

show:  $g_n \leq a_n$

STEPS

$$\{v : Tv = \lambda v\}$$

1. Basis  $\{v_1, v_2, \dots, v_k\}$

for  $E_n$  where  $k = g_n$

2. extend the basis to  $V$

$$B = \{v_1, v_2, \dots, v_n\} \text{ for } V$$

$$3. T : V \rightarrow V$$

$$[T]_B^B = \begin{bmatrix} \lambda I_k & A \\ 0 & B \end{bmatrix}$$

$$x_f = (a - r)^k x_B$$

$$\Rightarrow g_n \leq a_n$$

example:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$g_n = 1 \quad ; \quad a_n = 2$$

1. a calculate eigenvalues  
corresponding eigenvectors  
which gives you the  
eigenspace

6) read off the alg. and  
geom. multiplicities.

$$2. a) \quad A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

$$\chi_A(\lambda) = (\lambda - 1)(\lambda - 4) + 2$$

$$= \lambda^2 - 5\lambda + 4 + 2$$

$$= \lambda^2 - 5\lambda + 6$$

$$= (\lambda - 2)(\lambda - 3)$$

$$\lambda = 2 \text{ and } 3$$

$$a_2 = 1 \quad ; \quad a_3 = 1$$

$$A v = 2v \quad \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$\Rightarrow \quad x - y = 2x \Rightarrow x = -y$$

$$2x + 4y = 2y \Rightarrow x = -y$$

$$\begin{bmatrix} x \\ -x \end{bmatrix} = a \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$E_2 = \left\{ \begin{bmatrix} x \\ -x \end{bmatrix}, x \in \mathbb{Q} \right\}$$

$$g_2 = 1$$

can also argue diagonalizable because  
splits into linear factors (no exponents)

$$E_3 = \left\{ \begin{bmatrix} \alpha \\ -2\alpha \end{bmatrix}, \alpha \in \mathbb{Q} \right\}$$

$$g_3 = 1$$

$$P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$$

$$3. \quad f^2 = \text{id}$$

$-1$  is not an eigenvalue of  $f$

show  $f = \text{id}$

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### STEPS

a. decompose  $V$ .

$$b. \quad \ker(f + \text{id}) = \{0\}$$

$$c. \quad v = \frac{1}{2} (v + f v + v - f v)$$

$$d) \quad f = \text{id} \Leftrightarrow f v = v$$

$$\ker(f - \text{id}) = V$$

4.  $p \in K[x]$  minimum degree s.t.  
 $p(\tau) \vee = 0$

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a)  $p(x) = (x - \tau) q(x)$   
 $q(x) \in K[x]$

b)  $p(\tau) \vee = 0$  using (a)

(c) use minimality of  $p$ .

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$$T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad p(x) = x^3 + x^2$$

$$p = \sum a_i x^i, \quad a_i \in K$$

$$p(\tau) = \tau^3 + \tau^2$$



5.

$$T: C^\infty \rightarrow C^\infty$$
$$f \mapsto f'$$

$$Tf = \lambda f$$

$$Tf(x) = \lambda f(x) \quad \forall x$$

$$f'(x) = \lambda f(x)$$

$$6. a) \quad f(V_i) = V_i$$

$$\forall v \in V_i$$

$$f(v) \in V_i$$

i) Basis  $B_i$  for  $V_i$

$$\{T\}_R^R \quad V \rightarrow V$$

$$\text{where } B = \bigcup_{i=1}^r B_i$$

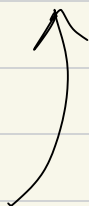
$T$  has block diagonal

$$\chi_T = \prod \chi_{T|V_i}$$

$$\forall v \in V, \quad v = v_1 + \dots + v_g$$

$$v_i \in V_i$$

use  $T$  on



b) use (a)

$$c) \quad P^{-1} A_f P = D_1$$

$$P^{-1} A_g P = D_2$$

same  $P$

$f$  and  $g$  simult. diag.

$\{v_1, v_2, \dots, v_n\}$  for  $v$

$$f : \lambda_1, \lambda_2, \dots, \lambda_n$$

$$g : \mu_1, \mu_2, \dots, \mu_n$$

$$b(g(\sum \alpha_i v_i)) =$$

$$g v_i = \mu_i v_i \quad ; \quad f v_i = \lambda_i v_i$$

same eigen vectors.  
different eigenvalues.

$$T: \mathbb{R}^\infty$$

$$T: (a_1, a_2, \dots)$$

$$\rightarrow (a_2, a_3, \dots)$$

- eigenvalues and eigenvectors?
- how does this help?

$$1. \quad T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$T(x, y) = (41x + 7y, -20x + 74y)$$

$$T = \begin{bmatrix} 41 & 7 \\ -20 & 74 \end{bmatrix}$$