

Exercises

- (1) If N is a nilpotent 3×3 matrix over \mathbb{C} , prove that $A = I + \frac{1}{2}N - \frac{1}{8}N^2$ satisfies $A^2 = I + N$, i.e., A is a square root of $I + N$. Use the binomial series for $(1+t)^{\frac{1}{2}}$ to obtain a similar formula for a square root of $I + N$, where N is any nilpotent $n \times n$ matrix over \mathbb{C} .
- (2) Use the above result to show that if c is a non-zero complex number and N is a nilpotent complex matrix, then $(cI + N)$ has a square root. Now use the Jordan form to prove that every non-singular complex $n \times n$ matrix has a square root.