A Survey on Temporal Reasoning in Artificial Intelligence

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Abstract

The notion of time is ubiquitous in any *activity* that requires intelligence. In particular, several important notions like change, causality, action are described in terms of time. Therefore, the representation of time and reasoning about time is of crucial importance for many Artificial Intelligence systems. Specifically during the last 10 years, it has been attracting the attention of many AI researchers. In this survey, the results of this work are analysed. Firstly, Temporal Reasoning is defined. Then, the most important representational issues which determine a Temporal Reasoning approach are introduced: the logical form on which the approach is based, the ontology (the units taken as primitives, the temporal relations, the algorithms that have been developed,...) and the concepts related with reasoning about action (the representation of change, causality, action,...). For each issue the different choices in the literature are discussed.

1 Introduction

The notion of time is ubiquitous in any *activity* that requires intelligence. Firstly, these activities concern mostly the real world, which is a dynamic world. The facts and phenomena which happen in it occur over time. Secondly, the human perception and understanding of the real world deeply incorporate the concept of time. Everything appears related by its temporal reference. Events occur temporally related ("before", "during", "after",...) between them. Things remain in a certain state for a while until a certain event happens. Time seems to be a fundamental entity which the rest of the objects in the world are related with; so it appears to play the role of a common universal reference.

In particular, time is fundamental to reason about *change* and *action*. The existence of time allows one to describe change and the characteristics of its occurrence (shape, interaction with other occurrences,...). As soon as we say that something changes, we are talking about different *states* or conditions of the same thing and we are forced to define how they are related. This relation will be temporal, either implicit or explicit. It also allows one to reason about the convenience of performing any action and when. Conversely, the passage of time is important only because changes are possible [104]. In a static world where nothing changes it would not make sense to talk about time.

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Therefore it seems that for many Artificial Intelligence (AI) systems it is necessary to consider the temporal dimension of information, the change of information over time and the knowledge about how it changes. Besides it, the *temporal reference* is an idea deeply integrated in human common-sense. It is notable how naturally and efficiently humans are able to manage it during everyday life when interacting with the environment. Despite this, agreeing on what time is and how it can be characterized has historically been a hard task, as well as its incorporation in automatic reasoning systems.

When thinking about the different areas in AI one easily finds the need for reasoning about time. For example:

- *Medical Diagnosis and Explanation.* Taking into account *when*, and in what order illness symptoms occurred, as well as their *evolution* can be critical in order to correctly determine the cause and the appropriate treatment.

- *Planning.* To develop a plan one has to consider the *duration* of the actions and tasks that can be performed and find out the most appropriate temporal ordering taking into account their *interaction over time.*

- *Industrial Process Supervision.* To correctly control a process it is necessary to consider the different *past states* of the process, the *historic evolution* of the variables, which and *when* operations have been performed and how the actions that can be caried out would affect the evolution of the process.

- *Natural Language Understanding.* A central element in the meaning of any natural language sentence is the verb *tense*, e.g. "Jordi will dance with Iolanda", or "Jordi will have danced with Iolanda".

Despite this evidence, most inference systems in use today ignore the representation of time. One explanation could be the complexity which formalizing this aspect of knowledge entails. However, as McDermott [80] claims "Dealing with time correctly would change everything in an AI program".

1.1 Reasoning Tasks

In order to provide solutions to the above applications, there is a number of basic tasks, namely *reasoning tasks*, which are required:

- *Temporal Consistency Maintenance.* It consists of i) incorporating new information to the knowledge base and ii) checking its consistency. In addition, in the case of inconsistency, it involves also localizing the subset of assertions responsible for it.

- *Temporal Question-Answering.* It consists of providing answers to queries involving temporal knowledge. The queries can range from simply *locating a fact in time* to determining when a set of assertions are simultaneously true.

A specially important type of knowledge which can be incorporated to a domain description is the set of *rules governing change.* Any approach oriented to reasoning about change and action puts special emphasis on these. Taking into account this knowledge, some additional tasks, specific for reasoning about change, are defined [104]:

- *Explanation.* To produce a description of the world at some *past* time which accounts for the world being the way it currently is.
- *Prediction.* To determine the state of the world at a given *future* time or, more generally, the evolution of the world until a given future time.
- *Learning about Physics.* Given a description of the world at different times, to produce a set of rules governing change which account for the regularities in the world.

1.2 What is Temporal Reasoning ?

Temporal Reasoning (TR) consists of formalizing the notion of time and providing means to represent and reason about the temporal aspects of knowledge. Hence, a TR framework should provide:

- An *extension to the language for representing the temporal aspects of the knowledge.* It is the addition of a further dimension to the truth of the information (see figure 1).

  On the one hand, the language has to allow for description of what is true or false over time, e.g. "Jordi is dancing with Iolanda from time A to time B". On the other hand, it has to provide means to express "pure" temporal expressions like "time A is before than time B". In addition, it should distinguish among different types of *temporal entities* one is going to talk about like *events* or *facts.* Let me provide initial definitions for them in simple terms. *Facts* or *properties* are, roughly speaking, "things that are true over time". Facts represent static aspects of the world like "the house is red", "I am happy". On the other hand, *events* are "things that happen" and they usually represent the dynamic aspects of the world like "to close the door" or "to start dancing".

- *A Temporal Reasoning System.* A method for reasoning about the above assertions formed using the extended language which allows one to determine the truth of any temporal logical assertion.

Usually TR is situated in the context of a more general reasoner. Ad hoc representations of time have been developed within some problem solving systems, but the goal of a TR system should be to embody generic temporal knowledge and provide the set of TR functionalities presented above to a problem solver.

1.3 Historical Overview

Work on TR in AI has to be situated in the context of the philosophical theories of time [96, 97, 117, 89, 47, 86, 109] intended to account for what time is. In AI the analysis and modelling of the notion of time started with several isolated application-oriented pieces of work like Bruce's Chronos [16] for natural language understanding and Kahn and Gorry's *Time Specialist* [54] in medical decision-making. Despite the fundamental role that time plays in common-sense reasoning, it was not until the early 80s that more general pieces of work aimed at providing general theories of time and action appeared like McDermott's temporal logic [80], Allen's theory of action and time [4], Vilain's theory of time [114]. These proposals were good to establish the two main contenders as temporal ontological primitive (the point and the interval), to make initial proposals on representational issues and reasoning algorithms,

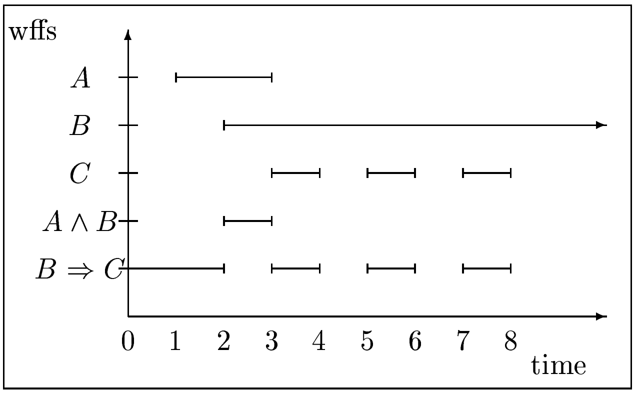

Figure 1: Time as a further dimension for the truth of formula.

to point out the general problems (the reasoning by default, the interaction of actions, the use of a temporal reasoner in an application) and for showing that a more basic machinery has to be built before defining a general theory of time. These results have been the starter of a prolific period of interest in the area by the AI community. A large amount of papers that have appeared during the last 10 years on specific topics. The most notable topics are:

- logical formalisms for time,
- ontological primitives
- algorithms for temporal reasoning (and their complexities)
- representation of change, causality and action

A lot of work has also been done on the application of these results. Temporal theories have been used either embodied within an application system or as the basis for a temporal reasoner in charge of providing temporal functionalities to an application system. TR has been applied in planning systems [8, 106, 92, 5], natural language understanding [120, 15, 17, 44] or expert systems [59, 31, 11, 88, 58, 94, 95], to cite few of them.

A set of ontological and representational issues has to be considered in defining a TR system. The rest of this article is devoted to presenting and analysing them. For each I discuss the advantages and problems of the different choices that one can make [1] . Therefore, in section 2 I present the philosophical distinction between absolute and relational theories of time, in section 3 the different logical formalisms that have been used for representing time, in section 4 the different choices for ontological decisions and in section 5, issues concerning the representation of change, causality and action. Finally section 6 provides general conclusions.

2 Absolute and Relational theories of time

Form a philosophical point of view, one can accord different status to time. Y. Lin [73] distinguishes between absolute and relational theories of time. *Absolute* theories give an autonomous status to time and define it independently of anything else. Time is axiomatized

[1]I believe that this is of major interest in order to describe, analyse and compare the existing TR systems in the literature -an analysis of each of the main TR approaches in AI can be found in [111].

according to the notion of time itself. Upon it, events can be said to happen and facts can be said to hold. On the other hand, *relational* approaches claim that time is relevant because our universe is made up of *events* that are temporally related. So time is determined by events and the properties of time must be defined by investigating the properties of events. Lin introduces a third, intermediate, approach called *moderate absolute*. It gives time an independent status but maintains the point that, as our direct experience is with events, temporal primitives must represent our view of events (periods, in general) and the temporal relations have to reflect the relations existing among events.

Despite the fact that, in Lin's opinion, the relational approach is more interesting because it is based on the human perception of time (both, events and their relations), the absolute approaches, either extreme or moderate, have gained wide acceptance in AI. The reason seems to be that most people are accustomed to thinking about time as an independent dimension in which individuals are placed and events occur.

3 The Logical Form

Let's now adress the logic on which the TR system is based. As Shoham remarks [102] it would be desirable for any proposal to be supplied with both a formal syntax and semantics. It would help to make clear what the proposal consists of, to see how it copes with the different problems and to compare it with other approaches. There are three main ways to introduce time in logic: first-order logic with temporal arguments, modal temporal logics and reified temporal logics.

3.1 The Temporal Arguments

The so-called method of *Temporal Arguments* (TA)[49] [2] simply consists of representing time just as another parameter in a first-order predicate calculus. Functions and predicates are extended with the additional temporal argument denoting the particular time at which they have to be interpreted. For example one can represent that "Jordi dances with Iolanda at April 11th" by $dance(Jordi, Iolanda, 11.april)$.

TA has been the classical treatment of time in mathematics and physics where time is just another variable. In philosophy, it was Russell [96] who initially introduced this idea. In AI, the *Situational Calculus* and the more recent work by Haugh [49] and by Bacchus, Tenenberg and Koomen [10] also follow this approach.

TA seems a natural and simple way to introduce time in our language. However, not giving a special status to time, neither notational nor conceptual can be problematic. For instance, there is nothing to disallow $dance(Jordi, Iolanda, Maria)$ as a legal formula. To avoid this problems it is convenient to move to a many-sorted logic where temporal objects are distinguished from nontemporal objects by partitioning both the constant and variable symbols and the domain objects. This approach leads to gain in the expressiveness as undesirable formula are disallowed. Moreover, expressions turn out to be more natural. Bacchus et al., in addition, claim for achieving considerably more flexibity by not limiting the number of temporal arguments that can appear in a function or predicate. Gains can be obtained

[2] Although it is not a new idea, Haugh is the first reference I know that uses the term Temporal Arguments. In Haugh's paper, it is defined as using additional arguments for time in the predicates and it is not constrained to any specific logic. In this paper the name Temporal Arguments means the Temporal Arguments on a first-order logic.

also in efficiency (the association variable-value is sought only among the appropriated sort) [19]. Another important advantage is that it benefits from having a well-known proof theory [116].

In order to perform temporal reasoning in TA, a temporal ordering relation $\leq$ also has to be introduced with its related axioms as well as a special temporal constant t_0 representing the present time [91]. Putting structure on the temporal sort is a delicate affair because it directly determines the properties of the logic and the complexity of reasoning [46]. The usual way taken by different approaches is not to take any *a priori* decision on the structure and let the axiom writer be free to define his particular set of axioms (see [10] for further discussion on axiomatizing diverse structures for time).

The main shortcoming of TA is its expressive power. The fact that time is introduced just as an argument makes the resulting formalism not very suitable to express certain kinds of expressions, as we shall see in next section. Moreover, it is not expressive enough to talk about generalities of the temporal aspects of other relations (it is discussed below in detail). The problem seems to be rooted in the fact that although a "special status is accorded to time", this is not enough. This is the problem that Haugh tries to overcome by providing a non-standard semantics to the TA method. This piece of work is also discussed after presenting the reified approach.

3.2 The Modal Temporal Logic

An alternative way to include the concept of time is by complicating the interpretation of the formula. The classical possible-worlds semantics [64] is re-interpreted in the temporal context by making each possible world represent a different time.

The language is an extension of the propositional or predicate calculus with modal temporal operators. Classically, the temporal operators introduced [89] are:

$$\begin{array}{lcl} F\phi & \equiv & \phi \text{ is true in some future time} \\ P\phi & \equiv & \phi \text{ is true in some past time} \\ \\ G\phi & \equiv & \phi \text{ is true in every future time} \\ H\phi & \equiv & \phi \text{ is true in every past time} \end{array}$$

In the semantics, each possible world is associated with an element of time. Time elements are related by a *temporal precedence relation* (akin to the accessibility relation in modal logics) and the standard modal operators of *possibility* and *necessity* are re-interpreted over future and past times. The interpretation mapping on semantic structures will depend on the world it is done. Hence, we are going to talk about a formula ϕ being either true or false in M at a given time t, written $M, t \models \phi$. In philosophy this approach has been called *tense logic* [93, 78].

Different logics are obtained by varying the properties attached to the temporal precedence relation $\prec$. These properties are syntactically expressed by an axiom set.

The *Minimal Temporal logic* K is obtained by imposing no restrictions on $\prec$. It is characterized (sound and complete) by the following set of axioms together with *modus ponens* as the rule of inference:

$$\begin{array}{l} \phi\text{, where } \phi \text{ is a tautology} \\ G(\phi_1 \Rightarrow \phi_2) \Rightarrow G\phi_1 \Rightarrow G\phi_2 \\ H(\phi_1 \Rightarrow \phi_2) \Rightarrow G\phi_1 \Rightarrow G\phi_2 \\ \phi \Rightarrow HF\phi \\ \phi \Rightarrow GP\phi \\ G\phi\text{, if } \phi \text{ is an axiom} \\ H\phi\text{, if } \phi \text{ is an axiom} \\ \text{(MP) if } \phi_1 \text{ and } \phi_1 \Rightarrow \phi_2 \text{ then } \phi_2 \end{array}$$

Particular systems are obtained by imposing further constraints on $\prec$. For instance, you can impose the *transitivity* of time (K_c system [18]) by adding the constraint

$$\forall t, t', t''.\ t \prec t' \wedge t' \prec t'' \Rightarrow t \prec t''$$

on $\prec$. It is axiomatized by

$$\begin{array}{l} FF\phi \Rightarrow F\phi \\ PP\phi \Rightarrow P\phi \end{array}$$

Backwards linearity defined by the condition

$$\forall t, t', t''.\ t \prec t'' \wedge t' \prec t'' \Rightarrow t \prec t' \vee t = t' \vee t \prec t'$$

rules out having different possible pasts. The correspondent axiom is

$$P\phi_1 \wedge P\phi_2 \Rightarrow P(\phi_1 \wedge \phi_2) \vee P(\phi_1 \wedge P\phi_2) \vee P(P\phi_1 \wedge \phi_2)$$

The *minimal* system with these two conditions forms the K_b system of Rescher and Urquhart. When the dual *forward linearity* is also imposed, a full linear system is obtained. One can also define the time to be *unbounded*, *dense*, *continuous*, etc. ...

Although no assumption is made about the nature of time (points or intervals), seeing temporal individuals as points of time appears to be more natural. Nevertheless some work has been done looking at temporal individuals as intervals and constructing an *Interval Modal Temporal Logic* (see [46]).

The Modal Temporal Logic (MTL) is a *relativist approach* to time in contrast with the *absolutist* TA. The statements are temporally qualified w.r.t. the present or to other events. For instance, (F $dance(Jordi, Iolanda, tango)$) means that there is a time where Jordi will dance a tango with Iolanda in the future and (P (F $dance(Jordi, Iolanda, tango)$)) is to mean that in the future it will have happened. A number of extensions have been proposed for coping with the possibility of referring to absolute precise times: Reichgelt [91] defines a modal logic that incorporates the operator $AT(t)$ where t is a temporal constant, $AT(t)\phi$ expressing that the formula ϕ is true at time t.

Concerning expresiveness, MTL turns out to be better than TA, at least for certain types of statements. For instance, the phrase used in the last example (belonging to the type of sentences referred to by Reichgelt as *repeated temporal expressions*), would be represented in Temporal Arguments by

$$\exists t.\ now < t \wedge \exists t'.\ t' < t \wedge dance(Jordi, Iolanda, tango, t)$$

which is less natural and concise. The situation has led to the suggestion of maintaining a two levels system [91, 33]: a *user level* based on the MTL approach where the user can express his statements and a *processing level* based on the temporal arguments approach where deductions can be made by taking advantage of the existing proof theory of first-order logic. The problem with this idea is that it requires the development of a translation program from one formalism to the other and vice versa. Achieving actual gains in efficiency will depend on this translation program. Also there can be some semantic problems for the translation of certain expressions (see [91] for further discussion).

The notational efficiency of MTL makes it very appealing for *Natural Language Understanding* applications where has been widely used [38]. But the area where probably has received the widest acceptance is programming theory. The idea is to specify a program by using this logic and to apply deduction methods for proving properties of the program like correctness, termination, the possibility of deadlock, ... For this objective the set of modal operators has been extended in a variety of ways, with additional connectives like *next* meaning "the next moment in time", *until*, etc. It has been applied in both, sequential and concurrent programs, for automatic verification but also for automatic synthesis. Temporal Logic has been proposed also as a programming language like the *Tempura language* [84] and the METATEM language [36, 13, 14]. This topic is not further analysed in this survey. For those interested I would recommend the section 1.3 in [38] as introductory text. Some relevant references are [68, 75, 45, 12, 37, 65].

A final good quality of MTL I would remark is *modularity*. It can directly and neatly be combined with other modal qualifications one could be interested in, like belief, knowledge [1], etc.

The disadvantages of MTL are on the side of efficiency. In general, theorem proving in modal logic is more difficult than in first-order logics. Recently there have been many attempts to provide efficient proof methods for modal logics [30, 53, 9] and some of them specially oriented to Temporal Modal Logics [37, 34]. The applicability of MTL depends on the advances on these theorem proving techniques.

3.3 The Reified Temporal Logic

Reification can be understood as an attempt of having a higher expressive power that allows one to talk about the truth of assertions while staying in a first-order logic. Reifying a logic involves moving into the meta-language where a formula in the initial language, i.e. the object language, becomes a term -namely a *propositional term*- in the new language. Thus, in this new logic one can reason about the particular aspects of the truth of expressions of the object language through the use of truth predicates like *true*. Usually the language of classical first-order logic has been taken as the object language -although one can take a modal logic for instance- while the reified language is many-sorted.

In the temporal case, one is interested in talking about *when* things are true. The "truth predicates" take as arguments a formula in the object language and an expression denoting a temporal object. Thus we have legal formula with the form

$$\mathrm{TRUE}\langle atemporal\ expression, temporal\ qualification\rangle$$

whose intended meaning is that the first argument is true "during" the time denoted by the second argument. For example $\textsc{Holds}(Dance(Jordi, Iolanda), 11th\_April)$. In the rough definition above I wrote "during" but, in general, the pattern of temporal occurrence for the atemporal expression induced from the temporal qualification attached to it admits many interpretations. In the example one could mean either that Jordi and Iolanda have been dancing the whole day, or just several times during the day, or one single durative time within the 24 hours of 11th of April. We can still imagine other possibilities. One may consider that if one says they have been dancing during “a period within the 11th of April” it is also true that thay have been dancing during “a period within April”. Or, if one considers the fact of dancing during a day as the *accomplishment* of dancing during an entire day, then, one can only say it happened for the period in its entirety but not for any subinterval. The truth predicates are used to express not only the time when something is true but also the pattern of its temporal occurrence. In the remaining of the papers we shall refer to them as *Temporal Occurrence Pattern* (TOP) predicates. Hence, one can introduce and axiomatize several TOP predicates according to the different types of temporal occurrence one is going to discuss.

Temporal reification has several advantages. On the one hand, this logic accords a special status to time. On the other, it allows to predicate and quantify over the propositional terms. It thus gives one the expressive power to discuss relationships between the propositional terms and about temporal aspects with a higher level of generalization; let's call this *general temporal knowledge*. One can be interested in representing general temporal knowledge about:

1. *The axiomatizations for the TOP predicates.* For instance the axiom for expressing the “homogeneity” of the HOLDS predicate wrt. facts

$$\textsc{Holds}(p,T) \iff \forall t.\ in(t,T) \Rightarrow \textsc{Holds}(p,t)$$

2. *Incompatible facts.* One can express the incompatible relation among facts e.g INCOMPATIBLE($dance(Jordi, Iolanda), dance(Jordi, Maria)$) and then state general axioms about the temporality of the incompatiblility relation, like “incompatible facts cannot overlap”

$$\forall f_1, f_2.\ \textsc{Holds}(f_1, time_1) \wedge \textsc{Holds}(f_2, time_2) \wedge \mathrm{INCOMPATIBLE}(f_1, f_2) \\ \Rightarrow \neg \mathrm{overlap}(time_1, time_2)$$

3. *Causality.* Similar to the previous case, one can express causal relationships. This is an important type of knowledge for many AI areas. A common case is one event causing the holding of a fact like CAUSES($start\_music, time_1, dance(Jordi, Iolanda), time_2$). As a general temporal rule example we can take “effects never precede their causes” that would be represented by

$$\forall event, fact, time_1, time_2.\ \mathrm{CAUSES}(event, time_1, fact, time_2) \Rightarrow time_1 < time_2$$

Due to these qualities, this approach has been favoured in TR. The most influential work can be seen as a reified approach [80, 4, 63, 24], although it is not completely clear what the underlying logic is because, although their formulate their logics as a many-sorted first-order one, no one provides a special formal semantics for their temporal features. Shoham [103] is an exception. He presents a formally defined reified logic and sets out that it is more general than McDermott and Allen systems. It is interesting to have a look in detail at Shoham's formalization. Let's see the first-order case:

Syntactically it has a vocabulary similar to a Temporal Arguments approach, being composed of a set of temporal symbols and a set of nontemporal ones. Furthermore, the separation is done also for temporal and nontemporal terms. He introduces a sort of meta-predicate TRUE that takes as arguments two temporal terms and one atomic propositional term made of a relation taking non-temporal terms as arguments.

Examples of wffs are:

$$\mathrm{TRUE}(23.00, 24.00, dance(Jordi, Iolanda))$$

$$\forall v_1.within(v_1, 23.00, 24.00) \wedge \Rightarrow \mathrm{TRUE}(v_1, 24.00, origin(partner(Jordi), Italy))$$

In the semantics, a separation is also defined between temporal objects and nontemporal ones. The way a term are interpreted depends on whether it is a temporal term or not. For *temporal terms* the meaning is the same regardless of when they are interpreted (e.g. 23:00.May.1992 denotes always the same time hour). For *nontemporal terms* the meaning is time-dependent (e.g. $partner(Jordi)$ can denote different individuals as it is interpreted at different times). The interpretation of the non-temporal terms and of the relations depends on the interpretation of the temporal terms in the first two arguments of the predicate TRUE.

From the point of view of expressiveness, it has several shortcomings. Firstly, the nontemporal part of the truth predicate is restricted to atomic propositions. As a consequence, you cannot cope with the problem of *changing ontologies* (the case where the individuals that exist, change throughout time) because the quantifiers have non-atomic temporal formula inside their scope. Secondly, the TRUE predicate makes everything in the nontemporal part within its scope (both the predicate and the functions possibly forming the nontemporal terms) to be interpreted under the same time points. Therefore, it is not clear how the sentence "Jordi has been dancing today with the girl that yesterday was wearing a red skirt" is expressed. Thirdly and most importantly, the nontemporal part is not a "*propositional term*", and one cannot, therefore, quantify over -as it is an atomic predicate and not the name of an atomic predicate. So, using this logic, one cannot express *general temporal knowledge* like "the effects never precede their causes" [3].

Form the point of view of logic, Shoham's formalism has a lot of similarities with a modal logic. The TRUE predicate, with regard to semantics, seems a modal operator scheme -it takes two time expressions and generates a new modal operator. The language is not dealing with the representation (i.e the naming, or the coding) of formula of another logic that allows explicit talk about the satisfaction of these formula at a certain time. Therefore Shoham's logic does not seem to deserve the qualification of reified [91, 10]. Moreover, since this logic moves away from the standard first-order language and its standard semantics, first-order proof theory no longer applies. Hence, it would require developent of a new reasoning procedure provided with, if possible, formal guarantees of soundness and completeness. However, there are no expressiveness gains that justify this effort. In [10] it is shown that the BTK system (a temporal arguments approach) subsumes Shoham's reified logic, in either words, anything expressed in Shoham's formalism can be expressed in the temporal arguments approach the inverse being not true.

After seeing a system which is not truly reified it is interesting to have a look at a real reified system. Next, I will sketch Reichgelt's reified temporal logic. Reichgelt [91] takes a modal temporal logic (a classical modal temporal language with the operator $AT(t)$) as the

[3] Argument which Shoham used as an argument for his proposal.

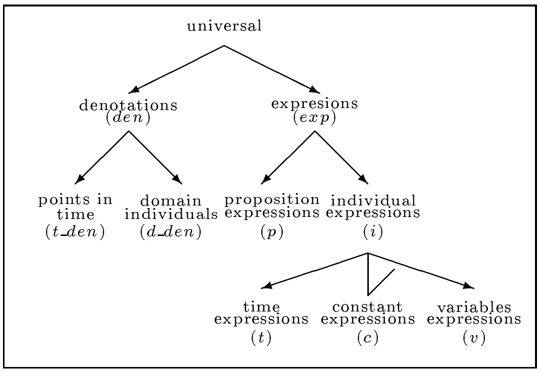

Figure 2: The sort hierarchy of Reichgelt's Temporal Reified Logic.

"object logic" to be reified, namely OL, and defines a Temporal Reified Logic (TRL) as a first-order language. The idea of reifying is based on the fact that the meta-language used to describe the satisfactibility in Reichgelt's modal logic is a first-order language. The reified logic is constructed by defining a language which represents symbols, formula and semantic objects of OL that will denote real symbols and formula of OL.

Syntax. The reified language is a meta-language of the OL where syntactic and semantic elements of OL are represented. In order to keep under control the definition of wffs, the vocabulary is organized into a sort structure (it is a many-sorted first-order logic). The two main sorts are *expressions* corresponding to the syntactic elements of OL (time constants, constants, variables and propositions) and *denotations* representing the semantic elements of OL (time elements and domain individuals). For every one of the above sorts the vocabulary contains both constants and variables.

Each n-ary function symbol in OL is an n-ary function symbol in TRL which arguments and value are of sort i. Each n-ary relation symbol in OL is also a n-ary function symbol in TRL which arguments are of sort i and value of sort p ("...predicates take individual expressions to make sentences..."). In addition, functions are defined for each connective and modal operator of OL to represent how a formula is built from other formula, e.g. the connective $\forall v.\psi$ is the origin of the function FORALL which arguments are of sort v and p and which value is of sort p ("the formula built from the variable and the quantified formula").

I would note that actually many applications do not require the expressive power obtained from reifying expressions containing connectives and temporal operators.

TRL has several predicate symbols representing the relationship between syntactic and semantic elements of OL. Among them the most important and characteristic is the predicate HOLDS that represents the satisfaction relation ($\models$) which links syntax and semantics of OL. HOLDS takes two arguments: the first is of sort p and the second is of sort t_den. Its intuitive reading is that the first argument is true at the time denoted by the second argument.

Semantics. The semantics of TRL is defined in a rather standard first-order way but, as Reichgelt notes, "TRL is a complicated first-order language". A TRL structure is defined to contain sets of individuals which are counterparts of each syntactic element. Symbols, functions and relations are interpreted over these sets according to their signature. The value-assignment and the satisfaction relation are defined in a rather standard way.

Reichgelt's system is fully reified but rather complex and baroque in nature. For example, one has to deal with constants and variables for individuals and for their symbols. And one also needs a predicate to express that some symbol refers to some individiual. Representing knowledge in the resulting logic is not trivial.

On the side of inference, while the resulting system allows one to use a theorem prover for first-order logic, its complicated nature makes it highly unlikely that such a theorem prover would achieve acceptable efficiency. One needs to introduce additional axioms to cope with the additional complexity in the semantics of the logic. For instance, additional axioms are needed to describe the behaviour of the reified predicates, or axioms relating the functions for the connectives with the connectives of the logic (e.g. $\textsc{Holds}(p \wedge q, t) \Leftrightarrow \textsc{Holds}(p, t) \wedge \textsc{Holds}(q, t)$). You have two choices for introducing these axioms:

1. to build the additional axioms into the theorem prover
2. you simply add the set of axioms to every knowledge base you build.

The second choice is simpler but inefficient. As an example of the first choice see [2].

In summary, reified temporal logics are an attempt to get higher expressive power than the temporal arguments method but without going beyond the boundaries of first-order logic. By allowing quantification over propositions it achieves higher expresiveness to represent *general temporal knowledge*. This is the reason they have been specially attractive for people working in AI. Nevertheless, recently they have come under attack. In the next section I will review some of the theories coming from of critical revisions on the reified approaches.

3.4 Further Approaches from Criticisms on the Reified Approach

I already mentioned the papers of Bacchus, Tenenberg and Koomen [10] which claim that the method of Temporal Arguments is more expressive than has historically been recognized and that it can be enough for many applications in AI.

Haugh [49] makes a similar claim. In Haugh's paper the Method of Temporal Arguments is defined as the use of a temporal argument in every predicate to establish temporal references. He notes that it is distinguished from the reified approach in that it admits, as predicates, ordinary properties and relations (e.g. is_red, is_heavier_than). These are excluded by the reified approach [4] . Haugh proposes a first-order logic with temporal arguments provided with a non-standard semantics. The non-standard elements are fundamentally two: i) the classification of predicates according to their Temporal Occurrence Pattern is introduced both in the syntax and in the semantics, and ii) an existence predicate is defined. By developing an explicit temporal semantics which includes these elements Haugh claims to provide the foundation for meaningful soundness and completeness. [5] The most remarkable shortcoming is still the expressiveness. In order to represent causality, Haugh needs to borrow the *propositional terms* from the reification technique to take advantage of its expressive power.

[4] This definition, as Haugh notices, leaves room for enormous variety of versions by taking different choices in the logic used as the basis, the set of temporal predicates, the basic temporal primitive, its structure,...

[5] The lack of meaningful soudness and completeness is a criticism which one can apply to those theories of time -including most of the previous ones- formulated in standard many-sorted first-order logics by giving a set of axioms and simply referring to the standard soundness and completeness proofs, although Haugh provides this semantics he does not provide any of the proofs.

Galton [40] argues that the reification is "philosophically suspect and technically unnecessary" and proposes a procedure for unreifying reified theories by following Davidson [20] in an approach similar to the one in the Event Calculus [63]. The idea is simply to introduce into a first-order logic an additional sort of elements ("tokens") which relate an atemporal expression to a particular time. For example $\textsc{Holds}(dance(Jordi, Iolanda), time_k)$ is rewritten as

$$\exists e.\ dance(Jordi, Iolanda, e) \land \textsc{Holds}(e, time_k)$$

Galton uses philosophical arguments to object to the artificial nature of terms with propositional content and to having types in the ontology. He claims that the expressiveness of reified approaches -Allen, McDermott- can be achieved by simply introducing tokens and applying truth predicates on them. He proposes to refer to types by quantifying over tokens. Nevertheless in Galton's work is not clear the way that Causality is represented and it is not clear how one expresses general statements like "effects never precede their causes".

Recently, Vila and Reichgelt [113] proposed a *Token Reified Logic* also based on the idea of introducing tokens, which they claim to achieve the desired expressiveness. It is based on a full reification of tokens by introducing "meaningful" names for tokens. This allows one to express statements about all the tokens that meet some condition, and this provides the needed expressive power as well as a more compact notation. For instance, "Jordi dances with Iolanda during $time_k$" is formalized by

$$\textsc{Holds}(dance(Jordi, Iolanda, time_k))$$

Notice that $time_k$ is now an argument of *dance* and *dance* is interpreted as a funcion that returns a token. One can quantify over any of the arguments of *dance* and also over the entire *dance* propositional term. Let's see how causality is represented. A causal relationship is understood as a token to token one. So the statement "whoever dances with Jordi, causes this person to be tired"

$$\forall I, time_1.\ \exists time_2.\ \textsc{Cause}(dance(Jordi, I, time_1), tired(I, time_2))$$

and the piece of general temporal knowledge "effects never precede their causes" would be expressed by

$$\forall e, s.\ \textsc{Cause}(e, s) \Rightarrow \texttt{begin}(e) < \texttt{begin}(s)$$

4 The Time Ontology

As mentioned above, any TR formalism establishes a link between atemporal assertions and a temporal reference. Let us, now, analyse the later in detail. The *temporal reference* is made up of a set of temporal elements related by one or a set of temporal relations. Different primitives can be considered for the temporal elements. The main candidates are time points and time intervals.

After deciding on the ontological primitive, one may want to give structure to this ontology by providing axioms describing the behaviour of the temporal relations -the definition of the modal temporal logic in section 3.2 illustrates this. Let's call it the *structure of time.* It wil determine the desired properties for time. I would emphasize the following:

- *Discrete vs. Dense.* Time is considered as a discrete collection of time elements or, instead, for any two elements there is always a third element between them.

- *Bounded vs. Unbounded.* This concerns the question of whether time is considered infinite in either or both directions.

- *Precedence (linear, branching, parallel, circular).*

It is very important to realize that the complexity of the reasoning strongly depends on the structure of time independently of the logic formalism (it is discussed for modal temporal logics in [46] and for many-sorted temporal logic in [10]).

Finally, one has to decide on the type of temporal relations and about the form of temporal expressions one is going to deal with. The well-formed forms accepted by the logics presented above provide an expressive power for the temporal expressions probably not affordable from a computational point of view. A lot of research has been done in defining more restrictive temporal expressions and developing algorithms for reasoning on them, mainly oriented to consistency checking.

Additionally, in most of the approaches on TR in AI, researchers include an analysis of temporal occurrence of the propositions and propose classifications of the propositions according to their different patterns of occurrence (it is closely related with the Temporal Occurrence Pattern predicates presented in section 3.3). I shall call these different classes *Temporal Entities*.

The ontology primitives and the structure of time are discussed in the first subsection, the different temporal expressions and its algorithmic properties in the second and the temporal entities in the third.

4.1 The Temporal Primitive

Which sort of elements are we going to take as the primitive for our ontology of time ? The two main contenders are *instants* (points of time) and *periods* (intervals of time). A third possibility is having both together.

On the one hand, the instant can be taken as the basic primitive. The early representations of time were made in terms of instants as in the Situational Calculus [79], in Bruce's *Chronos* [16] or in the *Time Specialist* [54]. McDermott also follows this way [80]. One of McDermott's main aims is to capture the continuity of time. Time is modelled as an infinite, dense, non-circular collection of instants. It is defined by a *dense* set of instants over which an ordering relation $\leq$ is defined which is *reflexive*, *anti-symmetric* and *transitive*. To represent things like the "the holding of a fact for a while" or "happening of an event over a period" an interval of time is simply defined as a pair of instants.

Other authors, on the other hand, claim that using periods is more in keeping with common sense of temporal concepts than to use the mathematical abstraction of points. Allen, for instance, argues that the period should be the only temporal primitive since it is the best concept for talking about both *properties* and *events*. Allen defined a calculus of intervals, i.e. *Interval Calculus* [3], based on the 13 relations that correspond to the simple definite mutually exclusive relationships that may exist between two periods (see figure 3).

Given these relationships, Allen formulates a set of axioms that define their behaviour:

- given any period, there exists another period related to it by each relationship in the set above

- the relationships are mutually exclusive

A before B	*B after A*
A meets B	*B met_by A*
A overlaps B	*B overlapped_by A*
A starts B	*B started_by A*
A during B	*B contains A*
A finishes B	*B finished_by A*
A equal B	*B equal A*

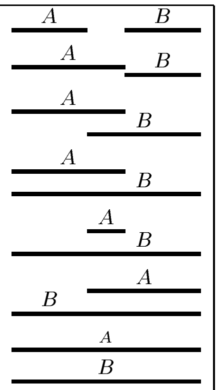

Figure 3: The 13 relations between temporal intervals.

- the relationships have a transitive behaviour, e.g. if I_1 is `Before` I_2 and I_2 `Meets` I_3 then I_1 is `Before` I_3.

Several authors model-theoretically studied the various interval theories. On the one hand, van Benthem [109, 110] and Ladkin-Maddux [67] formulated the theory of intervals over an unbounded, dense, countably categorical, complete and decidable (Ladkin proved that the theory admits also "elimination of quantifiers" and exhibited an explicit decision procedure). On the other hand, [7, 51] reformulated Allen's Interval Calculus as a formal theory in first-order logic in terms of the single relation *meets*. Allen and Hayes's axioms define precisely the theory of intervals over an unbounded linear order, not necessarily dense [66]. This weaker theory is still decidable, but does not admit elimination of quantifiers (I refer to Ladkin's several reports from Krestel Institute for further details).

Allen's arguments against time points [4] are: i) they are not necessary since instants can be represented as very short periods, and ii) their inclusion is the origin of some semantic problems that are discussed below. Therefore Allen completely refuses representing instants.

Nevertheless the notion of instant seems present in our common-sense and many situations suggest the need of including instants in our model of time as an entity different from periods:

- It is natural to talk about a proposition being true at a given instant (e.g. "the pressure of the valve at the moment the device stopped", "the patient temperature at 9.00", "has the block B any other block on it now ?", "has there been any instant where the object velocity was null ?"). This view is also apparent in the *Situational Calculus* and McDermott's temporal logic where world evolution is represented as a sequence of *states* ("instantaneous states").

- The so-called *accomplishment events* [85] like "I shoot the gun", "he turned off the light" or "to close the door" seem intuitively to be instantaneous.

- The most important argument, however, is the fact that *transitions* from one state to another one are usually seen as instantaneous (e.g. "when the light is turned off the room changes from being illuminated to dark", or "the door moves from being open to being closed when it is closed").

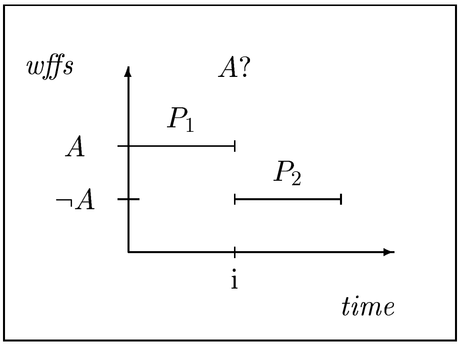

Figure 4: The Divided Instant Problem.

Some approaches propose to define an ontology made of both instants and periods. Vilain [114] extends Allen's interval logic with points, point-to-point and point-to-interval relations but the resulting theory and related problems are not analysed. Allen and Hayes reformulation of Allen's interval theory [7, 51] is extended with the concept of time-point (Allen seems to change his mind about instants) constructed upon the notion of interval. *Nests* are time-points defined as the set of those intervals that "touch" the point (i.e. that either contain or are bounded by it) and are used for the beginning and ending points of intervals. They introduce two functions START and END from periods to instants.

$$\forall P_1, P_2.\ \mathrm{MEETS}(P_1, P_2) \Longleftrightarrow \mathrm{END}(P_1) = \mathrm{START}(P_2)$$
$$\forall i.\ \exists P.\ i = \mathrm{END}(P)$$

Then, it can be shown that instants have the properties one would intuitively expect them to have (totally ordered, dense, unbounded).

Nevertheless, having both instants and periods in our model of time can lead to semantic problems like the *Divided Instant Problem* (DIP) [109]: Suppose the proposition A representing the fact "I am dancing" and $\neg A$ represents "I am not dancing". Suppose I am dancing during the period P_1, then I stop at the instant i and I am not dancing during the next period P_2. Oviously P_1 and P_2 meet. If we allow the existence of instants then we have the problem of deciding whether A is true of not at i. In terms of ontology it is a matter of deciding if the instant i belongs to either i) P_1 ii) P_2 iii) both or iv) to neither one (see figure 4). Deciding for either i) or ii) is just artificial. In case of iii) A and $\neg A$ would be true at i which is inconsistent. In case of iv) according to Allen, neither A nor $\neg A$ would be true at i which is equally unacceptable -I shall return to this argument lateron.

These problems are the reason why the instants that Allen and Hayes incorporate are judged as mere artificial entities and not as time elements that can be used to talk about the holding of facts and the occurrence of events. But this is the original motivation of incorporating instants. In order to escape to this quandary Allen and Hayes introduce the concept of *moment* [7]. A moment is a sort of instant long enough to be occupied by events, and a sort of period short enough to be not *decomposable* (contrasting with true periods that always are). Moments have many of the properties of points but, unlike time points, it is possible to distinguish between the "beginning" and the "ending" of a moment (so, even if very small, their duration is not null: as a result, they are closer to periods than to instants).

Unfortunately, enriching the ontology with moments makes the resulting model somewhat unintuitive and does not help too much since moments are not adequate for talking about things happening at truly instantaneous times. In [51] Hayes and Allen notice that the distinction between time points and moments is controversial and that the same short time period can be desired to be either a moment or a point according to the level of *granularity* you consider. Suppose you want to talk about the occurrence of an event at a time point (for instance "Jordi stops dancing" event). You could then try to insert a moment whenever a proposition needs to be true at a point. But, doing so makes the time periods separated by it (the period when Jordi is dancing and the period he is at rest) ceasing to meet anymore (the DIP again!).

The solution they propose consists of being able to treat moments as points and viceversa. They map moments into points (what Hobbs [52] calls a *change of granularity*) by defining the *indistinguishable* relation: two points are indistinguishable if they are either the same or the extremes of the same moment. To be an equivalence relation (as Hobbs points out a change of granularity is well-understood only in that case) they need to impose that their moments never meet. Then the axioms of the extended theory can be rewritten with instants equality replaced by instants indistinguishability. Hence, the extended theory applies uniformly at any level of coarseness in which moments are identified with their endpoints: "a moment can be treated as a point, or viceversa, without changing the axioms of the temporal theory (the extended theory is stable under changes of scale)". You can use this theory by having moments at those points where you are interested in asserting something while the rest are just time points where nothing can be directly asserted. Revisiting the DIP you just do not expand the transition point into a moment avoiding say whether P is true or is not at the moment i. If you are interested in asserting something at that point then you can introduce a moment and the periods I_1 and I_2 will still be meeting. Although it is an ingenious idea, it gives a non homogeneous structure to time what, from a phylosophical point of view seems rather unacceptable. You can find additional technical criticisms in [111].

Galton's revision or Allen's theory [39] is also oriented to introduce the instant in the time ontology. He notes that theories that treat instants by assuming that instants are part of intervals, or intervals are sets of instants, fall into the kind of problems illustrated by the DIP, and proposes to make a weaker definition just "retaining the idea of there being an instant at the point where the two intervals meet". The relations he is left with between instants and intervals are `Within` and `Limits`. He defines the following axioms (i,j,... denote instants; I,J,... denote intervals):

$$\forall I.\exists i.\mathtt{Within}(i,I)$$

$$\mathtt{Within}(i,I) \wedge \mathtt{In}(I,J) \Rightarrow \mathtt{Within}(i,J)$$

$$\mathtt{Within}(i,I) \wedge \mathtt{Within}(i,J) \Rightarrow \exists K.[\mathtt{In}(K,I) \wedge \mathtt{In}(K,J)]$$

$$\mathtt{Within}(i,I) \wedge \mathtt{Limits}(i,J) \Rightarrow \exists K.[\mathtt{In}(K,I) \wedge \mathtt{In}(K,J)]$$

This provides the basic means to define a theory of instants and intervals and an ontology of time with the right degree of commitment. For instance, the holding of a property on an interval can be defined without deciding whether it holds at the limiting points. This avoids the problems of Allen's arguments.

The criticism of Galton's approach could be that the kind of theory and the characterization of models of the set of axioms given above are not clear. In [112] Vila proposes a simple and clear theory made of both instants and periods as ontological primitives: the instants are a linear-ordered, unbounded, dense set and the periods are related to them by the functions

BEGIN(P) and END(P). The basic axioms ensure the i) the *Ordering of the extremes of a period* ii) given a period, the *existence* and *uniqueness* of instants and, iii) given two instants, the *existence* of periods and their *uniqueness*. Its models are the linear-ordered, unbounded, dense sets with its intervals. This is a simple construction that turns out to be a natural basis for temporal reasoning. It avoids the DIP-argument by simple taking the case *iv* of those mentioned above. It is based on the intuition that we cannot assert anything about what is happening at the transition point [6]. So, in terms of holding, if a property is holding during a period it holds at any of the instants that are within the period and the extreme instants are not [39]

$$\textsc{Holds}_{on}(p, P) \iff \forall i.\ \text{WITHIN}(i, P) \implies \textsc{Holds}_{at}(p, i)$$

So one is not able to infer whether Jordi is dancing or not at the meeting instant but we have instants which allow one to make assertions about what is happening at them and what is holding during the periods.

4.2 Temporal Relations, Expressions and Algorithms

The different choices about the type of *"pure" temporal expressions* –expressions completely composed of temporal relations– allowed inside our temporal logic, arise from considering the trade-off expressiveness/efficiency. Two main classes of approaches can be distinguished according to the nature of constraints used in the expressions: those approaches that are based on qualitative temporal relations and those that are based on metric information. Recently, some work has been also done on combining both.

4.2.1 Qualitative Relations

Allen develops an *algebra of intervals* [3] based on the 13 relations that correspond to the simple definite mutually exclusive relations that may exist between two intervals (see figure 3). The most general case allowing any arbitrary disjunctions on relations between temporal intervals that can be expressed in first-order logic is too complex to be considered for most AI applications. So Allen takes vectors of simple relations that are interpreted as the disjunction of relations. For instance, the vector (I_1 *Before Meets Overlaps* I_2) means that the interval I_1 either occurs before, meets or overlaps I_2. This is a way to represent *indefiniteness* on the interval temporal relation and allows to express any possible relation between two intervals. Two operations are introduced: *Addition* interpreted as the intersection of vectors (the least restrictive relation that the two vectors together admit) and *Multiplication* is the 3-elements transitivity (from $[I_1 V_1 I_2]$ and $[I_2 V_2 I_3]$, $V_1 * V_2$ is the least restrictive relation between I_1 and I_3). Allen provides a polynomial-time constraint propagation algorithm for computing the closure of a set of statements in the interval algebra [3]. The procedure obtained is simple and easy to implement.

Nevertheless the algorithm is sound but not complete. Completeness does not always have an affordable computational cost. Vilain and Kautz [115] prove that constraint consistency of statements (or determining their closure) in the *interval algebra is NP-hard*. They suggest several strategies to work in practical systems: i) to limit the number of statements, ii) to accept the incompleteness of the polynomial algorithm (it can be acceptable for applications that do not require much inference from the temporal reasoner), or iii) moving to a less expressive formalism, the point algebra.

[6] And, in fact, it is irrelevant for most of the applications.

INTERVAL VECTOR	POINT TRANSLATION	ILLUSTRATION
A (before meets overlaps) B	$A-$ precedes $B-$ $A-$ precedes $A+$ $A+$ precedes $B+$ $B-$ precedes $B+$	$A?$ B $A?$ $A?$
A (before after) B	No equivalent point form	$A?$ B $A?$

Figure 5: Moving from interval algebra to point algebra.

first-order logic ***wffs***
(arbitrary disjunctions $\equiv$ n-ary relations)

Full Interval Algebra [Allen83]
(I_1 before after I_2)
$(end(I_1) < begin(I_2) \ \vee \ end(I_2) < begin(I_1))$

Pointizable Interval Algebra $\equiv$ **Full Point Algebra** [vanBeek89]
$p_1 \neq p_2$

Continuous Point Algebra [Vilain&Kautz86]
$\equiv$ **Continuous Endpoint Interval**
$p_1 \leq p_2$

Figure 6: Temporal Algebras

The *point algebra* is defined in the same way that the interval algebra but the elements are points, the simple point relations are 3 (*precedes*, *equal* and *follows*) and relation vectors are composed of simple point relations, for instance (p_1 *precedes equal* p_2). The constraints closure can be computed in polynomial-time algorithms which are sound and complete.

Nevertheless, the notion of interval appears to be necessary in many cases (I would say in most of AI problems). The idea of representing an interval as a pair of points is not new (Khan and Gorry [54], McDermott, Shoham) and it benefits from the computational advantages of the point algebra. As expected, only a fragment of the interval algebra (the *restricted interval algebra*) can be translated to the point algebra. This fragment corresponds to the case where all constraints represent *convex* sets of intervals and, therefore, they can be expressed without the use of disjunctions between constraints on different pairs of endpoints, e.g. (p_2 *precedes* p_3) *or* (p_4 *precedes* p_1) (see in figure 5 an example from Vilain and Kautz).

Vilain and Kautz show that a constraint propagation algorithm excluding the constraints of the kind (p_1 *precedes follows* p_2) (which corresponds to the *continuous endpoint algebra*) operates in $O(n^3)$ time. Van Beek [108] includes also this type of constraints and gives an $O(n^4)$ algorithm. This result has been improved by [81] who establishes a $O(n^3)$ complexity. See Figure 6 for a picture of the different algebras for representing time.

An additional problem of the Interval Algebra is that the less one knows about relationship between two intervals, the longer the symbolic representation of that knowledge. Therefore,

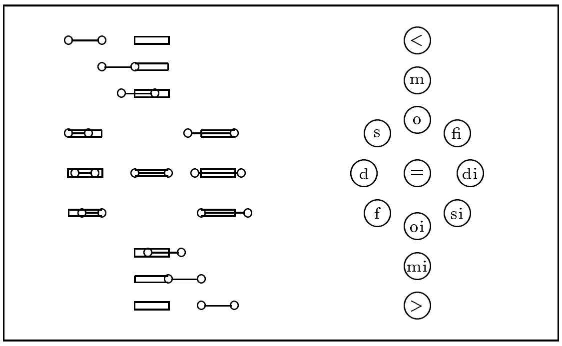

Figure 7: Interval relations arranged according to Freksa's conceptual neighborhood and their corresponding labels.

alternative sets of relations have been proposed for representing indefiniteness between intervals in a more adequate manner. Matuszec *et al* [77] approach is based on the partial information about the endpoints of the intervals (e.g. *X sbs Y* means that X starts before Y starts). Freksa [35] generalizes it with his *semi-intervals*. It is based on the concept of *neighborhood*. Fig. 7 shows how Allen's relations are arranged according to it. "This generalized temporal knowledge representation enables efficient, higher-level reasoning in terms of meaningful concepts". Freksa develops optimized transition tables for his neighborhood primitives to perform coarse reasoning. Freksa's neighborhood algebra is closely related to the *restricted interval algebra* [115] (i.e. the *convex interval algebra* [87]) in which global consistency is computationally tractable. Freksa's approach does not allow to include non-convex relations either, but directly representing indefiniteness as coarse knowledge can be more adequate. The required computational effort decreases when knowledge is coarser.

4.2.2 Metric Relations

The simplest case is when temporal information is available in terms of dates or another precise numeric form. In this case we have a numeric absolute temporal reference. Thus, assertions are timestamped to absolute numeric values. The durations can be numerically represented and easily computed (it is just the substraction of numeric values). In such case we have constant time algorithms that efficiently answer queries about occurrences by comparing numeric values.

Nevertheless, precise numeric information is not always available. Commonly the data available is about the temporal distance between events. In such cases a useful representation is an *acyclic directed graph* (nodes represent events and arcs represent distances between them) maintaining a partial ordering between events. For instance, it can be the case where distances are exactly known together with precise information about the occurrence of few special events. Then, an event can be represented by a constraint on its occurrence expressed as a pair $\langle$lower bound, upper bound$\rangle$. The earliest and latest bound of an unknown event occurrence is simply computed by adding up distances between known nodes. When there are several possible paths to an unknown node, its bounds are the minimal and maximal path distances respectively.

More complex is the case where information about distances is not precise but expressed

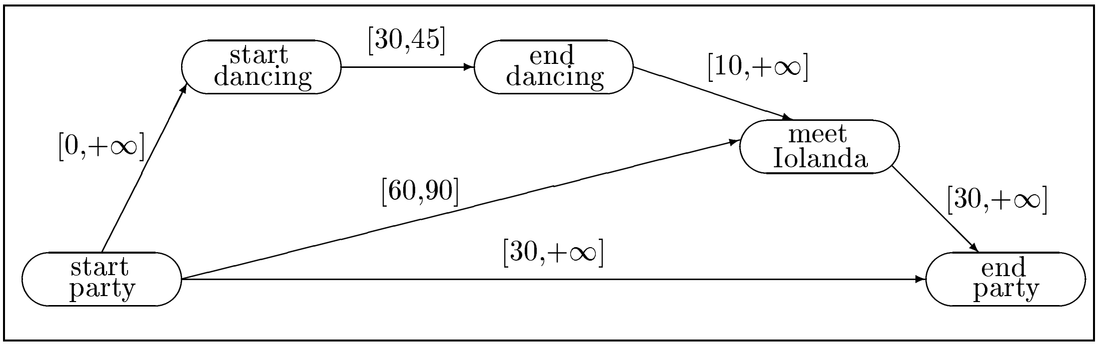

Figure 8: A *time map* example.

as a range of possible distances from precise distances up to completely qualitative distances (the $\langle 0 - \infty \rangle$ range). Its graphical representation (see an example in figure 8) is named *time map* [24].

By adding up the lower and upper distance bounds, distances which have not been explicitly given are derived. New available information can be added as temporal constraints which can change distance bounds. Path finding techniques are used to infer the new map state. A key point for the efficiency of this set of approaches is representing explicitly not everything but only the "interesting" information and means are needed to declare or identify it.

Dechter *et al* [25] adressed the same problem in a more formal and general way applying background from CSP. The *Temporal Constraint Satisfaction Problem* (TCSP) model is also based on points (each variable represents a time point). Different types of constraints are represented:

- **Unary constraints** : point-to-date constraint

simple case	a point p is represented as an interval $[a, b]$ $a < p < b$
general case	a point is represented as a set of intervals $I_1, \ldots, I_n$ $a_1 < p < b_1 \vee \ldots \vee a_n < p < b_n$

- **Binary constraints** : point-to-point constraint

simple case	The distance between two points p_1,p_2 is represented as an interval$[a, b]$ $a < p_2 - p_1 < b$ (if the values $-\infty$ and ∞ are included this case subsumes the continuous point algebra)
general case	the distance between two points p_1,p_2 is represented as a set of intervals $I_1, \ldots, I_n$ $a_1 < p_2 - p_1 < b_1 \vee \ldots \vee a_n < p_2 - p_1 < b_n$

The Simple Temporal Problem (STP), which includes only simple case constraints, can be solved in polynomial time applying the well-known *Floyd's all-pairs-shortest-paths* algorithm $O(n^3)$. For the general TCSP, Dechter et al. propose a *decomposition scheme* which may be improved by traditional constraint satisfaction techniques.

Allen's Interval algebra cannot be represented in time maps nor translated into binary TCPS. It would require 4-ary constraints for representing constraints like (I_1 `Before After` I_2). Dechter et al. remark that, unless metric constraints are specified, the representation suggested in [3] can be handled more conveniently. An open issue is addressing expresssions with these kind of higher-order constraints or others like "the event A has a longer duration than the event B". It would be formalized by

$$\text{end}(B)-\text{begin}(B) < \text{end}(A)-\text{begin}(A) \equiv 0 < (\text{end}(A)-\text{begin}(A))-(\text{end}(B)-\text{begin}(B)) < \infty$$

In summary, many different techniques can be used to process numeric information according to which information is available and its degree of precision. Their effectiveness is highly dependent on the degree of knowledge about the current case.

4.2.3 Qualitative and Metric Relations

The two different types of constraints discussed so far can be integrated in a single system in order to cope with the availability of knowledge at different degrees precision. This approach has been recently adressed in two different lines:

Kautz and Ladkin [56, 55] keep a metric and a qualitative interval based components separated and connect them by inter-component relationships. The reasoning tasks are based on solving each component independently and then circulating information between them to converge to a global solution.

Meiri [82] follows an alternative way by defining *general temporal networks* that integrate both types of constraints. The untractable problem of reasoning on general temporal networks is solved applying classical constraint satisfaction techniques, as

- by decomposition into STPs, each solvable in polynomial time, and a *backtracking* algorithm, or
- by applying *path consistency*, to compute an approximation to the minimal network.

Two classes of tractable networks are identified:

1. *Augmented Point Algebra networks*, networks with qualitative constraints between points and unary metric constraints on the values that a domain variable can take ("domain constraints"). One may distinguish between different augmented PA networks according to the type of the unary constraints: *discrete* (the variable takes a value form a discrete set), *single interval* (the variable takes a value from a single interval over a continuous set) and *multiple interval.* All three cases are tractable for the case of *Continuous Point Algebra* (CPA) and only the single interval for the case of the *full Point Algebra* (PA). They can also be solved applying path consistency.
2. Those networks which are equivalent to a STP network.

Meiri argues that his model is conceptually clearer the Kautz and Ladkin's and provide tighter bounds for various reasoning tasks.

4.2.4 Recent Advances

Recently, several pieces of work approached improvements of general case performances by exploiting the structure inherent in certain application domains. Such approaches enrich temporal knowledge representation with additional information structures to speed up query answering and some of them make use of incompletely connected graphs to reduce assertion time. There are different suplementary structures proposed: [7]

- **hierarchies**:
  - based on conventions for partitionning time according to the calendar and the clock (TMM [22])
  - made of *reference intervals* automatically generated based on *temporal containment* used in *TimeLogic* [61, 62] and by Davis and Carnes [21] for interval qualitative constraints
  - based in the inherent structure of interval constraints for *plan projection* (HIC) [118]
  - based in event subsets which are interleaved or encapsulated (Lin and Dean [72])
- **chains or sequences**:
  - the *IxTeT* system [43] uses a representation based on a maximum *spanning tree* of a lattice of time points related by the "resticted interval algebra", achieving an experimental average linear cost for both updating and retrieval
  - *Timegraph* [83] partitions a constraint networks into chains of point linked by ¡ and $\leq$ and handle both metric and qualitative information. *Timegraph II* [42, 41] extends its predecessor with $=$ and $\neq$ and automatic structuring though it does not yet handle metric information
  - Dorn's *sequence graphs* [26] based on domain *event sequences*

Concerning performance [119], in general, if one is facing large temporal datasets where information is added incrementally then the systems using incompletely connected graphs are better (like TMM or Timegraph). It is not the case in smaller databases where the criticallity of answering time or specific requirements from the application (such as the use of interval-based relations in planning applications or reasoning about the persistence in natural language) may make some systems more adequate.

4.2.5 Temporal Uncertainty and Imprecision

We have seen that in both approaches, qualitative and metric, the formalisms cope up to some extent with the representation of imprecision and uncertainty of the temporal knowledge. Some work has been done in the application of AI techniques for dealing with uncertainty and imprecision to the case of temporal representation. In this paper I just provide relevant references: [32, 23, 29, 28, 27, 60, 76].

[7] Since some of this pieces of work are very recent it is not always that clear how significant is their contribution wrt. previous results.

4.3 Temporal Entities

Most of the TR approaches include a classification of the propositions according to their different temporal patterns of occurrence on the basis of either cognitive, linguistic or intuitive criteria. We call the different classes *Temporal Entities*. We already have given in subsection 1.2 a rough definition for facts and events. Let's see now the temporal entities defined by different proposals.

McDermott distinguishes between *facts* and *events*. A *fact* is defined as a set of states (hence a set of points), intuitively those in which it is true. For example the fact "Jordi is near Iolanda" is the set of all states over time in which actually Jordi is near Iolanda. Hence, a fact can hold over either an instant or a time interval. However, there can be a problem of confusing coincident facts (e.g. Every time "Jordi is near Iolanda" it happens that "Jordi's cardiac rithm is higher than normal" so these are the same fact) but it might be argued that the branching structure of situations that McDermott defines in his logic to represent different possible futures individuate facts to the right degree of precision.

McDermott's definition of *events* is more subtle. The classical definition based on the fact changes resulting from the event[79] does not satisfy him because it implies that events have no duration (nothing can be said about when the event is happening) and does not cover *non effect events* (e.g. "Jordi is dancing" in the sense that it is something being accomplished). The alternative of identifying events with the fact that the "event is taking place" (so an event happens in a chronicle if any of the states of its related fact is in that chronicle) seems to be adequated for events "happening for a while" but has problems with the *inhomogeneity* of most of the events[8]. The final choice is to identify an event with the set of intervals over which one occurrence of the event takes place with no time left over. Intervals are denoted by the states marking their begin and end points. Although McDermott's classification has been widely accepted by subsequent approaches, the "circular" definitions given are not that satisfactory. This is part of the general criticism about the lack of a formal semantics definition.

Allen's classification is similar. He distinguishes between three types of temporal entities, namely *properties*, *events* and *processes*, and defines them by giving constraints between the truth of the proposition over one interval and its truth over related intervals. Properties roughly correspond to states that hold "homogeneously" over a particular interval of time. Thus, for example, if a property, such as a house being red, holds over an interval i then it holds over all the subintervals of i as well:

$$\textsc{Holds}(p, I) \Leftrightarrow \forall I'.\ \texttt{In}(I', I) \Rightarrow \textsc{Holds}(p, I')$$

Events occur over the smallest time possible for them to occur:

$$\textsc{Occurs}(e, t) \wedge \texttt{In}(t', t) \Rightarrow \neg\textsc{Occurs}(e, t')$$

For Allen, events describe an activity that involves a product or expected outcome (e.g. "Jordi built a bed"). *Processes* occur by an intermediate way between properties and events (e.g. Jordi has been dancing during the party). They seem to occur either over at least a *substantial number* of subintervals or over the subintervals greater than a certain interval size. Due to

[8]It is just the opposite case. It covers only those events characterized by a set of states which any subset cannot be considered as an occurrence of the event. In Davidson's example "John ran around the track three miles" such an event would be considered to happen also in a chronicle where a "2 miles around" event happens.

the problems in formalizing it Allen states that a process must be *occurring* over at least one subinterval:

$$\textsc{Occurring}(e, t) \Rightarrow \exists t'.\ \mathtt{In}(t', t) \wedge \textsc{Occurring}(e, t')$$

Allen's classification is based on the distinction between static and dynamic aspects of the world and problems arise when things can be described from either a dynamic or static perspective. For instance, the situation in which "Jordi is dancing" can be seen i) as a certain state for Jordi being for a while or ii) as an activity being carried out by Jordi. Concerning the events definition [98], we can consider the following three events: (1) I climb up the twenty-step stairs to the 1st floor, (2) I descend 3 steps to pick up my hand handkerchief just fallen from my hand, and, (3) I climb the remaining three steps to the 1st floor. I would like to regard the composite event as "to climb up the stairs to the first floor", but it is also occurring in a subinterval of it what is contradictory with the event definition. Finally, the definition of processes is too weak. If "I have been dancing during the party" it is also occurring according to this definition that "I have been dancing during my life" what is unintuitive (that party could be the only occasion I danced in my life). There is a problem in identifying the precise interval over which the process takes place, what is in fact a matter of *granularity in time*. Also Allen receives the criticism of not providing a formal semantics that clarifies the definition of temporal entities.

Shoham (1986) gives an exhaustive classification of temporal entities that subsumes previous ones by using the same criteria used by Allen of specifying how the truth of the proposition over one interval is related to its truth over related intervals. For instance, he defines a proposition to be:

- *downward-hereditary* iff it is the case that when it holds over an interval then it holds over all its subintervals (e.g. Jordi danced less than three pieces),
- *upward-hereditary* iff it is the case that if it holds over all proper subintervals of some nonpoint interval then it holds over the interval,
- *liquid* iff it is both upward-hereditary and downward-hereditary (e.g. Jordi is tired),
- *concatenable* iff whenever it holds over two consecutive intervals it holds also over their union (e.g. Iolanda is dancing samba),
- *gestalt* iff it never holds over two intervals one of which properly contains the other (e.g. Exactly six minutes passed),
- *solid* iff it never holds over two properly overlapping intervals (e.g. Jordi and Iolanda performed an spin turn), ...

Though we have an exhaustive set of definitions, from the point of view of the temporal logic, the permanent criticism of including these definitions in the semantics still holds. Haugh [49] distinguishes between *solid*, *liquid* and a third unconstrained sort of entites and borrows the definitions from Shoham. However, he introduces these definitions in the semantics of his logic. That makes the temporal entities not simple definitions part of a particular theory of time but part of the definition of the temporal logic with all its consequences. For instance, one gets meaningful soundness and completeness in the case that one proves them.

5 Change and Causality

Those researchers whose primary interest is providing a framework for building problem solvers in a dynamic world will be concerned to developing a theory that accounts for those concepts closely related with change like causality, actions, agents, etc. In this section I just introduce the main notions, I present some examples of the representations proposed and I discuss some of the problems that researchers encountered. There is not space in this survey to make a detailed analysis of this topics. I would recommend more specific surveys [69] and cited references to those interested.

5.1 Change

As I mentioned in the introduction, the concepts of *Change* and time are deeply related. Russell defines change as the difference, in respect of truth and falsehood between a proposition concerning an entity and a time t and the proposition concerning the same entity and a time t'. In the Situational Calculus change is defined *globally.* Instead of considering the variation on a single proposition, it considers the change from one complete description of the world to another one derived from the former. Oppositely, most of the later approaches follow Russell's idea and define change *locally*, as the change of truth value of one or a set of propositions. Another fundamental feature of change is whether it is discrete or continuous. Representing *continuous change* has been mentioned as a major objective in several relevant papers [80, 39, 101].

5.2 Causality

The representation of causality is a major issue for reasoning about change. In general, most of the changes are explained by causal relations. Diagnosis basically consists of determining the cause(s) that explain a set of findings. In planning, the actions are important because they "cause" effects. As we have seen in section 3, the representation of causality plays a prominent role in the evaluation of the expressivenes of a temporal logic.

In most of the TR approaches, causality is represented by causal predicates defined on temporal entities. Different predicates are defined to represent causal relations between different entities: an event that causes another event, an event that causes a fact, an agent that causes an event, an action causes another action, etc. Allen, for instance, introduces

$$\mathrm{ECAUSE}(event_1, time_1, event_2, time_2)$$

to express that an event causes another event and further constraints on their temporal occurrence can be expressed by referring to $time_1$ and $time_2$. In the Event Calculus [63], Kowalski and Sergot introduce the predicates *Initiates* and *Terminates* to implicitly represent an event causing a *fact* to hold towards the future and towards the past respectively. Causality is symetrically represented to both the future and the past. One can imagine more complex predicates like *project* defined in the TMM [24]:

(project *antecedent_conditions trigger_event delay consequent_effect duration*)

states that if an event of type *trigger-event* occurs, and the *antecedent-conditions* are true throughout the interval associated with the triggering event, then a fact of type *consequent-effect* holds following the triggering event by an interval of time determined by *delay* and lasting for a *duration* length.

All these approaches to reasoning about change and action have encountered the same kind of representational problems. Specially illustrative is the case of representing an event causing a fact. These kind of causal rules modelling the dynamics of the real systems use to be problematic because the complexity of the real world and the difficulty in getting the right abstraction of it. One may be convinced that one possesses the right knowledge about a causal relationship until one tries to make use of it to determine how the system evolves. McCarthy and Hayes [79] realized that one is not able to infer everything about a situation resulting from an action performed in a previous situation without having a large number of axioms of the form "$fact_i$ does not change in this transition". This is the so-called *frame problem*. Let's talk in more general terms about this problem and related ones.

5.3 The frame problem

The *frame problem* can be formulated as the problem of representing a change as performing an inference, i.e. which beliefs are true and false before and after, what remain unchanged and what changes. One requires a representation able to correctly account for every interesting behaviour but doing it efficiently, without having to make everything explicit. For example let consider the action of *start-dancing* performed by our dancer Jordi. You can formalize it by using a representation that explicits the conditions to be satisfied in order to carry out the action (e.g. "to have a partner who accepted to dance", "to have free space", etc.) and the consequences of that (e.g. "both dancers will be in movement"). Then you face the problem of specifying every possible relevant condition, namely the *qualification problem* [105] (e.g. "Jordi wants to start dancing", "Jordi has energy enough to dance", "she has not changed of opinion about dancing with Jordi", "Jordi knows dancing techniques", ...), the problem of specifying everything that changes, namely the *ramification problem* (e.g. "Jordi's shoes are in movement", "Jordi's shoes laces are in movement",...), the problem of specifying everything that does not change, properly called the *frame problem* (e.g. "the seat where Jordi's partner was sitting on is not in movement", "Jordi's shirt colour is still white",...). In either terms, it is the problem of representing explicitly the right things and considering the rest by default.

Solutions to these problems require a certain capability for *nonmonotonic reasoning*. It consist of being able to reason upon assumptions, i.e. making inferences based not only on known facts but also on assumed facts.

5.4 Nonmonotonic Temporal Logics

The first TR approach that attempts to capture these ideas and provide a solution to the above problems is McDermott's logic. McDermott incorporates the idea of *persistence*. A fact is said to persist until a certain time if it cannot be proven that it "ceases" to be true before that time. It allows to consider by default the holding of the truth over time of those many facts that are not going to change after an event. Reconsidering the formalization of an event causing a fact McDermott realizes that "...what events must cause directly is persistences, not the truth of facts. ...". This idea of persistence has been developed and implemented in the Event Calculus [63] by using PROLOG's negation by failure and in the TMM [24] by using a Reason Maintenance System.

A lot of work have been done on formalizing these concepts. The *Yale Shooting Problem* (YSP) [48] illustrated that the classical nonmonotonic logics (*Circumscription*, *Default logics*,...) give bad results in a temporal setting. The YSP concerns a situation where Fred is

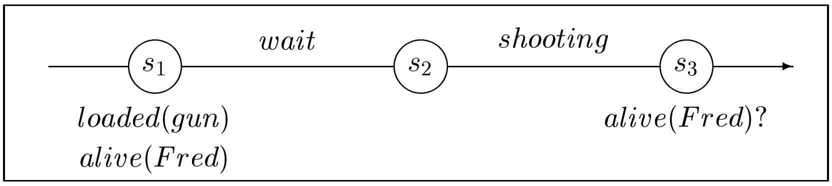

Figure 9: The Yale Shooting Problem scenario.

alive, a gun is loaded and shooting it would kill Fred. We have also a universal frame axiom that says that anything that is not *affected* by an action, persists through the execution of the action. From this initial situation there happens first a waiting action and then the gun is fired. Is Fred alive ? The YSP can be formalized as follows (see figure 9):

$$\begin{array}{l}
\textsc{Holds}(alive(fred), loaded(gun); S_1) \\
\forall S.\ \textsc{Holds}(loaded(gun); S) \Rightarrow \textsc{Holds}(\neg alive(fred); Result(shooting, S)) \\
S_2 = Result(wait, S_1) \\
S_3 = Result(shooting, S_2) \\
\textsc{Holds}(Alive(Fred); S_3)\ ?
\end{array}$$

While the intuitive answer is that Fred is either dead (assuming normality), any classical nonmonotonic system would only be able to conclude that either Fred is dead or the gun has been unloaded by the waiting action. In general, they would admit several possibilities, while intuitively some of them are more likely. One can overcome this problem by formalizing these intuitions as a criterion that expresses the preference for certain conclusions. Thus, one will restrict consideration to those conclusions that optimize the criterion.

Several solutions have been suggested based on performing the minimization[9] using a criterion based on temporal information (*pointwise circumscription* [70], *logic of persistence* [57] and *Chronological Minimization* [104]). Intuitively, Chronological Minimization consists of delaying as much as possible the occurrence of the abnormality. Other approaches has been studied in the line of minimizing abnormality but proposing criteria based on *causal information* [71, 50]. Sandewall [99, 100] proposes a preference relation that i) maximizes no change, ii) minimizes change when there are expectations to the opposite, and iii) minimizes the set of actions performed. It is a matter of research to study these approaches and analyse the weak points in order to find out new and better satisfactory solutions to the frame problem, at least for what concerns one's application tasks.

Besides it, the fact of dealing explicitly with time induces a new dimension of nonmonotonicity. We are able to deal with the temporal extension of the truth of propositions. *Assumptions*, which are the origin of nonmonotonicity, can be considered now also on these temporal extensions. Changes in the truth value of a proposition or in its temporal extension may affect the temporal extension of the truth of a related proposition. For example, if I know that Jordi started to dance with Iolanda then I can assume that they will be dancing for approximately the four minutes the musical piece lasts. But if, suddenly, an acute pain in the knee prevents Iolanda from moving her leg, then the dancing period estimated by me is reduced. The holding of a fact incompatible with the former *clips* the holding on another

[9] The notion of minimization is rendered precise by an extension to the logic semantics by incorporating the concept of *model preference* represented by a partial ordering on the set of models and *preferential entailment* [104].

fact, and it makes necessary to have the means to revise assumptions made on the temporal extension of the later by monitoring the temporal extension of the former.

6 Conclusions

Throughout this survey the most important representational issues for a TR System in AI have been presented and the choices taken by the different approaches in the literature have been discussed.

Concerning the logical formalism, the *Temporal Arguments* method has proven to be more useful than that which has been historically recognized [10]. It takes advantage of the well-known proof theory for first-order logics but we have seen some limitations in its expressive power. *Temporal Modal* approaches have been preferred for Natural Language and Program Specification applications due to their natural expression and the relative conception of temporal occurrence. However, the lack of representation of absolute time and the complexity of their theorem proving make them less attractive for problem solving. The incorporation of the modal operator $AT(t)$ is to make up for this shortcoming. In cases where *generic temporal knowledge* needs to be represented a *Reified* approach can be interesting, although may entail suspect ontological commitments. The *token reification* approach seems to be a good idea to avoid them, yet preserving the expressive power. An open issue is characterizing efficient specialized inference methods for first-order reified languages.

Concerning the ontology of time, it seems to be a general agreement that the *period* is the fundamental temporal unit representing the *duration* of things, although for several important applications (like those which include continuous change) a representation of instants is also needed. However, for many applications, the interval calculus may be too complex and the representation of intervals as a pair of points and the development of constraint-propagation algorithms on point-based representations is widely extended. The degree of precision in the information and the type of constraints on which the knowledge is expressed is crucial to assess the applicability of different representations and algorithms discussed in this survey [6].

Regarding the definition of Temporal Entities, the distinction between *facts* and *events* is widely accepted, but is not the same for the case of the *processes* proposed by Allen. Shoham's criterion of *how the truth of the proposition over one interval is related to its truth over related intervals* is more exhaustive yet simple. It cover many different cases possibilities although it is not clear that all of them are needed by real applications. The further diversification between instantaneous and durative occurrences recently introduced by Galton seems also interesting since instantaneous occurrences are needed, e.g. for representing continuous change.

If information is *incomplete* or *changing* and assumptions on the temporal occurrence of information are to be formulated then nonmonotonic capabilities are needed, i.e. means to state these assumptions and mechanisms to maintain the knowledge base from its consequences when assumptions have to be redrawn. A lot of work has been devoted to formalizing it but still there seems to be a long way to get a satisfactory solution that covers the many different cases.

The main conclusion of this survey is methodological. After considering the whole set of choices from which a TR approach is made, it seems rather unrealistic, as noticed by different reviewers [107, 90, 74], to claim for a general theory of time made on the basis of ideal choices, capable to achieve a satisfactory performance independently of the application we are faced with. Human beings seem able to behave efficiently in fairly different domains, without geting much in trouble. It has been the appealing argument for trying to discover the hidden key for representing time. Nevertheless, it is not pragmatic to state things in that way if our goal is to develop programs for solving problems in domains where time is relevant. The years of work on TR show us that it is more reasonable to consider the different choices for the TR approach issues as an open set over which a "rational decision" is to be taken in order to get the most appropriate formalism and reasoning devices for the intended application.

Hence, in order to build a TR system it is fundamental

1. to know the implications of the different choices for each design issue to be able to make *informed decisions*, and,

2. to determine the representational requisites from the intended application domain in order to take the *appropriate decisions*.

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