25$.

(2) Redo Problem 1 using the Runge Kutta method of order 2.

(3) Redo Problem 1 using the Trapezoidal Method.

(4) Using the Taylor's series method of order 2, Runge Kutta method of order 2 and the Trapezoidal Method to approximate the solutions of the following initial value problems and compare the results

(a) $y' = xe^{3x} - 2y$, $x \in [0, 1]$, $y(0) = 0$ with $h = 0.5$;

actual solution $y(x) = \frac{1}{5}xe^{3x} - \frac{1}{25}e^{3x} + \frac{1}{25}e^{-2x}$.

(b) $y' = 1 + (x - y)^2$, $x \in [2, 3]$, $y(2) = 1$, with $h = 0.5$;

actual solution $y(x) = x + \frac{1}{(1-x)}$.

(c) $y' = 1 + \frac{y}{x}$, $x \in [1, 2]$, $y(1) = 2$ with $h = 0.25$;

actual solution $y(x) = x\, ln\, x + 2x$.