Single Choice

(1) Let $\dim V = 4$. Then, there is a $\varphi \in V^*$ with $\dim \ker \varphi = 2$

(a) True

(b) False

(2) Every finite-dimensional vector space is the dual space of another finite-dimensional vector space.

(a) True

(b) False

(3) The set of invertible real $n \times n$ matrices is

(a) not a real subspace of $\mathbb{M}_n(\mathbb{R})$

(b) a real subspace of $\mathbb{M}_n(\mathbb{R})$

(4) Let $f : V \to W$ be an arbitrary homomorphism between two K-vectorspaces. Which of the following five statements is not equivalent to the others?

(a) f is injective

(b) The dual mapping $f^* : W^* \to V^*$ is surjective

(c) The zero element of V is the only element that is mapped to the zero element of W

(d) There is a homomorphism $g : W \to V$ with $f \circ g = \mathrm{id}_W$

(e) For every $v \in V \backslash \{0\}$ there exists an $l \in W^*$ with $l(f(v)) \neq 0$

(f) All five statements are equivalent.

Multiple Choice

(1) For which values of x is the matrix $A = \begin{bmatrix} 1 & x & 1 \\ 3 & 3 & x \\ 0 & 3 & 1 \end{bmatrix}$ not invertible?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

Write it out

(1) Calculate the determinant of the matrix

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 2 & 3 & 0 \\ 0 & 2 & 3 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \end{bmatrix}$$

over $\mathbb{R}$ and $\mathbb{F}_5$. Is it invertible?

(2) (a) Let K be a field, $\lambda \in K$ and let $A \in M_{n\times n}(K)$. Show that:

i. Let B be so that $A \xrightarrow{\lambda L_i \to L_i} B$. Then, $\det B = \lambda \det A$

ii. Let B be so that $A \xrightarrow{L_i \leftrightarrow L_i} B$. Then, $\det B = -\det A$

iii. Let B be so that $A \xrightarrow{\lambda L_i + L_j \to L_j} B$ with $i \neq j$. Then, $\det B = \det A$

(b) The numbers 2014, 1484, 3710 and 6996 are all divisible by 106. Show without calculating that

$$\det \begin{bmatrix} 2 & 1 & 3 & 6 \\ 0 & 4 & 7 & 9 \\ 1 & 8 & 1 & 9 \\ 4 & 4 & 0 & 6 \end{bmatrix}$$

is also divisible by 106

Hint: Read the numbers in each column from top to bottom.

(3) Compute the determinants of the matrices,

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -1 & 2 & 0 & 1 \\ 1 & 2 & -3 & 1 \\ 0 & -4 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & -3 & 5 & 1 & 4 \\ 2 & -3 & 1 & -6 & 18 \\ 4 & -3 & 9 & 6 & 10 \\ -2 & 4 & -6 & -1 & -1 \\ -6 & 11 & -23 & -14 & 9 \end{bmatrix}$$