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The cognitive adequacy of Allen's interval calculus for qualitative spatial representation and reasoning

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Abstract. Qualitative spatial reasoning (QSR) is often claimed to be cognitively more plausible than conventional numerical approaches to spatial reasoning, because it copes with the indeterminacy of spatial data and allows inferences based on incomplete spatial knowledge. The paper reports experimental results concerning the cognitive adequacy of an important approach used in QSR, namely the spatial interpretation of the interval calculus introduced by Allen (1983). Knauff, Rauh and Schlieder (1995) distinguished between the conceptual and inferential cognitive adequacy of Allen's interval calculus. The former refers to the thirteen base relations as a representational system and the latter to the compositions of these relations as a tool for reasoning. The results of two memory experiments on conceptual adequacy show that people use ordinal information similar to the interval relations when representing and remembering spatial arrangements. Furthermore, symmetry transformations on the interval relations were found to be responsible for most of the errors, whereas conceptual neighborhood theory did not appear to correspond to cognitively relevant concepts. Inferential adequacy was investigated by two reasoning experiments and the results show that in inference tasks where the number of possible interval relations for the composition is more than one, subjects ignore numerous possibilities and interindividually prefer the same relations. Reorientations and transpositions operating on the relations seem to be important for reasoning performance as well, whereas conceptual neighborhood did not appear to affect the difficulty of reasoning tasks based on the interval relations.

Key words: cognitive adequacy, conceptual neighborhood, interval relations, spatial knowledge, spatial memory, spatial reasoning, symmetry transformation

Reasoning based on the qualitative representation of spatial knowledge is an active research area of AI and has found growing interest from two quite different perspectives: from an engineering point of view and from a cognitive science perspective. From an engineer's standpoint, qualitative representations of spatial relationships together with appropriate reasoning mechanisms are a promising way to enable computers to make predictions, diagnoses, planing etc. in a qualitative manner, even if detailed quantitative knowledge is not available or inferences based on such knowledge are computationally intractable (Cohn 1997).[1] The cognitive science motivation for working on QSR is that it possibly allows us to gain insight into the way human beings represent and think about space. This hope goes back to numerous psychological findings indicating that physical space is not represented isomorphically in the human mind, but as a form of qualitative spatial knowledge. Based upon this assumption, QSR approaches are believed to be a promising possibility to understand how the human cognitive system is able to isolate spatial features that are unique or essential for the actual situation or problem from all those values that can be expressed with respect to a predefined unit (Hernández 1994). Accordingly, Hernández, for example, argues in his book on QSR "*... the motivation for the qualitative approach is a cognitive one. Introspection and the way we verbalize spatial knowledge suggest that our consciously retrievable long term representation of spatial configuration is qualitative and vague*" (Hernández 1994, p. 13).

The general cognitive interest of QSR is also reflected in researchers' preferences for specific approaches. For the well-known interval calculus introduced by Allen (1983), for instance, numerous researchers on QSR emphasized that the appeal of Allen's theory is not primarily based on its computational properties, but on its cognitive adequacy (e.g. Freksa 1991). Consequently, the approach, although developed for temporal reasoning originally, was soon transferred to the spatial domain and triggered numerous research enterprises (e.g. Güsgen 1989; Mukerjee and Joe 1990; Hernández 1994; Nebel and Bürckert 1995; Walischewski 1997). However, in this paper it is argued that such an assumption of cognitive adequacy must be based on experimental research comparing predictions from the formal approach with human behavior and the underlying cognitive processes. In this point it is in agreement with Cohn (1997), who wrote in the conclusion of his overview article on qualitative spatial reasoning: "*An issue which has not been much addressed yet in the QSR literature is the issue of cognitive validity – claims are often made that qualitative reasoning is akin to human reasoning, but with little or no empirical justification*" (Cohn 1997, p. 22). For Allen's interval calculus these demands made on cognitive adequacy mean that the proposed base relations should correspond to cognitively relevant concepts and the proposed reasoning mechanism should be similar to the way people reason about space.

In this paper, a number of experimental studies investigating the cognitive adequacy of Allen's interval calculus in the spatial domain are presented. The plan for the paper is as follows: at the beginning a more detailed definition of the term "cognitive adequacy" is given. It takes into account that cognitive adequacy of a formal approach can be claimed for the underlying knowledge representation or for the reasoning mechanism. Consequently, a distinction is made between conceptual cognitive adequacy and inferential cognitive adequacy. The following section gives a brief overview of the main representational and computational characteristics of Allen's interval calculus, that is the qualitative base relations and the algebra for reasoning about these relations. This part also summerizes more recent findings from conceptual neighborhood theory (e.g. Freksa 1992) and the analysis of symmetry transformations operating on the interval relations (e.g. Schlieder 1995; Ligozat 1990). Afterwards, in the main part of this paper, experimental findings with respect to both aspects of cognitive adequacy are reported: section 3 is concerned with the conceptual cognitive adequacy of Allen's approach, i.e., the base relations as a representational system (Experiment 1 and 2), whereas section 4 focuses on the inferential cognitive adequacy of the reasoning mechanism, i.e. the composition of the base relations (Experiment 3 and 4) as a tool for reasoning. Both parts also account for predictions coming from conceptual neighborhood theory and symmetry analyses. The last section summarizes the experiments and discusses the results with respect to cognitive theories of spatial knowledge and reasoning and its implications for QSR.

Cognitive adequacy in QSR

Charniak and McDermott (1985) in their well-known text book argued that the ultimate objective of AI is to build systems which think in a human-like fashion. Recently, the criteria made on "cognitively adequate" systems have become more modest. In Strube (1992), for instance, the concept ranges from an absolutely strong (idealized) meaning down to a very weak notion of conforming to well known ergonomic standards. Strong cognitive adequacy means that a formal approach (or implementation) is claimed to be an adequate model of human knowledge and reasoning mechanisms in all (absolutely strong cognitive adequacy) or some (relatively strong cognitive adequacy) relevant aspects. Weak cognitive adequacy is reduced to the assumption that an implementation of a formal approach can be characterized as ergonomic and user-friendly (Strube 1992). However, since much research on QSR aims at constructing systems that work similarly to the way people do, a more detailed definition of the term cognitive adequacy is needed. First, the question whether an approach to QSR can be claimed as cognitively adequate can be answered only on psychological experiments that allows a comparison between human cognitive processes and the characteristics of the formal approach. Second, a further distinction between the representational characteristics of formal approaches and their inferential aspects is also needed. For this reason, (Knauff, Rauh and Schlieder 1995; Knauff, Rauh and Renz 1997) proposed the notation of conceptual cognitive adequacy and inferential cognitive adequacy.

Conceptual cognitive adequacy

Conceptual cognitive adequacy (abbreviated in the following as conceptual adequacy) is concerned with the question of whether or not the proposed representational system is a model of people's conceptual knowledge of spatial relationships. Since Allen's approach, for instance, relies only on ordinal information of the startpoints and endpoints of intervals, experimental investigations with human subject must answer the question regarding the extent to which people use this kind of information when conceptualizing, encoding and remembering spatial arrangements. Only if the experimental results support this assumption can the approach be seen as conceptually more adequate than other approaches which lead to different predictions, for example which also account for metrical information or just deal with topological information. In fact, it must be shown that human subjects distinguish spatial arrangements with respect to the same features as proposed in the formal approach. On the other hand, a formal approach of spatial knowledge representation must be rejected as conceptually inadequate if the proposed kind of spatial information seems to be irrelevant in human knowledge.

Inferential cognitive adequacy

The second aspect of a cognitively adequate approach to QSR is inferential cognitive adequacy (abbreviated as inferential adequacy), which is concerned with the calculus as a reasoning mechanism. Inferential adequacy can be claimed if and only if the reasoning mechanism of the calculus is structurally similar to the way people reason about space (Knauff, Rauh and Schlieder 1995). As with conceptual adequacy, this question can be translated into a hypothesis that can be tested experimentally. In this way it is possible to predict that if the same spatial problem is given to human subjects and to a system using, for example, Allen's calculus, both come up with the same conclusion based on similar internal processes. Accordingly, it is only in this case that a formal reasoning algorithm can be claimed either as inferentially more adequate than alternative proposals or, of course, can be rejected as inferentially inadequate.

The distinction between conceptual and inferential adequacy is important from the psychological as well as from an engineering perspective and also has implication for strong and weak cognitive adequacy: empirical investigations on conceptual adequacy can be important for such parts of general psychology that are traditionally called the psychology of knowledge, whereas inferential adequacy overlaps a number of research topics from the psychology of reasoning. Both are related to the assumption of strong cognitive adequacy. However, empirical results related to the conceptual/inferential distinction are also crucial for weak cognitive adequacy and for the development of user-friendly software systems, because the reasons for ergonomic disadvantages can be traced back to users' problems with both parts of the systems, e.g. the knowledge representation or the reasoning algorithms. Empirical results therefore can also feed back into the development of new formal models.

Allen's interval calculus

Allen (1983) introduced a calculus based on intervals originally representing events, qualitative relations between these intervals and an algebra for reasoning about these relations. The 13 base relations are jointly exhaustive and pairwise disjoint (JEPD), i.e. exactly one relation holds between any two intervals. Further 2^{13} relations can be obtained as unions of the base relations. The base relations are illustrated in Table 1. The original names from Allen (1983; first column in Table 1) reflect that they were introduced for temporal reasoning. In the spatial domain the intervals represent segments on a line or curve and the second column shows how the relations can be translated into natural language expressions for the spatial domain. The symbolic notation (column 3) for the original temporal relations is maintained for the spatial relation. Note that the pictorial examples (column 4) with the two rectangles **X** (white) and **Y** (grey) are two dimensional only for graphical reasons but are in fact one-dimensional. In the last column the relations are defined on the basis of the startpoint **s** and endpoint **e** of the two intervals.

Reasoning with these relations relies on their composition. Given the qualitative relationships between X and Y, and Y and Z, such a composition is defined as the relation between X and Z. Since every relation can be combined with each other, there are 169 compositions and hence cells in the so-called composition table shown in Table 2 (The table has only 144 cells, because compositions with the trivial = relation were omitted). In this table it is important to note that there are 72 cells with only one entry, 42 cells with 3 entries, 24 cells with 5 entries, and 3 cells with 9 entries. There are also 3 cells with 13 entries (for example X < Y, Y > Z), which means that the relational expressions X r_1 Y and Y r_2 Z do not put any constraint on X r_3 Z. In his paper Allen (1983) proposed a constraint-satisfaction algorithm to compute the compositions and even if reasoning with the full calculus of 2^{13} relations is NP-hard, we know that reasoning with the 13 base relations can be performed in polynomial time (Vilain and Kautz 1986; Vilain, Kautz and van Beek 1990).

Out of the numerous publications related to the interval calculus, two investigations are in particular worth taking into account. The first proposal

Table 1. The 13 qualitative interval relations, their names introduced for temporal reasoning, natural language expressions for the spatial domain, one graphical realization, and ordering of startpoints and endpoints (adapted and augmented according to Allen 1983).

Names from Allen (1983) (temporal)	Natural language description for the spatial domain	Symbol	Graphical example	Point ordering (s = startpoint, e = endpoint)
before	X lies to the left of Y	X < Y		$s_X < e_X < s_Y < e_Y$
meets	X touches Y at the left	X m Y		$s_X < e_X = s_Y < e_Y$
overlaps	X overlaps Y from the left	X o Y		$s_X < s_Y < e_X < e_Y$
starts	X lies left-justified in Y	X s Y		$s_Y = s_X < e_X < e_Y$
during	X is completely in Y	X d Y		$s_Y < s_X < e_X < e_Y$
finishes	X lies right-justified in Y	X f Y		$s_Y < s_X < e_X = e_Y$
equals	X equals Y	X = Y		$s_X = s_Y < e_Y = e_X$
finishes-inverse	X contains Y right-justified	X fi Y		$s_X < s_Y < e_Y = e_X$
during-inverse	X surrounds Y	X di Y		$s_X < s_Y < e_Y < e_X$
starts-inverse	X contains Y left-justified	X si Y		$s_X = s_Y < e_Y < e_X$
overlaps-inverse	X overlaps Y from the right	X oi Y		$s_Y < s_X < e_Y < e_X$
meets -inverse	X touches Y at the right	X miY		$s_Y < e_Y = s_X < e_X$
after	X lies to the right of Y	X > Y		$s_Y < e_Y < s_X < e_X$

is given by the conceptual neighborhood theory by Freksa (1992), the second goes back to the symmetry transformations operating on the interval relations as analyzed by Schlieder (1995) and Ligozat (1990). In general, both are concerned with the relationships between the base relations, and can therefore be regarded as proposals to explain the similarity of these relations. From an engineer's viewpoint as well as from a cognitive scientist's perspective such similarities are important; for the former because they can be used to reduce the computational costs when the compositions must be computed, and for the latter, they could be helpful to understand interrelationships between spatial concepts.

Conceptual neighborhood theory

Conceptual neighborhood theory introduced by Freksa (1992) approaches the issue of reasoning with Allen's interval relations from a cognitive perspective and is motivated primarily by physical constraints on perception (Freksa, p. 200). In this theory two base relations are conceptual neighbors if they can be directly transformed into one another by continuous transformations.

The transformations can be seen as deformations of the intervals, i.e. moving, shortening or lengthening, implying the assumption that such transformations are the relevant aspects for defining the similarity between the interval relations. Accordingly, in the neighborhood graph shown in Figure 1, Freksa (1992) arranged the thirteen base relations in such a way that conceptual neighbors become neighbors in the graph.

It can be seen that depending on the type of transformations different neighborhood structures are obtained. If, for instance, one of the startpoints and endpoints is allowed to be moved, the A-neighborhood is obtained (lengthening, shortening). If complete intervals are moved, we obtain the B-neighborhood (moving). A few examples: the relations < and m are A-neighbors as well as B-neighbors because they can be transferred into one another by lengthening the intervals or by moving one of the intervals. The relations < and o are neither A-neighbors nor B-neighbors since they cannot be transferred directly into one another by lengthening or moving. The only possibility is via the relation m. The relations = and s are A-neighbors but not B-neighbors, because they can be transferred into one another by lengthening but not by moving one of the intervals. In general, a set of Allen's interval relations forms a conceptual neighborhood if they are connected in the diagrams. However, Freksa pointed out that his approach "*…provoke[s] the question of the cognitive significance of the neighborhoods. If the neighborhoods appear to correspond to cognitively relevant concepts, maybe the reasoning should be done based on these neighborhoods …rather than on individual relations themselves*" (Freksa 1992, p. 211).

Symmetry transformations on the interval relations

Another theoretical viewpoint for explaining the interrelationships between Allen's base relations is given in the work of Schlieder (1995), who analyzed the two possible symmetry transformations operating on the interval relations: reorientation transformation (abbreviated in the following as reorientation) and transposition transformation (abbreviated as transposition). For the temporal domain the former is also called "times reversal" by some authors (e.g. Ligozat and Bestougeff 1989). The geometrical interpretations of both transformations are shown in Table 3: if X and Y are two intervals on a line such that X r Y, the reorientation is characterized as the relation holding between X and Y after inversion of the linear order. Transposition is characterized as the relation holding after an exchange of the roles of X and Y.

In the table it is easy to see that each of the relations can be obtained by applying one of the symmetry transformations to another relation, but it is

Table 2. Composition table for the base relations from Allen (1983). The 72 compositions with only one entry are presented as white cells, whereas the 72 compositions with multiple entries are the shaded cells. The trivial = relation is omitted.

	<	m	o	fi	di	si	s	d	f	oi	mi	>
<	<	<	<	<	<	<	<	<, m, o, s, d	<, m, o, s, d,	<, m, o, s, d,	<, m, o, s, d,	<, . . . , >
m	<	<	<	<	<	m	m	o, s, d	o, s, d	o, s, d	fi, =, f	di, si, oi, mi, >
o	<	<	<, m, o	<, m, o	<, m, o, fi, di	o, fi, di	o	o, s, d	o, s, d	o, fi, di, si, =, s, d, f, oi	di, si, oi	di, si, oi, mi, >
fi	<	m	o	fi	di	di	o	o, s, d	fi, =, f	di, si, oi	di, si, oi	di, si, oi, mi, >
di	<, m, o, fi, di	o, fi, di	o, fi, di	di	di	di	o, fi, di	o, fi, di, si, =, s, d, f, oi	di, si, oi	di, si, oi	di, si, oi	di, si, oi, mi, >
si	<, m, o, fi, di	o, fi, di	o, fi, di	di	di	si	si, =, s	d, f, oi	oi	oi	mi	>

s	<	<	<, m, o	<, m, o	<, m, o, fi, di	si, =, s	s	d	d	d, f, oi	mi	>
d	<	<	<, m, o, s, d	<, m, o, s, d	<, . . . , >	d, f, oi, mi, >	d	d	d	d, f, oi, mi, >	>	>
f	<	m	o, s, d	fi, =, f	di, si, oi, mi, >	oi, mi, >	d	d	f	oi, mi, >	>	>
oi	<, m, o, fi, di,	o, fi, di	o, fi, di, si, =, s, d, f, oi	di, si, oi,	di, si, oi, mi, >	oi, mi, >	d, f, oi	d, f, oi	oi	oi, mi, >	>	>
mi	<, m, o, fi, di	si, =, s	d, f, oi	mi	>	>	d, f, oi	d, f, oi	mi	>	>	>
>	<, . . . , >†	d, f, oi, mi, >	d, f, oi, mi, >	>	>	>	d, f, oi, mi, >	d, f, oi, mi, >	>	>	>	>

†All 13 relations are possible.

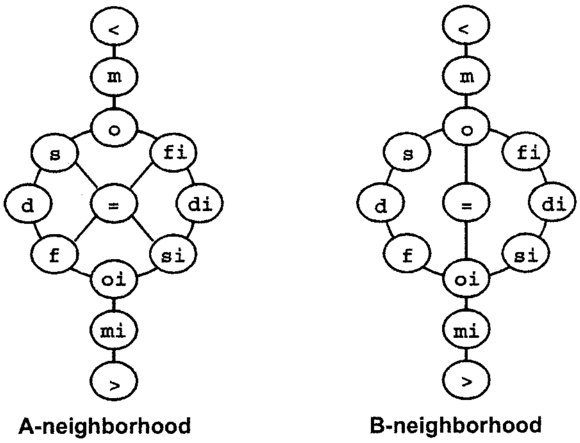

Figure 1. The conceptual neighborhood structures from Freksa (1992).

Table 3. Symmetry transformation on the interval relations from Schlieder (1995)

	<	m	o	fi	di	si	s	d	f	oi	mi	>
reorientation	>	mi	oi	si	di	fi	f	d	s	o	m	<
transposition	>	mi	oi	f	d	s	si	di	fi	o	m	<

important to notice, that in some cases reorientation and transposition of an interval relation produce the same relation.

So far we have outlined a brief review of formal investigations and we now come to the experimental studies comparing these approaches and the way people store and remember spatial arrangements or reason about such relations, respectively. The results of the experiments may in this way either support the assumption that people use similar information when conceptualizing spatial arrangements and reasoning about space, or the experimental outcomes reject the formal approaches as cognitively inadequate – conceptually, inferentially or both.

Conceptual adequacy of the interval relations

To investigate the conceptual adequacy of the thirteen base relations introduced by Allen (1983), a recognition experiment and a recall experiment in long-term-memory (LTM) were conducted. Subjects were invited to learn spatial arrangements where only the ordinal information was varied, followed by a test-phase in which subjects ability to remember and distinguish the learned arrangements was measured. The main hypothesis is: if the order of startpoints and endpoints of objects is an aspect of human conceptual spatial knowledge, subjects must be able to remember instances of different interval relations as different arrangements. If such information does not play a role in the conceptual representation, instances of different interval relations must be confused.

The experimental material was also varied taking the conceptual neighborhoods of Freksa (1992) and the symmetry transformations on the interval relations from Schlieder (1995) into account. Thus, we have two further, or more specifically four (when separating A neighborhood and B-neighborhood as well as reorientation and transposition) hypotheses that can be tested experimentally.

Note that the language-free memory experiment paradigm was used to refer the obtained results to the pure conceptual representation avoiding the influence of the semantic aspects of the associated natural language descriptions. The investigations were performed as LTM experiments, because in this case the influence of concept-driven processing and encoding is much greater than in short-term-memory. The experiment was computer controlled. Subjects could only participate if they were not familiar with Allen’s approach or any similar approaches.

Experiment 1: recognition of the interval relations

Based on the hypotheses mentioned above, a recognition experiment was conducted in which twenty-four subjects learned thirteen spatial arrangements of two rectangles which were instances of the interval relations. At the beginning each of the spatial arrangements was experimentally associated with a character that served as a cue stimulus in the recognition phase (paired associate learning). In the recognition phase (one hour later) the character was displayed together with two of the 13 arrangements, one of which was the associated arrangement (target item) and the other an arrangement that was associated with one of the other characters (distractor item). By pressing keys on the keyboard, subjects had to decide which of these two arrangements was previously associated with the current character. The accuracy was measured by the number of correct answers (hits and correct rejections) and by the reaction latencies. Due to the fact that such recognition tasks involve a degree of match between the original item and the current item, one task should become more difficult with an increasing similarity between the target and the distractor. Therefore, on the one hand, the material was varied systematically with respect to the conceptual neighborhood and the symmetry transformations. On the other hand, to enable the obtained results to be referred to the pure conceptual decision all other possibilities to distinguish the arrangements, for example sensory features, were controlled.

Results

With respect to the interdisciplinary readership of the journal we refrain from reporting the usually reviewed experimental details, so only statistically reliable effects are reported in the following sections. For this statistical decision a criterion for significance has been set at the 0.05 level, and therefore, all reported results are at or beyond this level.

Since subjects had just two alternatives to choose from, the expected number of correct answers by chance is 50%. On average, 92.1% of the learned arrangements were recognized correctly and no significant differences were found between the relations to be remembered (only the = relation was a little bit easier than the other 12 relations). However, the difficulty of the tasks was modulated by the presented distractors. By analyzing the pairs of targets and distractors separately, no reliable effect for the A-neighborhood ($\chi^2_{(1)} = 0.0499$, $p = 0.823$) or the B-neighborhood ($\chi^2_{(1)} = 0.1619$, $p = 0.203$), was found. On the other hand there was a strong and statistically significant effect of the symmetry transformations. As shown in Figure 2 the judgement of which of the two arrangements were originally associated with the cue stimulus was much harder if the distractor was a reorientation of the target arrangement. In such reoriented cases subjects made more errors (21.7% v. 6.9%; $\chi^2_{(1)} = 67.159$, $p < 0.001$), and the decision took more time than in the non-reoriented cases (4.23 sec. v. 2.99 sec.; $F(1,21) = 106.719$, $p < 0.001$).

We also analyzed the influence of transposition symmetry and found a similar and reliable effect for this factor. Separating the spatial arrangements, as shown in Figure 3, leads to more errors (24.2% v. 6.9%; $\chi^2_{(1)} = 93.722$, $p < 0.001$) and took more time (4.32 sec. v. 2.98 sec.; $F(1,21) = 24.744$, $p < 0.001$) if the distractor was a transposition of the target arrangement. This analysis takes into account that in some cases both symmetry transformations produce the same arrangement. In these cases the arrangement was only counted as reorientation. This was done due to the fact that a previous analysis, in which all ambiguous cases were eliminated, had shown that reorientation is the more important factor.

Experiment 2: recall of the interval relations

To validate the results of experiment 1 another experiment was conducted. Twenty-four subjects had to reproduce the previously learned items from

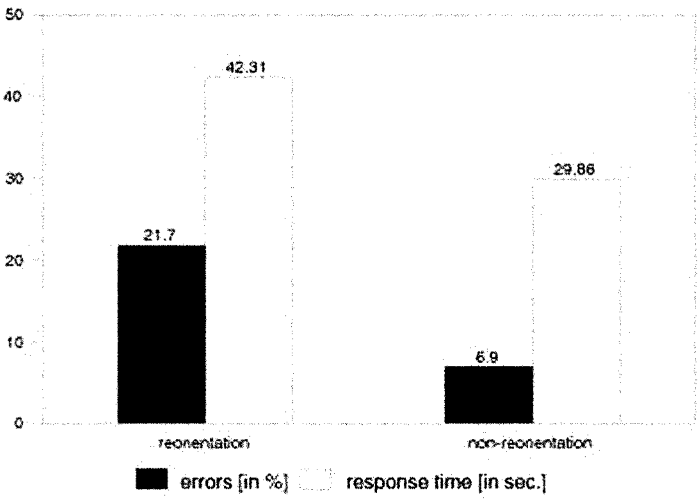

Figure 2. Error rates and response times in the recognition experiment as a function of reorientation v. non-reorientation from targets to distractors.

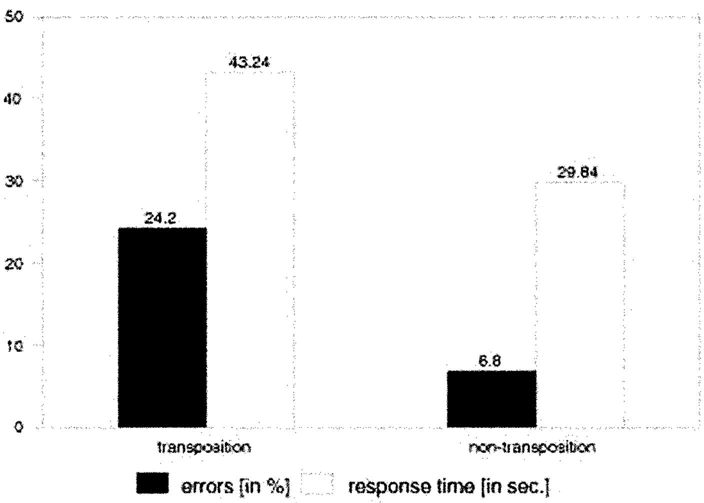

Figure 3. Error rates and response times in the recognition experiment as a function of transposition v. non-transposition.

memory without additional cues. Thus, all processes that may be based on the comparison of the sensory information of targets and distractors are eliminated. For this reason recall experiments are assumed to be more concept-driven than recognition experiments, but more difficult as well.

In the experiment subjects had to generate (draw) the previously learned spatial relations after receiving a cue stimulus. The cue stimuli and the learning phase were identical to the recognition experiment. This was followed by a recall phase where the characters were displayed on the screen and subjects had to draw the associated spatial arrangement with an easy drawing tool, as learned in a previous training. Under these circumstances it does not make sense to analyze reaction times, but it is possible to compute correct and incorrect answers.

Results

The results support the interpretation of experiment 1 in two ways. First, subjects remembered 60.4% of the learned arrangements, although the expected number of correct answers purely by chance was 7.7% (because subjects could generate 13 alternative relations). This supports the assumption that the ordinal information of startpoint and endpoint was encoded in memory. However, there were slight differences depending on which relation had to be remembered: correct reactions for the relation **=** were highest (100% correct), followed by the relations < (71.4%), **d** (71.4%) and > (66.7%). The most difficult relation was **f**, for which only 38.1% of the reproductions were correct.

Second, as in the first experiment, no significant effect of the conceptual neighborhoods could be found, but again there was a statistically reliable effect of reorientation and transposition. Under the hypothesis that the number of wrong relations is equally distributed, we would expect that in less than 8% of all errors a reorientation arrangement or a transposition would be generated, respectively. The results depicted in Figure 4 and Figure 5 show that in fact 23.14% ($\chi^2_{(1)}$ = 36.265, $p < 0.001$) reorientations and 15.7% ($\chi^2_{(1)}$ = 19.08, $p < 0.001$) transpositions were generated. Again, errors were only counted as reorientation if both symmetry transformations were possible. The results support the interpretation of the previous experiment and show that symmetry transformations are also responsible for most of the errors under recall conditions. We also analyzed whether or not the metrical information of the originally learned arrangements were reflected in subjects drawings in the recall phase, but the correlation of the previously learned and later reproduced arrangements with respect to this aspect was very low.

Inferential adequacy of interval-based reasoning

In order to investigate the inferential adequacy of Allen’s calculus, two experiments were conducted (Knauff, Rauh and Schlieder 1995; Rauh,

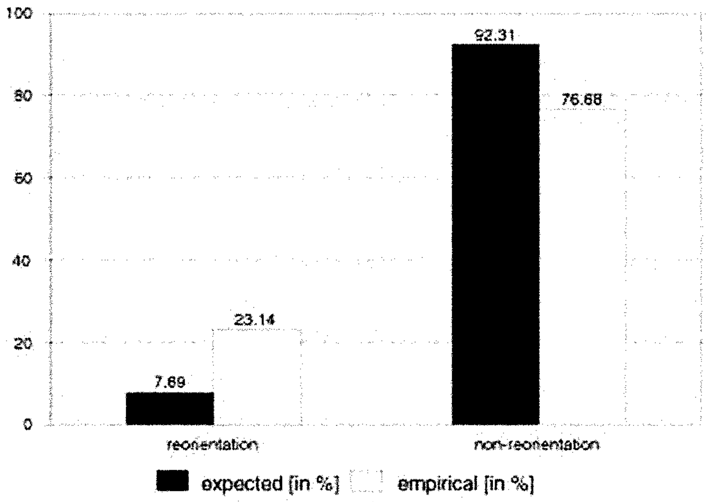

Figure 4. Distribution of expected and empirically recalled arrangements in reorientation v. non-reorientation.

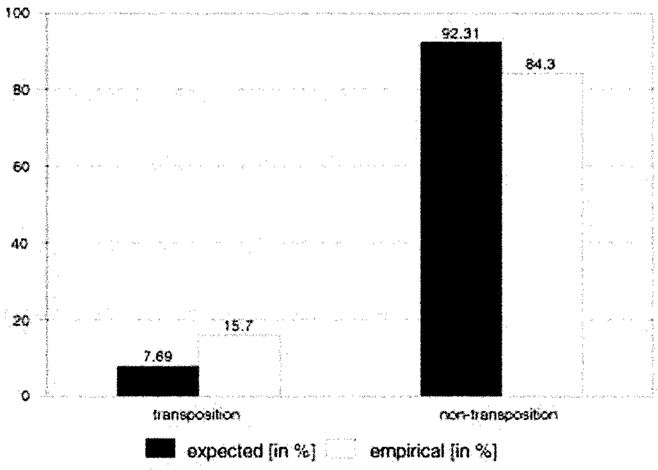

Figure 5. Distribution of expected and empirically recalled arrangements in transposition v. non-transpostition.

Schlieder and Knauff 1997) and a further one can be reinterpreted with respect to this issue (Rauh and Schlieder 1997). The main question, whether or not subjects produce compositions similar to those in Allen's composition table, was investigated with a variant of so-called n-term-series problems. In this important class of tasks studied in the psychology of reasoning, subjects have to find a conclusion on the basis of given premises. In the special case of a spatial three-term series problem (3ts-problem), two spatial relational terms X r_1 Y and Y r_2 Z are given as premises and subjects have to find a conclusion X r_3 Z (Johnson-Laird 1972). However, it is important to notice that there are two different kinds of inference tasks which are commonly used in reasoning research: in *active inference tasks* subjects have to generate possible conclusions which are compatible with the given premises, whereas in *inference verification tasks* conclusions are presented and subjects have to decide whether or not such conclusions are compatible with the premises.

Note that in our version of the tasks the term "conclusion" is not used in the usual logical sense, but only as the last term of a 3ts-problem. In fact, we did not ask for a conclusion in the strong sense, but for a relation between X and Z that is a possible model of the three propositions. This was done because we were interested in the cognitive aspects of the calculus and a comparison of experimental findings with the reasoning mechanism used in QSR.

Experiment 3: generating compositions

Thirty-three subjects were invited to solve 144 spatial 3ts-problems according to Allen's twelve relations (omitting the trivial = relation) in the active inference paradigm. The tasks were presented in the following way: The red interval **starts with** the green interval; The green interval **overlaps** the blue interval. What relationship can be held between the red and blue interval? (in German). Subjects had to specify one possible relation between the red and the blue intervals by specifying the startpoints and endpoints of the blue interval by pressing numbers on the keyboard as learned in a previous training. During the training they also learned the exact meaning of the natural language expressions. Although the conceptual adequacy experiments showed only small individual variation with respect to the base relations, this was done to be sure that not even a slight difference in the understanding of the tasks would affect our reasoning data and confuse the conceptual and inferential aspects. Thus the learning phase allows us to attribute the experimental results unambiguously to the inference process (for details see: Knauff, Rauh and Schlieder 1995).

Results

According to Allen's composition table, subjects generated on average more than 85% correct relations (resp. conclusions). This is the first important result with respect to inferential adequacy but a second finding seems even more interesting. Remember that in the composition table (shown in Table 2) half of the cells (white) have only one entry, whereas the other half (grey) have multiple entries (42 with three entries, 24 with 5 entries, 3 with 9 entries, and 3 with 13 entries). By analyzing all tasks related to these cells separately, we found that a significant majority of subjects were in agreement with respect to the generated relation. The most impressive example is the problem di – oi, where 84.8% of our subjects chose the relation oi, whereas the other two correct relations di and si were not used at all. Similarly in most of the problems with more than one possible relation in a table cell more than 60% of subjects generated the same solutions, whereas other correct relations were rarely used. To be sure about the reliability of this result, we tested the hypothesis that there are generally preferred relations against an equal distribution and obtained a χ^2-value of $\chi^2_{240} = 1848.04$, $p < 0.001$, which means that there were in fact statistically significant preferences. Therefore, we tested the 72 compositions with multiple possible relations separately, and obtained statistically significant χ^2-values in 59 out of the 72 tests. The preferred relations, with respect to the composition table (shown in Table 2), are presented in Table 4. If two relational symbols are in one cell (5 of the 72 cells), these relations were chosen by the subject with the same frequency, i.e. the preference was not quite clear.

Experiment 4: verifying compositions

Twenty-six subjects had to solve 156 inference tasks with respect to the composition table in the verification task paradigm. The material included three problems with one possible relation in conclusion, three with 3 possible relations, three with 5 and three with 9 possible relations and each of them were combined with all 13 base relations. Tasks were presented in the following way: "The red interval **overlaps** the green interval **from the left**"; "The green interval **overlaps** the blue interval **from the right**". After a short delay the question "Can the following relationship hold between the red and blue interval?" was presented followed by the relations that the subject had to verify; for example: The red interval **is on the left of** the blue interval. Subjects had to give simple YES/NO-answers. This experiment was conducted to investigate the causal influence of the preferred relations and to find out whether or not the compositions in verification tasks are similar to those in the generation task (for details see: Rauh, Schlieder and Knauff 1997).

Table 4. Empirically preferred relations (details in: Knauff, Rauh and Schlieder 1995).

	<	m	o	fi	di	si	s	d	f	oi	mi	>
<	<	<	<	<	<	<	<	<,d	o	o	o	=
m	<	<	<	<	<	m	m	o	o	o	=	oi
o	<	<	<	<	m	o	o	o	o,d	=	oi	>
fi	<	m	o	fi	di	di	o	d	=,f	oi	oi	>
di	<	o	o	di	di	di	o	=	oi	oi	oi	>
si	<	o	o	di	di	si	=	d	oi	oi	mi	>
s	<	<	o	o	fi	si	s	d	d	oi	mi	>
d	<	<	o	o	=	oi	d	d	d	oi	>	>
f	<	m	o	fi	di, oi	oi	d	d	f	oi	>	>
oi	<	o	=	oi	mi	mi	d	oi	oi	>	>	>
mi	<	si	oi	mi	>	>	oi	oi	mi	>	>	>
>	=	oi	oi	>	>	>	oi	>	>	>	>	>

Results

With an overall number of 89.7% correct reactions subjects performed the task very well. A more detailed analysis shows that 81.3% of correct relations were identified as valid conclusions and 91.12% of wrong relations were identified as invalid conclusions. Furthermore, subjects' performances were modulated by the number of possible relations, since they gave 93.6% correct answers for single-relation tasks but only 85.8% for multiple-relation tasks ($\chi^2_{(1)}$ = 7.3349, $p < 0.05$). However, the most important result refers to the preferred relations form the experiment in the generation tasks paradigm. As shown in Figure 6, conclusions that conformed to the preferred relations in generation tasks were faster (5.25 sec. v. 7.3 sec.; $F(1,25) = 27.38$, $p < 0.001$)

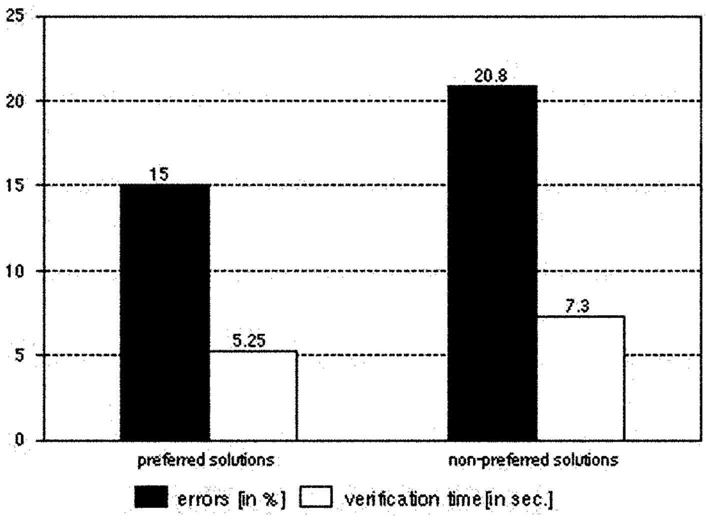

Figure 6. Error rates and verification times for preferred v. non-preferred relations.

and more often correctly verified (15% v. 20.8% errors; $\chi^2_{(1)} = 4.13$, $p < 0.05$) than other possible relationships.

Conceptual neighborhood and symmetries in interval-based reasoning

The empirical data from experiment 4 were also analyzed with respect to conceptual neighborhood theory and symmetry transformations. This was done by analyzing all tasks in which an incorrect relation was presented and therefore must be rejected as invalid by the subjects. If subjects reacted correctly the answer is called correct rejection, whereas an incorrect reaction is called false alarm. According to conceptual neighborhood theory one would predict more false alarms for conceptual neighbors of correct relations than for non-neighbors. However, remembering that neighborhood theory already proved to be conceptually inadequate, it is not surprising that neither an effect of A-neighborhood nor of B-neighborhood was found. In fact, as shown in Figure 7, the empirically observed frequency of 28.35% for A-neighbors and B-neighbors in the false alarms did not differ statistically significantly from the expected frequency purely by chance (27.52%). This result means that subjects did not choose such relations more often than other relations when giving incorrect answers ($\chi^2_{(1)} = 0.0977$, $p < 0.754$). A similar analysis was computed for the symmetry transformations and, as also shown in Figure 7, there was in fact a statistically significant effect of reorientation as well as for transposition. Indeed, reorientations and transpositions were

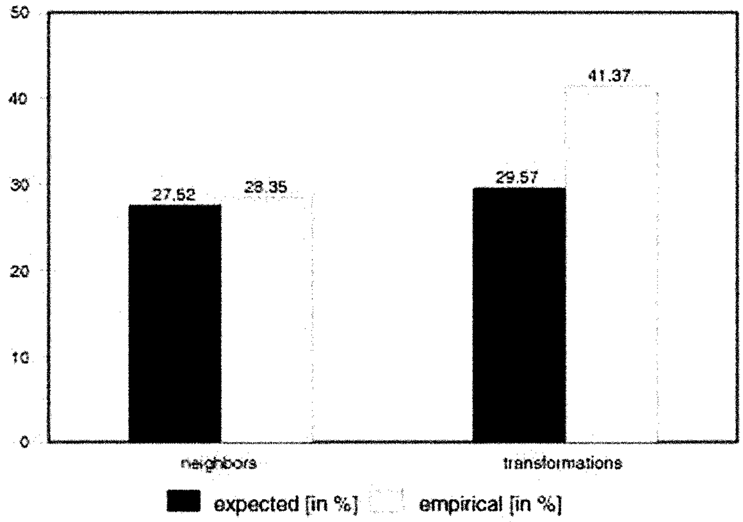

Figure 7. Distributions of expected and emirically observed conceptual neighbors and symmetry transformation responsible for false alarms in Experiment 4.

responsible for 41.37% of the false alarms, which is significantly higher than one would expect based on an equal distribution (29.57%; $\chi^2_{(1)} = 17.649$, $p < 0.001$). However, remember that in some cases reorientation and the transposition of an interval relation produce the same relation, whereas in other cases only one interpretation is possible. Therefore a further analysis tested these three possibilities separately and showed an interesting result: reorientation from the correct to the incorrect relations leads statistically reliable to more false alarms, whereas subjects produce more correct rejections for incorrect relations which were transpositions of correct relations. If we therefore count all false alarms in which we can not distinguish between reorientation and transposition as the former, reorientation alone was responsible for about 84% of all false alarms.

Supporting evidence for these results comes also from an experiment that was conducted by Rauh and Schlieder (1997). In this experiment, 4ts-problems were used and subjects had to infer three not explicitly given relations on the basis of three premises. For each original 4ts-problem there was a twin problem that differed only with respect to reorientation and an additional twin problem that differed from the original only with respect to the transposition. The main result of this study was that in 38.5% of the tasks, subjects constructed the same relations for original problems and their reorientations, whereas subjects constructed the same relations for original problems and their transpositions in only 10.8% of tasks (see Figure 8).

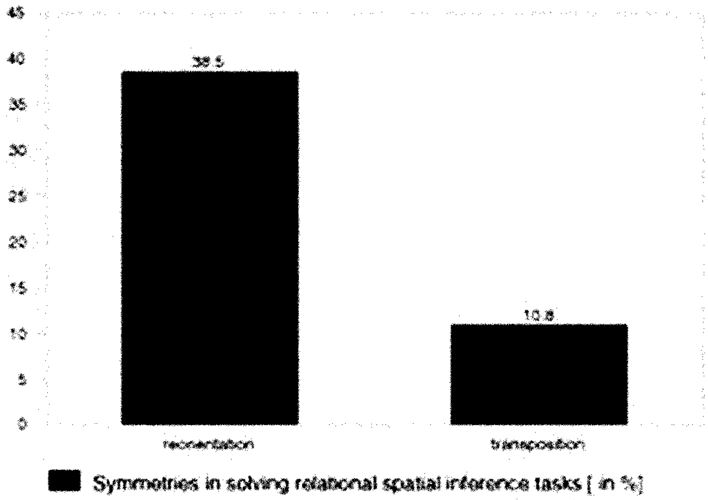

Figure 8. Frequency of reorientaiton and transposition symmetries in the experiment conducted by Rauh and Schlieder (1997).

Therefore, based on an chi-square value of $\chi^2_{(1)} = 27.26$, $p < 0.001$, we can conclude that the difficulty of a distinction between two relation depends more on reorientation rather than on transposition.

Summary of results and general discussion

This paper was mainly motivated by the lack of experimental data on the cognitive plausibility of the interval calculus introduced by Allen (1983). The reported experiments addressed both, conceptual cognitive adequacy and inferential cognitive adequacy, i.e., the thirteen base relations and/or the composition of these relations. The results will be discussed now with respect to cognitive theories of spatial knowledge and reasoning and their implications for QSR.

Ordinal information in human knowledge

To investigate conceptual adequacy a recognition experiment and a recall experiment in LTM were conducted and the experimental material was varied systematically with respect to the thirteen interval relations, their conceptual neighborhood and possible symmetry transformations. The experimental design was chosen to relate the results to the pure conceptual representation without confounding it with semantic aspects of the associated natural language descriptions and/or sensory features of the stimuli. The most important results with respect to conceptual adequacy are the following:

- Subjects use the order of startpoint and endpoint of intervals when conceptualizing spatial arrangements, i.e. the interval relations from Allen (1983) seem to describe some important aspects of human conceptual knowledge about spatial relationships.
- When subjects gave incorrect answers, they did not mistakenly remember conceptual neighbors of correct relations more often than other relations. Thus, conceptual neighborhood did not appear to correspond to a cognitively relevant conceptual relation.
- On the other hand, symmetry transformations such as reorientations and transpositions were responsible for most of the errors. Therefore, the analysis of symmetries operating on spatial relations seem to be important not only computationally but also from a cognitive science perspective.
- In the memory tasks we used, the metrical properties of the presented spatial arrangements were mostly ignored by the subjects and therefore do not see, to be cognitively important features.

Now it is possible to relate these results to psychological theories of human knowledge representation. First of all, the results are not compatible with an assumption that often can be found in the literature on spatial knowledge. This is that the left-right-axis is represented in human LTM only as two poles and one position in the middle of the poles (Pribbenow 1993). Supporters of this assumption often argue that the human body is symmetric with respect to this axis, whereas the other two body axes (before-behind; above-below) are asymmetric. Furthermore, they argue that natural language expressions such as “beside”, “at the side” etc. reflect the very rough conceptualization of the left-right-axis. In contrast to this, the reported results of experiments 1 and 2 suggested that people conceptualize spatial arrangements much more exactly. In fact, subjects were able to distinguish all possible configurations of objects on the left-right-axis. In the reported experiments this was possible only if the startpoints and endpoints of objects were taken into account separately and the order information was maintained. Another possibility would be an encoding that takes the metrical information into account, but we have seen that the empirical data do not support this hypothesis; in general subjects in experiment 2 ignored such information. Taken together, a cognitive theory of human spatial concepts that takes into account the main ideas of Allen’s calculus seems to be a promising foundation for further experimental analyses. Indeed, at the present time, such an approach could be claimed as conceptually more adequate than a pure left-right distinction, or theories that propose richer (e.g. metrical) representations.

A comparison of results with well-known theories of conceptual representation may shed light on their interconceptual organization. A useful data base for this shift from the base relations towards the question of their interrelationships is provided by the errors made by the subjects (20% errors in experiment 1 and 40% errors in Experiment 2). Of course, the most prominent theories of inter-conceptual organization are the so-called semantic networks, introduced firstly in computer science by Quillian (1968) and in psychology by Collins and Quillian (1969). Similar to this theory, the interval relations in the conceptual neighborhood approach by Freksa (1992) are represented as nodes and their interrelationships, defined as lengthening, shortening and moving, as edges. Despite this parallel, however, the experimental results did not support the assumption that such formal interrelationships between the interval relations play an important role in human spatial knowledge representation. If this would be true, subjects must confuse relations and their conceptual neighbors more often than other relations, but no statistically reliable effect in this direction was found in the empirical data.

Symmetry transformations, such as reorientation and/or transposition seem to make a more promising base for explaining the relationships between the interval relations. In fact, the experimental data show a strong effect of these symmetries on the difficulty of memory tasks. If the distractor arrangement was the reorientation or transposition of the target arrangement, it was much more difficult to distinguish both arrangements. If the probability of confusion can be seen as a function of the cognitive similarity, maybe as a path-connection between the two relations, this would be a strong argument for the importance of such symmetries in spatial knowledge. Following these ideas Figure 9 illustrates the conceptual neighborhood of the interval relations taking the reorientation symmetry and transposition symmetry into account. Unfortunately this graph is not connected as in Freksa's approach, because for such symmetries the whole arrangement must be taken into account, and not just local transformations. In fact, we have two graphs which are brought together in one colored graph with the colors **r** for reorientation and **t** for transposition.

The graph should be interpreted as follows: < and >, m and mi, o and oi are conceptual neighbors with respect to both symmetry transformation, i.e. reorientation and transposition; s and f as well as si and fi are conceptual neighbors with respect to reorientation, s and si, f and fi, d and di with respect to transposition. Furthermore by the reorientation of d, di, = the same relation is obtained for the transposition of = also.

The errors in the memory experiments also indicate that in those (a few) cases where the ordinal information was forgotten, the topological information was maintained. In this context it is important to be aware that

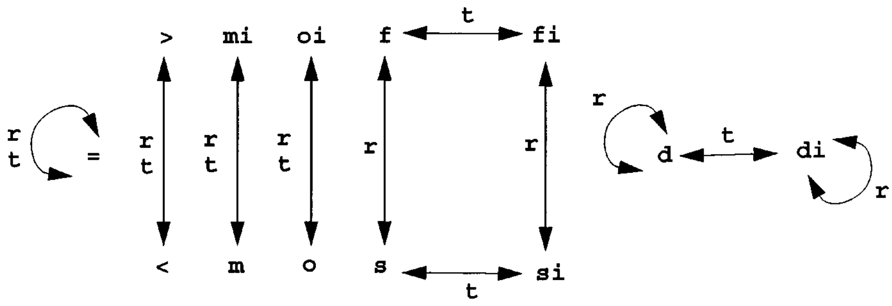

Figure 9. Neighborhood graph according to reorientation symmetries and transposition symmetries as found in the memory experiments.

topological information has the advantage of being invariant under reorientation and therefore is extremely useful for what is often realized as a change of perspective in our daily lives. Therefore Bartlett, Gernsbacher and Till (1987) and other cognitive psychologists have argued that visually perceived spatial arrangements and their reorientations are cognitively highly similar and therefore could be recognized as identical. Nevertheless, under circumstances in which ordinal information or perspective, respectively, is crucial for a given cognitive task it is also taken into account, but in general it is more likely to lead to errors.

Such a difference between ordinal information and topological information is supported by the reported experimental data. Of course, subjects were aware that ordinal information is crucial for the given task and therefore kept it in mind intentionally. Nevertheless, topological information that was encoded in memory less consciously (implicitly) comes into play if subjects were not able to remember the exact ordinal information. These ideas motivated colleagues and myself to investigate the cognitive plausibility of topological spatial calculi, such as the RCC-theory introduced by Randell, Cui and Cohn (1992) and the 9-intersection-model of Egenhofer (1991). Taking the results from this investigation (for details see: Knauff, Rauh and Renz 1997) together with the experimental findings presented here, the question arises how ordinal information and topological information are connected in human spatial knowledge. As a much more general problem, this issue is not discussed in this article, but the interested reader is referred to Knauff (1997). In this dissertation – at present only published in German – I proposed that ordinal information and topological information are represented seperately in human memory. Ordinal information is maintained only if it is essential for a given task, whereas, because of its importance, the representation of topological information could be automatic.

To investigate the inferential adequacy of Allen's calculus, two experiments, using experimental paradigms from the psychology of reasoning, were reported. In experiment 3 subjects received two spatial relational terms X r_1 Y and Y r_2 Z as premises and had to generate one possible relation X r_3 Z in conclusion; in experiment 4, a conclusion X r_3 Z was presented and subjects had to decide whether or not it was compatible with the premises X r_1 Y and Y r_2 Z. the most impressive findings with respect to inferential adequacy are the following:

- In active inference tasks subjects produced more than 85% correct relations or conclusions, respectively, which means it was not difficult to generate a composition of the interval relations from Allen (1983).
- Problems with more than one possible solution were more difficult than compositions with only one relation.
- When problems had more than one possible solution, a significant majority of subjects agreed with respect to the generated relation, which means this relation was strongly and interindividually preferred.
- If subjects had to decide in verification tasks whether or not a present conclusion was compatible with the premises, preferred relations were verified more quickly and more correctly than other correct relations.
- When verifying compositions, in disagreement with what one would expect based on conceptual neighborhood theory, subjects did not mistakenly choose conceptual neighbors of correct relations more often than other relations.
- In particular reorientation symmetries were responsible for most of the errors in verification tasks, whereas transpositions made such tasks easier to solve. These results agree nicely with what was found in the memory experiments, namely that such symmetries seem to be important not only computationally but also from a cognitive science viewpoint.

Taken together, these results have some important implications: First, it is reasonable to assume that the constraint satisfaction method proposed by Allen (1983) is not the appropriate method to model human spatial reasoning. This claim is in particular based on the observed preference for specific relations (Experiment 3) and the related verification bias (Experiment 4). Since all relations from a formal point of view are logically equivalent, one would not expect such preferred relations from a constraint-based approach (Schlieder 1998). Second, remembering that conceptual neighborhood theory seems to be conceptually inadequate, it is not surprising that we also found no evidence for its influence on spatial relational inference tasks, i.e. its inferential adequacy. Third, it interesting that subjects' performance in the memory experiments as well as in the inference tasks were strongly affected by reorientation symmetries. The last two points remind us what is often ignored by reasoning researchers, namely that thinking occurs not in a vacuum, but strongly relies on long-term representations of knowledge. The results of the inference experiments with respect to neighborhood theory and reorientation symmetry in this way are maybe not primarily affected by the reasoning processes, but rely on more fundamental characteristics of the used spatial knowledge.

Coming back to the empirically preferred relations and the implausibility of an underlying constraint-based reasoning mechanism, it is helpful to take psychological theories of human reasoning into account. Cognitive psychologists have proposed several theories that attempt to explain human reasoning, but only two of them had been articulated clearly (overview for example in Evans, Newstead and Byrne 1993). According to the first theory, which is often called theory of mental proof, subjects solve inference tasks by constructing proofs using general reasoning rules. The main assumption is that there is a set of language-like and context-independent formal rules of inference represented in human LTM, and inference tasks are solved by applying these rules to the given premises represented in working memory (e.g. Rips 1994). However, the argument against something like a constraint solver in the human mind is also true for rule-based approaches: since all relations are logically equivalent, in the framework of rule-based approaches the preferred relations are not explainable without further cognitively dubious assumptions.

The second theory is called mental model theory and in fact the reported results agree nicely with its main assumptions (Knauff, Rauh, Schlieder and Strube 1998). In general, the key idea of mental model theory is that people translate the information given by the premises into a mental model and use this representation to solve given inference tasks (Johnson-Laird 1983). According to this theory, spatial reasoning does not rely primarily on syntactic operations like in the rule-based approaches, but rather on the construction and manipulation of mental models (Johnson-Laird and Byrne 1991; Byrne and Johnson-Laird 1989). Such mental models are integrated representations of objects and relations ("structures") in working memory that constitute a model (in the usual logical sense) of the premises given in the reasoning task. That such models are not necessarily mental images, but more abstract spatial representations has been shown recently by Knauff and Johnson-Laird (2000).

Mental model theory now explains human spatial reasoning as the following three-stage process: in the first stage an integrated representation of the given premises is generated in working memory-the mental model.

This is seen as a possible state of affairs described by the premises. In the second stage people formulate a putative conclusion by inspecting this model to find new, not explicitly given information and the third stage is an optional search for alternative models, in which the premises are true but the putative conclusion is false. If such a counterexample is not found, the conclusion must be true.

Now the crucial point is that mental model theory explains the first stage of model construction as a serial process that always produces the same initial mental model (Johnson-Laird and Byrne 1991). According to this assumption, the preferred relations are easy to explain: when subjects read the premises, they build an integrated representation of the given information and this construction process forces an initial model, the inspection of which leads to the preferred relations. Furthermore, mental model theory would also predict that, if model variation is needed for verification tasks, problems with multiple solutions are more difficult than those with only one possible relation. This is exactly what was observed in experiment 4 and can be explained by the well-known limited capacity of working memory that makes accounting for all possible models of the premises difficult (for a more detailed explanation see: Knauff, Rauh, Schlieder and Strube 1998).

To summarize, the reported investigations indicate that Allen's interval approach could provide a promising foundation for the further development of spatial calculi and demonstrate in more detail what talking about the cognitive plausibility of formal approaches to QSR means. Thus, it also shows that AI research on QSR and psychological research on spatial cognition can learn from each other.

Acknowledgements

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Note

[1] Recent applications of QSR can be found for instance in computer vision (Fernyhough, Cohn and Hogg 1997), natural language processing (Aurnague and Vieu 1993), qualitative physics (Weld and De Kleer 1990), image information systems (Chang and Jungert 1996), robot navigation (Levitt and Lawton 1990), document analysis (Walischewski 1997) and geographic information systems (GIS; Egenhofer and Mark 1995).

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