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Region Connection Calculus: Its models and composition table ☆

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Abstract

Originating in Allen's analysis of temporal relations, the notion of *composition table* has become a key technique in providing an efficient inference mechanism for a wide class of theories in the field artificial intelligence. This paper is mainly about the consistency-based composition table (RCC8 **CT**) of the Region Connection Calculus (RCC) raised by Randell, Cui and Cohn. First we show each RCC model is a consistent model of the RCC8 **CT**. Then after an exhaustive analysis we show that no RCC model can be interpreted extensionally anyway and hence give a negative answer to a conjecture raised by Bennett. All these results are given in an 'extensional' RCC8 composition table, where we attach to each cell entry in the RCC8 **CT** a superscript to indicate in what circumstances an extensional interpretation is possible.

Keywords: Region Connection Calculus; Qualitative Spatial Reasoning; Weak composition table; Boolean connection algebra; Extensionality

1. Introduction

Qualitative Spatial Reasoning (QSR) has evolved in the last decade which is concerned with the qualitative aspects of representing and reasoning about spatial entities. The challenge of QSR is to "provide calculi which allow a machine to represent and reason with spatial entities of higher dimension, without resorting to the traditional

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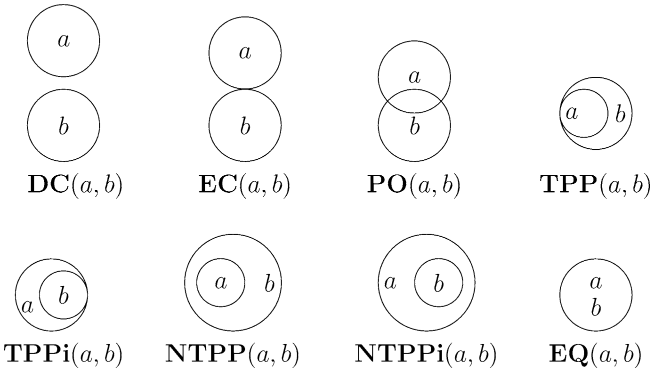


Fig. 1. Illustration of eight JEPD topological relations.

quantitative techniques prevalent in, for example, the computer graphics or computer vision communities" [11].

There are many possible applications of QSR, for examples, in Geographical Information Systems (GIS) [9,18,30], spatial query languages [10], natural languages [4] and many other fields. We invite the reader to consult [11] for an introduction and an overview of current trends.

This paper is mainly concerned with one of the most widely referenced formalisms for QSR, the *Region Connection Calculus* (RCC). RCC was initially described in [25,27], which is intended to provide a logical framework for incorporating spatial reasoning into **AI** systems.

RCC is a first order theory based on a primitive *connectedness* relation, **C**, which is a binary symmetric relation. Using this relation a set of binary relations are defined [27] (see Table 1 for some examples). Among the defined relations, the eight relations in the set {**DC**, **EC**, **PO**, **EQ**, **TPP**, **NTPP**, **TPPi**, **NTPPi**} (illustrated in Fig. 1) are identified as being of particular importance. The eight relations form a *Jointly Exhaustive and Pairwise Disjoint* (**JEPD**) set, which means that any two regions stand to each other in exactly one of these relations. These eight topological relations are known as RCC8 in the literature. Interestingly, the same set of relations has been independently identified as significant in the context of Geographical Information Systems (GIS) (see [16,17]). Their approach to spatial reasoning, known as 9-*intersection*, is based on the concepts of point-set topology. According to this model, each object p is represented as a closed homogeneous two-dimensional simply connected subset of $\mathbb{R}^2$. The topological relation between any two objects p and q is described by the nine intersections of p's interior, boundary and exterior, with the interior, boundary and exterior of q.

Unlike Egenhofer's 9-intersection approach, RCC takes regions rather than points as a fundamental notion. This region-based approach to spatial reasoning closely mirrors Allen's interval-based approach to temporal reasoning [2]—they both take extended entities, rather than points, as primitives. As a matter of fact, construction of the RCC theory of spatial regions was greatly influenced by the works of Allen and Hayes [3, 21–23] and consequently its development followed a similar pattern: a first order theory was presented and investigated, then to provide a reasoning mechanism useful constraint languages were identified within which composition based reasoning could be conducted [7].

Allen's interval calculus is the best known temporal logic within AI. In his interval-based theory, Allen identified a set of thirteen JEPD relations which can hold between two temporal intervals and studied reasoning procedures based on the composition of these relations [1,2]. More importantly, Allen introduced the idea of a transitivity table, which is known in RCC as a *composition table*. From that time on the use of composition tables has become a key technique in providing an efficient inference mechanism for a wide class of theories. This is particularly the case for Qualitative Spatial Reasoning. As far as the JEPD set of relations RCC8 is concerned, Cui, Cohn, Randell [13] and Egenhofer [16] independently established the composition table for these fundamental topological relations. For reasons that will become evident in the following we call this table the *weak composition table* for the RCC8 relations (RCC8 **CT** henceforth).

Given a fixed vocabulary of relations, **Rels** (normally this will constitute a JEPD set), such a table enables one to answer the following question by simple lookup: given two relational facts of the forms $\mathbf{R}(a,b)$ and $\mathbf{S}(b,c)$, what are the possible relations (from the set **Rels**) that can hold between a and c? This kind of computation is frequently very useful—for example, one can check the integrity of a database of atomic assertions (involving relations in some set for which we have a composition table) by testing whether every three relations are consistent with the table [12].

The present paper is mainly about the weakly composition table for the RCC8 relations. For the convenience of the reader, we summarize some basic notions concerned with composition table which have appeared in the literature, e.g., [7,8,14].

The precise meaning of a composition table (**CT**) depends to some extent on the context in which it is employed. Generally speaking, a **CT** is just a mapping $CT:\mathbf{Rels}\times\mathbf{Rels}\to 2^{\mathbf{Rels}}$, where **Rels** is a set of relation symbols. When **Rels** is finite, this table can be concisely represented in an $n\times n$ matrix, where $n=|\mathbf{Rels}|$. For three relation symbols **S, R** and **T**, if $\mathbf{T}\in CT(\mathbf{R},\mathbf{S})$, we say **T** is a cell entry in the cell specified by **R** and **S**.

A model[1] of $\langle\mathbf{Rels}, CT\rangle$ is a pair $\langle U, \upsilon\rangle$, where U is a set and υ is a mapping from **Rels** to the set of binary relations on U such that $\{\upsilon(\mathbf{R}):\mathbf{R}\in\mathbf{Rels}\}$ is a partition of $U\times U$ and $\upsilon(\mathbf{R})\circ\upsilon(\mathbf{S})\subseteq\bigcup_{\mathbf{T}\in CT(\mathbf{R},\mathbf{S})}\upsilon(\mathbf{T})$ for all $\mathbf{R},\mathbf{S}\in\mathbf{Rels}$, where $\circ$ is the usual relational composition. A model is called *consistent* if $\mathbf{T}\in CT(\mathbf{R},\mathbf{S})\Leftrightarrow(\upsilon(\mathbf{R})\circ\upsilon(\mathbf{S}))\cap\upsilon(\mathbf{T})\neq\emptyset$ for all $\mathbf{R},\mathbf{S},\mathbf{T}\in\mathbf{Rels}$. This means that, for any three relation symbols **T**, **R** and **S**, **T** is an entry of the cell specified by **R** and **S** if and only if there exist three regions a,b,c in U such that $\mathbf{R}(a,b)$, $\mathbf{S}(b,c)$ and $\mathbf{T}(a,c)$. We call a consistent model *extensional* if $\upsilon(\mathbf{R})\circ\upsilon(\mathbf{S})=\bigcup_{\mathbf{T}\in CT(\mathbf{R},\mathbf{S})}\upsilon(\mathbf{T})$ for all $\mathbf{R},\mathbf{S}\in\mathbf{Rels}$. In such a model, if **T** is an entry in the cell specified by **R** and **S**, then whenever $\mathbf{T}(a,c)$ holds, there must exist some b in U s.t. $\mathbf{R}(a,b)$ and $\mathbf{S}(b,c)$. In what follows, when the interpretation mapping υ is clear from the context, we also write U for this model.

But to give a precise meaning to claim that a theory entails or is equivalent to a **CT** one need to specify the meaning of a **CT** in terms of a theory. A weak specification, the *consistency-based* definition, is given by Bennett et al. [8].

[1] The definition is different from that of [14].

Given a theory Θ in which a set **Rels** of JEPD base relations is defined, the composition (entailed by Θ), $CT(\mathbf{R},\mathbf{S})$, where $\mathbf{R},\mathbf{S}\in\mathbf{Rels}$, is defined to be the unique smallest subset $\{\mathbf{T}_i\}\subseteq\mathbf{Rels}$ such that $\Theta\models\forall x\forall y\forall z[\mathbf{R}(x,y)\wedge\mathbf{S}(y,z)\to(\mathbf{T}_1(x,z)\vee\cdots\vee\mathbf{T}_n(x,z))]$. Semantically speaking, this ensures that whenever $\mathbf{R}(a,b)$ and $\mathbf{S}(b,c)$ hold, a and c must be related by some relation symbol $\mathbf{T_i}$ in $CT(\mathbf{R},\mathbf{S})$, where a,b,c are constants in a model R of Θ.

This consistency-based definition of composition is equivalent to that given in [26]. We also call such a table a *weak composition table*[2] (entailed by Θ). By definition, each model of the theory Θ is also a model of the weak composition table entailed by Θ. But a model of Θ is by no means necessarily a consistent model of the weak composition table entailed by Θ. For example, the Generalized Region Connection Calculus (GRCC) introduced in [24] entails the same weak composition table as the RCC theory does. But GRCC has a model containing only three regions, clearly this model is not consistent w.r.t. the weak composition table entailed by GRCC.

Bennett et al. call a weak composition table (entailed by Θ) *extensional* provided that the fact $CT(\mathbf{R},\mathbf{S})=\{\mathbf{T}_1,\ldots,\mathbf{T}_n\}$ always implies $\Theta\models\forall x\forall z[(\mathbf{T}_1(x,z)\vee\cdots\vee\mathbf{T}_n(x,z))\leftrightarrow\exists y[\mathbf{R}(x,y)\wedge\mathbf{S}(y,z)]]$ [8]. Semantically speaking, this assures that, for any Θ-model R and constants $a,c\in R$, the relational fact $\mathbf{T_i}(a,c)$ also implies that there exists some constant $b\in R$ such that $\mathbf{R}(a,b)$ and $\mathbf{S}(b,c)$ holds, where T_i is a relation symbol taken from $CT(\mathbf{R},\mathbf{S})$. That is to say, Θ entails an extensional weak composition table if and only if each of its model is also an extensional model of this composition table.

If a weak composition table is extensional then, as well as providing a means for consistency checking of ground relation sets, it can be employed to justify a certain kind of extensional inference: from a relation holding between two objects we deduce the existence of a third object related to the original two in a specific way. And if this is not the case, then information is lost when (composition-based) compositions are computed *via CT* [7].

After a long preliminary introduction to the general theory of composition table, we now phrase our questions concerning the RCC8 **CT**.

Note that the RCC theory entails the RCC8 **CT**, each RCC model is already a model of the RCC8 **CT**. But, when does an RCC model be consistent? And when does it be extensional? Questions like these are very important. As commented by Bennett [6], relational composition is "an area of research that is potentially very significant for AI because it is possible that composition tables—perhaps in conjunction with other forms of compiled logical information—may provide the key to effective reasoning in seemingly intractable theories". Thus, answers to the questions mentioned above would be one of the major challenges in the further development of the RCC theory.

An examination of the RCC8 **CT** (Table 2) reveals that an extensional interpretation is not compatible with the 1st-order RCC theory. This fact is pointed out by Bennett in [7] and [8].

To avoid such an objection and hence construct an extensional composition table, Bennett suggests [7] to remove the universal region u from the domain of possible referents of the region constants. He also writes

2 The same terminology used by Düntsch et al. [14] is model-dependent and hence different from ours.

> "All the exceptions to extensional composition that I am aware of involve u; so it seems that an extensional interpretation could be achieved with respect to a modified theory without a universal region."

In the present paper, we first show that each RCC model is a consistent RCC8 **CT** model and then an exhaustive investigation about extensional interpretation of the RCC8 **CT** is given. As a result, we show that Bennett's conjecture is not true. In fact, for each cell entry in the RCC8 **CT**, we attach a superscript to indicate in what circumstances an extensional interpretation is possible. There are all together 178 cell entries in the RCC8 **CT**. If R is an RCC model which satisfies the Interpolation Condition (see condition (3), Section 4.1), then there are 143 (about 80%) cell entries that can be interpreted extensionally. But for the rest 35 entries, extensional interpretations are impossible for any RCC model. The results obtained in the present paper suggest that, to get an extensional model of the RCC8 composition table, the domain of possible regions must be restricted greatly in the following sense: those regions with a hole in the same domain should be removed and the sum operation is generally disallowed. The resulting domain would be more homogeneous and more similar to that of Egenhofer [16].

To give such an investigation, an equivalent structure of RCC models introduced by Stell [28], *Boolean connection algebras* (BCAs), is used. This lattice theoretic approach to the study of spatial regions makes both description and proof easier.

Before using BCA as a substitute for RCC model, several issues should be settled.

An RCC model is called *strict* if the identity relation **EQ** in RCC8 is the logical equality '$=$' [28]. Note that BCAs are in essence *strict* RCC models. The situation in exploring non-strict RCC models is rather strange: on one hand, those who have developed RCC have been aware of the distinction between **EQ** and $=$ [12, p. 308, Note 11]; and on the other hand meaningful non-strict model has never been given before. Stell [28] points out that non-strict models do exist, but his example is quite artificial. It is showed in this paper that regular connected spaces provide *non-strict* RCC models by, roughly speaking, taking a region to mean a non-empty open set, where two non-empty open sets are identical if they have the same closure. Taking non-empty closed sets as regions also gives such a model. Comparing with Stell's one, our models are much more natural.

Although non-strict RCC models do exist, it seems to us that such models are not necessary. Next section we shall show each non-strict RCC model gives rise to a strict one, and the two models clearly play the same role in RCC. As a matter of fact, in the same paper, Cohn et al. also comment that [12]:

> "From within the RCC theory it is not possible to distinguish between regions that are open, closed or neither but have the same closure, and we argue that these distinctions are not necessary for qualitative spatial reasoning."

The rest of the paper is structured as follows. First we briefly summarize the basic concepts of the RCC theory and show certain distributive lattices give rise to non-strict RCC models. An alternative characterization of BCA dealing with the non-tangential proper part relation is also given. Then, in Section 3, we show each RCC model is a consistent model of the RCC8 **CT**. A complete check of extensional interpretation of the

Table 1
Some relations definable in terms of **C**

Relation	Interpretation	Definition of $R(x, y)$
$\mathbf{DC}(x, y)$	x is disconnected from y	$\neg\mathbf{C}(x, y)$
$\mathbf{P}(x, y)$	x is a part of y	$\forall z[\mathbf{C}(z, x) \to \mathbf{C}(z, y)]$
$\mathbf{PP}(x, y)$	x is a proper part of y	$\mathbf{P}(x, y) \wedge \neg\mathbf{P}(x, y)$
$\mathbf{EQ}(x, y)$	x is identical with y	$\mathbf{P}(y, x) \wedge \mathbf{P}(y, x)$
$\mathbf{O}(x, y)$	x overlaps y	$\exists z[\mathbf{P}(z, x) \wedge \mathbf{P}(z, y)]$
$\mathbf{PO}(x, y)$	x partially overlaps y	$\mathbf{O}(x, y) \wedge \neg\mathbf{P}(x, y) \wedge \neg\mathbf{P}(y, x)$
$\mathbf{EC}(x, y)$	x is externally connected to y	$\mathbf{C}(x, y) \wedge \neg\mathbf{O}(x, y)$
$\mathbf{DR}(x, y)$	x is discrete from y	$\neg\mathbf{O}(x, y)$
$\mathbf{TPP}(x, y)$	x is a tangential proper part of y	$\mathbf{PP}(x, y) \wedge \exists z[\mathbf{EC}(z, x) \wedge \mathbf{EC}(z, y)]$
$\mathbf{NTPP}(x, y)$	x is a non-tangential proper part of y	$\mathbf{PP}(x, y) \wedge \neg\exists z[\mathbf{EC}(z, x) \wedge \mathbf{EC}(z, y)]$

RCC8 **CT** is given in Section 4, where we attach to each cell entry in that table a superscript to indicate in what circumstances an extensional interpretation is possible. Conclusions are given in Section 5.

2. The region connection calculus

The Region Connection Calculus (RCC) is a first order theory based on a primitive *connectedness* relation, $\mathbf{C}(x, y)$. RCC is intended to provide a logical framework for incorporating spatial reasoning into **AI** systems. Using $\mathbf{C}(x, y)$, a basic set of binary relations are defined [27]. Definitions and intended meanings of those used here are given in Table 1. The relations **P, PP, TPP** and **NTPP** being non-symmetrical support inverses. For the inverses we use the notation $\boldsymbol{\Phi}\mathbf{i}$, where $\boldsymbol{\Phi} \in \{\mathbf{P}, \mathbf{PP}, \mathbf{TPP}, \mathbf{NTPP}\}$. Of the defined relations, those in the set $\{\mathbf{DC}, \mathbf{EC}, \mathbf{PO}, \mathbf{EQ}, \mathbf{TPP}, \mathbf{NTPP}, \mathbf{TPPi}, \mathbf{NTPPi}\}$ (illustrated in Fig. 1) are provably **JEPD** (*Jointly Exhaustive and Pairwise Disjoint*). We denote this set of RCC8 relations by $\mathcal{R}_8$.

2.1. RCC axioms

Definition 1.1 [20,28]. A *model of the Region Connection Calculus* consists of a set R, an element $\mathrm{u} \in R$, a singleton set $\{\mathrm{n}\}$ disjoint from R, a unary operation $\mathrm{compl}: R - \{\mathrm{u}\} \to R - \{\mathrm{u}\}$, binary operations $\mathrm{sum}: R \times R \to R$, and $\mathrm{prod}: R \times R \to R \cup \{\mathrm{n}\}$, and a primitive binary relation **C** on R. These data are required to satisfy the following axioms, which make use of the relations derived from **C** defined above.

R1. $(\forall x \in R)\mathbf{C}(x, x)$.
R2. $(\forall x, y \in R)[\mathbf{C}(x, y) \to \mathbf{C}(y, x)]$.
R3. $(\forall x \in R)\mathbf{C}(x, \mathrm{u})$.
R4a. $(\forall x \in R)(\forall y \in R - \{\mathrm{u}\})[\mathbf{C}(x, \mathrm{compl}\, y) \leftrightarrow \neg\mathbf{NTPP}(x, y)]$.
R4b. $(\forall x \in R)(\forall y \in R - \{\mathrm{u}\})[\mathbf{O}(x, \mathrm{compl}\, y) \leftrightarrow \neg\mathbf{P}(x, y)]$.
R5. $(\forall x, y, z \in R)[\mathbf{C}(x, \mathrm{sum}(y, z)) \leftrightarrow \mathbf{C}(x, y) \vee \mathbf{C}(x, z)]$.

R6. $(\forall x, y, z \in R)[\text{prod}(y, z) \in R \to$
$[\mathbf{C}(x, \text{prod}(y, z) \leftrightarrow (\exists w \in R)[\mathbf{P}(w, y) \wedge \mathbf{P}(w, z) \wedge \mathbf{C}(x, w)]]]$.
R7. $(\forall x, y \in R)[\text{prod}(x, y) \in R \leftrightarrow \mathbf{O(x,y)}]$.

Axioms R1 and R2 ensure that the connectedness relation **C** is a reflexive and symmetric binary relation; R3 reflects the university of the region u; the rest axioms are intended to capture the ideas of the complement of a region (R4a and R4b), the sum of two regions (R5) and the product of two regions (R6 and R7) respectively.

The original RCC system contains an additional axiom:

$$(\forall x \in R)(\exists y \in R)\mathbf{NTPP}(y, x).$$

However, Düntsch et al. [14] show that it is redundant. Here we give a new proof.

Lemma 2.1 [14]. *Let $\langle R, \{\mathrm{n}\}; \mathrm{u}, \text{sum}, \text{prod}, \mathbf{C}\rangle$ be a model of RCC. Then, for any region x in R, there exists another region y in R such that y is a non-tangential proper part of x.*

Proof. Suppose we have some region $x \neq \mathrm{u}$ such that for all $y \in R$, not $\mathbf{NTPP}(y, x)$. By R4a, this implies $(\forall y \in R)\mathbf{C}(y, \text{compl}\, x)$. Moreover, by R4b and $\mathbf{P}(x, x)$, we have $\neg\mathbf{O}(x, \text{compl}\, x)$, hence $\neg\mathbf{P}(x, \text{compl}\, x)$. By definition, there exists some region t such that $\mathbf{C}(t, x)$ and $\mathbf{DC}(t, \text{compl}\, x)$—a contradiction. □

Remark 2.1. Above lemma shows that each region is infinitely dividable, and as a result, RCC has nothing to do with discrete spaces. But discrete spaces are evidently important in implementations of spatial information systems. Noticing this, [24] introduces a generalized RCC theory, GRCC. This generalized theory is, however, based on two primitive relations: the connectedness relation **C** and the part of relation **P**. The major difference between RCC and GRCC is that the condition in Lemma 2.1 is given up in the GRCC theory. Furthermore, a GRCC model is an RCC model iff the condition given in above lemma holds.

The RCC axioms do not imply that the defined 'part of' relation **P** is antisymmetric. We call an RCC model R *strict* if R satisfies the following axiom:

R8. $(\forall x, y \in R)[\mathbf{P}(x, y) \wedge \mathbf{P}(y, x) \to x = y]$.

For strict RCC models, the following result substantially generalizes that obtained in Lemma 2.1. For simplicity, we write x' for $\text{compl}\, x$, $x + y$ for $\text{sum}(x, y)$ and $x - y$ for $\text{prod}(x, \text{compl}\, y)$.

Proposition 2.1. *Let $\langle R, \mathbf{C}\rangle$ be a strict RCC model. Then for each region $a \in R$ with $a \neq \mathrm{u}$, we have b_i $(i = 1, \ldots, 8)$ such that for each RCC8 relation $\mathbf{R}$ there exists a unique i with $\mathbf{R}(a, b_i)$.*

Proof. By Lemma 2.1, we can take p, q in R such that $\mathbf{NTPP}(p, a)$ and $\mathbf{NTPP}(q, a')$. Let $b_1 = q$, $b_2 = a' - q$, $b_3 = p + q$, $b_4 = a + q$, $b_5 = q'$, $b_6 = a - p$, $b_7 = p$ and $b_8 = a$. Then

$\mathbf{DC}(a, b_1)$, $\mathbf{EC}(a, b_2)$, $\mathbf{PO}(a, b_3)$, $\mathbf{TPP}(a, b_4)$, $\mathbf{NTPP}(a, b_5)$, $\mathbf{TPPi}(a, b_6)$, $\mathbf{NTPPi}(a, b_7)$ and $a = b_8$. $\square$

This fact shows that the domain (and the range) of each RCC8 relation contains all regions in $R - \{\mathrm{u}\}$. Moreover, for any two RCC8 relations, $\mathbf{R}$, $\mathbf{S}$, and any region $b \neq \mathrm{u}$, there are two regions a, c in R related to b in the forms of $\mathbf{R}(a, b)$ and $\mathbf{S}(b, c)$ respectively. In particular, the usual relational composition of any two RCC8 relations is non-empty.

2.2. Strict RCC models and Boolean connection algebras

Stell [28] introduces *Boolean connection algebras* (BCAs), and proves that these structures are equivalent to *strict* models of the RCC axioms. In this subsection, we first briefly recall this result, and then give another description of BCAs dealing with the non-tangential proper part relation.

Definition 1.2 [24,28]. Let $A = \langle A; \bot, \top, ', \vee, \wedge\rangle$ be a Boolean algebra with more than two elements, let R denote $A - \{\bot\}$, and let R_- denote $R - \{\top\}$. If $\mathbf{C}$ is a binary relation on R, then the structure $\langle A; \mathbf{C}\rangle$, is said to be a *generalized Boolean connection algebra* (GBCA) if it satisfies the following axioms.

A1. $\mathbf{C}$ is symmetric and reflexive.
A2. $(\forall x \in R_-)\mathbf{C}(x, x')$.
A3. $\forall x, y, z \in R$, $\mathbf{C}(x, y \vee z)$ iff $\mathbf{C}(x, y)$ or $\mathbf{C}(x, z)$.

GBCA $\langle A; \mathbf{C}\rangle$ is called a *Boolean connection algebra* if the following axiom is satisfied.

A4. $(\forall x \in R_-)(\exists y \in R)\neg\mathbf{C}(x, y)$.

Let $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ be a strict model of RCC. Define binary operations $\vee$ and $\wedge$ on the set $R \cup \{\mathrm{n}\}$ as follows.

$$x \vee y = \begin{cases} \mathrm{sum}(x, y) & \text{if } x, y \in R, \\ x & \text{if } y = \mathrm{n}, \\ y & \text{if } x = \mathrm{n}. \end{cases} \qquad x \wedge y = \begin{cases} \mathrm{prod}(x, y) & \text{if } x, y \in R, \\ \mathrm{n} & \text{if } \mathrm{n} \in \{x, y\}. \end{cases}$$

Also, define the unary operation $'$ on the set $R \cup \{\mathrm{n}\}$ by $x' = \mathrm{compl}\, x$ for $x \in R - \{\mathrm{u}\}$, and by $\mathrm{u}' = \mathrm{n}$, and $\mathrm{n}' = \mathrm{u}$.

For $x, y \in R \cup \{\mathrm{n}\}$, we say $x \leqslant y$ if either $\mathbf{P}(x, y)$ or $x = \mathrm{n}$.

Stell [28] shows that the structure $\langle R \cup \{\mathrm{n}\}; \mathrm{u}, \mathrm{n}, ', \wedge, \vee\rangle$ is a Boolean algebra, and accordingly, $\langle R \cup \{\mathrm{n}\}; \mathbf{C}\rangle$ is a BCA.

On the other hand, let $\langle A; \mathbf{C}\rangle$ be a BCA. Set $R = A - \{\bot\}$, $\mathrm{n} = \bot$, $\mathrm{u} = \top$, and for all $x, y \in R$, $z \in R - \{\top\}$, set $\mathrm{sum}(x, y) = x \vee y$, $\mathrm{prod}(x, y) = x \wedge y$, $\mathrm{compl}\, z = z'$. Then the structure $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is a strict model of RCC.

In what follows, we shall in consequence make no difference between BCAs and corresponding strict models of RCC. Given a strict RCC model R, we shall often write relations derived from $\mathbf{C}$ in the way of lattice theory; for example, we write $a \leqslant b$ for

$\mathbf{P}(a,b)$, $a < b$ for $\mathbf{PP}(a,b)$, $a \wedge b > 0$ for $\mathbf{O}(a,b)$ and $a \wedge b = 0$ for $\mathbf{DR}(a,b)$. We also write a' for the complement of a, $a \vee b$ for the sum of a and b, and $a - b$ for the product of a and the complement of b.

The non-tangential proper part relation **NTPP** plays an important role in RCC and deserves a careful study. In the following theorem we shall show that **NTPP** and **C** are inter-definable.

The proof of such a theorem depends upon a thorough analysis of the non-tangential proper part relation **NTPP**. For a more elegant presentation of the properties of **NTPP**, we write $a \ll b$ for $\mathbf{NTPP}(a,b)$.

Lemma 2.2. *Let* $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ *be a strict model of RCC. We have the following properties*:

(1) *For all* $a \in R - \{\mathrm{u}\}$, $a \ll \mathrm{u}$.
(2) *For all* $a \in R$, $a \not\ll a$.
(3) *For all* $a, b \in R - \{\mathrm{u}\}$, $a \ll b$ *iff* $b^* \ll a^*$.
(4) *For all* $a, b, c \in R$ *with* $a \ll c$ *and* $b \ll c$, *we have* $a \vee b \ll c$.
(5) *For all* $a, b, c \in R$ *with* $a \ll b$ *and* $a \ll c$, *we have* $a \ll b \wedge c$.
(6) *For all* $a, b, c, d \in R$ *with* $a \leqslant b \ll c \leqslant d$, *we have* $a \ll d$ *and* $b \leqslant c$.

From the above lemma, we easily prove:

Theorem 2.1. *Let* A *be an atomless Boolean algebra and let* $\prec$ *be a binary relation on* $A - \{\perp\}$. *Define a connection relation on* $A - \{\perp\}$ *as* $\mathbf{C}(x, y)$ *iff not* $x \prec y^*$, *then* $\langle A, \mathbf{C}\rangle$ *is a BCA iff* $\prec$ *satisfies conditions* (1)–(6) *of above lemma. Moreover, when this is the case,* $\prec$ *is precisely the non-tangential proper part relation* $\ll$ *generated by* $\mathbf{C}$.

This theorem shows that the primitive connectedness relation is definable form $\ll$ and consequently the theory of BCA can be based on the non-tangential proper part relation. The reformulated theory of BCA bears similarity to that of continuous lattice [19], where a continuous lattice is defined to be a complete lattice with an auxiliary relation, also denoted by $\ll$, satisfying certain property.

There is another theory in the literature which is also based on a connection relation **C** [29]. In this theory, $\ll$ and **C** are also definable from each other, where $\ll$ is defined as $x \ll y \Leftrightarrow \neg\mathbf{C}(x, y^*)$. This theory differs from the BCA theory mainly in two aspects: (1) $\ll$ satisfies the Interpolation Condition (see Condition (3), Section 4.1 of this paper); (2) a region is not necessarily connected to its complement.

2.3. A construction for strict RCC models

In this subsection we recall a construction for strict RCC models given in [28]. First we recall some definitions and notations.

A *pseudocomplemented distributive lattice* A is defined as a distributive lattice, A, equipped with a unary operation ${}^*: A \to A$, such that, for all $a \in A$, a^* is the pseudocomplement of a, namely, a^* is the greatest element of $\{x \in A \mid a \wedge x = \perp\}$.

The set of *skeletal* elements of A is defined by $\mathcal{S}(A) = \{a \in A \mid a^{**} = a\}$. It is well-known that $\mathcal{S}(A)$ is a Boolean algebra where $\bot$ and $\top$ are as in A, the complement is the restriction of the pseudocomplement to $\mathcal{S}(A)$, and where the meet, $\sqcap$, and the join, $\sqcup$, are defined by $x \sqcap y = x \wedge y$, and $x \sqcup y = (x \vee y)^{**}$ [5]. A lattice, A, is *connected* if it does not contain elements $a \neq \bot$ and $b \neq \bot$ such that $a \vee b = \top$ and $a \wedge b = \bot$. A pseudocomplemented distributive lattice A is *inexhaustible* if for every $b \in \mathcal{S}(A) - \{\bot\}$ there is some $a \in \mathcal{S}(A) - \{\bot\}$ such that $a^* \vee b = \top$ [28]. (Definition 26 in [28] is clearly misdefined for that it allows $a = \bot$ and therefore each pseudocomplemented distributive lattice will be inexhaustible.)

We summarize some well-known properties of pseudocomplemented distributive lattice.

Proposition 2.2 [28]. *The following equations hold in any pseudocomplemented distributive lattice.*

1. $(x \vee y)^* = x^* \wedge y^*$,
2. $(x \wedge y)^* = (x^* \vee y^*)^{**}$,
3. $(x \wedge y)^{**} = x^{**} \wedge y^{**}$,
4. $(x \vee y)^{**} = (x^{**} \vee y^{**})^{**}$,
5. $\bot^* = \top$,
6. $\top^* = \bot$,
7. $x^{***} = x^*$,
8. $x \vee x^{**} = x^{**}$,
9. $x \wedge x^* = \bot$,
10. $(x \vee x^*)^* = \bot$.

Let $\langle A; \bot, \top, ^*, \vee, \wedge\rangle$ be a pseudocomplemented distributive lattice with $\langle \mathcal{S}(A); \bot, \top, ', \sqcup, \sqcap\rangle$ as its Boolean algebra of skeletal elements, and let the relation $\mathbf{C}$ on $\mathcal{S}(A)$ be defined by $\mathbf{C}(x, y)$ iff $x^* \vee y^* \neq \top$. Stell shows that if A is connected and inexhaustible and if $\mathcal{S}(A)$ contains more than two elements, then $\langle \mathcal{S}(A); \mathbf{C}\rangle$ is a BCA [28]. Conversely, if $\mathcal{S}(A)$ contains more than two elements and if $\langle \mathcal{S}(A); \mathbf{C}\rangle$ is a BCA, then A must be connected and inexhaustible [24].

A topological space, X, is said to be *inexhaustible* if the lattice of its open sets is so.

Corollary 2.1 [20,24,28]. *Let X be a topological space, let R be the set of non-empty regular open sets of X, and assume that R contains more than two elements. Define the relation* $\mathbf{C}$ *on R by* $\mathbf{C}(H, K)$ *iff $\overline{H} \cap \overline{K} \neq \emptyset$. Define* $\mathrm{sum}(H, K)$ *to be the interior of $\overline{H \cup K}$, define* $\mathrm{prod}(H, K)$ *to be $H \cap K$, and* $\mathrm{compl}\, H$ *to be the interior of $X - H$. Then* $\langle R, \{\emptyset\}; X, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ *is a model of the RCC, if and only if X is connected and inexhaustible.*

This theorem completely characterized the variety of topological spaces which may provide a standard model of the RCC theory.

2.4. Examples of non-strict RCC models

Stell shows that non-strict models do exist [28], but his example is quite artificial. In this subsection we shall show that each *pseudocomplemented distributive lattices A* gives rise to a non-strict RCC models.

Given a pseudocomplemented distributive lattice $\langle A; \bot, \top, ^*, \vee, \wedge\rangle$, we set $A^d = A - \{a \neq \top\colon a^{**} = \top\}$, $R = A^d - \{\bot\}$. Define a binary operation $\sqcup$ on A^d as: $x \sqcup y = x \vee y$ if $(x \vee y)^{**} \neq \top$, otherwise $x \sqcup y = \top$.

Theorem 2.2. *Let $\langle A; \bot, \top, ^*, \vee, \wedge\rangle$ be a pseudocomplemented distributive lattice with $\langle \mathcal{S}(A); \bot, \top, ', \sqcup, \sqcap\rangle$ as its Boolean algebra of skeletal elements. Set $R = A^d - \{\bot\}$, $\mathrm{n} = \bot$, $\mathrm{u} = \top$, and for all $x, y \in R$, $z \in R - \{\top\}$, set $\mathrm{sum}(x, y) = x \sqcup y$, $\mathrm{prod}(x, y) = x \wedge y$, $\mathrm{compl}\, z = z^*$. Define a binary relation $\mathbf{C}$ on R by $\mathbf{C}(x, y)$ iff $x^* \vee y^* \neq \top$. Suppose that $\mathcal{S}(A)$ contains more than two elements. Then $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is an RCC model if and only if A is connected and inexhaustible.*

We need some lemmas:

Lemma 2.3. *Let A be a connected inexhaustible pseudocomplemented distributive lattice and R, $\mathbf{C}$ defined as above. Then $(\forall x, y \in R)[\mathbf{P}(x, y) \leftrightarrow y^* \leqslant x^*]$.*

Proof. Suppose $\mathbf{P}(x, y)$ and $y^* \nleqslant x^*$, then $y^* \wedge x^{**} > \bot$. Since A is inexhaustible, there exists some $b \in R$ such that $b^* \vee (y^* \wedge x^{**}) = \top$. By $b^* \vee y^* = \top$, $\mathbf{DC}(b, y)$. Note $b^* \vee x^* = (b^* \vee x^*) \wedge (b^* \vee (y^* \wedge x^{**})) = b^* \vee (x^* \wedge y^* \wedge x^{**}) = b^* < \top$. We also have $\mathbf{C}(b, x)$, this contradicts the definition of $\mathbf{P}$. The other direction is clear by definition. □

This lemma gives a lattice theoretic characterization of the part of relation on our domain R.

Lemma 2.4. *Let A be a connected inexhaustible pseudocomplemented distributive lattice and R, $\mathbf{C}$ defined as above. Then $(\forall x, y \in R)[\mathbf{O}(x, y) \leftrightarrow x \wedge y > \bot]$.*

Proof. Suppose $\mathbf{O}(x, y)$, then by definition, we have some $b \in R$ such that $\mathbf{P}(b, x)$ and $\mathbf{P}(b, y)$. Using Lemma 2.3, we have $b^{**} \leqslant x^{**} \wedge y^{**} = (x \wedge y)^{**}$ and therefore $x \wedge y > \bot$. The other hand is clear. □

This lemma characterizes the overlap relation on our domain R in the way of lattice theory.

Lemma 2.5. *Let A be a connected inexhaustible pseudocomplemented distributive lattice and R, $\mathbf{C}$ defined as above. Then $(\forall x, y \in R)[\mathbf{NTPP}(x, y) \leftrightarrow x^* \vee y^{**} = \top]$.*

Proof. Suppose $\mathbf{NTPP}(x, y)$ and $x^* \vee y^{**} < \top$. Then, by Lemma 2.3 and the definitions of $\mathbf{NTPP}$ and $\mathbf{C}$, we have $x^{**} < y^{**}$ and $\mathbf{C}(x, y^*)$. Since we always have $(\forall a, b \in R)\mathbf{C}(a, b) \leftrightarrow \mathbf{C}(a^{**}, b^{**})$, we have $\mathbf{C}(x^{**}, y^*)$ and $\mathbf{C}(y^{**}, y^*)$. Since $y^{**} \wedge y^* = \bot$ and $x^{**} < y^{**}$, we have by definition $\mathbf{EC}(x^{**}, y^*)$, $\mathbf{EC}(y^{**}, y^*)$, and so $\mathbf{EC}(y^*, x)$, $\mathbf{EC}(y^*, y)$. This contradicts the definition of $\mathbf{NTPP}$.

On the other hand, suppose $x^* \vee y^{**} = \top$. By $x^* \vee y^* = (x^* \vee y^*) \wedge (x^* \vee y^{**}) = x^* \vee (y^* \wedge y^{**}) = x^*$, we have $x^* > y^*$, namely $\mathbf{PP}(x, y)$ by Lemma 2.3. Suppose there is some $z \in R$ such that $\mathbf{EC}(z, x)$ and $\mathbf{EC}(z, y)$, then $z \wedge y = \bot$, hence $z \leqslant y^* < x^*$. But $\mathbf{EC}(z, y)$ also tells us that $\top > z^* \vee x^* \geqslant y^{**} \vee x^*$, a contradiction. □

This lemma gives a lattice theoretic characterization of the non-tangential proper part relation on our domain R.

The following lemma shows the necessary part of Theorem 2.2.

Lemma 2.6. *Let A be a pseudocomplemented distributive lattice and R, $\mathbf{C}$ defined as above. Suppose $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is an RCC model, then A is connected and inexhaustible.*

Proof. We first show that A is connected. Suppose not so, we have by definition $x, y \in A - \{\bot\}$ such that $x \vee y = \top$, $x \wedge y = \bot$. We claim that $x = y^*$ and $y = x^*$. Take $x = y^*$ as an example. By $x \wedge y = \bot$, we have $x \leqslant y^*$. Since $x = x \vee (y \wedge y^*) = (x \vee y) \wedge (x \vee y^*) = x \vee y^*$, we have $x \geqslant y^*$, hence $x = y^*$. Similarly we can prove that $y = x^*$. As a result, we have $x^* \vee y^* = y \vee x = \top$. By the definition of $\mathbf{C}$, we have $\mathbf{DC}(x, y)$. But since $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is an RCC model, we also have $\mathbf{C}(x, x^*)$, namely $\mathbf{C}(x, y)$, a contradiction.

For each $b \in \mathcal{S}(A) - \{\bot\}$, clearly b is also in R. Since $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is an RCC model, we have by Lemma 2.1 some $x \in R$ with $\mathbf{NTPP}(x, b)$. By Axiom R4a of Definition 1.1, $\mathbf{DC}(x, b^*)$, namely $x^* \vee (b^*)^* = \top$. Take $a = x^{**}$, clearly we have $a \in \mathcal{S}(A) - \{\bot\}$ and $a^* \vee b = x^* \vee b^{**} = \top$. Hence A is also inexhaustible. □

Proof of Theorem 2.2. The "only if" part follows from Lemma 2.6. As for the "if" part, we need only to verify Axioms A4b and A6.

For A4b, suppose $\mathbf{O}(x, y^*)$ and $\mathbf{P}(x, y)$. Then by $x \wedge y^* > \bot$ and $x^* \geqslant y^*$, we have $x \wedge x^* > \bot$, a contradiction. On the other hand, suppose $\neg\mathbf{O}(x, y^*)$. Then by $x \wedge y^* = \bot$, we have $y^* \leqslant x^*$, hence $\mathbf{P}(x, y)$ by Lemma 2.3. As for A6, suppose $y \wedge z > \bot$ and there exists some $w \in R$ such that $\mathbf{P}(w, y)$ and $\mathbf{P}(w, z)$ and $\mathbf{C}(x, w)$. By above lemmas, we have $y^*, z^* \leqslant w^*$ and $x^* \vee w^* < \top$. Hence $x^* \vee (y \wedge z)^* = x^* \vee (y^* \vee z^*)^{**} \leqslant x^* \vee w^* < \top$, accordingly we have $y \wedge z > \bot$ and $\mathbf{C}(x, y \wedge z)$. As for the other hand, note we have both $\mathbf{P}(y \wedge z, y)$ and $\mathbf{P}(y \wedge z, z)$ if $y \wedge z > \bot$. □

Interestingly, RCC model constructed from a connected inexhaustible pseudocomplemented distributive lattice is always *non-strict*.

Lemma 2.7. *Let A be a connected inexhaustible pseudocomplemented distributive lattice and R, $\mathbf{C}$ defined as above. Then the RCC model $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is non-strict.*

Proof. For each $x \in R$, since $x^* = (x^{**})^*$, we have $\mathbf{EQ}(x, x^{**})$ by definition. Suppose that $x = x^{**}$ for all $x \in R$, namely $R = \mathcal{S}(A) - \{\bot\}$. Take $a \in R - \{\top\}$, we have some $b \in R$ such that $b^* \vee a = \top$ since A is inexhaustible. Set $c = b^* \wedge (a \vee a^*)$. We have $c^* = (b^{**} \vee (a \vee a^*)^*)^{**} = (b \vee (a^* \wedge a^{**}))^{**} = b^{**} = b$, hence $c \neq \bot$, $c \in R$. By assumption, $c^{**} = c$, thereby $b^* \wedge (a \vee a^*) = b^*$, namely $b^* \leqslant a \vee a^*$. As a result, we have $a \vee a^* = \top$ for $b^* \vee a = \top$. Because a is connected to a^*, namely $a^* \vee a \neq \top$, this is a contradiction. $\square$

Since the open sets lattice $\Omega(X)$ of a topological space X is a complete Heyting algebra (hence a pseudocomplemented distributive lattice), by Theorem 2.2 and Lemma 2.7, we have the following

Corollary 2.2. *Let X be a topological space containing more than two regular open sets. Let $R = \{U \in \Omega(X) - \{\emptyset\}\colon U = X$ or $\overline{U} \neq X\}$. Define the relation $\mathbf{C}$ on R by $\mathbf{C}(H, K)$ iff $\overline{H} \cap \overline{K} \neq \emptyset$. Define $\mathrm{sum}(H, K) = H \cup K$ if the closure of $H \cup K$ isn't X, otherwise define $\mathrm{sum}(H, K) = X$. Define $\mathrm{prod}(H, K) = H \cap K$, and $\mathrm{compl}\, H$ to be the interior of $X - H$. Then $\langle R, \{\emptyset\}; X, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is a non-strict model of the RCC if and only if X is connected and inexhaustible.*

This result shows for the first time that natural examples of non-strict RCC model do exist.

Note that each non-strict RCC model always induce a strict RCC model. Suppose $\langle R, \{\mathrm{n}\}; \mathrm{u}, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is a non-strict RCC model, then by definition **EQ** is different from the identity relation '$=$'. Clearly **EQ** is an equivalent relation, for each $x \in R$, we denote by $[x]$ the equivalent class of x. Take $\widetilde{R} = R/\mathbf{EQ}$, $\tilde{\mathrm{n}} = \mathrm{n}$, $\tilde{\mathrm{u}} = [\mathrm{u}]$. For any $[x], [y], [z] \in \widetilde{R}$ with $[z] \neq \tilde{\mathrm{u}}$, set $\mathrm{sum}([x], [y]) = [\mathrm{sum}(x, y)]$, $\mathrm{compl}\,[z] = [\mathrm{compl}\, z]$; if $\mathrm{prod}(x, y) \neq \mathrm{n}$, set $\mathrm{prod}([x], [y]) = [\mathrm{prod}(x, y)]$, otherwise set $\mathrm{prod}([x], [y]) = \mathrm{n}$. Define a binary relation $\mathbf{C}$ on $\widetilde{R}$ by $\mathbf{C}([x], [y])$ iff $\mathbf{C}(x, y)$ for any $[x], [y] \in \widetilde{R}$. Then one can check that $\langle \widetilde{R}, \{\mathrm{n}\}; \tilde{\mathrm{u}}, \mathrm{compl}, \mathrm{sum}, \mathrm{prod}, \mathbf{C}\rangle$ is indeed a strict RCC model. Clearly, these two models play same role in the RCC theory.

Since the first order RCC theory make no distinction between two regions related by the relation **EQ**, one need only to consider the strict models. In consequence, we shall only discuss strict models in the rest of this paper. Recall the equivalence between strict models and BCAs, we shall often talk about RCC models lattice theoretically.

3. Composition table for the RCC8 relations

Originating in Allen's analysis of temporal relations, the notion of a *composition table*[3] (**CT**) has become a key technique in providing an efficient inference mechanism for a wide class of theories.

In this section, we investigate the weak composition table for the RCC8 relations, $\mathcal{R}_8$, entailed by the RCC theory. Recall $\mathcal{R}_8 = \{\mathbf{DC}, \mathbf{EC}, \mathbf{PO}, \mathbf{EQ}, \mathbf{TPP}, \mathbf{NTPP}, \mathbf{TPPi}, \mathbf{NTPPi}\}$. Table 2 gives the composition table for $\mathcal{R}_8$. Where **TPi** stands for **TPP** and $=$, $\top$ means that no base relation is excluded. We call this table, following Düntsch et al. [14], the *RCC8 weak composition table* (RCC8 **CT** henceforth). This table appears first in [13] and coincides with that of [16], who built an eight relation calculus, which, although based on point set topology, has many similarities to RCC8.

[3] The term 'composition table' is just what Allen called 'transitivity table'.

Table 2
Composition table for RCC8 relations

$\circ$	**DC**	**EC**	**PO**	**TPP**	**NTPP**	**TPPi**	**NTPPi**
DC	$\top$	**DR,PO,PP**	**DR,PO,PP**	**DR,PO,PP**	**DR,PO,PP**	**DC**	**DC**
EC	**DR,PO,PPi**	**DR,PO, TPP,TPi**	**DR,PO,PP**	**EC,PO,PP**	**PO,PP**	**DR**	**DC**
PO	**DR,PO,PPi**	**DR,PO,PPi**	$\top$	**PO,PP**	**PO,PP**	**DR,PO,PPi**	**DR,PO,PPi**
TPP	**DC**	**DR**	**DR,PO,PP**	**PP**	**NTPP**	**DR,PO, TPP,TPi**	**DR,PO,PPi**
NTPP	**DC**	**DC**	**DR,PO,PP**	**NTPP**	**NTPP**	**DR,PO,PP**	$\top$
TPPi	**DR,PO,PPi**	**EC,PO,PPi**	**PO,PPi**	**PO,TPP,TPi**	**PO,PP**	**PPi**	**NTPPi**
NTPPi	**DR,PO,PPi**	**PO,PPi**	**PO,PPi**	**PO,PPi**	**O**	**NTPPi**	**NTPPi**

We now give some models of the RCC8 **CT**.

Example 3.1. Let U be the set of planar regions bounded by Jordan curves. Two regions in U are connected if they have common parts, i.e., $\mathbf{C}(a,b) \Leftrightarrow a \cap b \neq \emptyset$ for all $a, b \in U$. As in Table 1, for each relation symbol in $\mathcal{R}_8$, we can define a corresponding relation. Then this domain U, together with the RCC8 relations, is a model of the RCC8 **CT**. Moreover, this model is consistent.

This model is given by Egenhofer [16]. We believe it is also extensional, and rigorous investigation will be given later.

Example 3.2 [15]. Let U be the set of closed disks of the Euclidean plane. Two regions in U are connected if they have common parts, i.e., $\mathbf{C}(a,b) \Leftrightarrow a \cap b \neq \emptyset$ for all $a, b \in U$. With RCC8 relations interpreted as in above example, U is an extensional model of the RCC8 **CT**.

Since the RCC theory entails the RCC8 **CT**, each RCC model is already a model of the RCC8 **CT**. But, when does an RCC model be consistent? And when does it be extensional?

The question about extensionality of RCC models will be discussed in next section. We end this section with an affirmative answer to the consistency question.

Definition 3.1. Let B_5 be the Boolean algebra with five atoms $X = \{a, b, c, d, e\}$. Define a binary relation $\mathbf{A}$ on X as: $\mathbf{A} = \{(a,b), (b,e), (c,e), (d,e)\} \cup \{(x,x) \mid x \in X\}$. Let $\mathbf{C}_5$ be the binary relation on B_5 defined as follows: $\forall s, t \in B_5$, $\mathbf{C}_5(s,t)$ iff $\exists x, y \in X$ s.t. $x \in s \wedge y \in t \wedge \mathbf{A}(x,y)$. Clearly $\langle B_5, \mathbf{C}_5 \rangle$ is a generalized Boolean connection algebra.

It is interesting that $B_5 - \{\bot\}$ will be a consistent model of the RCC8 **CT** if we interpret $\mathbf{P}(s,t)$ as $s \leqslant t$ and interpret each RCC8 relation by $\mathbf{C}_5$ and $\mathbf{P}$ as in Table 1. The verification is straightforward.

Theorem 3.1. *Each strict RCC model is a consistent model of the RCC8* **CT**.

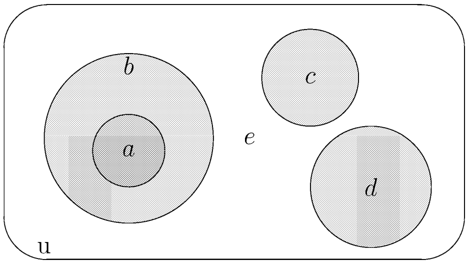


Fig. 2. An illustration of B_5.

Proof. Let $\langle A, \mathbf{C}\rangle$ be a Boolean connection algebra, we need only to show A contains B_5 as a sub-Boolean algebra and $\mathbf{C}_5$ is just the restriction of $\mathbf{C}$ on B_5.

Take $m, c, d \in A - \{\bot, \top\}$ with $\mathbf{DC}(m, c)$, $\mathbf{DC}(m, d)$ and $\mathbf{DC}(c, d)$. Take $a \ll m$ and let $b = m - a,\ e = (a \vee b \vee c \vee d)'$. Denoted by B the sub-Boolean algebra generated by $\{a, b, c, d, e\}$, then $\langle B, \mathbf{C}|_B\rangle$ is the same as $\langle B_5, \mathbf{C}_5\rangle$. $\square$

This theorem assures that for any RCC model R and for each entry $\mathbf{T}$ in the cell specified by $\mathbf{R}$ and $\mathbf{S}$, there are three regions a, b, c in R s.t. $\mathbf{R}(a, b) \wedge \mathbf{S}(b, c) \wedge \mathbf{T}(a, c)$.

4. Composition extensionality of RCC models

An examination of the RCC8 **CT** (Table 2) reveals that an extensional interpretation is not compatible with the 1st-order RCC theory. To avoid such problems and hence construct an extensional composition table, Bennett suggests to remove the universal region u from the domain of possible referents of the region constants [7].

This section will show that this is not true, however. We in fact give an exhaustively investigation on the RCC8 **CT**: for each cell entry, we first decide whether or not above extensional interpretation is possible and then, if this is not true, we decide for which RCC model does above extensional interpretation be possible.

4.1. Notations and basic lemmas

For any $\mathbf{R}, \mathbf{S}, \mathbf{T} \in \mathcal{R}_8$, we write by $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$ for convenience the fact that $\mathbf{T}$ is an entry in the cell specified by the ordered pair $\langle \mathbf{R}, \mathbf{S}\rangle$ in RCC8 weak composition table, i.e., $\mathbf{T} \in CT(\mathbf{R}, \mathbf{S})$. In the mean time, we call such $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$ a *composition triad* (a *triad* henceforth).

Our task now is to verify, for each RCC model R and each triad $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$, whether or not the following condition is right:

$$(\forall a, c \in R)\big[\mathbf{T}(a, c) \to (\exists b \in R)[\mathbf{R}(a, b) \wedge \mathbf{S}(b, c)]\big]. \tag{1}$$

Note if either $\mathbf{R}$ or $\mathbf{S}$ is the identity relation '$=$', then condition (1) is true.

However, a rough check will show condition (1) is not always true. For example, if $a \vee c = \mathrm{u}$, then there cannot exist any $b \in R$ such that b is disconnected from both a and c.

In particular, triad $\langle \mathbf{DC}, \mathbf{EC}, \mathbf{DC} \rangle$ doesn't satisfy condition (1). One plausible remedy, as suggested by Bennett [7,8], is to remove the universal region from the domain of possible referents of the region constants. He also writes

> "All the exceptions to extensional composition that I am aware of involve u; so it seems that an extensional interpretation could be achieved with respect to a modified theory without a universal region. The domain of this new theory would then be more homogeneous and more similar to that of the Allen relations, where intervals are always bounded."

Recall that Example 3.2 and, perhaps, Example 3.1 already provide extensional models of the RCC8 **CT**. So Bennett's suggestion is valid for obtaining an extensional model. But our intension here is, however, investigating the extensionality properties of the RCC8 **CT** in the framework of the RCC theory. In Section 4.5 we shall show that it is not enough to achieve an extensional interpretation by simply "removing the universal region of the possible referents of the region constants".

We rephrase the task as to verify for each triad, $\langle \mathbf{R}, \mathbf{T}, \mathbf{S} \rangle$, whether or not the following condition is right:

$$(\forall a, c \in R)\big[(a \vee c \neq \mathrm{u}) \wedge \mathbf{T}(a, c) \\ \rightarrow (\exists b \in R)[(a \vee b \vee c \neq \mathrm{u}) \wedge \mathbf{R}(a, b) \wedge \mathbf{S}(b, c)]\big]. \tag{2}$$

This equation, though not completely the same as removing the universal regions from the domain of the referents of the region constants, provides a formal, maybe weaker, description. An exhaustive analysis of condition (2) as well as condition (1) will be very helpful for determining the extensionality properties of the RCC8 **CT**.

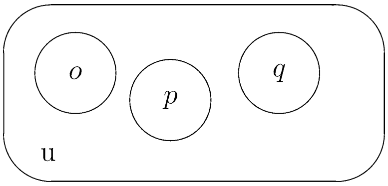
The following proposition shows that condition (1) is indeed stronger than condition (2).

Proposition 4.1. *For each triad $\langle \mathbf{R}, \mathbf{T}, \mathbf{S} \rangle$, if RCC model R satisfies condition* (1) *then R also satisfies condition* (2).

Proof. We suppose that neither **R** nor **S** is the identity relation. For all $a, c \in R$ with $a \vee c \neq \mathrm{u}$ and $\mathbf{T}(a, c)$, suppose there exists some $b \in R$ such that $\mathbf{R}(a, b)$ and $\mathbf{S}(b, c)$. Take $m \ll (a \vee c)'$ and let $\hat{b} = b - m$. Then one can check $\mathbf{R}(a, \hat{b})$ and $\mathbf{S}(\hat{b}, c)$. Clearly, $a \vee \hat{b} \vee c \neq \mathrm{u}$. □

Note triad $\langle \mathbf{DC}, \mathbf{EC}, \mathbf{DC} \rangle$ now satisfies condition (2): For all $a, c \in R$ with $\mathbf{EC}(a, c)$ and $a \vee c \neq \top$. Take some $b \ll (a \vee c)'$, then b is disconnected from $a \vee c$, hence disconnected from both a and c. But the following example shows that condition (2) is still not always true.

Example 4.1. Let R be an RCC model. Take $o, p, q \in R$ with $\mathbf{DC}(o, p)$, $\mathbf{DC}(p, q)$, $\mathbf{DC}(o, q)$, see Fig. 3. Let $a = o \vee p$, $c = p \vee q$. Then $\mathbf{PO}(a, c)$, there are no $b \in R$ s.t. both $\mathbf{EC}(a, b)$ and $\mathbf{TPP}(b, c)$. So the composition triad $\langle \mathbf{EC}, \mathbf{PO}, \mathbf{TPP} \rangle$ cannot be interpreted extensionally for any RCC model.



$$a = o \vee p,\ c = p \vee q$$

Fig. 3. A counter-example of condition (2).

In what follows, for each triad $\langle \mathbf{R}, \mathbf{T}, \mathbf{S} \rangle$, we say $\mathbf{T}$ is *below*[4] $CT(\mathbf{R}, \mathbf{S})$ w.r.t. RCC model R if condition (1) is satisfied. We also say $\mathbf{T}$ is *weakly below* $CT(\mathbf{R}, \mathbf{S})$ w.r.t. R if R satisfies condition (2) and $\mathbf{T}$ is not below $CT(\mathbf{R}, \mathbf{S})$.

Table 3 shows the result, where we append to each cell entry a superscript taken from $\{^{\oplus}, ^{+}, ^{\otimes}, ^{\times}, ^{?}, ^{??}\}$. The meanings of these superscripts are as follows:

1. $\langle \mathbf{R}, \mathbf{T}^{\oplus}, \mathbf{S} \rangle \Leftrightarrow_{\text{def}} \mathbf{T}$ is below $\mathbf{R} \circ \mathbf{S}$ w.r.t. each RCC model R.
2. $\langle \mathbf{R}, \mathbf{T}^{+}, \mathbf{S} \rangle \Leftrightarrow_{\text{def}} \mathbf{T}$ is weakly below $\mathbf{R} \circ \mathbf{S}$ w.r.t. each RCC model R.
3. $\langle \mathbf{R}, \mathbf{T}^{\otimes}, \mathbf{S} \rangle \Leftrightarrow_{\text{def}} \mathbf{T}$ is below $\mathbf{R} \circ \mathbf{S}$ for RCC model R if and only if R satisfies *Interpolation Condition*:
$$(\forall a, c \in R)\big[a \ll c \to (\exists b \in R)[a \ll b \ll c]\big]. \tag{3}$$
4. $\langle \mathbf{R}, \mathbf{T}^{\times}, \mathbf{S} \rangle \Leftrightarrow_{\text{def}} \mathbf{T}$ is *neither below nor weakly below* $\mathbf{R} \circ \mathbf{S}$ w.r.t. any RCC model R, i.e., the following condition (4) is satisfied for each R:
$$(\exists a, c \in R)\big[\mathbf{T}(a, c) \wedge (a \vee c \neq \mathrm{u}) \wedge (\forall b \in R)[\mathbf{R}(a, b) \to \neg \mathbf{S}(b, c)]\big]. \tag{4}$$
5. $\langle \mathbf{R}, \mathbf{T}^{?}, \mathbf{S} \rangle \Leftrightarrow_{\text{def}} \mathbf{T}$ is below $\mathbf{R} \circ \mathbf{S}$ w.r.t. RCC model R if and only if R satisfies the following condition (5):
$$(\forall a, c \in R)\big[(a \ll c) \to (\exists b \in R)[(b \ll c) \wedge \mathbf{EC}(a, b)]\big]. \tag{5}$$
6. $\langle \mathbf{R}, \mathbf{T}^{??}, \mathbf{S} \rangle \Leftrightarrow_{\text{def}} \mathbf{T}$ is below $\mathbf{R} \circ \mathbf{S}$ w.r.t. RCC model R if and only if R satisfies the following condition (6):
$$(\forall a, c \in R)\big[a \ll c \to (\exists b \in R)[\mathbf{TPP}(a, b) \wedge \mathbf{TPP}(b, c)]\big]. \tag{6}$$

We need some examples to explain the conditions listed above.

Example 4.2. Fig. 4 gives each situation an example.

Consider the triad $\langle \mathbf{DC}, \mathbf{DC}, \mathbf{DC} \rangle$. For any RCC model R and $a, c \in R$ with $\mathbf{DC}(a, c)$, taking some $b \ll (a \vee c)'$, we shall have $\mathbf{DC}(a, b)$ and $\mathbf{DC}(b, c)$. Therefore we append to $\mathbf{DC}$ a superscript $^{\oplus}$.

Consider the triad $\langle \mathbf{DC}, \mathbf{EC}, \mathbf{DC} \rangle$. For any RCC model R and $a, c \in R$ with $\mathbf{EC}(a, c)$, if $a = c'$, namely $a \wedge c = \bot$, $a \vee c = \top$, we cannot have a region b satisfying both $\mathbf{DC}(a, b)$

[4] In the same sense of [14].

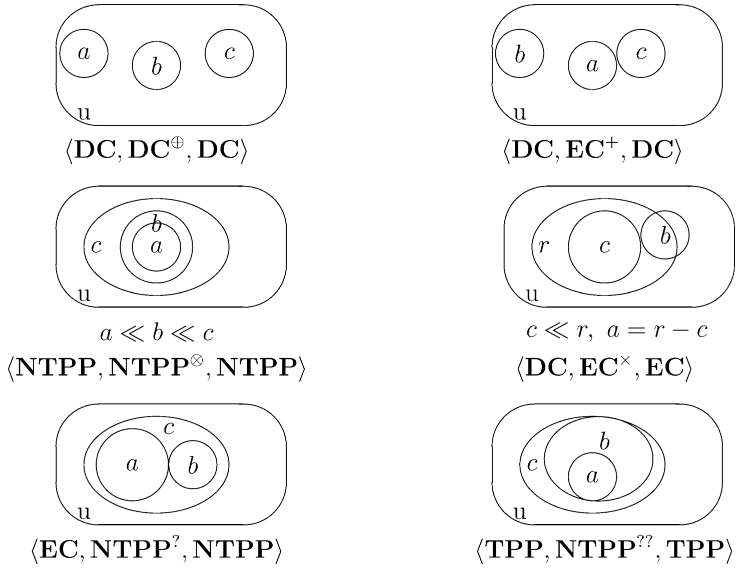


Fig. 4. Illustrations of the six situations of extensionality.

and $\mathbf{DC}(b,c)$. But if $a \vee c \neq \top$, taking some $b \ll (a \vee c)'$, we shall have $\mathbf{DC}(a,b)$ and $\mathbf{DC}(b,c)$. So we can append to **EC** a superscript $^+$.

Consider the triad $\langle \mathbf{DC}, \mathbf{EC}, \mathbf{EC}\rangle$. For any RCC model R. By Lemma 2.1, we can take two regions $r, c \in R$ with $c \ll r \neq \top$. Set $a = r - c$, clearly $\mathbf{EC}(a,c)$. Then for each $b \in R$, if $\mathbf{EC}(b,c)$, we must have $a \wedge b \neq \bot$. Therefore we append to **EC** a superscript $^\times$.

The rest three Triads $\langle \mathbf{NTPP}, \mathbf{NTPP}^{\otimes}, \mathbf{NTPP}\rangle$, $\langle \mathbf{EC}, \mathbf{NTPP}^{?}, \mathbf{NTPP}\rangle$ and $\langle \mathbf{TPP}, \mathbf{NTPP}^{??}, \mathbf{TPP}\rangle$ are clear form conditions (3), (5) and (6) respectively.

Note. We can show without much trouble that (*Interpolation Condition*) $\Rightarrow$ (5) $\Rightarrow$ (6). Moreover, Stell's graph example (see Remark 4.1) shows that Interpolation Condition is strictly stronger than (5). The least RCC model B_ω constructed in [24] also provides an example which fulfills condition (6) and violates condition (5) and Interpolation Condition (see Remark 4.2). But it is still unknown whether (6) is true for all RCC models.

For each triad $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$ with **R** or **T** being the identity relation, clearly **T** is below $CT(\mathbf{R}, \mathbf{S})$ (w.r.t. each RCC model R). That is why we do not include these situations in Table 3. Let **Tri** be the set of all triads in the RCC8 weak composition table with neither **R** nor **S** being the identity relation. There are altogether 178 triads in **Tri**, 92 triads with superscript $^\oplus$, 35 with $^+$, 4 with $^\otimes$, 35 with $^\times$, 8 with $^?$ and 4 with $^{??}$.

For a composition triad $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$ in **Tri**, clearly $\langle \mathbf{S}^{-1}, \mathbf{T}^{-1}, \mathbf{R}^{-1}\rangle$ is also in **Tri**, we call the latter the inverse triad of $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$. Moreover, if the inverse triad of $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$ is itself, namely $\mathbf{R} = \mathbf{S}^{-1}$, $\mathbf{T} = \mathbf{T}^{-1}$, then we call this triad a *self-dual* one. There are all together 24 self-dual triads in **Tri**.

The following lemma will be useful in the checking:

Lemma 4.1. *Triad* $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$ *has the same superscript with its inverse triad* $\langle \mathbf{S}^{-1}, \mathbf{T}^{-1}, \mathbf{R}^{-1}\rangle$.

Table 3
Extensional composition table for RCC8 relations

$\circ$	**DC**	**EC**	**PO**	**TPP**	**NTPP**	**TPPi**	**NTPPi**
DC	**DC**$^{\oplus}$ **EC**$^{+}$ **PO**$^{+}$ **TPP**$^{\oplus}$ **NTPP**$^{+}$ $=^{+}$ **TPPi**$^{\oplus}$ **NTPPi**$^{+}$	**DC**$^{?}$ **EC**$^{\times}$ **PO**$^{\times}$ **TPP**$^{\times}$ **NTPP**$^{+}$	**DC**$^{\oplus}$ **EC**$^{+}$ **PO**$^{+}$ **TPP**$^{\oplus}$ **NTPP**$^{+}$	**DC**$^{\oplus}$ **EC**$^{\times}$ **PO**$^{\times}$ **TPP**$^{\times}$ **NTPP**$^{?}$	**DC**$^{\oplus}$ **EC**$^{\oplus}$ **PO**$^{\oplus}$ **TPP**$^{\oplus}$ **NTPP**$^{\oplus}$	**DC**$^{\oplus}$	**DC**$^{\otimes}$
EC	**DC**$^{?}$ **EC**$^{\times}$ **PO**$^{\times}$ **TPPi**$^{\times}$ **NTPPi**$^{+}$	**DC**$^{\oplus}$ **EC**$^{\times}$ **PO**$^{\times}$ $=^{+}$ **TPP**$^{\oplus}$ **TPPi**$^{\oplus}$	**DC**$^{\oplus}$ **EC**$^{+}$ **PO**$^{+}$ **TPP**$^{\oplus}$ **NTPP**$^{+}$	**EC**$^{\oplus}$ **PO**$^{\times}$ **TPP**$^{\times}$ **NTPP**$^{+}$	**PO**$^{\times}$ **TPP**$^{\times}$ **NTPP**$^{?}$	**DC**$^{??}$ **EC**$^{+}$	**DC**$^{\oplus}$
PO	**DC**$^{\oplus}$ **EC**$^{+}$ **PO**$^{+}$ **TPPi**$^{\oplus}$ **NTPPi**$^{+}$	**DC**$^{\oplus}$ **EC**$^{+}$ **PO**$^{+}$ **TPPi**$^{\oplus}$ **NTPPi**$^{+}$	**DC**$^{\oplus}$ **EC**$^{\oplus}$ **PO**$^{\oplus}$ **TPP**$^{\oplus}$ **TPPi**$^{\oplus}$ $=^{+}$ **NTPP**$^{+}$ **NTPPi**$^{+}$	**PO**$^{\oplus}$ **TPP**$^{\oplus}$ **NTPP**$^{+}$	**PO**$^{\oplus}$ **TPP**$^{\oplus}$ **NTPP**$^{\oplus}$	**DC**$^{\oplus}$ **EC**$^{\oplus}$ **PO**$^{\oplus}$ **TPPi**$^{\oplus}$ **NTPPi**$^{\oplus}$	**DC**$^{\oplus}$ **EC**$^{\oplus}$ **PO**$^{\oplus}$ **TPPi**$^{\oplus}$ **NTPPi**$^{\oplus}$
TPP	**DC**$^{\oplus}$	**DC**$^{??}$ **EC**$^{+}$	**DC**$^{\oplus}$ **EC**$^{\oplus}$ **PO**$^{\oplus}$ **TPP**$^{\oplus}$ **NTPP**$^{\oplus}$	**TPP**$^{\oplus}$ **NTPP**$^{??}$	**NTPP**$^{\oplus}$	**DC**$^{\oplus}$ $=^{+}$ **EC**$^{\times}$ **PO**$^{\times}$ **TPP**$^{\oplus}$ **TPPi**$^{\oplus}$	**DC**$^{?}$ **EC**$^{\times}$ **PO**$^{\times}$ **TPPi**$^{\times}$ **NTPPi**$^{+}$
NTPP	**DC**$^{\otimes}$	**DC**$^{\oplus}$	**DC**$^{\oplus}$ **EC**$^{\oplus}$ **PO**$^{\oplus}$ **TPP**$^{\oplus}$ **NTPP**$^{\oplus}$	**NTPP**$^{+}$	**NTPP**$^{\otimes}$	**DC**$^{?}$ **EC**$^{\times}$ **PO**$^{\times}$ **TPP**$^{\times}$ **NTPP**$^{+}$	**DC**$^{\oplus}$ **EC**$^{\oplus}$ **PO**$^{\oplus}$ **TPP**$^{\oplus}$ **TPPi**$^{\oplus}$ $=^{\oplus}$ **NTPP**$^{\oplus}$ **NTPPi**$^{\oplus}$
TPPi	**DC**$^{\oplus}$ **EC**$^{\times}$ **PO**$^{\times}$ **TPPi**$^{\times}$ **NTPPi**$^{?}$	**EC**$^{\oplus}$ **PO**$^{\times}$ **TPPi**$^{\times}$ **NTPPi**$^{+}$	**PO**$^{\oplus}$ **TPPi**$^{\oplus}$ **NTPPi**$^{+}$	**PO**$^{\times}$ **TPP**$^{\oplus}$ **TPPi**$^{\oplus}$ $=^{+}$	**PO**$^{\times}$ **TPP**$^{\times}$ **NTPP**$^{\oplus}$	**TPPi**$^{\oplus}$ **NTPPi**$^{??}$	**NTPPi**$^{+}$
NTPPi	**DC**$^{\oplus}$ **EC**$^{\oplus}$ **PO**$^{\oplus}$ **TPPi**$^{\oplus}$ **NTPPi**$^{\oplus}$	**PO**$^{\times}$ **TPPi**$^{\times}$ **NTPPi**$^{?}$	**PO**$^{\oplus}$ **TPPi**$^{\oplus}$ **NTPPi**$^{\oplus}$	**PO**$^{\times}$ **TPPi**$^{\times}$ **NTPPi**$^{\oplus}$	**PO**$^{\oplus}$ $=^{\oplus}$ **TPP**$^{\oplus}$ **NTPP**$^{\oplus}$ **TPPi**$^{\oplus}$ **NTPPi**$^{\oplus}$	**NTPPi**$^{\oplus}$	**NTPPi**$^{\otimes}$

By this lemma, each triad has the same superscript with its inverse, consequently we need to check for only $24 + (178 - 24)/2 = 101$ triads.

We also need some basic results about RCC:

Lemma 4.2. *Let $\langle R, \mathbf{C}\rangle$ be an RCC model and let $a, b \in R - \{\mathrm{u}\}$. Then*

(1) $\mathbf{DC}(a, b) \Leftrightarrow a \ll b' \Leftrightarrow b \ll a'$;
(2) $\mathbf{EC}(a, b) \Leftrightarrow \mathbf{TPP}(a, b') \Leftrightarrow \mathbf{TPP}(b, a')$;
(3) $a \ll b \Leftrightarrow \mathbf{EC}(a, b - a) \wedge (\forall c \in R - \{\mathrm{u}\})[\mathbf{EC}(c, a) \rightarrow \mathbf{O}(c, b - a)]$.

The condition given in (3) is also equivalent to say a is, according to T. Mormann (in an unfinished paper), a 'hole' of $b - a$. But we may say alternatively $b - a$ circling a. We call this relation the Hole relation and denote it by $\mathbf{H}$. Note if a is a hole of b, then $\mathbf{EC}(a, b)$ and $a \neq b'$.

4.2. Cases of $\langle \mathbf{R}, \mathbf{T}^{\oplus}, \mathbf{S} \rangle$ and $\langle \mathbf{R}, \mathbf{T}^{+}, \mathbf{S} \rangle$

In these cases, we need to prove for each RCC model R the following

Theorem 4.1. (i) *For each triad $\langle \mathbf{R}, \mathbf{T}, \mathbf{S} \rangle$ in* **Tri**, *if $\langle \mathbf{R}, \mathbf{T}^{\oplus}, \mathbf{S} \rangle$ and suppose $\mathbf{T}(a, c)$, then there exist some $b \in R$, s.t. $\mathbf{R}(a, b)$ and $\mathbf{S}(b, c)$.*

(ii) *For each triad $\langle \mathbf{R}, \mathbf{T}, \mathbf{S} \rangle$ in* **Tri**, *if $\langle \mathbf{R}, \mathbf{T}^{+}, \mathbf{S} \rangle$ and suppose $\mathbf{T}(a, c)$, then $a \vee c \neq \mathrm{u}$ if and only if there exist some $b \in R - \{\mathrm{u}\}$ with $a \vee b \vee c \neq \mathrm{u}$, s.t. $\mathbf{R}(a, b)$ and $\mathbf{S}(b, c)$.*

Proof. For each triad $\langle \mathbf{R}, \mathbf{T}, \mathbf{S} \rangle$ with a superscript $^{\oplus}$ or $^{+}$ and each pair of regions $(a, c) \in \mathbf{T}$ (with $a \vee c \neq \mathbf{u}$ in the latter case), we show how to "construct" a region b such that $\mathbf{R}(a, b)$ and $\mathbf{S}(b, c)$. The procedures are summarized in Table 4.

We make some explanation of the meanings appeared in this table. In the first column we show the condition that a, c are subjected to; the second column gives some auxiliary regions, written by r, m, which are **NTPP** parts of certain regions composed from a and c; the third column gives a region b which is composed from a, c and, if necessary, r, m; the last column says that b satisfies the desired condition $\mathbf{R}(a, b) \wedge \mathbf{S}(b, c)$. Note that in this table we already synthesize some rows for brevity, e.g., the first row indeed justifies 8 triads, namely, $\langle \mathbf{DC}, \mathbf{T}^{\otimes}, \mathbf{DC} \rangle$ for $\mathbf{T} \in \{\mathbf{DC}, \mathbf{TPP}, \mathbf{TPPi}\}$ and $\langle \mathbf{DC}, \mathbf{T}^{+}, \mathbf{DC} \rangle$ for every other RCC8 relation $\mathbf{T}$. $\square$

There are all together 127 triads in **Tri** (about 71% of the total) with a superscript $^{\oplus}$ or $^{+}$. These composition triads can be seen as extensional in a sense.

4.3. Cases of $\langle \mathbf{R}, \mathbf{T}^{\otimes}, \mathbf{S} \rangle$

Theorem 4.2. *Given RCC model $\langle R, \mathbf{C} \rangle$, the following conditions are equivalent*:

(1) Interpolation Condition: $\forall a, c \in R$ *with* $a \ll c$, $\exists b \in R$ *s.t.* $a \ll b \ll c$.
(2) $\forall a, c \in R$ *with* $\mathbf{DC}(a, c)$, $\exists b \in R$ *s.t.* $\mathbf{DC}(a, b) \wedge \mathbf{NTPPi}(b, c)$.

Proof. We show only (1) $\Rightarrow$ (2). Since $\mathbf{DC}(a, c)$, we have $a \ll c'$, and by the interpolation condition we have some $m \in R$ s.t. $a \ll m \ll c'$. Let $b = m'$. Then we have $\mathbf{DC}(a, b)$ and $c \ll b$ for $a \ll b' \ll c'$. $\square$

By this theorem and Lemma 4.1, we have four triads in **TRi** with a superscript $^{\otimes}$: these are $\langle \mathbf{DC}, \mathbf{DC}^{\otimes}, \mathbf{NTPP} \rangle$, $\langle \mathbf{NTPP}, \mathbf{NTPP}^{\otimes}, \mathbf{NTPP} \rangle$ and their inverse triads.

Remark 4.1. In [28], Stell gives a Heyting algebra which is connected and exhaustible, but not regular. The Boolean algebra of its skeletal elements is a BCA which satisfies

Table 4
Extensional composition triads

$\mathbf{T}(a,c)$	r, m	b	$\mathbf{R}(a,b) \wedge \mathbf{S}(b,c)$
$a \vee c \neq \mathrm{u}$	$m \ll (a \vee c)'$	m	$\mathbf{DC}(a,b) \wedge \mathbf{DC}(b,c)$
$(a \vee c \neq \mathrm{u}) \wedge (c \nleq a)$	$m \ll (a \vee c)'$ $r \ll c - a$	$m \vee r$	$\mathbf{DC}(a,b) \wedge \mathbf{PO}(b,c)$
$\mathbf{DC}(a,c)$	$m \ll (a \vee c)'$	$m \vee c$	$\mathbf{DC}(a,b) \wedge \mathbf{TPPi}(b,c)$
$\mathbf{DC}(a,c)$	$m \ll c$	$c - m$	$\mathbf{DC}(a,b) \wedge \mathbf{TPP}(b,c)$
$c \nleq a$	$m \ll c - a$	m	$\mathbf{DC}(a,b) \wedge \mathbf{NTPP}(b,c)$
$\mathbf{DC}(a,c)$	$m \ll (a \vee c)'$	$(a \vee c)' - m$	$\mathbf{EC}(a,b) \wedge \mathbf{EC}(b,c)$
$(a \vee c \neq \mathrm{u}) \wedge (c \nleq a)$	$m \ll c - a$	$a' - m$	$\mathbf{EC}(a,b) \wedge \mathbf{PO}(b,c)$
$\mathbf{EC}(a,c)$	$m \ll c$	$c - m$	$\mathbf{EC}(a,b) \wedge \mathbf{TPP}(b,c)$
$\mathbf{NTPP}(a,c) \wedge (c \neq \mathrm{u})$		$c - a$	$\mathbf{EC}(a,b) \wedge \mathbf{TPP}(b,c)$
$\mathbf{EC}(a,c) \wedge (a \vee c \neq \mathrm{u})$	$m \ll (a \vee c)'$	$m \vee c$	$\mathbf{EC}(a,b) \wedge \mathbf{TPPi}(b,c)$
$\mathbf{DC}(a,c)$	$m \ll a'$	$a' - m$	$\mathbf{EC}(a,b) \wedge \mathbf{NTPPi}(b,c)$
$a \wedge c = \mathrm{n}$	$m \ll a$ $r \ll c$	$m \vee r$	$\mathbf{PO}(a,b) \wedge \mathbf{PO}(b,c)$
$(a \wedge c \neq \mathrm{n}) \wedge (a \vee c \neq \mathrm{u})$	$m \ll a \wedge c$ $r \ll (a \vee c)'$	$m \vee r$	$\mathbf{PO}(a,b) \wedge \mathbf{PO}(b,c)$
$\mathbf{PO}(a,c)$	$m \ll c - a$	$c - m$	$\mathbf{PO}(a,b) \wedge \mathbf{TPP}(b,c)$
$a < c$	$m \ll a$ $r \ll c - a$	$c - (m \vee r)$	$\mathbf{PO}(a,b) \wedge \mathbf{TPP}(b,c)$
$(a \wedge c > \mathrm{n}) \wedge (c \nleq a)$	$m \ll a \wedge c$ $r \ll c - a$	$m \vee r$	$\mathbf{PO}(a,b) \wedge \mathbf{NTPP}(b,c)$
$(a \vee c \neq \mathrm{u}) \wedge (a \nleq c)$	$m \ll a - c$ $r \ll (a \vee c)'$	$m \vee r \vee c$	$\mathbf{PO}(a,b) \wedge \mathbf{TPPi}(b,c)$
$(a \vee c \neq \mathrm{u}) \wedge (a \nleq c)$	$m \ll a - c$ $r \ll (a \vee c)'$	$(m \vee r)'$	$\mathbf{PO}(a,b) \wedge \mathbf{NTPPi}(b,c)$
$\mathbf{TPP}(a,c)$	$m \ll c - a$	$a \vee m$	$\mathbf{TPP}(a,b) \wedge \mathbf{TPP}(b,c)$
$\mathbf{NTPP}(a,c)$	$m \ll c - a$	$a \vee m$	$\mathbf{TPP}(a,b) \wedge \mathbf{NTPP}(b,c)$
$\mathbf{DC}(a,c)$		$a \vee c$	$\mathbf{TPP}(a,b) \wedge \mathbf{TPPi}(b,c)$
$\mathbf{TPP}(a,c)$	$m \ll c'$	$m \vee c$	$\mathbf{TPP}(a,b) \wedge \mathbf{TPPi}(b,c)$
$\mathbf{NTPPi}(a,c)$	$m \ll a'$	$m \vee a$	$\mathbf{TPP}(a,b) \wedge \mathbf{NTPPi}(b,c)$
$\mathbf{NTPP}(a,c) \wedge (c \neq \mathrm{u})$	$m \ll c - a$	$c - m$	$\mathbf{NTPP}(a,b) \wedge \mathbf{TPP}(b,c)$
$a \vee c \neq \mathrm{u}$	$m \ll (a \vee c)'$	m'	$\mathbf{NTPP}(a,b) \wedge \mathbf{NTPPi}(b,c)$
$\mathbf{TPP}(a,c) \vee \mathbf{TPPi}(a,c)$	$m \ll a \wedge c$	$(a \wedge c) - m$	$\mathbf{TPPi}(a,b) \wedge \mathbf{TPP}(b,c)$
$\mathbf{NTPP}(a,c)$	$m \ll a$	$a - m$	$\mathbf{TPPi}(a,b) \wedge \mathbf{NTPP}(b,c)$
$a \wedge c \neq \mathrm{n}$	$m \ll a \wedge c$	m	$\mathbf{NTPPi}(a,b) \wedge \mathbf{NTPP}(b,c)$

condition (5). But we can show that this model doesn't satisfy the interpolation condition. Using Stell's notation, $[1/8, 3/16)^{**} \ll [0, 1/2)^{**}$, if there is some b s.t. $[1/8, 3/16)^{**} \ll b \ll [0, 1/2)^{**}$, then $1/4 \in b$ and therefore $1/2 \in [0, 1/2)^{**}$, a contradiction.

4.4. *Cases of* $\langle \mathbf{R}, \mathbf{T}^?, \mathbf{S} \rangle$ *and* $\langle \mathbf{R}, \mathbf{T}^{??}, \mathbf{S} \rangle$

Using Lemma 4.2, we can easily prove the following two theorems.

Theorem 4.3. *The following conditions are equivalent to condition* (5):

(i) $\forall a, c \in R$ *with* $\mathbf{DC}(a, c)$, $\exists b \in R$ *s.t.* $\mathbf{DC}(a, b)$ *and* $\mathbf{EC}(b, c)$.
(ii) $\forall a, c \in R$ *with* $\mathbf{NTPP}(a, c)$, $\exists b \in R$ *s.t.* $\mathbf{DC}(a, b)$ *and* $\mathbf{TPP}(b, c)$.
(iii) $\forall a, c \in R$ *with* $\mathbf{DC}(a, c)$, $\exists b \in R$ *s.t.* $\mathbf{TPP}(a, b)$ *and* $\mathbf{NTPPi}(b, c)$.

By this theorem and Lemma 4.1, we have eight triads in **TRi** with a superscript $^?$: these are $\langle \mathbf{DC}, \mathbf{DC}^?, \mathbf{EC}\rangle$, $\langle \mathbf{DC}, \mathbf{NTPP}^?, \mathbf{TPP}\rangle$, $\langle \mathbf{EC}, \mathbf{NTPP}^?, \mathbf{NTPP}\rangle$, $\langle \mathbf{TPP}, \mathbf{DC}^?, \mathbf{NTPPi}\rangle$ and their inverse triads.

The following theorem also can be easily proved.

Theorem 4.4. *The following condition is equivalent to condition* (6):
$\forall a, c \in R$ *with* $\mathbf{DC}(a, c)$, $\exists b \in R$ *s.t.* $\mathbf{EC}(a, b)$ *and* $\mathbf{TPPi}(b, c)$.

Similarly, we have four triads in **Tri** with a superscript $^{??}$: these are $\langle \mathbf{TPP}, \mathbf{NTPP}^{??}, \mathbf{TPP}\rangle$, $\langle \mathbf{EC}, \mathbf{DC}^{??}, \mathbf{TPPi}\rangle$ and their inverse triads.

Remark 4.2. In [24], we construct a least RCC model B_ω and shows that this model doesn't satisfy condition (5). But this model satisfies condition (6). It is still unknown whether condition (6) is true for all RCC models.

4.5. Cases of $\langle \mathbf{R}, \mathbf{T}^\times, \mathbf{S}\rangle$

In this subsection, we need to prove for each RCC model R the following

Theorem 4.5. *For each triad* $\langle \mathbf{R}, \mathbf{T}, \mathbf{S}\rangle$ *in* **Tri**, *if* $\langle \mathbf{R}, \mathbf{T}^\times, \mathbf{S}\rangle$, *then there exist some* $a, c \in R$ *with* $a \vee c \neq \mathrm{u}$ *such that* $\mathbf{R}(a, b) \wedge \mathbf{S}(b, c)$ *is not true for all* $b \in R$.

Proof. There are all together 35 triads with superscript $^\times$ and 5 of these triads are self-dual. So we need only to verify 20 triads.

Take $m, s, r \in R$ with $m \ll s \ll r \neq \mathrm{u}$, then m is a hole of $s - m$ and s is a hole of r. Using these auxiliary regions, we can construct two regions a and c which provides instance for 31 triads with superscript $^\times$. The procedures are summarized in Table 5, where we listed only 18 cases, omitting the corresponding inverse triads.

Similarly, taking $o, p, q \in R$ such that $\mathbf{DC}(o, p)$, $\mathbf{DC}(p, q)$, $\mathbf{DC}(o, q)$, we set $a = o \vee p$ and $c = p \vee q$, then $\mathbf{PO}(a, c)$. This pair of regions provides instance for the rest 4 triads with superscript $^\times$. The procedures are summarized in Table 6, where we listed only 2 cases, omitting the corresponding inverse triads. $\square$

Above theorem gives a complete list of cases in which the extensionality conditions (conditions (1) and (2)) are violated. This result also holds for more general situations.

Suppose R is an RCC model and U is a subdomain of R. With the natural interpreting mapping v, U is also a model of the RCC8 **CT**.

In what follows, we give some necessary conditions for U to be extensional. Since each extensional model is also consistent, we assume U is a consistent model of the RCC8 **CT**.

Table 5
Non-extensional composition triads: hole cases

$\langle$**DC**, **EC**$^{\times}$, **EC**$\rangle$	$a = r - m$	$c = m$	**EC**(a, c)
$\langle$**DC**, **PO**$^{\times}$, **EC**$\rangle$	$a = (r - s) \vee m$	$c = s$	**PO**(a, c)
$\langle$**DC**, **TPP**$^{\times}$, **EC**$\rangle$	$a = r - m$	$c = r$	**TPP**(a, c)
$\langle$**DC**, **EC**$^{\times}$, **TPP**$\rangle$	$a = r - m$	$c = m$	**EC**(a, c)
$\langle$**DC**, **PO**$^{\times}$, **TPP**$\rangle$	$a = (r - s) \vee m$	$c = s$	**PO**(a, c)
$\langle$**DC**, **TPP**$^{\times}$, **TPP**$\rangle$	$a = r - m$	$c = r$	**TPP**(a, c)
$\langle$**EC**, **EC**$^{\times}$, **EC**$\rangle$	$a = r - m$	$c = m$	**EC**(a, c)
$\langle$**EC**, **PO**$^{\times}$, **EC**$\rangle$	$a = (r - s) \vee m$	$c = s$	**PO**(a, c)
$\langle$**EC**, **TPP**$^{\times}$, **TPP**$\rangle$	$a = r - m$	$c = r$	**TPP**(a, c)
$\langle$**EC**, **TPP**$^{\times}$, **NTPP**$\rangle$	$a = m$	$c = (r - s) \vee m$	**TPP**(a, c)
$\langle$**TPP**, **EC**$^{\times}$, **TPPi**$\rangle$	$a = m$	$c = r - m$	**EC**(a, c)
$\langle$**TPP**, **EC**$^{\times}$, **NTPPi**$\rangle$	$a = m$	$c = r - m$	**EC**(a, c)
$\langle$**TPP**, **PO**$^{\times}$, **TPPi**$\rangle$	$a = s$	$c = (r - s) \vee m$	**PO**(a, c)
$\langle$**TPP**, **PO**$^{\times}$, **NTPPi**$\rangle$	$a = s$	$c = (r - s) \vee m$	**PO**(a, c)
$\langle$**TPP**, **TPPi**$^{\times}$, **NTPPi**$\rangle$	$a = r$	$c = r - s$	**TPPi**(a, c)
$\langle$**TPPi**, **PO**$^{\times}$, **TPP**$\rangle$	$a = (r - s) \vee m$	$c = s$	**PO**(a, c)
$\langle$**TPPi**, **PO**$^{\times}$, **NTPP**$\rangle$	$a = s$	$c = (r - s) \vee m$	**PO**(a, c)
$\langle$**TPPi**, **TPP**$^{\times}$, **NTPP**$\rangle$	$a = r - s$	$c = (r - s) \vee m$	**TPP**(a, c)

m, s, r are three regions taken form R with $m \ll s \ll r \neq$ u.

Table 6
Non-extensional composition triads: union cases

$\langle$**EC**, **PO**$^{\times}$, **TPP**$\rangle$	$a = o \vee p$	$c = p \vee q$	**PO**(a, c)
$\langle$**EC**, **PO**$^{\times}$, **NTPP**$\rangle$	$a = o \vee p$	$c = p \vee q$	**PO**(a, c)

o, p, q are three pairwise disconnected regions taken from R.

Let a, b be two regions in U. Suppose $a \vee b \neq \top$. Recall that a is a hole of b if $\mathbf{EC}(a, b)$ and for all $c \in U$ with $\mathbf{EC}(a, c)$, we have $\mathbf{O}(b, c)$. We also write $\mathbf{H}$ for this relation and call it the Hole relation on U. Note this relation may be empty for some models, e.g., those given in Example 3.1 and Example 3.2.

Proposition 4.2. *If there exist $a, c \in U$ such that $\mathbf{H}(c, a)$, namely the Hole relation $\mathbf{H}$ is not empty, then U is not extensional.*

Proof. Since U is consistent, we have $\mathbf{EC} \in CT(\mathbf{DC}, \mathbf{EC})$. By $\mathbf{H}(c, a)$, we have $\mathbf{EC}(a, c)$. If b is a region in U with $\mathbf{EC}(b, c)$, then b overlaps with a by definition. Therefore U is not extensional. □

This fact shows that, to achieve an extensional interpretation, it would be not enough by simply "removing the universal region of the possible referents of the region constants": Regions with holes (in the same domain) should also be removed. Therefore Bennett's conjecture does not hold.

Proposition 4.3. *If there exist $o, p, q \in U$ such that o, p, q are pairwise disjoint and $o \vee p$, $p \vee q$ are also in U, then U is not extensional.*

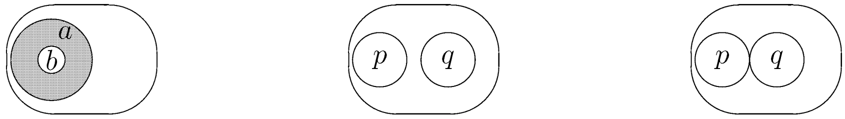


Fig. 5. Disallowed regions in an extensional model. (1) a has a hole b. (2) a has two disconnected components p, q. (3) a has two externally connected components p, q.

Proof. Since U is consistent, we have $\mathbf{PO} \in CT(\mathbf{EC}, \mathbf{TPP})$. Set $a = o \vee p$, $c = p \vee q$, then $\mathbf{PO}(a, c)$. For any region b in U with $\mathbf{TPP}(b, c)$, if $b \wedge a = \perp$, then $b \leqslant q$, hence $\mathbf{DC}(a, b)$. Therefore U is not extensional. □

This fact shows that, in an extensional model, the binary operation sum is normally disallowed.

We now consider a special case.

Example 4.3. Suppose R is the standard RCC model constructed from the real plane $\mathbb{R}^2$, namely R contains all non-empty regular closed sets on $\mathbb{R}^2$ and two regions A, B are said connected if they have non-empty intersection. Let U be a subset of R which contains all closed disks as well as other regions. Then if U is an extensional model, it cannot contain regions depicted in Fig. 5.

Case (1) is clear by Proposition 4.2. For case (2), take o as a closed disk such that $\mathbf{TPP}(q, o) \wedge \mathbf{DC}(p, o)$, then $\mathbf{PO}(o, a)$ and there is no region b in U which satisfies $\mathbf{EC}(o, b) \wedge \mathbf{NTPP}(b, a)$. Note that $\mathbf{PO} \in CT(\mathbf{EC}, \mathbf{NTPP})$, this contradicts with the assumption that U is extensional. For case (3), take o as a closed disk such that $\mathbf{TPP}(q, o) \wedge \mathbf{EC}(p, o)$, then, similar to case (2), we also have a contradiction.

This example suggests that regions which is composed of two discrete components are possibly disallowed in an extensional model.

5. Conclusions

This paper is mainly about the consistency-based composition table (RCC8 **CT**) of the RCC. We first show each RCC model is a consistent model of the RCC8 **CT**. Then after an exhaustive analysis we show that no RCC model can be interpreted extensionally anyway and hence give a negative answer to a conjecture raised by Bennett [7]. All these results are given in an 'extensional' RCC8 composition table, where we attach to each cell entry in the RCC8 **CT** a superscript to indicate in what circumstances an extensional interpretation is possible. There are all together 178 cell entries in this 'extensional' RCC8 **CT**. If R, which is an RCC model, satisfies moreover the Interpolation Condition (see condition (3), Section 4.1), then there are 143 (about 80%) cell entries that can be interpreted extensionally. But for the rest 35 entries, extensional interpretations are impossible for any RCC model.

The results obtained in the present paper suggest that, to get an extensional model of the RCC8 composition table, the domain of possible regions must be restricted greatly. In particular, regions with holes (in the same domain) are disallowed. As a consequence, to achieve an extensional interpretation, it would be not enough by simply "removing the universal region of the possible referents of the region constants". Therefore Bennett's conjecture is negatively answered.

We make some explanation here. Suppose U is an extensional model obtained by restricting the domain of regions from some RCC model R, then U is also consistent and **EC** is in particular an entry in the cell specified by the ordered pair $\langle \mathbf{DC}, \mathbf{EC} \rangle$ in the RCC8 **CT**. Suppose U also contains two regions a, c such that c is a hole of a, then there will be no region b in U with $\mathbf{DC}(a, b) \wedge \mathbf{EC}(b, c)$. Note that a is externally connected to c by the definition of the Hole relation. This will contradict the extensionality of the model U.

Proposition 4.3 shows that, in such an extensional model, the sum operation is normally disallowed. Moreover, the last example (Example 4.3) suggests that regions which has two discrete components are possibly disallowed.

As a result of above facts, the resulting domain of such an extensional model would be more homogeneous and more similar to that of Egenhofer [16] and careful study will be given later.

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