Allen's Interval Algebra Makes the Difference

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September 10, 2019

Qualitative Spatial & Temporal Reasoning

- QSTR is a major field of study in Knowledge Representation & Reasoning.
- QSTR abstracts from numerical quantities of space and time by using qualitative descriptions instead (e.g., *precedes*, *contains*, *is left of*).

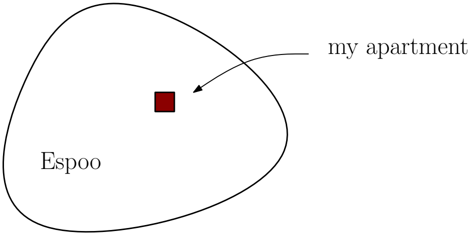


Figure: My apartment is located in the city of Espoo, Finland

Applications of QSTR

- Natural language processing
- Mobile robot navigation
- Image processing
- Geographical information systems (GIS)
- Qualitative spatio-temporal reasoning
- Querying linked geospatial data (e.g., GeoSPARQL)
- Neural-symbolic reasoning
- Data Mining
- Healthcare

Qualitative Constraint Language

Definition

A binary qualitative constraint language is based on a finite set B *of base relations such that:*

- *its base relations are jointly exhaustive and pairwise disjoint;*
- *its base relations are defined on an infinite domain* D*;*
- B *contains the identity relation* Id*;*
- B *is closed under the converse operation* ($^{-1}$)*;*
- 2^{B} *is equipped with the usual set-theoretic operations union and intersection, the converse operation, and the weak composition operation* ($\diamond$)*.*

Allen's Interval Algebra (IA) Constraint Language

precedes	p	pi
meets	m	mi
overlaps	o	oi
starts	s	si
during	d	di
finishes	f	fi
equals	eq	

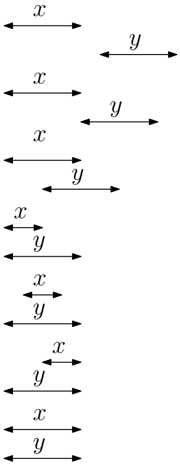


Figure: The thirteen base relations of Interval Algebra

The RCC-8 Constraint Language

disconnected	DC
externally connected	EC
partially overlapping	PO
tangential proper part	TPP $TPPi$
non-tangential proper part	$NTPP$ $NTPPi$
equal	EQ

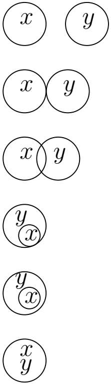
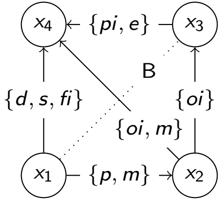


Figure: Two-dimensional examples for the eight base relations of RCC-8

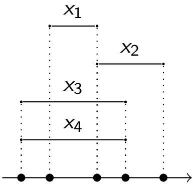
Qualitative Constraint Network (QCN)

Definition

A QCN *is a pair* $\mathcal{N} = (V, C)$ *where* V *is a non-empty finite set of variables, and* C *a mapping* $C : V \times V \to 2^{\mathsf{B}}$.



(a) A satisfiable QCN $\mathcal{N}$



(b) A solution σ of $\mathcal{N}$

Figure: Figurative examples of QCN terminology using Interval Algebra

Fundamental Reasoning Problems of QCNs

Definition

The satisfiability checking problem of a QCN $\mathcal{N}$ *is deciding whether* $\mathcal{N}$ *admits a solution.*

- Deciding the satisfiability of a QCN is NP-complete in general.

Definition

*The minimal labeling problem (*MLP*) of a* QCN $\mathcal{N}$ *is finding the strongest implied constraints of* $\mathcal{N}$*.*

- The satisfiability checking problem and the MLP are equivalent under polynomial Turing reductions [GS93].

Difference Constraints for Answer-Set Programming

- A *difference constraint* is an expression of the form

$$x - y \leq k$$

  where x and y are variables and k is a constant.
- Consistency checks can be performed using Bellman-Ford in time $O(|V| \cdot |E|)$.
- Inconsistency explanations are negative cycles.
- A difference constraint $x - y \leq k$ can be expressed as &diff$\{x - y\} \leq k$ in ASP.
- The implementation of ASP(DL) is known as the CLINGO-DL solver.[1]

[1] `https://potassco.org/labs/clingodl/`

Encoding Temporal Networks in ASP(DL)

```
% Domains
var(X) :- brel(X,Y,R).
var(Y) :- brel(X,Y,R).
arc(X,Y) :- brel(X,Y,R).

% Intervals for every variable X: sp(X) <= ep(X)
&diff{ sp(X)-ep(X) } <= 0 :- var(X).

% Choose base relations
{ chosen(X,Y,R): brel(X,Y,R) } = 1 :- arc(X,Y).
```

Example

```
% Relation eq(X,Y): sp(X) = sp(Y) and ep(X) = ep(Y)
&diff{ sp(X)-sp(Y) } <= 0 :- chosen(X,Y,eq).
&diff{ sp(Y)-sp(X) } <= 0 :- chosen(X,Y,eq).
&diff{ ep(X)-ep(Y) } <= 0 :- chosen(X,Y,eq).
&diff{ ep(Y)-ep(X) } <= 0 :- chosen(X,Y,eq).

% Relation during(X,Y): sp(Y) < sp(X) and ep(X) < ep(Y)
&diff{ sp(Y)-sp(X) } <= -1 :- chosen(X,Y,d).
&diff{ ep(X)-ep(Y) } <= -1 :- chosen(X,Y,d).
```

Evaluation (1/3)

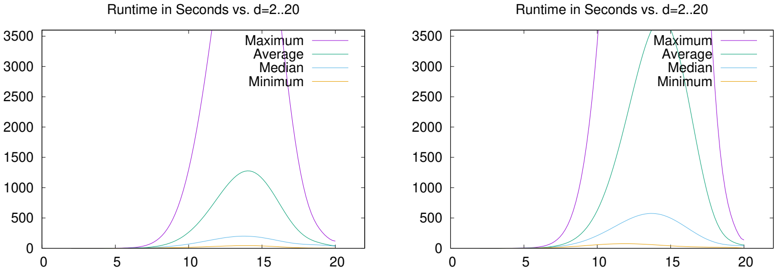


Figure: Runtime scaling: checking *satisfiability* vs computing *intersection of solutions*

Evaluation (2/3)

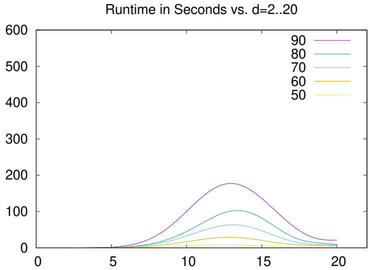


Figure: Runtime scaling (median): computing intersection of solutions vs computing union of solutions

Evaluation (3/3)

For hard instances, constraints are picked from a set of relations expressible in 3-CNF when transformed into first-order formulae.

d	9	10	11	12	13	14	15	16	17	18	19
Inters.	**4.8**	**8.7**	**19.8**	**50.8**	**122.3**	**940.7**	1738.0	**758.5**	**384.4**	258.0	**155.9**
Union	25.6	46.9	105.5	298.5	7226.3	5636.5	**749.8**	1585.5	438.9	**93.8**	169.3

Table: Median runtimes for hard IA instances with 50 variables

Conclusion

- We encoded QCNs based on Allen's Interval Algebra in ASP(DL), an extension of ASP by difference constraints.
- The transitive effects of relation composition are avoided when it comes to the space complexity of representing QCN instances.
- The presented encoding scales reasonably well.

Future Work

- We aim to investigate more thoroughly the performance characteristics of our ASP(DL) encoding.
- We would like establish collaborative frameworks among ASP-based and native QSTR tools.

Thank you for your interest and attention!