EECE 7220 / 8220: Scientific Computing

Fall 2018

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Homework: 2

Due: Oct 12, 2018

Submission Instructions:

- Upload your programs to the homework dropbox in http://elearn.memphis.edu
- Upload report of your results (solution vector, images, etc.) in a PDF report to the homework dropbox.

Learning Objective: To understand that every subspace admits an orthonormal basis

Homework Problem:

1. Write a program to implement the following *Gram-Schmidt Orthogonalization Procedure*

Input : A set of r linearly independent vectors $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_r\}$

Output: A set of r orthonormal vectors $\{\vec{q}_1, \vec{q}_2, \ldots, \vec{q}_r\}$ such that $\texttt{Span}\left(\{\vec{x}_i\}_{i=1}^{r}\right) = \texttt{Span}\left(\{\vec{q}_i\}_{i=1}^{r}\right)$

$r_{11} \leftarrow \|\vec{x}_1\|$;

if $r_{11} = 0$ **then**

    Stop

else

    $\vec{q}_1 \leftarrow \frac{x_1}{r_{11}}$

end

for $j \leftarrow 2$ **to** r **do**

    for $i \leftarrow 1$ **to** $j-1$ **do**

        $r_{ij} \leftarrow (\vec{x}_j, \vec{q}_i)$;

    end

    $\hat{q} \leftarrow \vec{x}_j - \sum_{i=1}^{j-1} r_{ij}\vec{q}_i$;

    $r_{jj} \leftarrow \|\hat{q}\|$;

    if $r_{jj} = 0$ **then**

        Stop;

    else

        $\vec{q}_j \leftarrow \frac{\hat{q}}{r_{jj}}$;

    end

end

Algorithm 1: Gram-Schmidt Orthogonalization Procedure

2. Write a program to implement the following *Modified Gram-Schmidt Orthogonalization Procedure*

Input : A set of r linearly independent vectors $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_r\}$

Output: A set of r orthonormal vectors $\{\vec{q}_1, \vec{q}_2, \ldots, \vec{q}_r\}$ such that $\texttt{Span}\,(\{\vec{x}_i\}_{i=1}^{r}) = \texttt{Span}\,(\{\vec{q}_i\}_{i=1}^{r})$

$r_{11} \leftarrow \|\vec{x}_1\|$;
if $r_{11} = 0$ **then**
    Stop
else
    $\vec{q}_1 \leftarrow \frac{x_1}{r_{11}}$
end
for $j \leftarrow 2$ **to** r **do**
    $\hat{q} \leftarrow \vec{x}_j$;
    for $i \leftarrow 1$ **to** $j-1$ **do**
        $r_{ij} = (\hat{q}, \vec{q}_i)$;
        $\hat{q} = \hat{q} - \sum_{i=1}^{j-1} r_{ij} \vec{q}_i$;
    end
    $r_{jj} \leftarrow \|\hat{q}\|$;
    if $r_{jj} = 0$ **then**
        Stop;
    else
        $\vec{q}_j = \frac{\hat{q}}{r_{jj}}$;
    end
end

Algorithm 2: Modified Gram-Schmidt Orthogonalization Procedure