

EECE 7220 / 8220: Scientific Computing

Fall 2018

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Homework: 1

Due: Oct 4, 2018

Submission Instructions:

- Upload your programs to the homework dropbox in <http://elearn.memphis.edu>
- Upload report of your results (solution vector, images, etc.) in a PDF report to the homework dropbox.

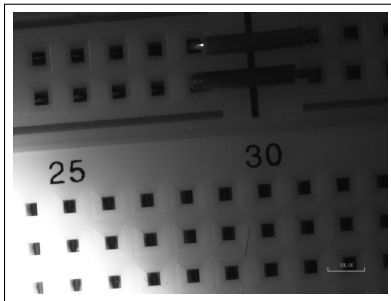
Homework Problem:

1. (a) Write a computer program to implement the spectral method for solving $A\vec{x} = \vec{b}$.
(b) In your program, test whether the eigenvectors of A are orthogonal.
(c) Program inputs: For any m : 1. matrix $A \in \mathbb{R}^{m \times m}$; 2. vector $\vec{b} \in \mathbb{R}^{m \times 1}$.
(d) Program output: 1. unknown vector $\vec{x} \in \mathbb{R}^{m \times 1}$.
(e) Test your program with the following inputs:

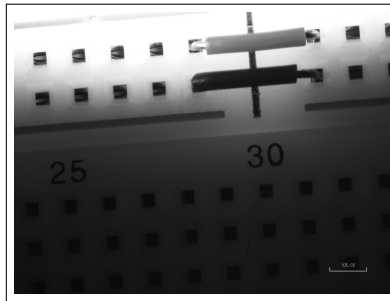
$$\text{i. } A = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix}_{9 \times 9}$$

$$\text{ii. } \vec{b} = \begin{bmatrix} -0.01 \\ -0.01 \\ -0.01 \\ -0.01 \\ -0.01 \\ -0.01 \\ -0.01 \\ -0.01 \\ -0.01 \end{bmatrix}_{9 \times 1}$$

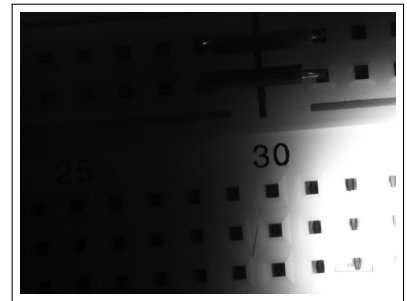
2. (a) Write a general computer program to implement the *reduced singular value decomposition* (reduced SVD) method of a matrix $A \in \mathbb{R}^{m \times n}$ with $n \ll m$.
- (b) Input: Use images of a scene under three different illumination to form the data matrix A as follows. Recall that images can be read using MATLAB function `imread()` and that a grayscale image is in a matrix form. Let I_1, I_2, I_3 are the three image matrices. Vectorize the image matrices (using the colon operator, e.g. $\vec{a}_1 = I_1(:)$). Form A with $\vec{a}_1, \vec{a}_2, \vec{a}_3$ as column entries of A .
- (c) Example Input: Use the attached images `Image1.jpg`, `Image2.jpg`, `Image3.jpg` to test your reduced SVD method and to answer the following questions. You are also encouraged to collect your own images (of various scenes, various illuminations, and various number of images).



(a) `Image1.jpg`



(b) `Image2.jpg`



(c) `Image3.jpg`

- (d) Show that the left singular vectors in the U matrix form an orthogonal set.
- (e) Show that the left singular vectors in U span the images in the data matrix A .
- (f) Create at least six new images using the left singular vectors in U that are not in the original data matrix A . Save your images using `imwrite()` as `Image4.jpg`, `Image5.jpg`, ..., `Image9.jpg`.
Hint: Use the concepts of basis, span and vector space.