Show: $g_\lambda \leq a_\lambda$

STEPS

$\{v : Tv = \lambda v\}$

1. Basis $\{v_1, v_2, \ldots, v_k\}$

for E_λ where $k = g_\lambda$

2. extend the basis to V

$B = \{v_1, v_2, \ldots, v_n\}$ for V

3. $T : V \to V$

$$[T]_B^B : \left[\begin{array}{c|c} \lambda I_k & A \\ \hline O & B \end{array}\right]$$

$$x_T = (\lambda - \lambda)^k \, x_B$$

$$\Rightarrow g_\lambda \le a_\lambda$$

example:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$g_\lambda = 1 \quad ; \quad a_\lambda = 2$$

1. a calculate eigenvalues
corresponding eigenvectors
which gives you the
eigenspace

b) read off the alg. and
geom. multiplicities.

2. a)

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

$$\chi_A(x) = (x-1)(x-4) + 2$$

$$= x^2 - 5x + 4 + 2$$

$$= x^2 - 5x + 6$$

$$= (x-2)(x-3)$$

$\lambda = 2$ and 3

$a_2 = 1$; $a_3 = 1$

$$Av = 2v \quad \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$\Rightarrow \quad x - y = 2x \Rightarrow x = -y$$

$$2x + 4y = 2y \Rightarrow x = -y$$

$$\begin{bmatrix} x \\ -x \end{bmatrix} = x \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$E_2 = \left\{ \begin{bmatrix} x \\ -x \end{bmatrix}, \ x \in \mathbb{Q} \right\}$$

$$g_2 = 1$$

can also argue diagonalizable because splits into linear factors (no exponents)

$$E_3 = \left\{ \begin{bmatrix} x \\ -2x \end{bmatrix}, \; x \in \mathbb{Q} \right\}$$

$$g_3 = 1$$

$$P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$$

3. $f^2 = id$

-1 is not an eigenvalue of f

show $f = id$

STEPS

a. decompose V.

b. $\ker(f + id) = \{0\}$

c. $v = \frac{1}{2}(v + fv + v - fv)$

d) $f = id \Leftrightarrow fv = v$

$\ker(f - id) = V$

4. $p \in K[x]$ minimum degree s.t

$p(T)v = 0$

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a) $p(x) = (x - \lambda) q(x)$

$q(x) \in K[x]$

b) $p(T)v = 0$ using (a)

(c) use minimality of P.

$T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $p(x) = x^3 + x^2$

$p(T) = T^3 + T^2$

$p = \sum a_i x^i$ $a_i \in K$

5.

$$C^\infty \longrightarrow C^\infty$$

$$T: f \to f'$$

$$Tf = \lambda f$$

$$Tf(x) = \lambda f(x) \quad \forall x$$

$$f'(x) = \lambda f(x)$$

6. a) $f(V_i) = V_i$

$\forall v \in V_i$

$f(v) \in V_i$

i) Basis B_i for V_i

$[T]_B^B \quad V \to V$

where $B = \bigcup_{i=1}^{r} B_i$

T has block diagonal

$$\chi_T = \prod \chi_{T|V_i}$$

$\forall\, v \in V,\quad v = v_1 + \dots + v_r$

$v_i \in V_i$

use T on

b) use (a)

c) $P^{-1} A_f P = D_1$

$P^{-1} A_g P = D_2$

same P

f and g simult. diag.

$\{v_1, v_2, \dots, v_n\}$ for V

$f : \lambda_1, \lambda_2, \ldots, \lambda_n$

$g : \mu_1, \mu_2, \ldots, \mu_n$

$f(g(\sum \alpha_i v_i) =$

$g v_i = \mu_i v_i \; ; \; f v_i = \lambda_i v_i$

same eigen vectors.
different eigenvalues.

$T_o \quad K^\infty$

$$T : (a_1, a_2, \dots) \to (a_2, a_3, \dots)$$

- eigenvalues and eigenvectors?

- how does this help?

1. $T: \mathbb{R}^2 \to \mathbb{R}^2$

$$T(x, y) = (41x + 7y, -20x + 74y)$$

$$T = \begin{bmatrix} 41 & 7 \\ -20 & 74 \end{bmatrix}$$