Qualitative Spatial Reasoning about Cardinal Directions[1]

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Abstract

Spatial reasoning is very important for cartography and GIS. Most known methods translate a spatial problem to an analytical formulation in order to solve it quantitatively. This paper shows a method for formal, qualitative reasoning about cardinal directions. The problem addressed is how to deduce the direction from A to C, given the direction from A to B and B to C. It first analyzes properties that a formal cardinal direction system should have. It then constructs an algebra with the direction symbols (e.g. N, E, S, and W) and a combination operation which connects two directions. Two examples for such algebras are given, one formalizing the well-known triangular concept of directions (here called cone-shaped directions) and a projection-based concept. Completing the algebra with an identity element to represent the direction from a point to itself simplifies reasoning and permits one to deduce a direction for any combination of two direction values. The results of the deductions for the two systems essentially agree, but the projection bases system produces more 'euclidian exact' results, in a sense defined in the paper.

1. Introduction

Humans reason in various ways about space and spatial properties. The most common examples are navigational tasks in which the problem is to find a route between a given starting point and an end point, and to determine the route with certain properties. Many other examples, such as decisions about the location of a resource, which translates into a mundane household question like "where should the telephone be placed?", or the major problem of locating a nuclear waste facility. Military applications frequently use spatial reasoning for terrain analysis, route selection in terrain etc. (Piazza and Pessaro 1990). Indeed, spatial reasoning is so intuitive that it is often not recognized as a special case of reasoning.

Spatial reasoning is a major requirement for a comprehensive GIS and several research efforts are underway to address this need (Abler 1987, p. 306, NCGIA 1989, p. 125). It is important that a GIS can carry out spatial tasks, including specific inferences based on spatial properties, in a manner similar to a human expert and that there are capabilities that explain the conclusion to users in terms they can follow (Try and Benton 1988, p. 10). In current GIS systems, such spatial reasoning tasks are most often formalized by translating the situation to Euclidian geometry, using then an analytical treatment for finding a solution. This is admittedly not an appropriate model for human reasoning (Kuipers 1978, p.143) and thus does not lead to acceptable explanations, but euclidian geometry is a convenient and sometimes the only known model of space available for rigorous analytical approaches. A similar problem was found in

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physics, where the well known equations from textbooks were not usable to build expert systems and a formalization of the physical laws we use in our everyday lives, so called 'naive physics' was started (Hayes 1985) using more qualitative than quantitative approaches (Hobbs and Moore 1985, Weld and de Kleer 1990).

This paper addresses a small subset of spatial reasoning, namely qualitative reasoning with cardinal directions between point-like objects. We assume a 2-dimensional space and exclude radial reference frames, as e.g. customary in Hawaii (Bier 1976). We want to establish rules for inference from a set of directional data about some points to conclude other directional relations between these. This follows (McDermott and Davis 1984, p. 107) in assuming that such basic capabilities are necessary for solving the more complex spatial reasoning problems. A previous paper with the terms 'qualitative reasoning' in its title (Dutta 1990) is essentially based on analytical geometry. In contrast, our treatment is entirely qualitative and we use euclidian geometry only as a source of intuition in section 4 to determine desirable properties of the reasoning with cardinal directions.

The problem described in practical terms, is the following: In an unknown country, one is informed that the inhabitants use 4 cardinal directions, by the names of 'al' 'bes' 'cel' and 'des', equally spaced around the compass in this order. One also receives information of the type

> Town Alix is al of Beta, Celag is cel of Diton, Beta is des of Diton, Efag is cel of Beta etc.

We show how one can assert that this is sufficient information to conclude that Alix is al of Efag.

Our concern is different from (Peuquet and Zhan 1987), where 'an algorithm to determine the directional relationship between arbitrarily-shaped polygons in the plane' is given. They start with two descriptions of the shape of an object given in coordinate space and determine the directional relationship (we say the cardinal direction) between the two objects. Instead we are concerned with given cardinal directions between a set of objects and determine what rules of inference are used to deduce other cardinal directions.

This work is part of a larger effort to understand how we describe and reason about space and spatial situations. Within the research initiative 2, 'Languages of Spatial Relations' of the NCGIA (NCGIA 1989) a need for multiple formal descriptions of spatial reasoning—both quantitative-analytical and qualitative—became evident (Frank 1990, Frank and Mark 1991, Mark and Frank 1990, Mark et al. 1989). Terence Smith presented some simple examples during the specialist meeting .

> "The direction relation NORTH. From the transitive property of NORTH one can conclude that if A is NORTH of B and B is NORTH of C then A must be NORTH of C as well (Mark et al. 1989)"

The organization of the paper is as follows. First we discuses a number of previous approaches to spatial reasoning, most of them using analytical geometry. In section 3 we introduce the concept of qualitative reasoning and relate it to spatial reasoning using analytical geometry; we define 'euclidian exact' qualitative reasoning based on a homomorphism. In the following section, we list the desirable properties of cardinal directions, introduce the concept of an 'identity' direction and show how the desirable properties are in contradiction. In section 5 we discuss two different systems for reasoning with directions and compare them, one based on cone shaped cardinal directions, the other based on projections and we observe that they allow essentially the same inferences (the projection-based system is slightly more powerful). We conclude the paper with some suggestions for future research.

2. Previous Approaches

A standard approach to modelling human spatial reasoning is to use Euclidian geometry in the plane or 3-dimensional space and represent the task using analytical geometry formulae. Many problems can be expressed as an optimization problem with a set of constraints, such as location of a resource and shortest path.

Similarly, the important field of geographic reference frames in natural language (Mark, Svorou, and Zubin 1987) has mostly been treated using an analytical geometry approach. Typically, spatial positions are expressed relative to positions of other objects. Examples occur in everyday speech in forms like "the church is west of the restaurant". In the past these descriptions were translated into Cartesian coordinate space and the mathematical formulations analyzed. A special problem is posed by the inherent uncertainties in these descriptions and the translation of uncertainty into an analytical format. (McDermott and Davis 1984) introduced a method using 'fuzz' and in (Dutta 1988, Dutta 1990) fuzzy logic (Zadeh 1974) is used to combine such approximately metric data.

An approach that is entirely qualitative, and thus similar to the thrust in this paper is the work on symbolic projections. It translates exact metric information (primarily about objects in pictures) in a qualitative form (Chang, Jungert, and Li 1990, Chang, Shi, and Yan 1987). The order in which object appear, projected vertically and horizontally, is encoded in two strings and spatial reasoning, especially spatial queries, are executed as fast substring searches (Chang et al. 1988).

3. Qualitative approach

3.1. Qualitative reasoning

This paper gives a set of qualitative deduction rules for a subset of spatial reasoning, namely reasoning with cardinal directions. We do not rely on quantitative calculations (e.g. square roots, trigonometric functions) or analytical geometry. In qualitative reasoning a situation is characterized by variables which 'can only take a small, predetermined number of values' (de Kleer and Brown 1985, p. 116) and the inference rules use these values and not numerical quantities approximating them. It is clear that the qualitative approach loses some information, but this may simplify reasoning. We assume that a set of propositions about the relative positions of objects in a plane is given and we have to deduce other spatial relationships. This is the same problem definition as in (Dutta 1990, p. 351)

"Given: A set of objects (landmarks) and
A set of constraints on these objects.
Find: The induced spatial constraints".

The relations we are interested in are the directions, expressed as symbols representing the cardinal direction.

Without debating whether human reasoning follows the structure of propositional logic , we understand that there is some evidence that human thinking is at least partially symbolic and qualitative (Kosslyn 1980, Lakoff 1987, Pylyshyn 1981). Formal, qualitative spatial reasoning is crucial for the design of flexible methods to represent spatial knowledge in GIS and for constructing usable GIS expert systems (Buisson 1990, McDermott and Davis 1984). Spatial knowledge is currently seldom included in expert systems and is considered 'difficult' (Bobrow, Mittal, and Stefik 1986, p.887).

In terms of the example given in the introduction, the following chain of reasoning deduces a direction from Alix to Efag:

1. Use 'Alix is al of Beta' and 'Efag is cel of Beta', two statements which establish a sequence of directions Alix - Beta - Efag.
2. Deduce 'Beta is al of Efag' from 'Efag is cel of Beta'
3. Use a concept of transitivity: 'Alix is al of Beta' and 'Beta is al of Efag' thus conclude 'Alix is al of Efag'.

This paper formalizes such rules and make them available for inclusion in an expert system. It also explores how such rule based reasoning agrees with analytical geometry and discover contradiction between the different desirable properties of such rules.

3.2. Advantage of qualitative reasoning

A qualitative approach uses less precise data and therefore yields less precise results than the quantitative one. This is highly desirable (Kuipers 1983, NCGIA 1989, p. 126), because

- precision is not always desirable, and
- precise, quantitative data is not always available.

Qualitative reasoning has the advantage that it can deal with imprecise data and need not translate it to a quantitative form. Verbal descriptions are typically not metrically precise, but are sufficient for a task. Imprecise descriptions are necessary in query languages where one specifies some property that the requested data should have, for example a building about 3 miles from town. It is difficult to show this as a figure, because the figure does necessarily overspecify or is then very complex. Qualitative reasoning can also be used for query simplification to transform a query from the form it is posed to another, equivalent one, that is easier to execute.

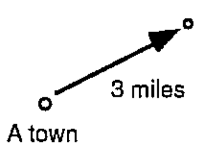

Figure 1: Overspecific visualization

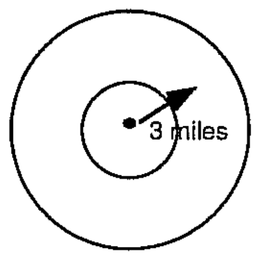

Figure 2: Complex visualization

In other cases, the available data is in qualitative form, most often text documents. For example (Tobler and Wineberg 1971) tried to reconstruct spatial locations of historic places from scant descriptions in a few documents. Verbal information about locations of places can leave certain aspects imprecise and we should be able to simulate the way humans deduce information from such descriptions (for example in order to automatically analyze description of location in natural science collections (McGranaghan 1988, McGranaghan 1989a, McGranaghan 1989b).

3.3. Exact and approximately reasoning

We compare the result of a qualitative reasoning rule with the result we obtain by translating the data to analytical geometry and applying the equivalent functions to them. If the results are always the same, i.e. if we have a homomorphism, we call the qualitative rule **euclidian exact.** If the qualitative rule produces results, at least for some data values, which are different from the ones obtained from analytical geometry, we call it **euclidian approximate.**

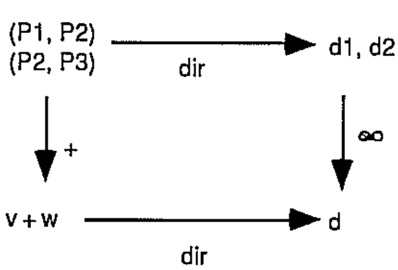

Figure 3: Homomorphism

This is a general definition, which applies to the operation to combine two directions and deduce the direction of the resultant (introduced in 4.3, see figure 5). We establish a mapping from analytical geometry to symbolic directions using a function dir (P1, P2), which maps from a pair of points in euclidian space to a symbolic direction (e.g. west). Vector addition, with the regular properties is carried to symbolic combination ∞.

DEFINITION: a rule for qualitative reasoning on directions is called **euclidian exact** (or short exact) if dir (P_1, P_2) is a homomorphism.

$$\mathrm{dir}(P_1,P_2) \infty \mathrm{dir}\ (P_2, P_3) = \mathrm{dir}\ ((P_1, P_2) + (P_2, P_3))$$

3.4. Formalism used

Our method is algebraic and thus different from others that concentrated on relations. The objects we operate on are the direction symbols, e.g. S for south, E for west, not the points in the plane. Arguments involving pairs of points, representing line segments between them, are to justify the desirable properties we list only.

For the formalism we generally use an algebra oriented style, consistent with the method of study. An algebra consists of

- a set of symbols D, called the domain of the algebra - comparable to the concept of data type in computer programming languages (e.g. D = {N, E, W, S})
- a set of operations over D, comparable to functions in a computer program (primarily operations to reverse and to combine directions), and
- a set of axioms that set forth the basic rules explaining what the operations do (Gill 1976, p. 94).

Specifically, we write (P_1, P_2) for the line segment from P_1 to P_2, and dir (P_1, P_2) = d_1 for the operation that determines the direction between two points P_1 and P_2, with d_1 the direction from P_1 to P_2 expressed as one of the cardinal direction symbols. We then define operations on the cardinal direction symbols.

4. General properties of directions between points

We are interested in two types of operations applicable to direction:

- the reversing of the order of the points and thus the direction of the line segment, and
- the combination of two directions between two pairs of consecutive points.

Using geometric figures and conclusions from manipulations of line segments, we can deduce properties of these two operations. These properties then form the base for the qualitative reasoning systems defined in the next two sections, where we will not use arguments based on euclidian geometry or line segments. The rules found in this section will be expressed without reference to points or line segments, only in terms of direction symbols and rules for their manipulation and are defining manipulations of directional symbols without reference to points or euclidian geometry.

One can formally show that all the desirable properties form a contradictory system and not all of them can be fulfilled at once. As we will see in the following section, this does not affect practical deductions much, but requires further research.

4.1. Cardinal directions

We define direction as a function between two points in the plane that maps to a symbolic direction:

dir: p x p -> D, with p the points of a plane and D the directional symbols.

The symbols available for describing the direction are given as a set; they depend on the specific system of directions used, e.g. {N, E, S, W} or more extensive {N, NE, E, SE, S, SW, W, NW}.

In the literature, it is often assumed that the two points must not be the same, but we introduce a special symbol, which means 'two points to close that a meaningful direction can be determined' and call it an identity element. This makes the function total (i.e. it has a result for all values of its arguments) and greatly enhance the deductive power of the system.

for all P: dir (P, P) = 0.

4.2. Reversing direction

Cardinal directions depend on the order in which one travels from one point to the other. If a direction is given for a line segment between points P_1 and P_2, we need to be able to deduce the direction from P_2 to P_1. Already (Peuquet and Zhan 1987) and (Freeman 1975) stressed the importance of this operation: "Each direction is coupled with a semantic inverse". We call this 'inverse' (this name will be justified in 4.3.5) and write 'inv' .

inv: d -> d such that inv (dir (P_1, P_2)) = dir (P_2, P_1)

An we ask that the inversing twice is the identity function

inv (inv (d)) = d because inv (inv (P1,P2)) = inv (P_2, P_1) = (P1,P2).

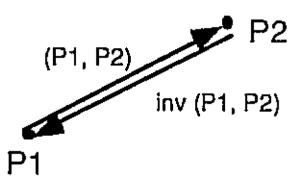

Figure 4: Inverse

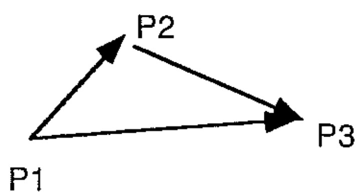

Figure 5: Combination

4.3. Combination

Two directions between two contiguous line segments can be combined in a single one. We define an operation with the following properties:

4.3.1. Definition

The combination operation is defined, such that the end point of the first direction is the start point of the second one.

comb : d x d -> d , always written in infix format: $d_1 \infty d_2 = d_3$

with the meaning:

dir (P_1,P_2) ∞ dir (P_2, P_3) = dir (P_1, P_3)

4.3.2. Associativity

Combinations of more than two directions should be independent of the order in which they are combined (**associative** law) and we need not use parenthesis:

$$a \infty (b \infty c) = (a \infty b) \infty c = a \infty b \infty c \text{ (associative law)}$$

This rule follows immediately from a figure or from the definition of combination:

$$\text{dir}(P_1, P_2) \infty (\text{dir}(P_2, P_3) \infty \text{dir}(P_3, P_4)) =$$
$$\text{dir}(P_1, P_2) \infty \text{dir}(P_2, P_4) = \text{dir}(P_1, P_4)$$
$$(\text{dir}(P_1, P_2) \infty \text{dir}(P_2, P_3)) \infty \text{dir}(P_3, P_4) =$$
$$\text{dir}(P_1, P_3) \infty \text{dir}(P_3, P_4) = \text{dir}(P_1, P_4)$$

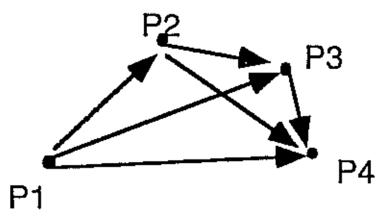

Figure 6: Associativity

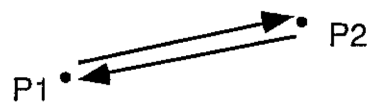

Figure 7: Inverse

4.3.3. Identity

Adding the direction from a point to itself $d(P_1, P_1)$ to any other direction should not change it. We call this direction identity element and postulate

$$d \infty 0 = 0 \infty d = d \text{ for any } d.$$

This follows from the definition of combination:

$$\text{dir}(P1, P2) \infty \text{dir}(P2, P2) = \text{dir}(P1, P_2)$$
$$\text{dir}(P_1, P_1) \infty \text{dir}(P_1, P_2) = \text{dir}(P_1, P_2).$$

One could argue that natural language uses an 'empty' symbol to indicate the direction from a point to itself or the more general case that two points are too close for a meaningful determination of a direction within their frame of reference. For formal reasoning it is more convenient to introduce an explicit symbol 0 for identity.

4.3.4. Algebraic definition of inverse

In algebra, an inverse to a binary operation is defined, such that a value, combined with its inverse results in the identity value.

From Figure 7 follows that this is just the inverses of the given line segment:

$$\text{dir}(P_1, P_2) \infty \text{dir}(P_2, P_1) = \text{dir}(P_1, P_1)$$

In case that two line segments are selected as in figure 8, such that

$$\text{dir}(P_1, P_2) = d_1 \quad \text{and dir}(P_2, P_3) = d_2 = \text{inv}(d_1)$$

computing the combination

$$\text{dir}(P_1, P_2) \infty \text{dir}(P_2, P_3) = d_1 \infty \text{inv}(d_1) = 0$$

is an approximation and not euclidian exact. The degree of error depends on the definition of 0 used and the difference in the size of the line segments - if they are the same, the inference rule is exact.

This represents a type of reasoning like Baltimore is east of San Francisco, San Francisco is west of Washington DC, thus the direction from Baltimore to Washington is 'too close' (but different from 'we cannot make a determination').

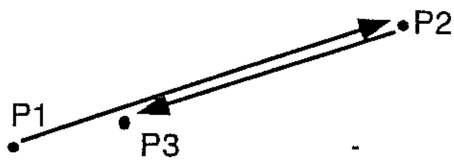

Figure 8: d ∞ inv (d)

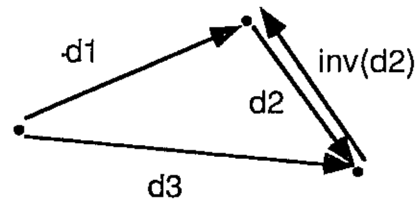

Figure 9: $d_1 = d_3$ ∞ inv (d_2)

4.4. Group properties for algebra of direction with combination

The algebra over the directions between pairs of points in a 2-dimensional space together with the combination ∞ form a group (if the operations are suitably completed). The necessary criteria are:

- ∞ is associative,
- a identity element exists, and
- for every element dir (P_1,P_2) we have an inverse element (dir (P_2, P_1))

The group formed by <dir(P_1,P_2); ∞> is not an Abelian group which would require commutativity, which we did not assert.

4.5. Transitivity for cardinal directions

If one combines two line segments with the same direction, one expects that the result maintains the same direction.

dir (P_1, P_2) = dir (P_2, P_3) = d then dir (P_1, P_3) = d
or short: d ∞ d = d, for any d.

4.6. Properties of cardinal directions

In summary, we found three fundamental rules for cardinal directions and the operations of inverse and combination:

- The direction between a point and itself is a special symbol, called *identity*.
- The direction between a point and another is the *inverse* of the direction between the other point and the first.
- Combining two equal directions results in the same direction (*transitivity* for directions).
- The result of combining several directions does not depend on the order (*associativity*).

We propose that a property that maps from a pair of points to a set of values (finite or infinite) which has these properties, should be called cardinal direction. The symbols for cardinal directions D and the combination operation form a group.

dir (P_1, P_1) = 0	d ∞ (d ∞ d) = (d ∞ d) ∞ d
dir (P_1, P_2) = inv (dir (P_2, P_1))	d ∞ 0 = 0 ∞ d = d
d ∞ d = d	d ∞ inv (d) = 0
inv (inv (d))= d	
Properties of direction	Group properties

4.7 Contradiction in rules

The desired properties lead, unfortunately, to a contradiction and no algebraic system can fulfill all of them at once. The problem results from the demand for associativity and transitivity:

d ∞ (d ∞ inv (d)) = d ∞ 0 = d   but
(d ∞ d) ∞ inv (d) = d ∞ inv (d) = 0.

Formalizing concepts from natural language, one should not be surprised to find formal inconsistencies. The problem remaining to be solved is to find a method to cope with these differences between formal and natural language reasoning.

5. Cardinal directions as cones

The most often used, prototypical concept of cardinal directions is related to the angular direction between the observer's position and a destination point. This direction is rounded to the next established cardinal direction. The compass is usually divided in 4 major cardinal directions, often with subdivisions for a total of 8 or more directions (Svorou 1988). This results in cone shaped areas for which a symbolic direction is applicable. We limit the investigation here to the case of 4 and 8 directions. This model of cardinal directions has the property that 'the area of acceptance for any given direction increases with distance' (Peuquet and Zhan 1987, p. 66) (with additional references) and is sometimes called 'triangular'.

5.1. Definitions with 4 directional symbols

We define 4 cardinal directions as cones, such that for every line segment, exactly one direction from the set of North, East, South or West applies.

for all P_1, P_2 ($P_1 \neq P_2$): exist dir (P_1, P_2) = d, with d in D_4 ={N, S, E, W}.

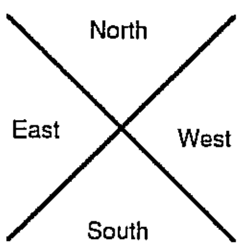

Figure 9: Cone-shaped directions

An obvious operation on these directions is a quarter-turn anti-clock-wise (mathematically positive) q, such that

q: d -> d, with q(N) = E, q(E) = S, q(S) = W, q (W) = N

and four quarter turns are an identity:

$q\ (q\ (q\ (q\ (d)))) = q^4\ (d) = d$

These quarter turns order the directional symbols in a cyclic order. Reversing a direction is equal to 2 quarter turns (or one half turn)

inv (d) = q^2 (d)

Finally, we just define the combination of two directions, such that transitivity holds

$d \infty d = d$

but every other combination remains undefined.

This definitions would fulfill the requirements for the direction except that we did not define a symbol for identity for the direction between a point and itself (see exclusion $P_1 \neq P_2$ above). This system allows hardly any inferences.

5.2. Completion with identity

Introducing an identity element, we eliminate the restriction in the input values for the direction function for all P_1, P_2 exist d : dir (P_1, P_2) = d, with d in D_5 = {N, S, E, W, 0}

In addition to the previously defined axioms, a quarter turn on the identity element 0 is 0

$q(0) = 0$

and thus

$inv(0) = 0$	from $q(q(0)) = q(0) = 0$
$d \infty 0 = 0 \infty d = d$	from group properties
$0 \infty 0 = 0$	from $d \infty d = d$.

The inverse must further have the property that a direction combined with its inverse is 0

$d \infty inv(d) = 0$

These definitions contain the previously listed ones as subset D_4. Both the set D_5 and the subset D_4 is closed under the operations 'inverse' and 'combination'.

From the total of 25 different combinations, one can only infer 13 cases exact and 4 approximate. Summarized in a table (italics indicate approximate reasoning):

	N	E	S	W	0
N	N		*0*		N
E		E		*0*	E
S	*0*		S		S
W		*0*		W	W
0	N	E	S	W	0

5.4. Directions with 8 directional symbols

One may use a set of 8 cardinal directions D_9 = {N, NE, E, SE, S, SW, W, NW, 0}, using exactly the same formulae. In lieu of a quarter turn, we define a turn of an eighth:

e (N) = NE, e (NE) = E, e (E) = SE, ... , e (NW) = N, e (0) = 0

with 8 eighth turns being the identity

$e^8(d) = d$

and inverse is now equal to 4 eighth turns

$inv(d) = e^4(d)$.

All the rules about combination of direction etc. remain the same and one can also form a subset {N, NE, E, SE, S, SW, W, NW} without 0.

An approximate averaging rule combines two directions that are each one eighth off. For example, SW combined with SE should result in S, or N combined with E should result in NE.

$e(d) \infty -e(d) = d$

with $-e(d) = e^7(d)$, or one eight turn in the other direction)

One could also assume that if two directions are combined that are just be one eights turn apart, one selects one of the two (S combined with SE results in S, N combined with NW results in NW).

$e(d) \infty d = d$ and $d \infty e(d) = d$

Human beings would probably round to the simple directions N, E, W, S, but formalizing is easier, if preference is given to the direction which is second in the turning direction. This is another rule of approximate reasoning.

This rule can then be combined with other rules, for example to yield (approximate)

$e(d) \infty inv\ d = 0$ or and $e(d) \infty e(inv(d)) = 0$

In this system, from all the 81 pairs of values (64 for the subset without 0) combinations can be inferred, but most of them only approximately. Only 24 cases (8 for the subset) can be inferred exactly, 25 result in a value of 0 and another 32 give approximate results. We can write it as a table, where italics denote euclidian approximate inferences:

	N	NE	E	SE	S	SW	W	NW	0
N	N	*NE*	*NE*	*0*	*0*	*0*	*NW*	*N*	N
NE	*NE*	NE	*E*	*E*	*0*	*0*	*0*	*N*	NE
E	*NE*	*E*	E	*SE*	*SE*	*0*	*0*	*0*	E
SE	*0*	*E*	*SE*	SE	*S*	*S*	*0*	*0*	SE
S	*0*	*0*	*SE*	*S*	S	*SW*	*SW*	*0*	S
SW	*0*	*0*	*0*	*S*	*SW*	SW	*W*	*W*	SW
W	*NW*	*0*	*0*	*0*	*SW*	*W*	W	*NW*	W
NW	*N*	*N*	*0*	*0*	*0*	*W*	*NW*	NW	NW
0	N	NE	E	SE	S	SW	W	NW	0

6. Cardinal directions defined by projections

6.1. Directions in 4 half-planes

Four directions can be defined, such that they are pair-wise opposites and each pair divides the plane into two half-planes. The direction operation assigns for each pair of points a combination of two directions, e.g. South and East for a total of 4 different directions. This is an alternative semantic for the cardinal direction, which can be related to Jackendoff's principles of centrality, necessity and typicality (Jackendoff 1983, p. 121). Peuquet pointed out, that directions defined by half-planes are related to the necessary conditions, whereas the cone-shaped directions give the typical condition (Mark et al. 1989, p. 24).

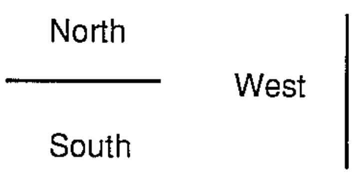

Figure 10: Two sets of half-planes

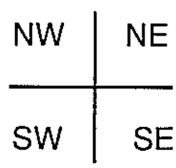

Figure 11: Directions defined by half-planes

Another justification for this type of reasoning is found in the structure geographic longitude and latitude imposes on the globe. Cone-shaped directions better represent the direction of 'going towards', whereas the 'half-planes' (or equivalent parts of the globe) better represents the relative position of points on the earth. However, the two coincide most of the time. To reach an object which is north$_{half\text{-}plane}$ on the globe one has to go north$_{cone}$.

For half-plane directions, one defines the cardinal directions as different from each other and E - W and N - S pair-wise inverse (Peuquet and Zhan 1987, p. 66). In this system, the two projections can be dealt with individually. Each of them has the exact same structure and we describe first one case separately and then show how it combines with the other.

6.1.1. One projection - 2 directions

The N-S case, considered the prototype for the two cases E-W and N-S has the following axioms:

for all P_1, P_2 ($P_1 \neq P_2$): dir_{ns} (P_1, P_2)= d_{ns} with d_{ns} in {N,S} (1)

The inverse operation is defined as

inv (N) = S, inv (S)= N

and one can easily show that the inv (inv (d)) = d rule holds.

Next we define the combination of two directions, such that transitivity holds:

for all d in {N,S}: d ∞ d = d (which is N ∞ N = N, S ∞ S = S)

With these definitions, one can derive our example from the introduction:

al (Alix, Beta) and cel (Efag, Beta) => al (Alix, Efag) because
al ∞ inv (cel)= al ∞ inv (cel) = al ∞ al = al.

6.1.2. Combinations to four directions

We now combine the two projections in N-S and E-W to form a single system, in which we have for each line segment one of 4 combinations of directions assigned.

D_4 = { NE, NW, SE, SW}

We label the projection operations by the directions they include (not the direction of the projection):

p_{ns}: d_4 -> d_{ns}, d_{ns} in {N, S}
p_{ew}: d_4-> d_{ew}, d_{ew} in {E, W}

and a composition operation

c: dns x dew -> d_9 such that c (p_{ns} (d), p_{ew} (d)) = d.

The rules for d_{ew} are the same as for d_{ns} explained above, replacing N by E and S by W:

inv (E) = W, inv (E) = W
E ∞ E = E, W ∞ W = W.

The inverse operation is defined as the inverse applied to each projection:

inv (d) = c (inv (d_{ns}), inv (d_{ew})),

which gives:

inv (NE) = SW inv (NW) = SE
inv (SE) = NW inv (SW) = NE.

One can immediately verify by substitution that

inv (inv (d)) = d.

Combination is similarly defined as combination of each projection

d_1 ∞ d_2 = c (d_{ns} (d_1) ∞ d_{ns} (d_2), d_{ew} (d_1) ∞ d_{ew} (d_2))

which respects transitivity

d ∞ d = c (d_{ns} (d) ∞ d_{ns} (d), d_{ew} (d) ∞ d_{ew} (d))
c (d_{ns} (d) , d_{ew} (d)) = d.

Combination is defined only for the four cases

NE ∞ NE = NE NW ∞ NW = NW
SE ∞ SE = SE SW ∞ SW = SW

can be determined and combinations, like

NE ∞ NW

which should approximately result in N, can not be computed. This system is not very powerful, as only 4 of the 16 combinations can be inferred.

6.2. Directions with neutral zone

We can define the directions such that points which are near to due north (or west, east, south) are not assigned a second direction, i.e. one does not decide if such a point is more east or west. This results in a division of the plane in 9 regions, a central neutral area, four regions where only one direction letter applies and 4 regions where two are used.We define for N-S three values for direction d_{ns} {N, P, S} and for the E - W direction the values d_{ew} {E, Q, W}.

NW	N	NE
W	U	E
SW	S	SE

Figure 12: Directions with neutral zone

6.2.1. Size of neutral zone

It is important to note, that there is no determination of the width of the 'neutral zone' made. Its size is effectively decided when the directional values are assigned and a decision is made that P_2 is north (not north-west or north-east) of P_1. We only assume that these decisions are consistently made. Similar arguments apply to the neutral zone of cone shaped directions, but they are not as important.

6.2.2. Tolerance geometry

Allowing a neutral zone, either for the cone-shaped or projection-based directions, introduces an aspect of 'tolerance geometry'. Strictly, whenever we assign an identity $dir(P_1, P_2) = 0$ for cases where $P_1 \neq P_2$ we violate the transitivity assumption of equality.

$$dir\ (P_1, P_2) = 0 \text{ and } dir\ (P_1, P_3) = 0 \text{ need not imply } dir\ (P_2, P_3) = 0$$

A tolerance space (Zeeman 1962) is mathematically defined as a set (in this case the points P) and a tolerance relation. The tolerance relation relates objects which are close, i.e. tol (A, B) can be read A is sufficiently close to B that we can or need not differentiate between them. A tolerance relation is similar to an equality, except that it admits small differences. It is reflexive and symmetric, but not transitive (as an equality would be)

tol (A, B)
tol (A, B) = tol (B, A)

A tolerance relation can be applied to space and geometric problems (Robert 1973).

6.2.3. One projection

Again, we use the d_{ns} projection as the prototype for the two cases:

for all P_1, P_2: $dir_{ns}\ (P_1, P_2) = d_{ns}$ with d_{ns} in {N, P, S}

The inverse operation is defined as

inv (N) = S, inv (S)= N, inv (P) = P

which obviously fulfills the inv (inv (d)) = d rule.

Next we define combination of two directions, such that transitivity holds

$d \infty d = d$, for all d in {N, S, P}
(which is $N \infty N = N$, $S \infty S = S$, $P \infty P = P$)

Combining any direction with the identity direction is resulting in the first one (that is the definition of identity - see section 4.3.4)

$d \infty P = P \infty d = d$, for all d in {N, S, P}

We also state the rule that combining a direction with its reverse results in the identity direction.

d ∞ inv (d) = inv (d) ∞ d = P, for all d in {N,S, P}

6.2.4. Combination of the two projection

The combination of the two projections is such that for each pair of points we assign a pair of direction letters

for all P_1, P_2: dir (P_1, P_2)= p,
with p in {NE, NQ, NW, PE, PW, PQ, SE, SQ, SW}

and we abbreviate NQ with N, PE with P, PQ with 0, etc. The methods for combination from section 6.1.2 apply.

The reverse operations is combined from the reverse for each projection, written as a table:

d=	NE	N	NW	E	W	0	SE	S	SW
inv(d)=	SW	S	SE	W	E	0	NW	N	NE

And last but not least, the combination operation, again defined as combination for each projection. Using the three rules we can compute the values for each combination. Written as a table (again, italics indicate approximate reasoning):

	N	NE	E	SE	S	SW	W	NW	0
N	N	N	NE	*E*	*0*	*W*	NW	*N*	N
NE	*N*	NE	*E*	*E*	*E*	*0*	*N*	*N*	NE
E	NE	E	E	*E*	SE	*S*	*0*	*N*	E
SE	*E*	*E*	*SE*	SE	*S*	*S*	*S*	*0*	SE
S	*0*	*E*	SE	S	S	S	SW	*W*	S
SW	*W*	*0*	*S*	*S*	*S*	SW	*W*	*W*	SW
W	NW	*N*	*0*	*S*	SW	*W*	W	*W*	W
NW	*N*	*N*	*N*	*0*	*W*	*W*	*N*	NW	NW
0	N	NE	E	SE	S	SW	W	NW	0

We have

- an identity element 'PQ' or 0 which denotes the direction from a point to itself:
  dir (P_1, P_2) = PQ
- an inverse operation that fulfills the requirements, and
- a combination operation which is defined for all possible pairs of directions.

The system, however, is not associative, as

(N ∞ N) ∞ S = N ∞ S = 0 but N ∞ (N ∞ S) = N ∞ N = N.

In the half-plane based system of directions with a neutral zone, we can deduce a value for all input values for combination (81 total), 56 cases are exact reasoning, not resulting in 0, 9 cases yield a value of 0, and another 16 cases are approximate.

7. Assessment

The power of the 4 direction cone-shaped and the 4 half-plane directional system are both very limited. Similarly, the 8 direction cone-shaped and the 4 projection-based directional system are comparable. Each system uses 9 directional symbols, 8 cone directions plus identity on one hand, the Cartesian product of 3 values (2 directional symbols and 1 identity symbol) for each projection on the other hand. The reasoning process in the half-plane based system uses less rules, as each projection is handled separately with only two rules. The cone-shaped system uses two additional approximate rules which are then combined with the other ones.

Both the cone-shaped and the projection-based directions without the identity element, are not powerful and the addition of an identity increased the deductive power of the system significantly. The systems with identity element allows conclusions for any pair of input values (81 different pairs) at least approximately.

Comparing we find, that the projection-based directions result in more cases of exact deductions than the cone-shaped (32 vs. 16) and that it yields more often the value 0 (25 vs. 9). Considering the actual values (other than 0) deduced, we see differences for 8 pairs, but the resulting approximate values differ by one eighth turn only. These are cases in which the cone shaped system uses the approximate rule $d \infty e(d) = d$ and the projection-based deduces each projection separately and arrives at an exact result. Not counting this insignificant difference, we observe that both system produce the same deductions.

8. Future work

There is little previous work on qualitative spatial reasoning and several different directions for work remain open.

8.1. Symbolic reasoning using distances

There is a good, mathematically based definition for distance measures expressed as real numbers. This can probably be carried over to qualitative distance expression, e.g. {Near, Far} or {Near, Intermediate and Far} and rules for symbolic combinations similar to the one listed here deduced.

8.2. Combining reasoning with distances and directions

Combining the reasoning with directions and distances can be more than just combining two orthogonal systems; there are certainly interesting interactions between them (Hernández 1990). Most of the approximate reasoning rules are based on the assumption that the distances between the points discussed are about equal.

8.3. Hierarchical systems for qualitative reasoning

A system for reasoning with distances differentiating only two or three steps of distance is quite limited. Depending on the circumstances a distance appears far or near compared to others. One could thus construct a system of hierarchically nested neighbourhoods, wherein all points are about equally spaced. Such a system can be formalized and may quite adequately explain some forms of human spatial reasoning.

8.4. Directions of extended objects

The discussion in this paper has dealt exclusively with point-like objects. This is a severe limitation and avoided the difficult problem of explaining directions between extended objects. In (Peuquet and Zhan 1987) there was an attempt to find an algorithm that gives the same result as 'visual inspection'; however, visual inspection does not yield consistent results. It might be useful to see if sound rules, like the above developed ones, may be used to resolve some of the ambiguities.

8.5. What system of qualitative reasoning do humans use?

We can also ask, which one of the systems proposed do humans use. For this, one has to see in which cases different systems produce different results and then test human subjects to see which one they employ. This may be difficult for the cone and projection-based direction system, as their deduction results are very similar. Care must be applied to control for the area of application, as we suspect that different types of problems suggest different types of spatial reasoning.

9. Conclusions

This paper introduces a system for inference rules for completely symbolic, qualitative spatial reasoning with cardinal distances. We have first stressed the need for symbolic, qualitative reasoning for spatial problems, that does not translate the problem to analytical geometry as most of the past work did. The systems investigated are capable of resolving any combination of directional inference using a few rules. Returning to our example in the introduction, we cannot only assert that Alix is al of Efag, but also that Alix is al-des from Diton and Celag al-des from Beta, etc.

We have used geometric intuition and the definition of a direction as linking two points. From this we deduced a number of desirable properties for a system to deal with cardinal directions. We use an algebraic approach and define two operations namely inverse and combination. We found several properties e.g.

- the direction from a point to itself is a special value, meaning 'too close to determine a direction'
- every direction has an inverse, namely the direction from the end point to the start point of the line segment
- the combination of two lines segments with the same direction result in a line segment with the same direction.

These rules are somewhat similar to the well known requirements for a distance function in geometry. People also assume that reasoning with directions should be associative ((a+b)+c= a+b(+c)), but this causes contradictions.

We defined the notion of 'euclidian exact' and 'euclidian approximate' as properties of a qualitative spatial reasoning system. A deduction rule is called 'euclidian exact' if it produces the same results as euclidian geometry operations would.

We then investigated two systems for cardinal directions, both fulfilling the requirements for directions. One is based on cone-shaped (or triangular) directions, the other deals with directions in two orthogonal projections. Both systems, if dealing with 4 cardinal directions are very limited and when dealing with 8 directions, still weak. The introduction of the identity element simplifies the reasoning rules in both cases and increases the power for both, cone and projection-based directional systems. The deductions in this section use only the algebraic properties and do not rely on geometric intuition or properties of line segments.

Both system yield results for all the 81 different inputs for the combination operation, but the projection-based system does more often yield an euclidian exact result than the cone-shaped one (56 vs. 24 cases). It also produces less often the value 0 (9 vs. 25 cases). The two systems do not differ substantially in their conclusions - if definite conclusions can be drawn (i.e. not the value 0). The small differences observed are due to an arbitrary choices in the rules. There are cases of approximate reasoning, where the cone-shaped system deduces the direction 0 whereas the projection-based system deduces a regular cardinal direction. This reduces the potential for extensive testing with human subjects to find out which system they use, by observing cases where the conclusion using one or the other line of reasoning differ. Both systems violate the assumption of associativity.

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