

# STAT206

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## 1. DESCRIPTIVE AND INFERENCE STATISTICS TERMINOLOGY

- (1) Statistics: Science of conducting studies to collect, organize, summarize, analyze and draw conclusions from data.
- (2) variable: attribute can be diff values
- (3) Categorical variables (Gender), Quantitative variables (age weight)
- (4) Discrete var: values that can be counted
- (5) continuous var: assume all values between any two specific values, measured
- (6) random: vals determined by chance
- (7) Data set: collection of data values
- (8) descriptive statistics: describe situation in data ( collect, organize summarize, present data)
- (9) inferential statistics: make inference from samples to population
- (10) population: all individuals under study
- (11) sample group selected
- (12) statistical hyp: claim
- (13) hypothesis testing: decision making process for evaluating a claim about population from samples

study samples of pop and use inferential stats to make inference about pop

**1.1. Freq distribution and graphs.** raw data in original form organized in some way (i.e. freq distribution). can then be presented in graphs or charts

freq. dist organizes raw data into classes and frequencies.

range of data is large (lower class limit, upper class limit, class boundaries, class width, class midpoint, open-ended distribution)

**1.2. General rules for grouped freq. dist.** 5-20 classes, class width should be odd to ensure midpoints are integers, mutually exclusive (non overlapping), continuous, exhaustive, equal in width

**1.3. general procedure.**

- (1) determine classes
  - (a) find highest (H) and lowest (L) values,
  - (b)  $\text{range} = H - L$
  - (c) number (N) of classes desired
  - (d)  $\text{width} = R/N$  (round to whole)

- (e) lowest class limit, convenient value less than or equal to the smallest data value.
- (f) add W to get lower limit of next class
- (g) class boundaries
- (2) tally data
- (3) find freq. of tallies
- (4) cumulative freq

1.4. **Histogram.** histogram uses vertical bars to rep. freq of classes  
different shapes bell shaped, uniform, j-shaped reverse j-shaped, right and left skewed, bimodal, u-shaped

1.5. **Stem and leaf plots.** uses part of data value as stem and part as leaf to form groups  
more informative because shows values  
first arrange data in order, separate into classes then plot  
look for peaks and gaps, shape of distribution, variability by looking at spread  
can put back to back to compare with other data set

1.6. **Descriptive Measures.** describe data with numerical measures  
characteristic or measure from population is a parameter, from sample data is called a statistic

Measures of central tendency (mean median, mode) **don't round until done**

1.6.1. *Average.* sum divided by total

$$\mu = \bar{X} = \frac{\sum X}{N}$$

rounding rule: final answer should be rounded to one or more decimal place than original finding mean from grouped frequency distribution

$$(1) \quad \bar{X} = \frac{\sum f \cdot X_m}{n}$$

f is frequency and  $X_m$  are midpoints of class

1.6.2. *Median.* ordered so called data array  
half smaller half larger, position  $\frac{n+1}{2}$   
when even number of values, b/w two given data values

1.6.3. *Mode.* value that appears the most frequently  
data set can have multiple or no modes

#### 1.6.4. *properties of central tendency.*

- (1) mean needs all values of data, varies less than media and mode, unique, cannot be computed for open-ended freq. distribution, highly affected by outliers
- (2) median find middle, whether data falls into upper or lower half of the distribution, can be calc for open data dist. less affected by outlier
- (3) most typical case req., easier to compute, can be computed for categorical or nominal data, not unique

## 2. VARIATION

### 2.1. **Measure of Variation.** range is simple measure of variability

**Variance** is measure of variability that uses all the data points, avg. deviation of values from mean.

**population variance**  $\sigma^2$  is avg. of squared deviations of values from population mean  $\mu$

$$(2) \quad \sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

where N represents total no. of values in pop

**sample variance** denoted by  $s^2$  sum of squared deviations of the values from sample mean  $\bar{X}$  divided by  $(n - 1)$

$$(3) \quad s^2 = \frac{\sum (X - \bar{X})^2}{n - 1}$$

where n is no. values in sample

divided by n-1 b/c unbiased estimate of  $\sigma^2$ . standard deviation is square root of variance

**population standard deviation**

$$(4) \quad \sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum (X - \bar{\mu})^2}{n - 1}}$$

sample standard deviation

$$(5) \quad s = \sqrt{s^2} = \sqrt{\frac{\sum (X - \bar{X})^2}{n - 1}}$$

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n - 1}$$

$$s^2 = \frac{\sum f \cdot X_m^2 - \frac{(\sum f X_m)^2}{n}}{n - 1}$$

### 2.2. **Note.**

- (1) value of  $s^2$  always +
- (2) sum of  $(X - \bar{X})$  is always 0
- (3) larger value of  $s^2$  or s, larger variability of data ( same with  $\sigma$  )

### 3. EMPIRICAL(NORMAL) RULE

distro. bell shaped

some % of vals fall within 1 standard deviation of mean ( $X-nS$ ,  $X+nS$ )

### 4. CHEBYSHEV'S THEOREM

in an dist regardl of shape, proportion of value that fall within  $k$  standard deviations will be at least  $1 - \frac{1}{k^2}$  where  $k > 1$

subtract average from larger value, divide diff by standard deviation to get  $k$ , use cheb to get %

### 5. MEASURE OF POSITION

standard score / z-score subtract mean from observation and divide result by standard deviation  $z = \frac{X-\bar{X}}{s}$  or  $\frac{X-\mu}{\sigma}$

number of standard deviations falls above or below mean

#### 5.1. Partition Values.

5.1.1. *percentile*. 100 equal groups

5.1.2. *decile*. 10 equal groups

5.1.3. *quartiles*. 4 equal groups

5.2. **Percentile Equations.** find the percentile of a piece of data

$$(6) \quad \text{Percentile} = \frac{\text{number of data value below } X + 0.5}{n} * 100\%$$

find the position of a percentil

$$(7) \quad c = \frac{n \cdot p}{100}$$

if  $c$  is not a whole number round to next whole number, and position is the number. if  $c$  is a whole number average  $c$  and  $c+1$ . this is position of  $X$

### 6. BOX-PLOTS

show distrivution of data

helpful for finding outliers in the data based on the min and max relative the  $Q_1$  and  $Q_3$  measurements

modified box plots dont show outliers

### 7. PRODUCT RULE

total number of possibilities are  $k_1 \cdot k_2 \cdot \dots k_n$

## 8. PERMUTATION RULE

permutation: arrangement of  $n$  objects in specific order

Rule 1: arrangement of  $n$  objects in specific order using  $r$  object of time, denoted by  $nPr$ :

$$(8) \quad nP_r = \frac{n!}{(n-r)!}$$

arrangement of  $n$  objects in specific order taking all  $n$  at a time is  $n!$

## 9. COMBINATION RULE

Combination: selection of objects with no order

the selection of  $r$  objects out of  $n$  objects can be done in  $nC_r$  ways

$$(9) \quad \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

## 10. COMBINATORICS EX

$$(10) \quad n = \frac{8!}{5! * 3!} = \frac{5! * 6 * 7 * 8}{1 * 2 * 3} = \frac{6!}{3!3!} + \frac{6!}{5!} = \frac{13}{28}$$

## 11. BAYES THEOREM

$$(11) \quad \begin{aligned} P(A \cap B) &= P(A) \cdot P(B|A) \\ P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ P(A|B) &= \frac{P(Ai) * P(D|Ai)}{P(Ai)P(D|Ai) + \dots + P(Ak)P(D|Ak)} \end{aligned}$$

## 12. PROBABILITY DISTRIBUTIONS

- (1) variable: characteristic or attribute that can assume different values
- (2) Random Variable: variable whose value is determined by chance
- (3) have requirement that  $\sum P(X) = 1$ ,  $0 \leq P(X) \leq 1$
- (4) Expectation:  $E(X) = \mu = \sum X \cdot P(X)$

## 13. THE BINOMIAL DISTRIBUTION

binomial experiment: probability experiment that satisfies:

- (1) each trial can have only two outcomes or can be reduced to success or failure
- (2) fixed # of trials
- (3) outcomes independent of each other
- (4) probability of success must remain same for each trial

$$\text{Mean} : \mu = \sum (X \cdot P(X))$$

$$\text{Variance} : \sigma^2 = \sum [X^2 \cdot P(X)] - \mu^2$$

$$\text{StandardDeviation} : \sigma = \sqrt{\sigma^2}$$

$$\text{Expectation} : E(X) = \mu = \sum (X \cdot P(X))$$

#### 14. POISSON DISTRIBUTION

discrete probability distribution that is useful in large number of trials with small success rate

$$(12) \quad P(X; \lambda) = \frac{e^{-\lambda} \lambda^x}{X!} \text{ where } X = 0, 1, 2$$

where  $\lambda = n \cdot p$  is the parameter of the poisson distribution

- (1) Mean:  $\mu = \lambda$
- (2) Variance:  $\sigma^2 = \lambda$
- (3) Standard Deviation:  $\sigma = \sqrt{\lambda}$

#### 15. POISSON PROCESS

events occur randomly in time and space

- (1) Independence: number of occurrences in disjoint intervals are independent
- (2) individuality: events occur singly  $P(\text{two or more events occur simultaneously}) = 0$
- (3) Homogeneity: events occur according to a uniform rate of intensity

if events occur with average rate of  $\lambda$  per unit of time, and  $X$  is num of events which occur in  $t$  units of time, the  $X \sim \text{Poisson}(\lambda \cdot t)$

$$(13) \quad f(x) = \frac{e^{-\lambda t} (\lambda t)^x}{x!}$$

$$(14) \quad E(x) = \lambda t$$

$$(15) \quad \text{Var}(x) = \lambda t$$

#### 16. GEOMETRIC DISTRIBUTION

Bernoulli trials completed until successful.

let  $X$  be no of independant Bernoulli trials until the first success including first success then  $X$  follow geo dist.  $X \sim \text{Geom}(p)$

$$\begin{aligned} f(x) &= p(1-p)^{x-1}, x = 1, 2, \dots \\ E(X) &= \frac{1}{p} \\ \text{Var}(X) &= \frac{1-p}{p^2} \end{aligned} \quad (16)$$

**16.1. Memoryless Property of Geometric Random Variable.** let  $X$  be a Geometric(p) R.V. and  $t_1, t_2 \in \mathbb{R}$ . then  $P(X = t_1 + t_2 \mid x = t_1) = P(X > t_2)$

buy a car and travel  $x$  kilometers. reliability does not depend on previous examples

$$LS = \frac{P(X \geq t_1 + t_2 \text{ and } x \geq t_1)}{P(X \geq t_1)} \quad (17)$$

## 17. CHEBYSHEV'S THEOREM

Let  $X$  be a R.V with  $E(X)$  and  $\text{Var}(x) = \sigma^2$  Then for any  $\epsilon > 0$

$$P(|X - E(X)| \geq \epsilon) \leq \frac{\text{Var}(x)}{\epsilon^2} \quad \forall \epsilon > 0 \quad (18)$$

$$P(|X - E(X)| < \epsilon) = 1 - P(|X - E(X)| \geq \epsilon) \geq 1 - \frac{\text{VAR}(X)}{\epsilon^2} \quad (19)$$

Let  $\epsilon = k\sigma$ , where  $k \geq 0$ ,  $\sigma > 0$  - stdeviation

$$P(|X - E(X)| < k\sigma) \geq 1 - \frac{\sigma^2}{k^2\sigma^2} \quad (20)$$

$$P(|X - E(X)| < k\sigma) \geq 1 - \frac{1}{k^2} \quad (21)$$

**17.1. Example.** The # of students who miss a friday class is a Random variable  $X$  with mean 15 and st. deviation 2. Find/estimate the probability.

$$P(9 < x < 21) = P(-6 < x - 15 < 6) = P(|X - 15| \leq 6) = 1 - \frac{1}{3^2} \quad (22)$$

**17.2. Example 2.**

$$\begin{aligned} X &\sim \text{Bin}(5, 1/2); \text{ Find } P(|X - E(X)| < 2\sigma) \\ E[X] &= n \cdot p = 5 \cdot 1/2 = 2.5 \\ \text{Var}[X] &= np(1-p) = 5 \cdot 1/2 \cdot 1/2 = 5/4 = 1.25 \\ P(|X - 2.5| < 2 \cdot \sqrt{1.25}) &\geq 1 - 1/4 = 0.75 \end{aligned} \quad (23)$$

actual probability

$$P(|x - 2.5| < 2\sqrt{1.25}) = P(-2.236 < x - 2.5 < 2.236) = P(0.264 < X < 4.37) = P(1 \leq x \leq 4) = F(4) - F(0) =$$

## 18. CONTINUOUS RANDOM VARIABLES AND DISTRIBUTIONS

function from sample space to real numbers

$$(27) \quad \text{def} : X : S \rightarrow R[a, b] \in R$$

where  $R(X)$  is continuous and individual points in  $R$  must have 0 probability

$$P(X = x) = 0 \text{ for any } x \in \mathfrak{R}$$

$$P(a \leq X \leq b) = P(a < X < b)$$

18.1. **P.** .d.f probabiility density function

for random variable  $X$ , denoted  $f(x)$ , assigns probability to  $x \in \mathfrak{R}(X)$

$$(1) \quad P(a < X < b) = \int_a^b f(x)dx, (a, b) \subseteq \mathfrak{R}(x)$$

$$(2) \quad f(x) \geq 0, \forall x \in \mathfrak{R}(x)$$

$$(3) \quad \int_{-\infty}^{\infty} f(x)dx = 1$$

18.2. **Cumulative distribution function(cdf).** cdf of continuous random variable  $X$ , denoted  $F(x)$  gives prob that  $X$  takes on value less than or equal to  $x$ .

$$(28) \quad F(x) = P(X \leq x) = P(X < x)$$

18.2.1. *Properties.*

$$(1) \quad F(-\infty) = 0$$

$$(2) \quad F(\infty) = 1$$

$$(3) \quad F(x) \text{ non decreasing}$$

18.3. **Relationship b/w pdf and cdf.**

$$(29) \quad \begin{aligned} \int_a^b f(x)dx &= P(a < X < b) \\ &= P(X < b) - P(X < a) \\ &= F(b) - F(a) \end{aligned}$$

fundamental theorem of calc  $\frac{dF(x)}{dx} = f(x)$

## 19. CONTINUOUS UNIFORM DISTRIBUTION

the probability of any subinterval of the range is proportional to the length of the interval (two sub-interv. with same length must have same probability)

$$(30) \quad \begin{aligned} f(X) &= \frac{1}{b-a}, a \leq x \leq b \\ F(x) &= \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b < x \end{cases} \end{aligned}$$



### 19.1. Mean/expected value.

$$\begin{aligned} E(X) &= \int_x x f(x) dx = \mu \\ E[g(x)] &= \int_x g(x) f(x) dx \end{aligned} \quad E[aX + bY] = aE[X] + bE[Y]$$

(31)

### 19.2. Variance.

$$\begin{aligned} \text{Var}(X) &= E[(X - E(X))^2] \\ \text{Var}[aX + bY] &= a^2 \text{Var}(X) + b^2 \text{Var}(Y) \end{aligned}$$

(32)

**19.3. Exponential Distribution.** Events occur according to a Poisson process, measure the inter arrival time s b/w events. If X is the amount of time until next event in a poisson process,  $X \sim \text{Exp}(\theta)$  where  $\theta = \frac{1}{\lambda}$

$$f(x) = (1/\theta)e^{-x/\theta} \quad F(x) = 1 - e^{-x/\theta} \quad E(X) = \theta \quad \text{Var}(X) = \theta^2$$

(33)

## 20. NORMAL DISTRIBUTION

range - to + infinity  
denote x  $X \sim N(\mu, \sigma^2)$

$$pdf = f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-1/2(\frac{x-\mu}{\sigma})^2}$$

(34)

mean and variance are parameters

- (1) linear combination of independantly normally distributed parameters is normally-distributed.
- (2)  $P(Z > z) = P(Z < -z)$  because of symmetry
- (3) probably density function for normally distributed randome variable is not integrable for anyfinite limits a,b.

$$\begin{aligned} E(X) &= \mu \\ \text{Var}(X) &= \sigma^2 \end{aligned}$$

(35)

**20.1. Central Limit Theorem.** use normal distrubtion to approximate probabilities for non-normal distributions

independant random variables: have no influence on one another's values

$$f(x, y) = P(X = x \cap Y = y) = P(X = x)P(Y = y) = f_x(x)f_y(y)$$

(36)

20.1.1. *Sum.* let  $X_1, X_2, \dots, X_n$  be independent random variables with same distribution  $E(X_i) = \mu$  and  $Var(X_i) = \sigma^2$

$$(37) \quad \text{as } n \rightarrow \infty, f_{\sum_i X_i} \text{ approaches cdf for } N(n\mu, n\sigma^2)$$

$$(38) \quad \text{cdf of } \frac{\sum_i X_i - n\mu}{\sigma\sqrt{n}} \rightarrow (0, 1)$$

20.1.2. *Average.* let  $X_1, \dots, X_n$  be independent random variables, with same distribution  $E(X_i) = \mu$  and  $Var(X_i) = \sigma^2$

as  $n \rightarrow \infty$  cdf of random variable  $\bar{X} \equiv \sum_i X_i / n$  approaches cdf  $N(\mu, \frac{\sigma^2}{n})$

cdf of random variable  $\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$  approaches cdf for  $N(0, 1)$

use it when  $n$  is large but finite.

usually need  $n \geq 30$

works for any variables unless  $\mu$  or  $\sigma^2$  do not exist

20.1.3. *Continuity Correction.* improve approximation to sum or average of discrete random variables using normal random variable.

go  $\pm 0.5$  for bar of width one as the integer value will be centered

## 21. CI FOR $\mu$ WHEN $\sigma$ KNOWN OR $n \geq 30$

**point estimate** of a parameter is a specific numeric value.

**interval estimate** : interval or range of values used to estimate the parameter

$$(39) \quad \bar{X} - z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right) < \mu < \bar{X} + z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$$

point estimate  $\pm$  maximum error  $z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$  maximum error  $z_{\alpha/2}$  critical value  $\alpha$  level of significance

21.1. **Confidence level.** probability that interval estimate will contain the parameter

21.2. **confidence interval.** specific interval estimate of a parameter determined by using data obtained from sample and by using specific confidence interval of estimate.

21.3. **Sample size.** formula for minimum sample size needed for an interval estimate of population mean.  $E$  is maximum error

$$(40) \quad n = \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

22. CI FOR  $\mu$  WHEN  $\sigma$  KNOWN OR  $n < 30$ 

$$(41) \quad \bar{X} - t_{\alpha/2} \left( \frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\alpha/2} \left( \frac{s}{\sqrt{n}} \right)$$

$t_{\alpha/2} \left( \frac{s}{\sqrt{n}} \right)$  maximum error

where  $s$  is sample standard deviation and  $t_{\alpha/2}$  are found in t-table for  $n-1$  degree of freedom

## 23. CI AND SAMPLE SIZE FOR PROPORTIONS

(1)  $p$  = population proportion

(2)  $\hat{p}$  = sample proportion or  $\hat{p} = \frac{X}{n}$   $X$  is of sample units with characteristic  $n$  sample size

23.1. **sample size for proportions.** formula for minimum sample size needed for interval estimate of population is

$$(42) \quad n = \hat{p}\hat{q} \left( \frac{z_{\alpha/2}}{E} \right)^2, \text{ E is maximum error for proportion}$$

## 24. MAXIMUM LIKELIHOOD ESTIMATOR

find derivative of the multiplication of the equation for all  $x_i$ 's; when multiple unknown parameters, take partial derivatives and solve for unknown.

## 25. INDEPENDANT SMALL SAMPLES

pooled estimator

$$(43) \quad S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$