UMC 202
PROBLEM SET 11

(1) Use Gaussian elimination with backward substitution with tolerance 10^{-2} to solve the following linear system

$$\begin{aligned} 4x_1 - x_2 + x_3 &= 8, \\ 2x_1 + 5x_2 + 2x_3 &= 3, \\ x_1 + 2x_2 + 4x_3 &= 11. \end{aligned}$$

The exact solution of the system is $x_1 = 1$, $x_2 = -1$, $x_3 = 3$.

(2) The following linear system

$$\begin{aligned} 10x_1 - x_2 + 2x_3 &= 6, \\ -x_1 + 11x_2 - x_3 + 3x_4 &= 25, \\ 2x_1 - x_2 + 10x_3 - x_4 &= -11, \\ 3x_2 - x_3 + 8x_4 &= 15 \end{aligned}$$

has the unique solution $x = (1, 2, -1, 1)^T$. Use Gauss Jacobi's iterative technique to find the approximations $x^{(k)}$ to x with $x^0 = (0, 0, 0, 0)^T$ until

$$\frac{\|x^{(k)} - x^{(k-1)}\|_\infty}{\|x^{(k)}\|_\infty} < 10^{-3}$$

where $\|x\|_\infty = \max_{1\leq j\leq 4} |x_j|$.

(3) Solve Problem 2 by Gauss Seidel iterative Technique.

(4) Use Gauss-Jacobi Iterations to attempt solving the linear system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ 2x_1 - x_2 + 2x_3 &= 1 \\ 3x_1 + x_2 - 2x_3 &= -1. \end{aligned}$$

(Whether the method converges?)

(5) Use Gauss-Seidel Iterations to attempt solving the linear system

$$\begin{aligned} 2x_1 + 8x_2 + 3x_3 + x_4 &= -2 \\ 2x_2 - x_3 + 4x_4 &= \qquad 4 \\ 7x_1 - 2x_2 + x_3 + 2x_4 &= 3 \\ -x_1 + 5x_3 + 2x_2 &= 5. \end{aligned}$$

(Whether the method converges?)