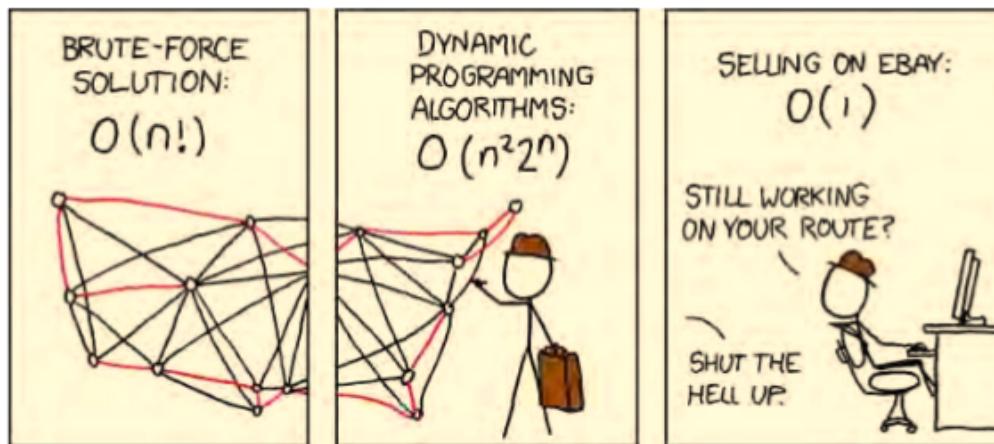


♥ Python & Algorithms 2.0 ♥

A Guide to Learn how to Fly

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*“There’s nothing to fear but the fear itself.
That’s called recursion, and that would lead you to
infinite fear.”*

Hello, human! Welcome to the second edition of my book on (flying with) Python. This new revision was written during my (amazing) time at Hacker School. It contains some improvements and some updates. You might notice a great difference in the last chapters, about graphs and trees. Hope you enjoy and have fun!

Marina, Hacker School, NYC
Summer/2014

Hello, human! Welcome to my book on Python and algorithms! If you are reading this you probably agree with me that those two can be a lot of fun together (or you might be lost, and in this case I suggest you give it a try anyway!). Also, many of the examples shown here are available in my git repository, together with several other (more advanced) examples for abstract data structures, trees, graphs, and solutions for the Euler Project and the Topcoder website. Dont forget to check them out!

This text was written purely for fun (I know, I know, this is a broad definition of the word fun...) with no pretensions for anything big, so please forgive me (or better, let me know) if you find any typo or mistake. I am not a computer scientist by formation (I am actually an almost-I-swear-it-is-close-Ph.D. in Physics) so this maybe makes things a little less usual (or risky?). I hope you have fun!

Marina, Stony Brook, NY
Summer/2013

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Part I

Get your wings! Python is a general-purpose, high-level programming language, which supports multiple programming paradigms, including object-oriented, imperative and functional programming or procedural styles. In the first part of this book, we will learn all these fancy words.

Chapter 1

Oh Hay, Numbers!

When you learn a new language, the first thing you usually do is scream `Hello World!` Because we all need to be noticed. The second thing we do is check if the math makes sense, playing around with `numbers` and arithmetic operations. Numbers can be `integers`, `float`, or `complex`. Because humans have 10 fingers, we have learned to represent numbers as `decimals`. Computers, however, are much more Hamletian. `Binary` believers have a point: why waste all these bytes if we can just state that either things `are` (`True`) or they `are not` (`False`)? In addition, since computers care about equality for extraterrestrial beings, they also let you represent things in other basis such as `hexadecimal` and `octal`.

1.1 Integers

Python represents integers (positive and negative whole numbers) using the `int` (`immutable`) type. For immutable objects, there is no difference between a variable and an *object reference*.

The size of Python's integers is limited only by the machine memory, not by a fixed number of bytes (the range depends on the C or Java compiler that Python was built with). Usually plain integers are at least 32-bit long (4 bytes)¹. To see how many bytes an integer needs to be represented, starting in Python 3.1, the `int.bit_length()` method is available:

¹To have an idea of how much this means: 1K of disk memory has 1024×8 bits = 2^{10} bytes.

```
>>> (999).bit_length()
10
```

To cast a string to an integer (in some base) or to change the base of an integer, we use `int(s, base)`:

```
>>> s = '11'
>>> d = int(s)
>>> print(d)
11
>>> b = int(s, 2)
>>> print(b)
3
```

The optional base argument must be an integer between 2 and 36 (inclusive). If the string cannot be represented as the integer in the chosen base, this method raises a `ValueError` exception. For example, this will happen if we try to obtain a binary representation with `s='12'`.

1.2 Floats

Numbers with a fractional part are represented by the (`immutable`) type `float`. When we use *single precision*, a 32-bit float is represented by: **1 bit for sign** (negative being 1, positive being 0) + **23 bits for the significant digits** (or *mantissa*) + **8 bits for the exponent**. When we use *double precision*, the mantissa has 53 bits instead. Also, the exponent is usually represented using the *biased notation*, where you add the number 127 to the original value².

Comparing Floats

We should never compare floats for equality nor subtract them. The reason for this is that floats are represented in binary fractions. There are several numbers that are exact in a decimal base but not exact in a binary base (for

²Biassing is done because exponents have to be signed values to be able to represent tiny and huge values, but the usual representation makes comparison harder. To solve this problem, the exponent is adjusted to be within an unsigned range suitable for comparison. Learn more: <http://www.doc.ic.ac.uk/~eedwards/compsys/float>

example, the decimal 0.1). Equality tests should instead be done in terms of some predefined precision. For example, we could employ the same approach as the Python's `unittest` module: `assert AlmostEqual`:

```
>>> def a(x , y, places=7):
...     return round(abs(x-y), places) == 0
```

Float numbers can also be compared by their bit patterns in memory. First we need to handle sign comparison separately: if both numbers are negative, we may compare them by flipping their signs, returning the opposite answer. Patterns with the same exponent are compared according to their mantissa.

Methods for Floats and Integers

In Python, the division operator `/` always returns a float. A `floor` division (truncation) is made with the operator `//`. A module (remainder) operation is given by the operator `%`. In addition, the method `divmod(x,y)` returns both the quotient and remainder when dividing `x` by `y`:

```
>>> divmod(45,6)
(7, 3)
```

The method `round(x, n)` returns `x` rounded to `n` integral digits if `n` is a negative `int` or returns `x` rounded to `n` decimal places if `n` is a positive `int`. The returned value has the same type as `x`:

```
>>> round(100.96,-2)
100.0
>>> round(100.96,2)
100.96
```

The method `as_integer_ratio()` gives the integer fractional representation of a float:

```
>>> 2.75.as_integer_ratio()
(11, 4)
```

1.3 Complex Numbers

The *complex data type* is an (**immutable**) type that holds a pair of floats: $z = 3 + 4j$. It has methods such as: `z.real`, `z.imag`, and `z.conjugate()`.

Complex numbers are imported from the `cmath` module, which provides complex number versions of most of the trigonometric and logarithmic functions that are in the `math` module, plus some complex number-specific functions such as: `cmath.phase()`, `cmath.polar()`, `cmath.rect()`, `cmath.pi`, and `cmath.e`.

1.4 The fraction Module

Python has the `fraction` module to deal with parts of a fraction. The following snippet shows the basics methods of this module:³

```
[general_problems/numbers/testing_floats.py]

from fractions import Fraction

def rounding_floats(number1, places):
    ''' some operations with float()'''
    return round(number1, places)

def float_to_fractions(number):
    return Fraction(*number.as_integer_ratio())

def get_denominator(number1, number2):
    a = Fraction(number1, number2)
    return a.denominator

def get_numerator(number1, number2):
    a = Fraction(number1, number2)
    return a.numerator
```

³All of the codes shown in this book are in the designed directory structure in the git repository: <https://github.com/mariwahl/Python-and-Algorithms-and-Data-Structures>. Note that the PEP 8 (Python Enhancement Proposal) guidelines recommend four spaces per level of indentation, and only spaces (no tabs). This is not evident here because of the way Latex formats the text.

```

def test_testing_floats(module_name='this module'):
    number1 = 1.25
    number2 = 1
    number3 = -1
    number4 = 5/4
    number6 = 6
    assert(rounding_floats(number1, number2) == 1.2)
    assert(rounding_floats(number1*10, number3) == 10)
    assert(float_to_fractions(number1) == number4)
    assert(get_denominator(number2, number6) == number6)
    assert(get_numerator(number2, number6) == number2)

    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_testing_floats()

```

1.5 The decimal Module

In the cases when we need **exact** decimal floating-point numbers, Python includes an additional (**immutable**) float type, the `decimal.Decimal`. The method takes an integer or a string as the argument (and starting from Python 3.1, also floats, with the `decimal.Decimal.from_float()` function). This is an efficient alternative when we do not want to deal with the rounding, equality, and subtraction problems that `floats` come with:

```

>>> sum (0.1 for i in range(10)) == 1.0
False
>>> from decimal import Decimal
>>> sum (Decimal ("0.1") for i in range(10)) == Decimal("1.0")
True

```

While the `math` and `cmath` modules are not suitable for the `decimal` module, its built-in functions, such as `decimal.Decimal.exp(x)`, are enough to most of the cases.

1.6 Other Representations

The `bin(i)` method returns the binary representation of the `int i` as a string:

```
>>> bin(999)
'0b1111100111'
```

The `hex(i)` method returns the hexadecimal representation of `i` as a string:

```
>>> hex(999)
'0x3e7'
```

The `oct(i)` method returns the octal representation of `i` as a string:

```
>>> oct(999)
'0o1747'
```

1.7 Doing Some Math

Converting Between Different Bases

We can write our own functions to change bases in numbers. The snippet bellow converts a number in any base smaller than 10 to the decimal base:

```
[general_problems/numbers/convert_to_decimal.py]

def convert_to_decimal(number, base):
    multiplier, result = 1, 0
    while number > 0:
        result += number%10*multiplier
        multiplier *= base
        number = number//10
    return result

def test_convert_to_decimal():
    number, base = 1001, 2
    assert(convert_to_decimal(number, base) == 9)
    print('Tests passed!')
```

```
if __name__ == '__main__':
    test_convert_to_decimal()
```

By swapping all the occurrences of 10 with any other base in our previous example, we can create a function that converts from a decimal number to another number ($2 \leq \text{base} \leq 10$):

```
[general_problems/numbers/convert_from_decimal.py]

def convert_from_decimal(number, base):
    multiplier, result = 1, 0
    while number > 0:
        result += number%base*multiplier
        multiplier *= 10
        number = number//base
    return result

def test_convert_from_decimal():
    number, base = 9, 2
    assert(convert_from_decimal(number, base) == 1001)
    print('Tests passed!')

if __name__ == '__main__':
    test_convert_from_decimal()
```

If the `base` is above 10 then we must use non-numeric characters to represent larger digits. We can let ‘A’ stand for 10, ‘B’ stand for 11 and so on. The following code will convert a number from a decimal base to any other base (up to 20):

```
[general_problems/numbers/convert_from_decimal_larger_bases.py]

def convert_from_decimal_larger_bases(number, base):
    strings = "0123456789ABCDEFGHIJ"
    result = ""
    while number > 0:
        digit = number%base
        result = strings[digit] + result
        number = number//base
```

```

        return result

def test_convert_from_decimal_larger_bases():
    number, base = 31, 16
    assert(convert_from_decimal_larger_bases(number, base) == '1F')
    print('Tests passed!')

if __name__ == '__main__':
    test_convert_from_decimal_larger_bases()

```

Finally, the snippet bellow is a very general base-conversion module, using a recursive method:

```

[general_problems/numbers/convert_dec_to_any_base_rec.py]

def convert_dec_to_any_base_rec(number, base):
    ''' convert an integer to a string in any base'''
    convertString = '012345679ABCDEF'
    if number < base: return convertString[number]
    else:
        return convert_dec_to_any_base_rec(number//base, base) +
               convertString[number%base]

def test_convert_dec_to_any_base_rec(module_name='this module'):
    number = 9
    base = 2
    assert(convert_dec_to_any_base_rec(number, base) == '1001')

    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_convert_dec_to_any_base_rec()

```

Greatest Common Divisor

The following module calculates the greatest common divisor (gcd) between two given integers:

```
[general_problems/numbers/finding_gcd.py]

def finding_gcd(a, b):
    ''' implements the greatest common divider algorithm '''
    while(b != 0):
        result = b
        a, b = b, a % b
    return result

def test_finding_gcd():
    number1 = 21
    number2 = 12
    assert(finding_gcd(number1, number2) == 3)
    print('Tests passed!')

if __name__ == '__main__':
    test_finding_gcd()
```

The Random Module

The follow snippet runs some tests on the Python's `random` module:

```
[general_problems/numbers/testing_random.py]

import random

def testing_random():
    ''' testing the module random'''
    values = [1, 2, 3, 4]
    print(random.choice(values))
    print(random.choice(values))
    print(random.choice(values))
    print(random.sample(values, 2))
    print(random.sample(values, 3))

    ''' shuffle in place '''
    random.shuffle(values)
```

```

print(values)

''' create random integers '''
print(random.randint(0,10))
print(random.randint(0,10))

if __name__ == '__main__':
    testing_random()

```

Fibonacci Sequences

The module bellow shows how to find the n^{th} number in a *Fibonacci sequence* in three different ways: (a) with a recursive $\mathcal{O}(2^n)$ runtime; (b) with a iterative $\mathcal{O}(n^2)$ runtime; and (c) using a formula that gives a $\mathcal{O}(1)$ runtime (but that is not precise after around the 70^{th} element):

```

[general_problems/numbers/find_fibonacci_seq.py]

import math

def find_fibonacci_seq_rec(n):
    if n < 2: return n
    return find_fibonacci_seq_rec(n - 1) + find_fibonacci_seq_rec(n - 2)

def find_fibonacci_seq_iter(n):
    if n < 2: return n
    a, b = 0, 1
    for i in range(n):
        a, b = b, a + b
    return a

def find_fibonacci_seq_form(n):
    sq5 = math.sqrt(5)
    phi = (1 + sq5) / 2
    return int(math.floor(phi ** n / sq5))

```

```
def test_find_fib():
    n = 10
    assert(fibonacci_seq_rec(n) == 55)
    assert(fibonacci_seq_iter(n) == 55)
    assert(fibonacci_seq_form(n) == 55)
    print('Tests passed!')

if __name__ == '__main__':
    test_find_fib()
```

Primes

The following program finds whether a number is a prime in three different ways: (a) using brute force; (b) rejecting all the candidates up to the square root of the number; and (c) using the Fermat's theorem with probabilistic tests:

```
[general_problems/numbers/finding_if_prime.py]

import math
import random

def finding_prime(number):
    num = abs(number)
    if num < 4 : return True
    for x in range(2, num):
        if num % x == 0:
            return False
    return True

def finding_prime_sqrt(number):
    num = abs(number)
    if num < 4 : return True
    for x in range(2, int(math.sqrt(num)) + 1):
        if number % x == 0:
            return False
    return True
```

```

def finding_prime_fermat(number):
    if number <= 102:
        for a in range(2, number):
            if pow(a, number- 1, number) != 1:
                return False
        return True
    else:
        for i in range(100):
            a = random.randint(2, number - 1)
            if pow(a, number - 1, number) != 1:
                return False
        return True

def test_finding_prime():
    number1 = 17
    number2 = 20
    assert(finding_prime(number1) == True)
    assert(finding_prime(number2) == False)
    assert(finding_prime_sqrt(number1) == True)
    assert(finding_prime_sqrt(number2) == False)
    assert(finding_prime_fermat(number1) == True)
    assert(finding_prime_fermat(number2) == False)
    print('Tests passed!')

if __name__ == '__main__':
    test_finding_prime()

```

The following program uses Python's `random` module to generate n-bit prime numbers:

```

[general_problems/numbers/generate_prime.py]

import math
import random
import sys
from finding_prime import finding_prime_sqrt

def generate_prime(number=3):

```

```

while 1:
    p = random.randint(pow(2, number-2), pow(2, number-1)-1)
    p = 2 * p + 1
    if finding_prime_sqrt(p):
        return p

if __name__ == '__main__':
    if len(sys.argv) < 2:
        print ("Usage: generate_prime.py number")
        sys.exit()
    else:
        number = int(sys.argv[1])
        print(generate_prime(number))

```

1.8 The NumPy Package

The **NumPy** package⁴ is an extension to the Python programming language, adding support for large, multi-dimensional arrays and matrices, along with a large library of high-level mathematical functions to operate on these arrays.

Arrays in NumPy can have any arbitrary dimension. They can be generated from a list or a tuple with the array-method, which transforms sequences of sequences into two dimensional arrays:

```

>>> x = np.array( ((11,12,13), (21,22,23), (31,32,33)) )
>>> print x
[[11 12 13]
 [21 22 23]
 [31 32 33]]

```

The attribute `ndim` tells us the number of dimensions of an array:

```

>>> x = np.array( ((11,12,13), (21,22,23)) )
>>> x.ndim
2

```

Some other neat examples are shown below:

⁴<http://www.numpy.org>

```
[general_problems/numbers/testing_numpy.py]

import numpy as np

def testing_numpy():
    ''' tests many features of numpy '''
    ax = np.array([1,2,3])
    ay = np.array([3,4,5])
    print(ax)
    print(ax*2)
    print(ax+10)
    print(np.sqrt(ax))
    print(np.cos(ax))
    print(ax-ay)
    print(np.where(ax<2, ax, 10))

    m = np.matrix([ax, ay, ax])
    print(m)
    print(m.T)

    grid1 = np.zeros(shape=(10,10), dtype=float)
    grid2 = np.ones(shape=(10,10), dtype=float)
    print(grid1)
    print(grid2)
    print(grid1[1]+10)
    print(grid2[:,2]*2)

if __name__ == '__main__':
    testing_numpy()
```

NumPy arrays are also much more efficient than Python's lists, as we can see in the benchmark tests below:

```
[general_problems/numbers/testing_numpy_speed.py]

import numpy
import time

def trad_version():
    t1 = time.time()
```

```
X = range(10000000)
Y = range(10000000)
Z = []
for i in range(len(X)):
    Z.append(X[i] + Y[i])
return time.time() - t1

def numpy_version():
    t1 = time.time()
    X = numpy.arange(10000000)
    Y = numpy.arange(10000000)
    Z = X + Y
    return time.time() - t1

if __name__ == '__main__':
    print(trad_version())
    print(numpy_version())

...
Results:
3.23564291
0.0714290142059
...
```


Chapter 2

Built-in Sequence Types

The next step in our studies is learning how Python represents *sequence data types*. A sequence type is defined by having the following properties:

- * **membership operator** (for example, the ability of using the keyword `in`);
- * **a size method** (given by the method `len(seq)`);
- * **slicing properties** (for example, `seq[:-1]`); and
- * **iterability** (we can iterate the data inside loops).

Python has five built-in sequence types: **strings**, **tuples**, **lists**, **byte arrays**, and **bytes**:¹

```
>>> l = []
>>> type(l)
<type 'list'>
>>> s = ''
>>> type(s)
<type 'str'>
>>> t = ()
>>> type(t)
<type 'tuple'>
>>> ba = bytearray(b'')
```

¹A **named tuple** is also available in the standard library, under the `collections` package.

```
>>> type(ba)
<type 'bytearray'>
>>> b = bytes([])
>>> type(byte)
<type 'type'>
```

Mutability

In the last chapter we learned that numbers are immutable types in Python. Now it's time to learn about objects that are *mutable*. In Python, tuple, strings, and bytes are immutable, while lists and byte arrays are mutable. Immutable types are in general more efficient than mutable. In addition, some *collection data types*² can only be indexed by immutable data types.

In Python any variable is an object reference, so copying mutable objects can be tricky. When you say $a = b$ you are actually pointing a to where b points to. For this reason, it's important to understand the concept of **deep copying**:

```
To make a deep copy of a list:
>>> newList = myList[:]
>>> newList2 = list(myList2)

To make a deep copy of a set (we will see in the next chapter),
use:
>>> people = {"Buffy", "Angel", "Giles"}
>>> slayers = people.copy()
>>> slayers.discard("Giles")
>>> slayers.remove("Angel")
>>> slayers
{'Buffy'}
>>> people
{'Giles', 'Buffy', 'Angel'}

To make a deep copy of a dict (also in the next chapter), use the
following:
>>> newDict = myDict.copy()
```

²Collection data types, such as sets and dictionaries, are reviewed in the next chapter.

```
To make a deep copy of some other object, use the copy module:  
>>> import copy  
>>> newObj = copy.copy(myObj)      # shallow copy  
>>> newObj2 = copy.deepcopy(myObj2) # deep copy
```

The Slicing Operator

In Python's sequence types, the slicing operator have the following syntax:

```
seq[start]  
seq[start:end]  
seq[start:end:step]
```

If we want to start counting from the right, we can represent the index as negative:

```
>>> word = "Let us kill some vampires!"  
>>> word[-1]  
'!'  
>>> word[-2]  
's'  
>>> word[-2:]  
's!'  
>>> word[:-2]  
'Let us kill some vampire'  
>>> word[-0]  
'L'
```

2.1 Strings

Python represents **strings**, *i.e.* a sequence of characters, using the immutable `str` type. In Python, every object has two output forms: while *string forms* are designed to be human-readable, *representational forms* are designed to produce an output that if fed to a Python interpreter, reproduces the represented object. When we write classes in Python, it is important to define their string representation.

Unicode Strings

Python's *Unicode encoding* is used to include a special characters in the string (for example, whitespace). Starting from Python 3, all strings are Unicode, not just plain bytes. To create a Unicode string, we use the 'u' prefix:

```
>>> u'Goodbye\u0020World !'
'Goodbye World !'
```

In the example above, the escape sequence indicates the Unicode character with the ordinal value 0x0020. It is also useful to remember that in general ASCII representations are given by only 8-bits while the Unicode representation needs 16-bits.

Methods for Strings

The `join(list1)` Method:

Joins all the strings in a list into one single string. While we could use + to concatenate these strings, when a large volume of data is involved, this becomes much less efficient:

```
>>> slayer = ["Buffy", "Anne", "Summers"]
>>> ".join(slayer)
'Buffy Anne Summers'
>>> "->-".join(slayer)
'Buffy->-Anne->-Summers',
>>> "".join(slayer)
'BuffyAnneSummers'
```

`join()` can also be used with the built-in `reversed()` method:

```
>>> "".join(reversed(slayer))
'SummersAnneBuffy'
```

The `rjust(width[, fillchar])` and `ljust(width[, fillchar])` Methods:

Some formation (aligning) can be obtained with the methods `rjust()` (add only at the end), `ljust()` (add only at the start):

```
>>> name = "Agent Mulder"
>>> name.rjust(50, '-')
'-----Agent Mulder'
```

The format() Method:

Used to format or add variable values to a string:

```
>>> "{0} {1}".format("I'm the One!", "I'm not")
"I'm the One! I'm not"
>>> "{who} turned {age} this year!".format(who="Buffy", age=17)
"She turned 88 this year"
>>> "The {who} was {0} last week".format(12, who="boy")
'Buffy turned 17 this year!'
```

From Python 3.1 it is possible to omit field names, in which case Python will in effect put them in for us, using numbers starting from 0. For example:

```
>>> "{} {} {}".format("Python", "can", "count")
'Python can count'
```

However, using the operator + would allow a more concise style here. This method allows three specifiers: **s** to force string form, **r** to force representational form, and **a** to force representational form but only using ASCII characters:

```
>>> import decimal
>>> "{0} {0!s} {0!r} {0!a}".format(decimal.Decimal("99.9"))
"99.9 99.9 Decimal('99.9') Decimal('99.9')"
```

String (Mapping) Unpacking

The mapping unpacking operator is ****** and it produces a key-value list suitable for passing to a function. The local variables that are currently in scope are available from the built-in **locals()** and this can be used to feed the **format()** method:

```
>>> hero = "Buffy"
>>> number = 999
```

```
>>> "Element {number} is a {hero}".format(**locals())
'Element 999 is a Buffy'
```

The `splitlines(f)` Method:

Returns the list of lines produced by splitting the string on line terminators, stripping the terminators unless `f` is True:

```
>>> slayers = "Buffy\nFaith"
>>> slayers.splitlines()
['Buffy', 'Faith']
```

The `split(t, n)` Method:

Returns a list of strings splitting at most `n` times on string `t`. If `n` is not given, it splits as many times as possible. If `t` is not given, it splits on whitespace:

```
>>> slayers = "Buffy*Slaying-Vamps*16"
>>> fields = slayers.split("*")
>>> fields
['Buffy', 'Slaying-Vamps', '16']
>>> job = fields[1].split("-")
>>> job
['Slaying', 'Vamps']
```

We can use `split()` to write our own method for erasing spaces from strings:

```
>>> def erase_space_from_string(string):
...     s1 = string.split(" ")
...     s2 = "".join(s1)
...     return s2
```

A similar method, `rsplit()`, splits the string from right to left.

The `strip('chars')` Method:

Returns a copy of the string with leading and trailing whitespace (or the characters `chars`) removed:

```
>>> slayers = "Buffy and Faith999"
>>> slayers.strip("999")
'Buffy and Faith'
```

The program below uses `strip()` to list every word and the number of the times they occur in alphabetical order for some file:³

```
[general_problems/strings/count_unique_words.py]

import string
import sys

def count_unique_word():
    words = {} # create an empty dictionary
    strip = string.whitespace + string.punctuation + string.digits
    + "\\\\""
    for filename in sys.argv[1:]:
        with open(filename) as file:
            for line in file:
                for word in line.lower().split():
                    word = word.strip(strip)
                    if len(word) > 2:
                        words[word] = words.get(word,0) +1
    for word in sorted(words):
        print("{0} occurs {1} times.".format(word, words[word]))
```

Similar methods are: `lstrip()`, which returns a copy of the string with all whitespace at the beginning of the string stripped away; and `rstrip()`, which returns a copy of the string with all whitespace at the end of the string stripped away.

The `swapcase('chars')` Method:

The `swapcase()` method returns a copy of the string with uppercase characters lowercased and lowercase characters uppercased.

```
>>> slayers = "Buffy and Faith"
```

³A similar example is shown in the Default Dictionaries section.

```
>>> slayers.swapcase()
'bUFFY AND fAITH'
```

Similar methods are:

- ★ `capitalize()` returns a copy of the string with only the first character in uppercase;
- ★ `lower()` returns a copy of the original string, but with all characters in lowercase;
- ★ `upper()` returns a copy of the original string, but with all characters in uppercase.

The `index(x)` and `find(x)` Methods:

There are two methods to find the position of a string inside another string. `index(x)` returns the index position of the substring `x`, or raises a `ValueError` exception on failure. `find(x)` returns the index position of the substring `x`, or -1 on failure:

```
>>> slayers = "Buffy and Faith"
>>> slayers.find("y")
4
>>> slayers.find("k")
-1
>>> slayers.index("k")
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ValueError: substring not found
>>> slayers.index("y")
4
```

Extensions of the previous methods are: `rfind(string)`, which returns the index within the string of the last (from the right) occurrence of ‘string’; and `rindex(string)`, which returns the index within the string of the last (from the right) occurrence of ‘string’ (causing an error if it cannot be found).

The `count(t, start, end)` Method:

Returns the number of occurrences of the string `t` in the string `s`:

```
>>> slayer = "Buffy is Buffy is Buffy"
>>> slayer.count("Buffy", 0, -1)
2
>>> slayer.count("Buffy")
3
```

The `replace(t, u, n)` Method:

Returns a copy of the string with every (or a maximum of `n` if given) occurrences of string `t` replaced with string `u`:

```
>>> slayer = "Buffy is Buffy is Buffy"
>>> slayer.replace("Buffy", "who", 2)
'who is who is Buffy'
```

2.2 Tuples

A **tuple** is an Python *immutable* sequence type consisting of values separated by commas:

```
>>> t1 = 1234, 'hello!'
>>> t1[0]
1234
>>> t1
(12345, 'hello!')
>>> t2 = t2, (1, 2, 3, 4, 5) # nested
>>> u
((1234, 'hello!'), (1, 2, 3, 4, 5))
```

Where strings have a character at every position, tuples have an object reference at each position. Therefore, it its possible to create tuples that contain mutable objects, such as lists. Empty tuples are constructed by an empty pair of parentheses. A tuple with one item is constructed by following a value with a comma (it is not sufficient to enclose a single value in parentheses):

```
>>> empty = ()
>>> t1 = 'hello',
>>> len(empty)
0
>>> len(t1)
1
>>> t1
('hello',)
```

Methods for Tuples

The `count(x)` method counts how many times `x` appears in the tuple:

```
>>> t = 1, 5, 7, 8, 9, 4, 1, 4
>>> t.count(4)
2
```

The `index(x)` method returns the index position of the element `x`:

```
>>> t = 1, 5, 7
>>> t.index(5)
1
```

Tuple Unpacking

In Python, any iterable object can be unpacked using the *sequence unpacking operator*, `*`. When used with two or more variables on the left-hand side of an assignment, one of which preceded by `*`, items are assigned to the variables, with all those left over assigned to the starred variable:

```
>>> x, *y = (1, 2, 3, 4)
>>> x
1
>>> y
[2, 3, 4]
```

Named Tuples

Python's package `collections`⁴ contains a sequence data type called *named tuple*. These objects behave just like the built-in tuple, with the same performance characteristics, but they also carry the ability to refer to items in the tuple by name as well as by index position. This allows the creation of aggregates of data items:

```
>>> import collections
>>> MonsterTuple = collections.namedtuple("Monsters", "name age
   power")
>>> MonsterTuple = ('Vampire', 230, 'immortal')
>>> MonsterTuple
('Vampire', 230, 'immortal')
```

The first argument to `collections.namedtuple` is the name of the custom tuple data type to be created. The second argument is a string of space-separated names, one for each item that the custom tuple will take. The first argument and the names in the second argument must be valid Python identifiers.

```
[general_problems/tuples/namedtuple_example.py]

from collections import namedtuple

def namedtuple_example():
    ''' show an example for named tuples
    >>> namedtuple_example()
    slayer
    '''
    sunnydale = namedtuple('name', ['job', 'age'])
    buffy = sunnydale('slayer', '17')
    print(buffy.job)

if __name__ == '__main__':
    namedtuple_example()
```

⁴We are going to explore `collections` in the following chapters.

2.3 Lists

In general, in computer science, *arrays* are a very simple data structure where elements are sequentially stored in continued memory and *linked lists* are structures where several separated nodes link to each other. Iterating over the contents of the data structure is equally efficient for both kinds, but *directly accessing* an element at a given index has $\mathcal{O}(1)$ (complexity) runtime⁵ in an array, while it has $\mathcal{O}(n)$ in a linked list with n nodes (you would have to transverse the list from the beginning). Furthermore, in a linked list, once you know where you want to insert something, *insertion* is $\mathcal{O}(1)$, no matter how many elements the list has. For arrays, an insertion would have to move all elements that are to the right of the insertion point or moving all the elements to a larger array if needed, being then $\mathcal{O}(n)$.

In Python, the closest object to an array is a **list**, which is a dynamic resizing array and it does not have anything to do with the formal concept of linked lists. Linked lists are a very important *abstract data structure* (we will see more about them in a following chapter) and it is fundamental to understand what makes them different from arrays (or Python's lists).

Lists in Python are created by comma-separated values, between square brackets. List items do not need to have all the same data type. Unlike strings which are immutable, it is possible to change individual elements of a list (lists are mutable):

```
>>> q = [2, 3]
>>> p = [1, q, 4]
>>> p[1].append("buffy")
>>> p
[1, [2, 3, 'buffy'], 4]
>>> q
[2, 3, 'buffy']
>>> q
[2, 3, 'buffy']
```

To insert items, lists perform best ($\mathcal{O}(1)$) when items are added or removed at the end, using the methods `append()` and `pop()`, respectively. The worst performance ($\mathcal{O}(n)$) occurs with operations that need to search

⁵The Big-O notation is a key to understand algorithms! We will learn more about it in the following chapters and extensively use them in our studies. For now just keep in mine that $\mathcal{O}(1)$ times $\ll \mathcal{O}(n) \ll \mathcal{O}(n^2)$, etc...

for items in the list, for example, using `remove()` or `index()`, or using `in` for membership testing.⁶

If fast searching or membership testing is required, a collection type such as a *set* or a *dictionary* may be a more suitable choice (as we will see in the next chapter). Alternatively, lists can provide fast searching if they are kept in order by being sorted (we will see searching methods that perform on $\mathcal{O}(\log n)$ for sorted sequences, particular the *binary search*, in the following chapters).

Methods for Lists

The `append(x)` Method:

Adds a new element at the end of the list. It is equivalent to `list[len(list):]= [x]:`

```
>>> people = ["Buffy", "Faith"]
>>> people.append("Giles")
>>> people
['Buffy', 'Faith', 'Giles']
>>> people[len(people):] = ["Xander"]
>>> people
['Buffy', 'Faith', 'Giles', 'Xander']
```

The `extend(c)` Method:

This method is used to extend the list by appending all the iterable items in the given list. Equivalent to `a[len(a):]=L` or using `+=`:

```
>>> people = ["Buffy", "Faith"]
>>> people.extend("Giles")
>>> people
['Buffy', 'Faith', 'G', 'i', 'l', 'e', 's']
>>> people += "Willow"
>>> people
['Buffy', 'Faith', 'G', 'i', 'l', 'e', 's', 'W', 'i', 'l', 'l',
 'o', 'w']
```

⁶This explains why `append()` is so much more efficient than `insert()`.

```
>>> people += ["Xander"]
>>> people
['Buffy', 'Faith', 'G', 'i', 'l', 'e', 's', 'W', 'i', 'l', 'l',
 'o', 'w', 'Xander']
```

The `insert(i, x)` Method:

Inserts an item at a given position `i`: the first argument is the index of the element *before which* to insert:

```
>>> people = ["Buffy", "Faith"]
>>> people.insert(1, "Xander")
>>> people
['Buffy', 'Xander', 'Faith']
```

The `remove()` Method:

Removes the first item from the list whose value is `x`. Raises a `ValueError` exception if `x` is not found:

```
>>> people = ["Buffy", "Faith"]
>>> people.remove("Buffy")
>>> people
['Faith']
>>> people.remove("Buffy")
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ValueError: list.remove(x): x not in list
```

The `pop()` Method:

Removes the item at the given position in the list, and then returns it. If no index is specified, `pop()` returns the last item in the list:

```
>>> people = ["Buffy", "Faith"]
>>> people.pop()
'Faith'
>>> people
```

```
[‘Buffy’]
```

The del Method:

Deletes the object reference, not the contend, *i.e.*, it is a way to remove an item from a list given its index instead of its value. This can also be used to remove slices from a list:

```
>>> a = [-1, 4, 5, 7, 10]
>>> del a[0]
>>> a
[4, 5, 7, 10]
>>> del a[2:3]
>>> a
[4, 5, 10]
>>> del a      # also used to delete entire variable
```

When an object reference is deleted and if no other object refers to its data, Python schedules the data item to be *garbage-collected*.⁷

The index(x) Method:

Returns the index in the list of the first item whose value is x:

```
>>> people = ["Buffy", "Faith"]
>>> people.index("Buffy")
0
```

The count(x) Method:

Returns the number of times x appears in the list:

```
>>> people = ["Buffy", "Faith", "Buffy"]
>>> people.count("Buffy")
2
```

⁷Garbage is a memory occupied by objects that are no longer referenced and garbage collection is a form of automatic memory management, freeing the memory occupied by the garbage.

The sort() Method:

Sorts the items of the list, in place:

```
>>> people = ["Xander", "Faith", "Buffy"]
>>> people.sort()
>>> people
['Buffy', 'Faith', 'Xander']
```

The reverse() Method:

Reverses the elements of the list, in place:

```
>>> people = ["Xander", "Faith", "Buffy"]
>>> people.reverse()
>>> people
['Buffy', 'Faith', 'Xander']
```

List Unpacking

Similar to tuple unpacking:

```
>>> first, *rest = [1,2,3,4,5]
>>> first
1
>>> rest
[2, 3, 4, 5]
```

Python also has a related concept called *starred arguments*, that can be used as a passing argument for a function:

```
>>> def example_args(a, b, c):
...     return a * b * c  # here * is the multiplication operator
>>> L = [2, 3, 4]
>>> example_args(*L)
24
>>> example_args(2, *L[1:])
24
```

List Comprehensions

A *list comprehension* is an expression and loop (with an optional condition) enclosed in brackets:

```
[item for item in iterable]
[expression for item in iterable]
[expression for item in iterable if condition]
```

Some examples of list comprehensions:

```
>>> a = [y for y in range(1900, 1940) if y%4 == 0]
>>> a
[1900, 1904, 1908, 1912, 1916, 1920, 1924, 1928, 1932, 1936]
>>> b = [2**i for i in range(13)]
>>> b
[1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096]
>>> c = [x for x in a if x%2==0]
>>> c
[0, 4, 16, 36, 64]
>>> d = [str(round(355/113.0,i)) for i in range(1,6)]
>>> d
['3.1', '3.14', '3.142', '3.1416', '3.14159']
>>> words = 'Buffy is awesome and a vampire slayer'.split()
>>> e = [[w.upper(), w.lower(), len(w)] for w in words]
>>> for i in e:
...     print(i)
...
[['BUFFY', 'buffy', 5]
['IS', 'is', 2]
['AWESOME', 'awesome', 7]
['AND', 'and', 3]
['A', 'a', 1]
['VAMPIRE', 'vampire', 7]
['SLAYER', 'slayer', 6]]
```

List comprehensions should only be used for simple cases, when each portion fits in one line (no multiple `for` clauses or `filter` expressions):

```
[Good]
result = []
for x in range(10):
    for y in range(5):
        if x * y > 10:
            result.append((x, y))

for x in range(5):
    for y in range(5):
        if x != y:
            for z in range(5):
                if y != z:
                    yield (x, y, z)

return ((x, complicated_transform(x))
        for x in long_generator_function(parameter)
        if x is not None)

squares = [x * x for x in range(10)]

eat(jelly.Bean for jelly.Bean in jelly_beans
    if jelly.Bean.color == 'black')

[Bad]
result = [(x, y) for x in range(10) for y in range(5) if x * y >
10]

return ((x, y, z)
        for x in xrange(5)
        for y in xrange(5)
        if x != y
        for z in xrange(5)
        if y != z)
```

Runtime Analysis for Lists

To understand better the performance of Python's lists, we benchmark some lists' methods. In the snippet below, we use Python's `timeit` module to

create a `Timer` object whose first parameter is what we want to time and the second parameter is a statement to set up the test. The `timeit` module will time how long it takes to execute the statement some number of times (one million times by default). When test is done, it returns the time as a floating point value representing the total number of seconds:

```
[general_problems/lists/runtime_lists_with_timeit_module.py]

def test1():
    l = []
    for i in range(1000):
        l = l + [i]
def test2():
    l = []
    for i in range(1000):
        l.append(i)

def test3():
    l = [i for i in range(1000)]

def test4():
    l = list(range(1000))

if __name__ == '__main__':
    import timeit
    t1 = timeit.Timer("test1()", "from __main__ import test1")
    print("concat ",t1.timeit(number=1000), "milliseconds")
    t2 = timeit.Timer("test2()", "from __main__ import test2")
    print("append ",t2.timeit(number=1000), "milliseconds")
    t3 = timeit.Timer("test3()", "from __main__ import test3")
    print("comprehension ",t3.timeit(number=1000), "milliseconds")
    t4 = timeit.Timer("test4()", "from __main__ import test4")
    print("list range ",t4.timeit(number=1000), "milliseconds")

"""
The results are:
(concat , 2.366791009902954, 'milliseconds')
	append , 0.16743111610412598, 'milliseconds'
(comprehension , 0.06446194648742676, 'milliseconds')
```

```
('list range ', 0.021029949188232422, 'milliseconds')
```

So we see the following pattern for lists:

Operation	Big-O Efficiency
index []	$O(1)$
index assignment	$O(1)$
append	$O(1)$
pop()	$O(1)$
pop(i)	$O(n)$
insert(i,item)	$O(n)$
del operator	$O(n)$
iteration	$O(n)$
contains (in)	$O(n)$
get slice [x:y]	$O(k)$
del slice	$O(n)$
set slice	$O(n+k)$
reverse	$O(n)$
concatenate	$O(k)$
sort	$O(n \log n)$
multiply	$O(nk)$

2.4 Bytes and Byte Arrays

Python provides two data types for handling raw bytes: `bytes` which is immutable, and `bytearray`, which is mutable. Both types hold a sequence of zero or more 8-bit unsigned integers in the range 0 ... 255. The `byte` type is very similar to the `string` type and the `bytearray` provides mutating methods similar to `lists`.

Bits and Bitwise Operations

Bitwise operations can be very useful to manipulate numbers represented as bits (for example, reproduce an division without using the division operation). We can you quickly compute 2^x by the *left-shifting* operation: $1 \gg x$.

We can also quickly verify whether a number is a power of 2 by checking whether $x \& (x - 1)$ is 0 (if x is not an even power of 2, the highest position of x with a 1 will also have a 1 in $x - 1$, otherwise, x will be 100...0 and $x - 1$ will be 011...1; add them together they will return 0).

2.5 Further Examples

Reversing Words in a String, without Reversing the Words

Here we want to invert the words in a string, without reverting the words. It is important to remember that Python strings are immutable, so we might want to convert to lists in some cases.

There are many ways we can solve this problems. We can use Python's builtin methods for lists and strings, or we can work with pointers. In the second case, the solution consists of two loops. The first loop will revert all the characters utilizing two pointers. The second loop will search for spaces and then revert the words.

Further caveats of this problems is that we can represent space as ' ' or as unicodes (u0020). We also might have to pay attention in the last word, since it does not end with a space. Finally, an extension of this problem is to look for symbols such as "!?;-.".

```
[arrays_strings_lists/reversing_words_string.py]

# EXAMPLE NUMBER 1

def reverser(string1, p1=0, p2=None):
    if len(string1) < 2:
        return string1
    p2 = p2 or len(string1)-1
    while p1 < p2:
        aux = string1[p1]
        string1[p1] = string1[p2]
        string1[p2] = aux
        p1 += 1
        p2 -= 1
```

```
def reversing_words_setence_logic(string1):
    reverser(string1)
    p = 0
    start = 0
    final = []
    while p < len(string1):
        if string1[p] == u"\u0020":
            reverser(string1,start,p-1)
            start = p+1
        p += 1
    reverser(string1,start,p-1)

    return "".join(string1)

# EXAMPLE NUMBER 2 AND 3 USING PYTHON AWESOMESAUCE

def reversing_words_setence_py(str1):
    ''' reverse the words in a sentence'''
    words = str1.split()
    rev_set = " ".join(reversed(words))
    return rev_set

def reversing_words_setence_py2(str1):
    """
    Reverse the order of the words in a sentence.
    :param string: the string which words will be reversed.
    :return: the reversed string.
    """
    words = str1.split(' ')
    words.reverse()
    return ' '.join(words)

# EXAMPLE 4, VERY SILLY, USING BRUTE FORCE
#
```

```
def reverse_words_brute(string):
    """
    Reverse the order of the words in a sentence.
    :param string: the string which words will be reversed.
    :return: the reversed string.
    """
    word, sentence = [], []
    for character in string:
        if character != ' ':
            word.append(character)
        else:
            # So we do not keep multiple whitespaces. An empty list
            # evaluates to False.
            if word:
                sentence.append(''.join(word))
            word = []
    # So we do not keep multiple whitespaces. An empty list
    # evaluates to False.
    if word != '':
        sentence.append(''.join(word))
    sentence.reverse()
    return ' '.join(sentence)

# TESTS

def test_reversing_words_sentence():
    str1 = "Buffy is a Vampire Slayer"
    assert(reversing_words_setence_py(str1) == "Slayer Vampire a is
        Buffy")
    assert(reversing_words_setence_py2(str1) == "Slayer Vampire a
        is Buffy")
    assert(reverse_words_brute(str1) == "Slayer Vampire a is Buffy")
    assert(reversing_words_setence_logic(list(str1)) == "Slayer
        Vampire a is Buffy")

    print("Tests passed!")
```

```
if __name__ == '__main__':
    test_reversing_words_sentence()
```

Chapter 3

Collection Data Structures

Differently from the last chapter’s sequence data structures, where the data can be ordered or sliced, *collection data structures* are containers which aggregates data without relating them. Collection data structures also have some proprieties that sequence types have:

- * **membership operator** (for example, using `in`);
- * **a size method** (given by `len(seq)`); and
- * **iterability** (we can iterate the data in loops).

In Python, built-in collection data types are given by **sets** and **dicts**. In addition, many useful collection data are found in the `collections` package, as we discuss in the last part of this chapter.

3.1 Sets

In Python, a **Set** is an unordered collection data type that is iterable, mutable, and has no duplicate elements. Sets are used for *membership testing* and *eliminating duplicate entries*. Sets have $\mathcal{O}(1)$ *insertion*, so the runtime of *union* is $\mathcal{O}(m + n)$. For *intersection*, it is only necessary to transverse the smaller set, so the runtime is $\mathcal{O}(n)$.¹

¹Python’s `collection` package has supporting for *Ordered sets*. This data type enforces some predefined comparison for their members.

Frozen Sets

Frozen sets are immutable objects that only support methods and operators that produce a result without affecting the frozen set or sets to which they are applied.

Methods for Sets

The `s.add(x)` Method:

Adds the item `x` to set if it is not already in the set:

```
>>> people = {"Buffy", "Angel", "Giles"}
>>> people.add("Willow")
>>> people
{'Willow', 'Giles', 'Buffy', 'Angel'}
```

The `s.update(t)` or `s |= t` Methods:

They both return a set `s` with elements added from `t`.

The `s.union(t)` or `s|t` Methods:

They both perform union of the two sets.

The `s.intersection(t)` or `s&t` Methods:

They both return a new set that has each item from the sets:

```
>>> people = {"Buffy", "Angel", "Giles", "Xander"}
>>> people.intersection({"Angel", "Giles", "Willow"})
{'Giles', 'Angel'}
```

The `s.difference(t)` or `s - t` Methods:

They both return a new set that has every item that is not in the second set:

```
>>> people = {"Buffy", "Angel", "Giles", "Xander"}
>>> vampires = {"Spike", "Angel", "Drusilia"}
```

```
>>> people.difference(vampires)
{'Xander', 'Giles', 'Buffy'}
```

The `clear()` Method:

Removes all the items in the set:

```
>>> people = {"Buffy", "Angel", "Giles"}
>>> people.clear()
>>> people
set()
```

The `discard(x)`, `remove(x)`, and `pop()` Methods:

`discard(x)` removes the item `x` from the set. `remove(x)` removes the item `x` from the set or raises a `KeyError` exception if the element is not in the set. `pop()` returns and removes a random item from the set or raises a `KeyError` exception if the set is empty.

Sets with Lists and Dictionaries

You can cast a set from a list. For example, the snippet below shows some of the available set operations on lists:

```
[general_problems/sets/set_operations_with_lists.py]

def difference(l1):
    """ return the list with duplicate elements removed """
    return list(set(l1))

def intersection(l1, l2):
    """ return the intersection of two lists """
    return list(set(l1) & set(l2))

def union(l1, l2):
    """ return the union of two lists """
    return list(set(l1) | set(l2))

def test_sets_operations_with_lists():
```

```

l1 = [1,2,3,4,5,9,11,15]
l2 = [4,5,6,7,8]
l3 = []
assert(difference(l1) == [1, 2, 3, 4, 5, 9, 11, 15])
assert(difference(l2) == [8, 4, 5, 6, 7])
assert(intersection(l1, l2) == [4,5])
assert(union(l1, l2) == [1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15])
assert(difference(l3) == [])
assert(intersection(l3, l2) == 13)
assert(sorted(union(l3, l2)) == sorted(l2))
print('Tests passed!')


if __name__ == '__main__':
    test_sets_operations_with_lists()

```

We can also use sets' properties in dictionaries:²

```

[general_problems/dicts/set_operations_dict.py]

from collections import OrderedDict

def set_operations_with_dict():
    pairs = [('a', 1), ('b', 2), ('c', 3)]
    d1 = OrderedDict(pairs)
    print(d1) # {'a': 1}, {'b': 2}, {'c': 3}

    d2 = {'a':1, 'c':2, 'd':3, 'e':4}
    print(d2) # {'a': 1, 'c': 2, 'e': 4, 'd': 3}

    union = d1.keys() & d2.keys()
    print(union) # {'a', 'c'}

    union_items = d1.items() & d2.items()
    print(union_items) # {('a', 1)}

    subtraction1 = d1.keys() - d2.keys()
    print(subtraction1) # {'b'}

```

²Sets properties can be used on the dict's attributes `items()` and `keys()` attributes, however `values()` do not support set operations.

```

subtraction2 = d2.keys() - d1.keys()
print(subtraction2) # {'d', 'e'}

subtraction_items = d1.items() - d2.items()
print(subtraction_items) # {('b', 2), ('c', 3)}

''' we can remove keys from a dict doing: '''
d3 = {key:d2[key] for key in d2.keys() - {'c', 'd'}}
print(d3) {'a': 1, 'e': 4}

if __name__ == '__main__':
    set_operations_with_dict()

```

3.2 Dictionaries

Dictionaries in Python are implemented using *hash tables*³. Hashing functions compute some random integer value from an arbitrary object in constant time, that can be used as an index into an array:

```

>>> hash(42)
42
>>> hash("hello")
355070280260770553

```

A **dict** is a collection *mapping type* that is iterable and supports the membership operator `in` and the size function `len()`. Mappings are collections of key-value items, providing methods for accessing items and their keys and values. When iterated, unordered mapping types provide their items in an arbitrary order.

Accessing dictionaries has runtime $\mathcal{O}(1)$ so they are used to keep counts of unique items (for example, counting the number of each unique word in a file) and for fast membership test. Dictionaries are mutable, so we can easily add or remove items, but since they are unordered, they have no notion of

³A hash table is a data structure used to implement an associative array, a structure that can map keys to values.

index position (so that they cannot be sliced or striped):

```
>>> tarantino = {}
>>> tarantino['name'] = 'Quentin Tarantino'
>>> tarantino['job'] = 'director'
>>> tarantino
{'job': 'director', 'name': 'Quentin Tarantino'}
>>>
>>> sunnydale = dict({"name": "Buffy", "age": 16, "hobby": "slaying"})
>>> sunnydale
{'hobby': 'slaying', 'age': 16, 'name': 'Buffy'}
>>>
>>> sunnydale = dict(name="Giles", age=45, hobby="Watch")
>>> sunnydale
{'hobby': 'Watch', 'age': 45, 'name': 'Giles'}
>>>
>>> sunnydale = dict([("name", "Willow"), ("age", 15), ("hobby",
    "nerding")])
>>> sunnydale
{'hobby': 'nerding', 'age': 15, 'name': 'Willow'}
```

Methods for Dictionaries

The `setdefault(key[, default])` Method:

The `setdefault()` method is used when we want to access a key in the dictionary without being sure that this key exists (if we simply try to access a non-existent key in a dictionary, we will get an exception). With `setdefault()`, if key is in the dictionary, we get the value to it. If not, we successfully insert the new key with the value of default:

```
[general_problems/dicts/setdefault_example.py]

def usual_dict(dict_data):
    newdata = {}
    for k, v in dict_data:
        if k in newdata:
            newdata[k].append(v)
        else:
```

```

        newdata[k] = [v]
    return newdata


def setdefault_dict(dict_data):
    newdata = {}
    for k, v in dict_data:
        newdata.setdefault(k, []).append(v)
    return newdata


def test_setdef(module_name='this module'):
    dict_data = (('key1', 'value1'),
                 ('key1', 'value2'),
                 ('key2', 'value3'),
                 ('key2', 'value4'),
                 ('key2', 'value5'),)
    print(usual_dict(dict_data))
    print(setdefault_dict(dict_data))

    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))


if __name__ == '__main__':
    test_setdef()

```

The update([other]) Method:

Updates the dictionary with the key/value pairs from other, overwriting existing keys. .

The get(key) Method:

Returns the key's associated value or `None` if the key is not in the dictionary:

```

>>> sunnydale = dict(name="Xander", age=17, hobby="winning")
>>> sunnydale.get("hobby")
'winning'

```

The `items()`, `values()`, and `keys()` Methods:

The `items()`, `keys()`, and `values()` methods all return dictionary views. A dictionary view is effectively a read-only iterable object that appears to hold the dictionary's items or keys or values:

```
>>> sunnydale = dict(name="Xander", age=17, hobby="winning")
>>> sunnydale.items()
dict_items([('hobby', 'winning'), ('age', 17), ('name', 'Xander')])
>>> sunnydale.values()
dict_values(['winning', 17, 'Xander'])
>>> sunnydale.keys()
dict_keys(['hobby', 'age', 'name'])
```

The `pop()` and `popitem()` Methods:

The `pop()` method removes an arbitrary item from the dictionary, returning it. The `popitem()` method removes an arbitrary (key, value) from the dictionary, also returning it.

The `clear()` Method:

Removes all the items in the dictionary:

```
>>> sunnydale.clear()
>>> sunnydale
{}
```

Runtime Analysis for Dictionaries

We analyze Python's dictionary performance by benchmarking some available methods. The snippet below shows that the membership operation for lists is $\mathcal{O}(n)$ and for dictionaries is $\mathcal{O}(1)$:

```
[general_problems/dicts/runtime_dicts_with_timeit_module.py]

import timeit
import random
```

```
for i in range(10000,1000001,20000):
    t = timeit.Timer("random.randrange(%d) in x"%i, "from __main__"
                     "import random,x")
    x = list(range(i))
    lst_time = t.timeit(number=1000)
    x = {j:None for j in range(i)}
    d_time = t.timeit(number=1000)
    print("%d,%10.3f,%10.3f" % (i, lst_time, d_time))

"""
There results are:
10000,      0.192,      0.002
30000,      0.600,      0.002
50000,      1.000,      0.002
70000,      1.348,      0.002
90000,      1.755,      0.002
110000,     2.194,      0.002
130000,     2.635,      0.002
150000,     2.951,      0.002
170000,     3.405,      0.002
190000,     3.743,      0.002
210000,     4.142,      0.002
230000,     4.577,      0.002
250000,     4.797,      0.002
270000,     5.371,      0.002
290000,     5.690,      0.002
310000,     5.977,      0.002
```

So we can see the linear time for lists, and constant for dict!

Big-O Efficiency of Python Dictionary Operations	
Operation	Big-O Efficiency
copy	$O(n)$
get item	$O(1)$
set item	$O(1)$
delete item	$O(1)$
contains (in)	$O(1)$
iteration	$O(n)$

Iterating over Dictionaries

A loop over a dictionary iterates over its keys by default. The keys will appear in an arbitrary order but we can use `sorted()` to iterate over the items in a sorted way. This also works for the attributes `keys()`, `values()`, and `items()`:

```
>>> for key in sorted(dict.keys()):
...     print key, dict[key]
```

An useful tool to create a list of key-items for a dictionary is by using generators:

```
def items_in_key_order(d):
    for key in sorted(d):
        yield key, d[key]
```

Dictionaries also support reverse iteration using `reversed()`. Finally, default iterators should be used for types that support them:

<pre>[Good] for key in adict: ... if key not in adict: ...</pre>
<pre>[Bad] for key in adict.keys(): ... if not adict.has_key(key): ...</pre>

Branching using Dictionaries

We can use dictionaries to write a branching menu:

<pre>if action == "a": add_to_dict(db) elif action == "e": edit_dict(db)</pre>
--

And a more efficient way:

```
functions = dict(a=add_to_dict, e=edit_dict,...)
functions[actions](db)
```

3.3 Python's collection Data Types

Python's `collections` module implements specialized container data types providing high-performance alternatives to the general purpose built-in containers.

Default Dictionaries

Default dictionaries are an additional unordered mapping type provided by Python's `collections.defaultdict`. They have all the operators and methods that a built-in dictionary has, but, in addition, they handle missing keys:

```
[general_examples/dicts/defaultdict_example.py]

from collections import defaultdict

def defaultdict_example():
    ''' show some examples for defaultdicts '''
    pairs = {('a', 1), ('b',2), ('c',3)}

    d1 = {}
    for key, value in pairs:
        if key not in d1:
            d1[key] = []
        d1[key].append(value)
    print(d1)

    d2 = defaultdict(list)
    for key, value in pairs:
        d2[key].append(value)
    print(d2)
```

```
if __name__ == '__main__':
    defaultdict_example()
```

Ordered Dictionaries

Ordered dictionaries are an ordered mapping type provided by Python's `collections.OrderedDict`. They have all the methods and properties of a built-in dict, but in addition they store items in the *insertion order*:

```
[general_examples/dicts/OrderedDict_example.py]

from collections import OrderedDict

pairs = [('a', 1), ('b', 2), ('c', 3)]
d1 = {}
for key, value in pairs:
    if key not in d1:
        d1[key] = []
    d1[key].append(value)
for key in d1:
    print(key, d1[key])

d2 = OrderedDict(pairs)
for key in d2:
    print(key, d2[key])

if __name__ == '__main__':
    OrderedDict_example()

"""
a [1]
c [3]
b [2]
a 1
b 2
c 3
"""
```

We can create ordered dictionaries incrementally:

```
>>> tasks = collections.OrderedDict()
>>> tasks[8031] = "Backup"
>>> tasks[4027] = "Scan Email"
>>> tasks[5733] = "Build System"
>>> tasks
OrderedDict([(8031, 'Backup'), (4027, 'Scan Email'), (5733, 'Build
System')])
```

If we change a key value, the order is not changed. To move an item to the end we should delete and re-insert it. We can also call `popitem()` to remove and return the last key-value item in the ordered dictionary.

In general, using an ordered dictionary to produce a sorted dictionary makes sense only if we *expect to iterate over the dictionary multiple times*, and if we *do not expect to do any insertions* (or very few).

Counter Dictionaries

A specialised **Counter** type (subclass for counting hashable objects) is provided by Python's `collections.Counter`:

```
[general_examples/dicts/Counter_example.py]

from collections import Counter

def Counter_example():
    ''' show some examples for Counter '''
    ''' it is a dictionary that maps the items to the number of
    occurrences '''
    seq1 = [1, 2, 3, 5, 1, 2, 5, 5, 2, 5, 1, 4]
    seq_counts = Counter(seq1)
    print(seq_counts)

    ''' we can increment manually or use the update() method '''
    seq2 = [1, 2, 3]
    seq_counts.update(seq2)
    print(seq_counts)
```

```
seq3 = [1, 4, 3]
for key in seq3:
    seq_counts[key] += 1
print(seq_counts)

''' also, we can use set operations such as a-b or a+b '''
seq_counts_2 = Counter(seq3)
print(seq_counts_2)
print(seq_counts + seq_counts_2)
print(seq_counts - seq_counts_2)

if __name__ == '__main__':
    Counter_example()

"""
Counter({5: 4, 1: 3, 2: 3, 3: 1, 4: 1})
Counter({1: 4, 2: 4, 5: 4, 3: 2, 4: 1})
Counter({1: 5, 2: 4, 5: 4, 3: 3, 4: 2})
Counter({1: 1, 3: 1, 4: 1})
Counter({1: 6, 2: 4, 3: 4, 5: 4, 4: 3})
Counter({1: 4, 2: 4, 5: 4, 3: 2, 4: 1})
"""
```

3.4 Further Examples

Counting Frequency of Items

In the example below we use `collections.Counter()`'s `most_common()` method to find the top N recurring words in a sequence:

```
[general_problems/dicts/find_top_N_recurring_words.py]

from collections import Counter

def find_top_N_recurring_words(seq, N):
    dcounter = Counter()
    for word in seq.split():
        dcounter[word] += 1
    return dcounter.most_common(N)

def test_find_top_N_recurring_words(module_name='this module'):
    seq = 'buffy angel monster xander a willow gg buffy the monster
          super buffy angel'
    N = 3
    assert(find_top_N_recurring_words(seq, N) == [('buffy', 3),
                                                   ('monster', 2), ('angel', 2)])
    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_find_top_N_recurring_words()
```

The program below counts all the unique words in a file:

```
[general_problems/dicts/count_unique_words.py]

import collections
import string
import sys
```

```

def count_unique_word():
    words = collections.defaultdict(int)
    strip = string.whitespace + string.punctuation + string.digits
    + "\n\r\t"
    for filename in sys.argv[1:]:
        with open(filename) as file:
            for line in file:
                for word in line.lower().split():
                    word = word.strip(strip)
                    if len(word) > 2:
                        words[word] += 1
    for word in sorted(words):
        print("{} occurs {} times.".format(word, words[word]))

```

Anagrams

The following program finds whether two words are anagrams. Since sets do not count occurrence, and sorting a list is $\mathcal{O}(n \log n)$, hash tables can be the best solution in this case. The procedure we use is: we scan the first string and add all the character occurrences. Then we scan the second string, decreasing all the character occurrences. In the end, if all the entries are zero, the string is an anagram:

```

[general_problems/dicts/verify_two_strings_are_anagrams.py]

def verify_two_strings_are_anagrams(str1, str2):
    ana_table = {key:0 for key in string.ascii_lowercase}

    for i in str1:
        ana_table[i] += 1

    for i in str2:
        ana_table[i] -= 1

    # verify whether all the entries are 0
    if len(set(ana_table.values())) < 2: return True
    else: return False

```

```
def test_verify_two_strings_are_anagrams():
    str1 = 'marina'
    str2 = 'aniram'
    assert(verify_two_strings_are_anagrams(str1, str2) == True)
    str1 = 'google'
    str2 = 'gougle'
    assert(verify_two_strings_are_anagrams(str1, str2) == False)
    print('Tests passed!')

if __name__ == '__main__':
    test_verify_two_strings_are_anagrams()
```

Another way to find whether two words are anagrams is using the hashing function's properties, where every different amount of characters should give a different result. In the following program, `ord()` returns an integer representing the Unicode code point of the character when the argument is a unicode object, or the value of the byte when the argument is an 8-bit string:

```
[general_problems/dicts/find_anagram_hash_function.py]

def hash_func(astring, tablesize):
    sum = 0
    for pos in range(len(astring)):
        sum = sum + ord(astring[pos])
    return sum%tablesize

def find_anagram_hash_function(word1, word2):
    tablesize = 11
    return hash_func(word1, tablesize) == hash_func(word2,
        tablesize)

def test_find_anagram_hash_function(module_name='this module'):
    word1 = 'buffy'
    word2 = 'bffyu'
    word3 = 'bffya'
    assert(find_anagram_hash_function(word1, word2) == True)
    assert(find_anagram_hash_function(word1, word3) == False)
```

```

s = 'Tests in {name} have {con}!'
print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_find_anagram_hash_function()

```

Sums of Paths

The following program uses two different dictionary containers to determine the number of ways two dices can sum to a certain value:

```

[general_problems/dicts/find_dice_probabilities.py]

from collections import Counter, defaultdict

def find_dice_probabilities(S, n_faces=6):
    if S > 2*n_faces or S < 2: return None

    cdict = Counter()
    ddict = defaultdict(list)

    for dice1 in range(1, n_faces+1):
        for dice2 in range(1, n_faces+1):
            t = [dice1, dice2]
            cdict[dice1+dice2] += 1
            ddict[dice1+dice2].append(t)

    return [cdict[S], ddict[S]]

def test_find_dice_probabilities(module_name='this module'):
    n_faces = 6
    S = 5
    results = find_dice_probabilities(S, n_faces)
    print(results)
    assert(results[0] == len(results[1]))

if __name__ == '__main__':
    test_find_dice_probabilities()

```

Finding Duplicates

The program below uses dictionaries to find and delete all the duplicate characters in a string:

```
[general_problems/dicts/delete_duplicate_char_str.py]

import string
def delete_unique_word(str1):
    table_c = { key : 0 for key in string.ascii_lowercase}
    for i in str1:
        table_c[i] += 1
    for key, value in table_c.items():
        if value > 1:
            str1 = str1.replace(key, "")
    return str1

def test_delete_unique_word():
    str1 = "google"
    assert(delete_unique_word(str1) == 'le')
    print('Tests passed!')

if __name__ == '__main__':
    test_delete_unique_word()
```


Chapter 4

Python's Structure and Modules

4.1 Modules in Python

In Python, modules are defined using the built-in name `def`. When `def` is executed, a *function object* is created together with its *object reference*. If we do not define a `return` value, Python automatically returns `None` (like in C, we call the function a *procedure* when it does not return a value).

An *activation record* happens every time we invoke a method: information is put in the *stack* to support invocation. Activation records process in the following order:

Activation Records

1. the actual parameters of the method are pushed onto the stack,
2. the return address is pushed onto the stack,
3. the top-of-stack index is incremented by the total amount required by the local variables within the method,
4. a jump to the method.

The process of unwinding an activation record happens in the following order:

1. the top-of-stack index is decremented by the total amount of memory consumed,
2. the returned address is popped off the stack,
3. the top-of-stack index is decremented by the total amount of memory by the actual parameters.

Default Values in Modules

Whenever you create a module, remember that mutable objects should not be used as default values in the function or method definition:

```
[Good]
def foo(a, b=None):
    if b is None:
        b = []
[Bad]
def foo(a, b=[]):
```

The `__init__.py` File

A *package* is a directory that contains a set of modules and a file called `__init__.py`. This is required to make Python treat the directories as containing packages, preventing directories with a common name (such as “string”) from hiding valid modules that occur later on the module search path:

```
>>> import foldername.filemodulename
```

In the simplest case, it can just be an empty file, but it can also execute initialization code for the package or set the `__all__` variable: `__init__.py` to:

```
__all__ = ["file1", ...]
```

(with no `.py` in the end).

Moreover, the statement:

```
from foldername import *
```

means importing every object in the module, except those whose names begin with `__`, or if the module has a global `__all__` variable, the list in it.

Checking the Existence of a Module

To check the existence of a module, we use the flag `-c`:

```
$ python -c "import decimas"
Traceback (most recent call last):
  File "<string>", line 1, in <module>
ImportError: No module named decimas
```

The `__name__` Variable

Whenever a module is *imported*, Python creates a variable for it called `__name__`, and stores the module's name in this variable. In this case, everything below the statement:

```
if __name__ == '__main__':
```

will not be executed. In the other hand, if we run the `.py` file directly, Python sets `__name__` to `__main__`, and every instruction following the above statement will be executed.

Byte-coded Compiled Modules

Byte-compiled code, in form of `.pyc` files, is used by the compiler to *speed-up the start-up time* (load time) for short programs that use a lot of standard modules.

When the Python interpreter is invoked with the `-O` flag, optimized code is generated and stored in `.pyo` files. The optimizer removes assert statements.

This also can be used to distribute a library of Python code in a form that is moderately hard to reverse engineer.

The `sys` Module

The variable `sys.path` is a list of strings that determines the interpreter's search path for modules. It is initialized to a default path taken from the

environment variable `PYTHONPATH`, or from a built-in default. You can modify it using standard list operations:

```
>>> import sys
>>> sys.path.append( /buffy/lib/python )
```

The variables `sys.ps1` and `sys.ps2` define the strings used as primary and secondary prompts. The variable `sys.argv` allows us to use the arguments passed in the command line inside our programs:

```
import sys

def main():
    ''' print command line arguments '''
    for arg in sys.argv[1:]:
        print arg

if __name__ == "__main__":
    main()
```

The built-in method `dir()` is used to find *which names a module defines* (all types of names: variables, modules, functions). It returns a sorted list of strings:

```
>>> import sys
>>> dir(sys)
[ '__name__' ,   argv ,   builtin_module_names ,   copyright ,
  exit ,   maxint ,   modules ,   path ,   ps1 ,
  ps2 ,   setprofile ,   settrace ,   stderr ,
  stdin ,   stdout ,   version ]
```

It does not list the names of built-in functions and variables. Therefore, we can see that `dir()` is useful to find all the methods or attributes of an object.

4.2 Control Flow

if

The `if` statement substitutes the `switch` or `case` statements in other languages:¹

```
>>> x = int(input("Please enter a number: "))
>>> if x < 0:
...     x = 0
...     print "Negative changed to zero"
>>> elif x == 0:
...     print "Zero"
>>> elif x == 1:
...     print "Single"
>>> else:
...     print "More"
```

for

The `for` statement in Python differs from C or Pascal. Rather than always iterating over an arithmetic progression of numbers (like in Pascal), or giving the user the ability to define both the iteration step and halting condition (as C), Python's `for` statement iterates over the items of any sequence (*e.g.*, a list or a string), in the order that they appear in the sequence:

```
>>> a = ["buffy", "willow", "xander", "giles"]
>>> for i in range(len(a)):
...     print(a[i])
buffy
willow
xander
giles
```

¹Note that colons are used with `else`, `elif`, and in any other place where a suite is to follow.

False and True in Python

`False` is defined by the predefined constant `False`, or the special object `None`, or by an empty sequence of collection (empty string `' '`, list `[]`, or tuple `()`). Anything else is `True`. It is also possible to assign the result of a comparison or other Boolean expression to a variable:

```
>>> string1, string2, string3 = '', 'monsters', 'aliens'
>>> non_null = string1 or string2 or string3
>>> non_null
'monsters'
```

The Google Python Style guide sets the following rules for using implicit `False` in Python:

- ★ Never use `==` or `!=` to compare *singletons*, such as the built-in variable `None`. Use `is` or `is not` instead.
- ★ Beware of writing `if x:` when you really mean `if x is not None`.
- ★ Never compare a boolean variable to `False` using `==`. Use `if not x:` instead. If you need to distinguish `False` from `None` then chain the expressions, such as `if not x and x is not None`:
- ★ For sequences (strings, lists, tuples), use the fact that empty sequences are `False`, so `if not seq:` or `if seq:` is preferable to `if len(seq):` or `if not len(seq):`.
- ★ When handling integers, implicit `False` may involve more risk than benefit, such as accidentally handling `None` as 0:

<pre>[Good] if not users: print 'no users' if foo == 0: self.handle_zero() if i % 10 == 0: self.handle_multiple_of_ten() [Bad] if len(users) == 0: print 'no users' if foo is not None and not foo: self.handle_zero() if not i % 10: self.handle_multiple_of_ten()</pre>
--

yield vs. return

In Python, generators are a convenient way to write an iterator. If an object has both `__iter__()` and `__next__()` methods defined, it is said to implement the iterator protocol. In this context, `yield` comes in handy.

The difference between the keywords `yield` and `return` is that the former returns each value to the caller and then *only returns to the caller when all values to return have been exhausted*, and the latter *causes the method to exit and return control to the caller*.

One great feature in Python is how it handles iterability. An *iterator* is a container object that implements the iterator protocol and is based on two methods: `next`, which returns the next item in the container, and `__iter__` which returns the iterator itself. Since all methods in Python are virtual, we are free to modify how iterability works in our functions (and classes) the way we like.

The `yield` paradigm becomes important in the context of *generators*, which are a powerful tool for creating iterators. Generators are like regular functions but instead of returning a final value in the end, they use the `yield` statement to return data *during execution*. In other words, values are extracted from the iterator one at time by calling its `__next__()` method and at each of these calls, the `yield` expression's value is returned. This happens until the final call, when a `StopIteration` is raised:

```
>>> def f(a):
...     while a:
...         yield a.pop()
```

Generators are very robust and efficient and they should be considered every time you deal with a function that returns a sequence or creates a loop. For example, the following program implements a Fibonacci sequence using the iterator paradigm:

```
def fib_generator():
    a, b = 0, 1
    while True:
        yield b
        a, b = b, a+b

if __name__ == '__main__':
    fib = fib_generator()
```

```
print(next(fib))
print(next(fib))
print(next(fib))
print(next(fib))
```

break vs. continue

The command `break`, *breaks out of the smallest enclosing* `for` or `while` loop, and switches control to the statement *following the innermost loop in which the break statement appears* (it breaks out of the loop).

In the other hand, `continue`, *continues with the next iteration of the loop*, and switches control to the *start of the loop* (continues with the next iteration of the loop).

Loop statements may have an `else` clause which is executed when the loop terminates through exhaustion of the list (with `for`) or when the condition becomes false (with `while`), but not when the loop is terminated by a `break` statement.

The `range()` Method

This method generates lists containing arithmetic progressions. It is useful when you need to iterate over a sequence of numbers:

```
>>> range(10)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
>>> range(4, 10)
[4, 5, 6, 7, 8, 9]
>>> range(0, 10, 3)
[0, 3, 6, 9]
```

The `enumerate()` Method

Returns a tuple with the item and the index values from an iterable. For example, we can use `enumerate` to write our own `grep` function, which gets a word and files from the command line and outputs where the word appears:

```
[general_problems/modules/grep_word_from_files.py]

import sys

def grep_word_from_files():
    word = sys.argv[1]
    for filename in sys.argv[2:]:
        with open(filename) as file:
            for lino, line in enumerate(file, start=1):
                if word in line:
                    print("{0}:{1}:{2:.40}".format(filename, lino,
                        line.rstrip()))

if __name__ == '__main__':
    if len(sys.argv) < 2:
        print("Usage: grep_word_from_files.py word infile1
              [infile2...]")
        sys.exit()
    else:
        grep_word_from_files()
```

The zip() Method

The `zip` function takes two or more sequences and creates a new sequence of tuples where each tuple contains one element from each list:

```
>>> a = [1, 2, 3, 4, 5]
>>> b = ['a', 'b', 'c', 'd', 'e']
>>> zip(a, b)
<zip object at 0xb72d65cc>
>>> list(zip(a,b))
[(1, 'a'), (2, 'b'), (3, 'c'), (4, 'd'), (5, 'e')]
```

The filter(function, sequence) Method

This method returns a sequence consisting of those items from the sequence for which function (item) is true:

```
>>> def f(x): return x % 2 != 0 and x % 3 != 0
>>> f(33)
False
>>> f(17)
True
>>> filter(f, range(2, 25))
[5, 7, 11, 13, 17, 19, 23]
```

The `map(function, list)` Function

A convenient way of turning a pair of lists into a list of pairs. It applies a function to every item of an iterable and then returns a list of the results:

```
>>> def cube(x): return x*x*x
>>> map(cube, range(1, 11))
[1, 8, 27, 64, 125, 216, 343, 512, 729, 1000]
>>> seq = range(8)
>>> def square(x): return x*x
>>> map(None, seq, map(square, seq))
[(0, 0), (1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36), (7,
49)]
```

The `lambda` Function

The `lambda` function is a dynamic way of compacting functions inside the code. For example, the function:

```
>>> def area(b, h):
...     return 0.5 * b * h
...
>>> area(5,4)
10.0
```

could be rather written as

```
>>> area = lambda b, h: 0.5 * b * h
>>> area(5, 4)
10.0
```

Lambda functions are very useful for creating keys in dictionaries:

```
>>> import collections
>>> minus_one_dict = collections.defaultdict(lambda: -1)
>>> point_zero_dict = collections.defaultdict(lambda: (0, 0))
>>> message_dict = collections.defaultdict(lambda: "No message")
```

4.3 File Handling

File handling is very easy in Python. For example, the snippet bellow reads a file and delete all the blank lines:

```
[general_problems/modules/remove_blank_lines.py]

import os
import sys

def read_data(filename):
    lines = []
    fh = None
    try:
        fh = open(filename)
        for line in fh:
            if line.strip():
                lines.append(line)
    except (IOError, OSError) as err:
        print(err)
    finally:
        if fh is not None:
            fh.close()
    return lines

def write_data(lines, filename):
    fh = None
    try:
        fh = open(filename, "w")
        for line in lines:
            fh.write(line)
```

```

except (EnvironmentError) as err:
    print(err)
finally:
    if fh is not None:
        fh.close()

def remove_blank_lines():
    if len(sys.argv) < 2:
        print ("Usage: noblank.py infile1 [infile2..."])

    for filename in sys.argv[1:]:
        lines = read_data(filename)
        if lines:
            write_data(lines, filename)

if __name__ == '__main__':
    remove_blank_lines()

```

Methods for File Handling

The `open(filename, mode)` Method:

Returns a file object. The mode argument is optional and `read` will be assumed if it is omitted, the other options are:

- ★ `r` for reading,
- ★ `w` for writing (an existing file with the same name will be erased),
- ★ `a` for appending (any data written to the file is automatically added to the end),
- ★ and `r+` for both reading and writing.

```

fin = open(filename, encoding="utf8")
fout = open(filename, "w", encoding="utf8")

```

The `read(size)` Method:

Reads some quantity from the data and returns it as a string. Size is an optional numeric argument, when it is omitted or negative, the entire contents of the file will be read and returned. If the end of the file has been reached, `read()` will return an empty string:

```
>>> f.read()
'This is the entire file.\n'
>>> f.read()
''
```

The `readline()` Method:

Reads a single line from the file. A newline character is left at the end of the string, and is only omitted on the last line of the file if the file. This makes the return value unambiguous.

The `readlines()` Method:

Return a list containing all the lines of data in the file. If given an optional parameter `size`, it reads that many bytes from the file and enough more to complete a line, and returns the lines from that. This is often used to allow efficient reading of a large file by lines, but without having to load the entire file in memory. Only complete lines will be returned:

```
>>> f.readlines()
['This is the first line of the file.\n', 'Second line of the
file\n']
```

The `write()` Method:

Writes the contents of a string to the file, returning `None`. Write bytes/bytarray object to the file if opened in binary mode or a string object if opened in text mode:

```
>>> f.write('This is a test\n')
```

The tell() and seek() Methods:

The `tell()` method returns an integer giving the file object's current position in the file, measured in bytes from the beginning of the file.

To change the file object's position, use `seek(offset, from-what)`. The position is computed from adding offset to a reference point and the reference point is selected by the from-what argument. A from-what value of 0 measures from the beginning of the file, 1 uses the current file position, and 2 uses the end of the file as the reference point.

The close() Method:

Closes the file and free up any system resources taken up by the open file. It returns `True` if the file is closed.

The input() Method:

Accepts input from the user. This function takes an optional string argument (which will be printed in the console), then it will wait for the user to type in a response and to finish by pressing Enter (or Return).

If the user does not type any text but just presses Enter, the function returns an empty string; otherwise, it returns a string containing what the user typed, without any line terminator.

```
>>> def get_int(msg):
...     while True:
...         try:
...             i = int(input(msg))
...             return i
...         except ValueError as err:
...             print(err)
>>> age = get_int("Enter your age: ")
```

The peek(n) Method:

Returns `n` bytes without moving the file pointer position.

The fileno() Method:

Returns the underlying file's *file descriptor* (available only for file objects that have file descriptors).

The shutil Package

This package is useful for manipulating files in the system. For example, the following snippet gets a file and an extension from the command line and produces a copy of this file with its extension changed to the given string:

```
[general_problems/files/change_ext_file.py]

import os
import sys
import shutil

def change_file_ext():
    if len(sys.argv) < 2:
        print("Usage: change_ext.py filename.old_ext 'new_ext'")
        sys.exit()

    name = os.path.splitext(sys.argv[1])[0] + "." + sys.argv[2]
    print (name)

    try:
        shutil.copyfile(sys.argv[1], name)
    except OSError as err:
        print (err)

if __name__ == '__main__':
    change_file_ext()
```

The pickle Module

The `pickle` module can take almost any Python object (even some forms of Python code!), and convert it to a string representation. This process is

called *pickling*, where reconstructing the object from the string representation is called *unpickling*.

If you have an object `x`, and a file object `f` that has been opened for writing, the simplest way to pickle the object takes only one line of code:

```
>>> pickle.dump(x, f)
```

Then, to unpickle this object:

```
>>> x = pickle.load(f)
```

Exporting Data with Pickle

The example below shows an use for `pickle`:

```
[general_problems/files/export_pickle.py]

import pickle

def export_pickle(data, filename='test.dat', compress=False):
    fh = None
    try:
        if compress:
            fh = gzip.open(filename, "wb") # write binary
        else:
            fh = open(filename, "wb") # compact binary pickle format
            pickle.dump(data, fh, pickle.HIGHEST_PROTOCOL)

    except(EnvironmentError, pickle.PicklingError) as err:
        print("{0}: export error:\n{1}".format(os.path.basename(sys.argv[0]), err))
        return False

    finally:
        if fh is not None:
            fh.close()

def test_export_pickle():
    mydict = {'a': 1, 'b': 2, 'c': 3}
    export_pickle(mydict)
```

```
if __name__ == '__main__':
    test_export_pickle()
```

In general, booleans, numbers, and strings, can be pickled as can instances of classes and built-in collection types (providing they contain only pickleable objects, i.e., their `__dict__` is pickleable).

Reading Data with Pickle

The example below shows how to read a pickled data:

```
[general_problems/files/import.py]

import pickle

def import_pickle(filename):
    fh = None
    try:
        fh = open(filename, "rb")
        mydict2 = pickle.load(fh)
        return mydict2

    except (EnvironmentError) as err:
        print ("{}: import error:
               {}".format(os.path.basename(sys.argv[0]), err))
        return False

    finally:
        if fh is not None:
            fh.close()

def test_import_pickle():
    pkl_file = 'test.dat'
    mydict = import_pickle(pkl_file)
    print(mydict)

if __name__ == '__main__':
    test_export_pickle()
```

```
test_import_pickle()
```

The struct Module

We can convert Python objects to and from suitable binary representation using `struct`. This object can handle only strings with a specific length. `struct` allows the creation of a function that takes a string and return a byte object with an integer length count and a sequence of length count UTF-8 encoded bytes for the text.

Some methods are: `struct.pack()` (takes a struct format string and values and returns a byte object), `struct.unpack()` (takes a format and a bytes or bytearray object and returns a tuple of values), and `struct.calcsize()` (takes a format and returns how many bytes a struct using the format will occupy):

```
>>> data = struct.pack("<2h", 11, -9)
>>> items = struct.unpack("<2h", data) # little endian
```

4.4 Error Handling in Python

There are two distinguishable kinds of errors when we compile a program in Python: *syntax errors* (parsing errors) and *exceptions* (errors detected during execution, not unconditionally fatal). While syntax errors will never allow the program to be compiled, exceptions can only be detected in some cases and for this reason they should be carefully handled.

Handling Exceptions

When an exception is raised and not handled, Python outputs a *traceback* along with the exception's error message. A traceback (sometimes called a backtrace) is a list of all the calls made from the point where the unhandled exception occurred back to the top of the call *stack*.

We can handle predictable exceptions by using the `try-except-finally` paradigm:

```
try:
    try_suite
except exception1 as variable1:
    exception_suite1
...
except exceptionN as variableN:
    exception_suiteN
```

If the statements in the try block's suite are all executed without raising an exception, the except blocks are skipped. If an exception is raised inside the try block, control is immediately passed to the suite corresponding to the first matching exception. This means that any statements in the suite that follow the one that caused the exception will not be executed:

```
while 1:
    try:
        x = int(raw_input("Please enter a number: "))
        break
    except ValueError:
        print "Oops! That was no valid number. Try again..."
```

The `raise` statement allows the programmer to force a specified exception to occur:

```
import string
import sys
try:
    f = open('myfile.txt')
    s = f.readline()
    i = int(string.strip(s))
except IOError, (errno, strerror):
    print "I/O error(%s): %s" % (errno, strerror)
except ValueError:
    print "Could not convert data to an integer."
except:
    print "Unexpected error:", sys.exc_info()[0]
    raise
```

We also can use else:

```
for arg in sys.argv[1:]:
    try:
        f = open(arg, 'r')
    except IOError:
        print 'cannot open', arg
    else:
        print arg, 'has', len(f.readlines()), 'lines'
        f.close()
```

The **Google** Python Style Guide for Exceptions

Exceptions must be used carefully and must follow certain conditions:

- ★ Raise exceptions like this: `raise MyException('Error message')` or `raise MyException`. Do not use the two-argument form.
- ★ Modules or packages should define their own domain-specific base exception class, which should inherit from the built-in `Exception` class. The base exception for a module should be called `Error`.

```
class Error(Exception):
    pass
```

- ★ Never use catch-all `except:` statements, or catch `Exception` or `StandardError`, unless you are re-raising the exception or in the outermost block in your thread (and printing an error message).
- ★ Minimize the amount of code in a `try/except` block. The larger the body of the try, the more likely that an exception will be raised by a line of code that you didn't expect to raise an exception. In those cases, the try/except block hides a real error.
- ★ Use the `finally` clause to execute code whether or not an exception is raised in the try block. This is often useful for clean-up, *i.e.*, closing a file.
- ★ When capturing an exception, use `as` rather than a comma. For example:

```
try:  
    raise Error  
except Error as error:  
    pass
```


Chapter 5

Object-Oriented Design

Suppose we want to define an object in Python to represent a circle. We could remember about Python's `collections` module and create a **named tuple** for our circle:

```
>>> circle = collections.namedtuple("Circle", "x y radius")
>>> circle
<class '__main__.Circle'>
>>> circle = circle(13, 84, 9)
>>> circle
Circle(x=13, y=84, radius=9)
```

However, many things are missing here. First, there are no guarantees that anyone who uses our circle data is not going to type an invalid input value, such as a negative number for the radius. Second, how could we also associate to our circle some operations that are proper from it, such as its area or perimeter?

For the first problem, we can see that the inability to validate when creating an object is a really bad aspect of taking a purely *procedural* approach in programming. Even if we decide to include many exceptions handling the invalid inputs for our circles, we still would have a data container that is not intrinsically made and validated for its real purpose. Imagine now if we had chosen a **list** instead of the named tuple, how would we handle the fact that lists have sorting properties?

It is clear from the example above that we need to find a way to create an object that has **only** the proprieties that we expect it to have. In other words, we want to find a way to package data and restrict its methods. That

is what *object-oriented programming* allows you to do: to *create your own custom data type*, which in this example would be a circle *class*.

5.1 Classes and Objects

Classes are the way we can gather special predefined data and methods together. We use them by creating *objects*, which are *instances* of a particular class. The simplest form of a class in Python looks like the following snippet:

```
class ClassName:
    <statement-1>
    .
    .
    <statement-N>

>>> x = ClassName() # class instantiation
```

Class Instantiation

Class instantiation uses function notation to create objects in a known initial state. The instantiation operation creates an empty object which has individuality. However, multiple names (in multiple scopes) can be bound to the same object (also know as *aliasing*). In Python, when an object is created, first the special method `__new__()` is called (the *constructor*) and then `__init__()` initializes it.

Attributes

Objects have the *attributes* from their Classes, which are *methods* and *data*. Method attributes are functions whose first argument is the instance on which it is called to operate (which in Python is conventionally called `self`).

Attributes are any name following a dot. References to names in modules are attribute references: in the expression `modname.funcname`, `modname` is a module object and `funcname` is one of its attribute. Attributes may be read-only or writable. Writable attributes may be deleted with the `del` statement.

Namespaces

A *namespace* is a mapping from names to objects. Most namespaces are currently implemented as Python dictionaries. Examples of namespaces are: the set of built-in names, the global names in a module, and the local names in a function invocation. The statements executed by the top-level invocation of the interpreter, either reading from a script file or interactively, are considered part of a module called `__main__`, so they have their own global namespace.

Scope

A *scope* is a textual region of a Python program where a namespace is directly accessible. Although scopes are determined statically, they are used dynamically. Scopes are determined textually: the global scope of a function defined in a module is that module's namespace. When a class definition is entered, a new namespace is created, and used as the local scope.

5.2 Principles of OOP

Specialization

Specialization (or inheritance) is the procedure of creating a new class that *inherits* all the attributes from the super class (also called *base class*). Any method can be overridden (re-implemented) in a subclass (in Python, all methods are virtual). Inheritance is described as an **is-a** relationship.

Furthermore [Google Python Style Guide](#) advises that if a class inherits from no other base classes, we should explicitly inherit it from Python's highest class, `object`:

```
class OuterClass(object):
    class InnerClass(object):
```

Polymorphism

Polymorphism (or dynamic method binding) is the principle where methods can be redefined inside subclasses. In other words, if we have an object of a subclass and we call a method that is also defined in the superclass,

Python will use the method defined in the subclass. If, for instance, we need to recover the superclass's method, we can easily call it using the built-in `super()`.

For example, all instances of a custom class are *hashable* by default in Python. This means that the `hash()` attribute can be called, allowing them to be used as dictionary keys and to be stored in sets. However, if we reimplement the attribute `__eq__()`, we change this propriety (what can result on our instances no longer being hashable).

Aggregation

Aggregation (or composition) defines the process where a class includes one or more instance variables that are from other classes. It is a **has-a** relationship. In Python, every class uses inheritance (they are all custom classes from the object base class), and most use aggregation since most classes have instance variables of various types.

A first Example of a Class

We now have the tools for writing our first class in Python. The example below illustrate how we could write a circle data container¹. using the object-oriented design paradigm. First, we created a class called Point with general data and methods attributes. Then we use *inheritance* to create a Circle subclass from it:

```
[general_problems/oop/ShapeClass.py]

import math

class Point:
    def __init__(self, x = 0, y = 0):
        self.x = x # data attribute
        self.y = y

    def distance_from_origin(self): # method attribute
        return math.hypot(self.x, self.y)
```

¹ *containers*, which is a generic data structure that permits storage and retrieval of data items independent of content.

```
def __eq__(self, other):
    return self.x == other.x and self.y == other.y

def __repr__(self):
    return "point ({0.x!r}, {0.y!r})".format(self)

def __str__(self):
    return "({0.x!r}, {0.y!r})".format(self)

class Circle(Point):

    def __init__(self, radius, x=0, y=0):
        super().__init__(x,y) # creates/initializes
        self.radius = radius

    def edge_distance_from_origin(self):
        return abs(self.distance_from_origin() - self.radius)

    def area(self):
        return math.pi*(self.radius**2)

    def circumference(self):
        return 2*math.pi*self.radius

    def __eq__(self, other): # avoid infinite recursion
        return self.radius == other.radius and super().__eq__(other)

    def __repr__(self):
        return "circle ({0.radius!r}, {0.x!r})".format(self)

    def __str__(self):
        return repr(self)

>>> import ShapeClass as shape
>>> a = shape.Point(3,4)
>>> a
point (3, 4)
>>> repr(a)
'point (3, 4)'
>>> str(a)
```

```
'(3, 4)'
>>> a.distance_from_origin()
5.0
>>> c = shape.Circle(3,2,1)
>>> c
circle (3, 2)
>>> repr(c)
'circle (3, 2)'
>>> str(c)
'circle (3, 2)'
>>> c.circumference()
18.84955592153876
>>> c.edge_distance_from_origin()
0.7639320225002102
```

5.3 Python Design Patterns

Design patterns are an attempt to bring a formal definition for correctly designed structures to software engineering. There are many different design patterns to solve different general problems.

Decorator Pattern

Decorators (also known as the @ notation) are a tool to elegantly specify some transformation on functions and methods. The decorator pattern allows us to *wrap* an object that provides core functionality with other objects that alter that functionality. For example, the snippet below was copied from the [Google](#) Python Style guide:

```
class C(object):
    def method(self):
        method = my_decorator(method)
```

can be written as

```
class C(object):
    @my_decorator
```

```
def method(self):
```

A full example of a decorator, for a custom benchmarking function, is shown bellow, :

```
[general_problems/oop/do_benchmark.py]

import random

def benchmark(func):
    import time
    def wrapper(*args, **kwargs):
        t = time.clock()
        res = func(*args, **kwargs)
        print("\t%s" % func.__name__, time.clock()-t)
        return res
    return wrapper

@benchmark
def random_tree(n):
    temp = [n for n in range(n)]
    for i in range(n+1):
        temp[random.choice(temp)] = random.choice(temp)
    return temp

if __name__ == '__main__':
    random_tree(10000)

"""
python3 do_benchmark.py
    random_tree 0.04999999999999999
"""
```

The most common decorators are `@classmethod` and `@staticmethod`, for converting ordinary methods to class or static methods.

Observer Pattern

The *observer pattern* is useful when we want to have a core object that maintains certain values, and then having some observers to create serialized copies of that object. This can be implemented by using the `@properties` decorator, placed before our functions (before `def`). This will control attribute access, for example, to make an attribute to be read-only. Properties are used for accessing or setting data instead of *simple accessors or setters*:

```
@property
def radius(self):
    return self.__radius
```

Singleton Pattern

A class follows the *singleton pattern* if it allows exactly one instance of a certain object to exist. Since Python does not have private constructors, we use the `__new__` class method to ensure that only one instance is ever created. When we override it, we first check whether our singleton instance was created. If not, we create it using a super class call:

```
>>> class SinEx:
...     __sing = None
...     def __new__(self, *args, **kwargs):
...         if not self.__sing:
...             self.__sing = super(SinEx, self).__new__(self, *args,
... **kwargs)
...     return self.__sing

>>> x = SinEx()
>>> x
<__main__.SinEx object at 0xb72d680c>
>>> y = SinEx()
>>> x == y
True
>>> y
<__main__.SinEx object at 0xb72d680c>
```

The two objects are equal and are in the same address, so they are the same object.

Chapter 6

Advanced Topics

6.1 Multiprocessing and Threading

Each program in the operational system is a separate *process*. Each process has one or more *threads*. If a process has several threads, they appear to run simultaneously. Two approaches can be used to spread the workload in programs:

Multiple processes Multiple processes have separate regions of memory and can only communicate by special mechanisms. The processor loads and saves a separate set of registers for each thread. It is inconvenient for communication and data sharing. This is handled by the `subprocess` module.

Multiple threads Multiple threads in a single process have access to the same memory. They communicate simply by sharing data, providing ensure of one thread at time, handled by the `threading` module. Threads share the process's resources, including the heap space. But each thread still has its own stack.

Although Python has a threading mechanism, it does not support true parallel execution. However, it is possible to use parallel processes, which in modern operating systems are really efficient.

The subprocess Module

Used to create a pair of “parent-child” programs. The parent program is started by the user and this in turn runs instances of the child program, each with different work to do. Using child processing allow us to take maximum advantage of multicore processor and leaves concurrency issues to be handled by the operational system.

The threading Module

The complexity arises when we want to separate threads to share data, and we need to be careful with a policy for locks and for avoiding *deadlocks*. Each Python program has at least one thread, the main thread. To create multiple threads we use the `threading` module:

- ★ calling `threading.Thread()` and passing a callable object,
- ★ calling `threading.ThreadClass` and `threading.ThreadSubclass`.

The `queue.Queue` class can handle all the locking internally: we can rely on it to serialize accesses, meaning that only one thread at time has access to the data (FIFO). The program will not terminate while it has any threads running.

It might create a problem since once the worker threads have done their work, they are finished but they are technically still running. The solution is to transform threads into *daemons*. In this case, the program will terminate as soon as there is no daemon threads running. The method `queue.Queue.join()` blocks the end until the queue is empty.

Mutexes and Semaphores

A *mutex* is like a lock. Mutexes are used in parallel programming to ensure that only one thread can access a shared resource at a time. For example, say one thread is modifying an array. When it has gotten halfway through the array, the processor switches to another thread. If we were not using mutexes, the thread could try to modify the array as well, at the same time.

Conceptually, a mutex is an integer that starts at 1. Whenever a thread needs to alter the array, it “locks” the mutex. This causes the thread to wait until the number is positive and then decreases it by one. When the thread

is done modifying the array, it “unlocks” the mutex, causing the number to increase by 1. If we are sure to lock the mutex before modifying the array and to unlock it when we are done, then we know that no two threads will modify the array at the same time.

Semaphores are more general than mutexes. A semaphore’s integer may start at a number greater than 1. The number at which a semaphore starts is the number of threads that may access the resource at once. Semaphores support “wait” and “signal” operations, which are analogous to the “lock” and “unlock” operations of mutexes.

Deadlock and Spinlock

Deadlock is a problem that sometimes arises in parallel programming, when two threads are stuck indefinitely. We can prevent deadlock if we assign an order to our locks and require that locks will always be acquired in order (this is a very general and a not precise approach).

Spinlock is a form of *busy waiting*, that can be useful for high-performance computing (HPC) (when the entire system is dedicated to a single application and exactly one thread per core). It takes less time than a semaphore.

The **Google** Python Style Guide for Threading

Do not rely on the atomicity of built-in types (data types such as dictionaries appear to have atomic operations, but they are actually based on private methods such as `__hash__` or `__eq__`). Queue module’s data type as the preferred way to communicate data between threads. Otherwise, use the `threading` module and its locking primitives.

6.2 Good Practices

Virtual Environments

The more projects you have, the more likely it is that you will be working with different versions of Python itself, or at least different versions of Python libraries. For this reason, we use virtual environments (`virtualenvs` or `venvs`).

Virtualenv

Following <http://docs.python-guide.org/en/latest/dev/virtualenvs/>:

```
Create a virtual environment:  
$ virtualenv venv  
  
To begin using the virtual environment, it needs to be activated:  
$ source venv/bin/activate  
  
If you are done working in the virtual environment for the moment,  
you can deactivate it:  
$ deactivate  
  
To delete a virtual environment, just delete its folder.
```

Virtualenvwrapper

Virtualenvwrapper provides a set of commands and also places all your virtual environments in one place. Following <http://virtualenvwrapper.com>:

```
$ pip install virtualenvwrapper  
  
In your bashrc file:  
export WORKON_HOME=$HOME/.virtualenvs  
export PROJECT_HOME=$HOME/Devel  
source /usr/local/bin/virtualenvwrapper.sh  
  
Basic Usage:  
Create a virtual environment:  
$ mkvirtualenv test  
  
Check if it is working:  
$ which python  
  
Work on a virtual environment:  
$ workon test  
  
Deactivating is still the same:  
$ deactivate
```

```
To delete:  
$ rmvirtualenv test  
  
Check the configuration:  
$ pip freeze  
  
Other useful commands  
lsvirtualenvList  
cdvirtualenv  
cdsitepackages  
lssitepackages
```

Debugging

The Python debugger, pdb, can be found at <http://pymotw.com/2/pdb/>.

Interactive Running:

If you have some code in a source file and you want to explore it interactively, you can run Python with the `-i` switch, like this: `python -i example.py`.

It also can be used in the command line:

```
>>> python3 -m pdb program.py
```

or added as a module as the first statement of the function we want to examine:

```
import pdb  
pdb.set_trace()
```

To perform the inspection, type: `s` for step, `p` for point, and `n` for next line, `list` to see the next 10 lines, and `help` for help.

Profiling

If a program runs very slowly or consumes far more memory than expected, the problem is most often due to our choice of algorithms or data structures

or due to some inefficient implementation. Some performance verification:

- ★ prefer tuples to list with read-only data;
- ★ use *generators* rather than large lists or tuples to iteration;
- ★ when creating large strings out of small strings, instead of concatenating the small, accumulate them all in a list and `join` the list of strings in the end. A good examples is given by the [Google Python Style guide](#):

```
[Good]
items = ['<table>']
for last_name, first_name in employee_list:
    items.append('<tr><td>%s, %s</td></tr>' % (last_name,
        first_name))
items.append('</table>')
employee_table = ''.join(items)

[Bad]
employee_table = '<table>'
for last_name, first_name in employee_list:
    employee_table += '<tr><td>%s, %s</td></tr>' % (last_name,
        first_name)
employee_table += '</table>'
```

The `cProfile` Package:

Provides a detailed breakdown of call times and can be used to find performance bottlenecks.

```
import cProfile
cProfile.run('main()')
```

You can run it by typing:

```
$ python3 -m cProfile -o profile.day mymodule.py
$ python3 -m pstats
```

The timeit Package:

Used for timing small pieces of the code:

```
>>> import timeit
>>> timeit.timeit("x = 2 + 2")
0.034976959228515625
>>> timeit.timeit("x = sum(range(10))")
0.92387008666992188
> python -m timeit -s "import mymodule as m" "m.myfunction()"
```

The following code shows a simple example of how to time a function:

```
[general_problems/modules/using_time_module.py]

import time

def sumOfN2(n):
    ''' a simple example of how to time a function '''
    start = time.time()
    theSum = 0
    for i in range(1,n+1):
        theSum = theSum + i
    end = time.time()
    return theSum,end-start

if __name__ == '__main__':
    n = 5
    print("Sum is %d and required %10.7f seconds"%sumOfN2(n))
    n = 200
    print("Sum is %d and required %10.7f seconds"%sumOfN2(n))
```

6.3 Unit Testing

It is good practice to write tests for individual functions, classes, and methods, to ensure they behave to the expectations. Python's standard library provides two unit testing modules: `doctest` and `unittest`. There are also third part testing tools: `nose` and `py.test`.

Test Nomenclature

Test fixtures The code necessary to set up a test (for example, creating an input file for testing and deleting afterwards).

Test cases The basic unit of testing.

Test suites Collection of test cases, created by the subclass `unittest.TestCase`, where each method has a name beginning with “test”.

Test runner An object that executes one or more test suites.

doctest

Use it when writing the tests inside the modules and functions’ docstrings. Then just add three line in the end:

```
if __name__ == "__main__":
    import doctest
    doctest.testmod()
```

To run the program’s `doctest`, there are two approaches:

- ★ Importing the `doctest` module and then run the program:

```
$ python3 program.py -v
```

- ★ Creating a separate test program using the `unittest` module, which is modeled on Java’s JUnit unittesting library.

```
import doctest
import unittest
import module_to_be_tested

suite = unittest.TestSuite()
suite.addTest(doctest.DocTestSuite(module_to_be_tested))
runner = unittest.TextTestRunner()
print(runner.run(suite))
```

pytest

Very easy to use: just include a function that starts with `test` in a file that starts with `test`:

```
Install with:  
pip install pytest  
Example:  
def func(x):  
    return x + 1  
def test_answer():  
    assert func(3) == 51  
Run with  
py.test  
python -m pytest  
In case of more than one test:  
py.test -q test_class.py  
Create a pytest standalone script:  
py.test --genscript=runtests.py  
Dropping to pdb:  
py.test --pdb  
\begin
```


Part II

Algorithms are Fun! It's time
to add some sauce into our
flight! In this second part we
will learn how to make the
computer become our awesome
spaceship!

Chapter 7

Abstract Data Structures

An *abstract data type* (ADT) is a mathematical model for a certain class of data structures that have similar behavior. Different classes of abstract data types have many different but functionally equivalent data structures that implement them.

Data structures can be classified as either *contiguous* or *linked*, depending whether they are based on arrays or pointers. In Python, for instance, contiguously-allocated structures (composed of single slabs of memory) include strings, lists, tuples, and dictionaries. In the following sections we will see examples of some more specialized continuous structures and examples of some linked data structures (distinct chunks of memory bound together by pointers).

7.1 Stacks

A *stack* is a linear data structure that can be accessed only at one of its ends (which we will refer as the top) for either storing or retrieving. In other words, array access of elements in a stack is restricted and they are an example of a last-in-first-out (LIFO) structure. You can think of a stack as a huge pile of books on your desk. Stacks need to have the following operations running at $\mathcal{O}(1)$:

`push` Insert an item at the top of the stack.

`pop` Remove an item from the top of the stack.

`top/peek` Look up the element on the top.

`empty/size` Check whether the stack is empty or return its size.

Stacks in Python can be implemented with lists and the methods `append()` and `pop()` (without an explicit index):

```
[adt/stacks/stack.py]

class Stack(list):
    ''' define the stack class '''
    def __init__(self):
        self.items = []

    def isEmpty(self):
        return self.items == []

    def push(self, items):
        self.items.append(items)

    def pop(self):
        if not self.isEmpty():
            return self.items.pop()
        else:
            raise Exception('Stack is empty!')

    def peek(self):
        return self.items[-1]

    def size(self):
        return len(self.items)

def main():
    stack = Stack()
    stack.push(1)
    stack.push(2)
    stack.push(3)
    print(stack.size())
    print(stack.peek())
    print(stack.pop())
    print(stack.peek())
```

```
if __name__ == '__main__':
    main()
```

Another approach we can use to implement a stack (or any abstract structure) is by thinking of it as a container of *nodes* (objects) following a LIFO order.¹

```
class Node(object):
    def __init__(self, value=None):
        self.value = value
        self.next = None
```

and then building the stack as a collection of nodes:

```
[adt/stacks/linked_stack.py]

class Node(object):
    def __init__(self, value=None):
        self.value = value
        self.next = None

class StackwithNodes(object):
    ''' Define a Stack with nodes'''
    def __init__(self):
        self.top = None

    def isEmpty(self):
        return bool(self.top)

    def pop(self):
        node = self.top
        if node:
            self.top = node.next
            return node.value
        else:
            raise Exception('Stack is empty.')

    def push(self, value):
```

¹We will use similar a Node Class in many examples in the rest of these notes.

```

node = Node(value)
node.next = self.top
self.top = node

def size(self):
    node = self.top
    if node not None: num_nodes = 1
    else: return 0
    node = node.next
    while node:
        num_nodes += 1
        node = node.next
    return num_nodes

def peek(self):
    return self.top.value

def main():
    stack = StackwithNodes()
    stack.push(1)
    stack.push(2)
    stack.push(3)
    print(stack.size())
    print(stack.peek())
    print(stack.pop())
    print(stack.peek())

if __name__ == '__main__':
    main()

```

Stacks are suitable for *depth-first traversal* algorithms in *graphs*, as we will see in future chapters.

7.2 Queues

A *queue*, differently of a stack, is a structure where the first enqueued element (at the back) will be the first one to be dequeued (when it is at the front),

i.e., a queue is a *first-in-first-out* (FIFO) structure. You can think of a queue as a line of people waiting for a roller-coaster ride. Array access of elements in queues is also restricted and queues should have the following operations running at $\mathcal{O}(1)$:

`enqueue` Insert an item at the back of the queue.

`dequeue` Remove an item from the front of the queue.

`peek/front` Retrieve an item at the front of the queue without removing it.

`empty/size` Check whether the queue is empty or give its size.

The example below shows a class for a queue in Python:

```
[adt/queues/queue.py]

class Queue(object):
    ''' a class for a queue '''
    def __init__(self):
        self.items = []

    def isEmpty(self):
        return self.items == []

    def enqueue(self, item):
        self.items.insert(0, item)

    def dequeue(self):
        return self.items.pop()

    def size(self):
        return len(self.items)

    def peek(self):
        if not self.isEmpty():
            return self.items[-1]
        else:
            raise Exception('Queue is empty.')

    def size(self):
        return len(self.items)
```

```

def main():
    queue = Queue()
    queue.enqueue(1)
    queue.enqueue(2)
    queue.enqueue(3)
    print(queue.size())
    print(queue.peek())
    print(queue.dequeue())
    print(queue.peek())

if __name__ == '__main__':
    main()

```

However, we have learned that the method `insert()` for lists in Python is very inefficient (remember, lists only work on $\mathcal{O}(1)$ when we append or pop at/from their end, because otherwise all of the other elements would have to be shifted in memory). We can be smarter than that and write an efficient queue using two stacks (two lists) instead of one:

```

[adt/queues/queue_from_two_stacks.py]

class Queue(object):
    ''' an example of a queue implemented from 2 stacks '''
    def __init__(self):
        self.in_stack = []
        self.out_stack = []

    def enqueue(self, item):
        return self.in_stack.append(item)

    def dequeue(self):
        if self.out_stack:
            return self.out_stack.pop()
        while self.in_stack:
            self.out_stack.append(self.in_stack.pop())
        if not self.out_stack:
            raise Exception("Queue empty!")
        return self.out_stack.pop()

```

```
def size(self):
    return len(self.in_stack) + len(self.out_stack)

def peek(self):
    if self.out_stack:
        return self.out_stack[-1]
    while self.in_stack:
        self.out_stack.append(self.in_stack.pop())
    if self.out_stack:
        return self.out_stack[-1]
    else:
        return None

def main():
    queue = Queue()
    queue.enqueue(1)
    queue.enqueue(2)
    queue.enqueue(3)
    print(queue.size())
    print(queue.peek())
    print(queue.dequeue())
    print(queue.peek())

if __name__ == '__main__':
    main()
```

Another approach is to implement a queue as a container for nodes, as we have done for stacks, but now the nodes are inserted and removed in a FIFO order:

```
[adt/queues/linked_queue.py]

class Node(object):
    def __init__(self, value):
        self.value = value
        self.next = None

class LinkedQueue(object):
```

```
''' Queue acts as a container for nodes (objects) that are
   inserted and removed according FIFO'''
def __init__(self):
    self.front = None
    self.back = None

def isEmpty(self):
    return bool(self.front) and bool(self.back)

def dequeue(self):
    if self.front:
        value = self.front.value
        self.front = self.front.next
        return value
    raise Exception('Queue is empty, cannot dequeue.')

def enqueue(self, value):
    node = Node(value)
    if self.front:
        self.back.next = node
    else:
        self.front = node
    self.back = node
    return True

def size(self):
    node = self.front
    if node:
        num_nodes = 1
        node = node.next
        while node:
            num_nodes += 1
            node = node.next
    return num_nodes

def peek(self):
    return self.front.value

def main():
    queue = LinkedQueue()
```

```
queue.enqueue(1)
queue.enqueue(2)
queue.enqueue(3)
print(queue.size())
print(queue.peek())
print(queue.dequeue())
print(queue.peek())

if __name__ == '__main__':
    main()
```

Queues are necessary for *breadth-first traversal* algorithms for graphs, as we will see in future chapters.

7.3 Deques

A *deque* is a double-ended queue, which can roughly be seen as an union of a stack and a queue:

```
[adt/queues/dequeue.py]

class Deque(object):
    ''' a class for a double ended queue '''
    def __init__(self):
        self.items = []

    def isEmpty(self):
        return self.items == []

    def addFront(self, item):
        self.items.append(item)

    def addRear(self, item):
        self.items.insert(0, item)

    def removeFront(self):
        return self.items.pop()
```

```

def removeRear(self):
    return self.items.pop(0)

def size(self):
    return len(self.items)

def __repr__(self):
    return '{}'.format(self.items)

def main():
    dq = Deque()
    dq.addFront(1)
    dq.addFront(2)
    dq.addFront(3)
    dq.addRear(40)
    dq.addRear(50)
    print(dq.size())
    print(dq)

if __name__ == '__main__':
    main()

```

We see again the problem of inserting/removing items in Python's lists in any positions that are not the end. The good news is that Python is your friend and its `collections.deque` gives us an efficient deque, with fast appending and popping from both ends:

```

>>> from collections import deque
>>> q = deque(["buffy", "xander", "willow"])
>>> q
deque(['buffy', 'xander', 'willow'])
>>> q.append("giles")
>>> q
deque(['buffy', 'xander', 'willow', 'giles'])
>>> q.popleft()
'buffy'
>>> q.pop()
'giles'
>>> q
deque(['xander', 'willow'])

```

```
>>> q.appendleft('angel')
>>> q
deque(['angel', 'xander', 'willow'])
```

Note that we can also specify the size of our deque. For example, we could have written `q = deque(maxlen = 4)` in the example above. Another interesting method for deques is `rotate(n)`, which rotated the deque n steps to the right or, if n is negative, to the left.

Interestingly, deques in Python are based on a *doubly linked list*,² not in dynamic arrays. It means that operations such as inserting an item anywhere are fast ($\mathcal{O}(1)$), but arbitrary index accessing can be slow ($\mathcal{O}(n)$).

7.4 Priority Queues and Heaps

A *priority queue* is an abstract data type which is similar to a regular queue or stack, but where each element has a *priority* associated with it. If two elements have the same priority, they are served according to their order in the queue.

A sensible implementation of a priority queue is given by a *heap* data structure and we will use it for most of our examples.

Heaps

Conceptually, a heap is a *binary tree* where each node is smaller (larger) than its children. We will learn about trees in the next chapters but we should already keep in mind that when modifications are made in a *balanced tree*, we can repair its structure with $\mathcal{O}(\log n)$ runtimes. Heaps are generally useful for applications that repeatedly access the smallest (largest) element in the list. Moreover, min-(max-)heap will let you to find the smallest (largest) element in $\mathcal{O}(1)$ and to extract/add/replace it in $\mathcal{O}(\ln n)$.

²Linked lists are another abstract data structure that we will learn about at the end of this chapter. Doubly here means that their nodes have links to the next and to the previous node.

Python's heapq Package

A very efficient heap implementation in Python is given by the `heapq` module, which provides functions to insert and remove items while keeping the sequence as a heap.

We can use the `heapq.heapify(x)` method to transform a list into a heap, in-place, and in $\mathcal{O}(n)$ time:

```
>>> list1 = [4, 6, 8, 1]
>>> heapq.heapify(list1)
>>> list1
[1, 4, 8, 6]
```

Once we have a heap, the `heapq.heappush(heap, item)` method is used to push the item onto it:

```
>>> import heapq
>>> h = []
>>> heapq.heappush(h, (1, 'food'))
>>> heapq.heappush(h, (2, 'have fun'))
>>> heapq.heappush(h, (3, 'work'))
>>> heapq.heappush(h, (4, 'study'))
>>> h
[(1, 'food'), (2, 'have fun'), (3, 'work'), (4, 'study')]
```

The method `heapq.heappop(heap)` is used to pop and return the smallest item from the heap:

```
>>> list1
[1, 4, 8, 6]
>>> heapq.heappop(list1)
1
>>> list1
[4, 6, 8]
```

The method `heapq.heappushpop(heap, item)` is used to push the item on the heap, then it pops and returns the smallest item from the heap. In a similar way, `heapq.heapreplace(heap, item)` will pop and return the smallest item from the heap, and then push the new item. The heap size does not change in any of these methods and they are more efficient than using each method separately.

In addition, many operations can be made using the heap's propriety. For example `heapq.merge(*iterables)` will merge multiple sorted inputs into a single sorted output (returning a iterator):

```
>>> for x in heapq.merge([1,3,5],[2,4,6]):
...     print(x,end="\n")
...
1
2
3
4
5
6
```

The methods `heapq.nlargest(n, iterable[, key])` and `heapq.nsmallest(n, iterable[, key])` will return a list with the n largest and smallest elements from the dataset defined by iterable.

A Class for a Heap

If we want to write ourselves a class for a heap, the fist thing to do is defining a method to create the heap propriety (heapify). The following code does that, implementing a max-heap for a given list:

```
[heap/heapify.py]

class Heapify(object):
    def __init__(self, data=None):
        self.data = data or []
        for i in range(len(data)//2, -1, -1):
            self.__max_heapify__(i)

    def __repr__(self):
        return '{}'.format(self.data)

    def parent(self, i):
        return i >> 1

    def left_child(self, i):
```

```
    return (i << 1) + 1

def right_child(self, i):
    return (i << 1) + 2 # +2 instead of +1 because it's
    0-indexed.

def __max_heapify__(self, i):
    largest = i
    left = self.left_child(i)
    right = self.right_child(i)
    n = len(self.data)
    largest = (left < n and self.data[left] > self.data[i]) and
        left or i
    largest = (right < n and self.data[right] >
        self.data[largest]) and right or largest
    if i is not largest:
        self.data[i], self.data[largest] = self.data[largest],
            self.data[i]
        self.__max_heapify__(largest)

def extract_max(self):
    n = len(self.data)
    max_element = self.data[0]
    self.data[0] = self.data[n - 1]
    self.data = self.data[:n - 1]
    self.__max_heapify__(0)
    return max_element

def test_Heapify():
    l1 = [3, 2, 5, 1, 7, 8, 2]
    h = Heapify(l1)
    assert(h.extract_max() == 8)
    print ("Tests Passed!")

if __name__ == '__main__':
    test_Heapify()
```

A Class for a Priority Queue

To conclude this section, the following example shows how to use the `heapq` package to implement a priority queue class:

```
[heap/PriorityQueueClass.py]

import heapq

class PriorityQueue(object):
    ''' implements a priority queue class '''
    def __init__(self):
        self._queue = []
        self._index = 0 # comparying same priority level

    def push(self, item, priority):
        heapq.heappush(self._queue, (-priority, self._index, item))
        self._index += 1

    def pop(self):
        return heapq.heappop(self._queue)[-1]

class Item:
    def __init__(self, name):
        self.name = name
    def __repr__(self):
        return "Item({!r})".format(self.name)

def test_PriorityQueue():
    ''' push and pop are all O(logN) '''
    q = PriorityQueue()
    q.push(Item('test1'), 1)
    q.push(Item('test2'), 4)
    q.push(Item('test3'), 3)
    assert(str(q.pop()) == "Item('test2')")
    print('Tests passed!'.center(20, '*'))

if __name__ == '__main__':
    test_PriorityQueue()
```

7.5 Linked Lists

A *linked list* is like a stack (new elements are added to the head) or a queue (new elements are added to the tail), except that we can peek any element in the structure on $\mathcal{O}(1)$ (not only the elements at the ends). In general, a linked list is simply a linear list of nodes containing a value and a pointer (a reference) to the next node. The last node is an exception, having a `null` reference (which in Python is `None`).

To understand these concepts better, let us start defining a node class, with some *get* and *set* methods:

```
[abstract_structures/linked_lists/node.py]

class Node(object):
    def __init__(self, value=None, pointer=None):
        self.value = value
        self.pointer = pointer

    def getData(self):
        return self.value

    def getNext(self):
        return self.pointer

    def setData(self, newdata):
        self.value = newdata

    def setNext(self, newpointer):
        self.pointer = newpointer

if __name__ == '__main__':
    L = Node("a", Node("b", Node("c", Node("d"))))
    assert(L.pointer.pointer.value=='c')

    print(L.getData())
    print(L.getNext().getData())
    L.setData('aa')
    L.setNext(Node('e'))
```

```
print(L.getData())
print(L.getNext().getData())
```

We can obtain a LIFO linked list as a collection of these nodes:

```
''' Implement a unordered linked list, i.e. a LIFO linked list
    (like a stack) '''

from node import Node

class LinkedListLIFO(object):

    def __init__(self):
        self.head = None
        self.length = 0

    # print each node's value, starting from the head
    def _printList(self):
        node = self.head
        while node:
            print(node.value)
            node = node.pointer

    # delete a node, given the previous node
    def _delete(self, prev, node):
        self.length -= 1
        if not prev:
            self.head = node.pointer
        else:
            prev.pointer = node.pointer

    # add a new node, pointing to the previous node
    # in the head
    def _add(self, value):
        self.length += 1
        self.head = Node(value, self.head)

    # locate node with some index
    def _find(self, index):
        prev = None
```

```
node = self.head
i = 0
while node and i < index:
    prev = node
    node = node.pointer
    i += 1
return node, prev, i

# locate node by value
def _find_by_value(self, value):
    prev = None
    node = self.head
    found = 0
    while node and not found:
        if node.value == value:
            found = True
        else:
            prev = node
            node = node.pointer
    return node, prev, found

# find and delete a node by index
def deleteNode(self, index):
    node, prev, i = self._find(index)
    if index == i:
        self._delete(prev, node)
    else:
        print('Node with index {} not found'.format(index))

# find and delete a node by value
def deleteNodeByValue(self, value):
    node, prev, found = self._find_by_value(value)
    if found:
        self._delete(prev, node)
    else:
        print('Node with value {} not found'.format(value))

if __name__ == '__main__':
    ll = LinkedListLIFO()
    for i in range(1, 5):
```

```
ll._add(i)
print('The list is:')
ll._printList()
print('The list after deleting node with index 2:')
ll.deleteNode(2)
ll._printList()
print('The list after deleting node with value 3:')
ll.deleteNodeByValue(2)
ll._printList()
print('The list after adding node with value 15')
ll._add(15)
ll._printList()
print("The list after deleting everything...")
for i in range(ll.length-1, -1, -1):
    ll.deleteNode(i)
ll._printList()
```

Finally, we can also write a class for a FIFO linked list:

```
[adt/linked_lists/likedlist_fifo.py]

''' A class for a linked list that has the nodes in a FIFO order
(such as a queue)'''

from node import Node

class LinkedListFIFO(object):

    def __init__(self):
        self.head = None
        self.length = 0
        self.tail = None # this is different from ll.lifo

    # print each node's value, starting from the head
    def _printList(self):
        node = self.head
        while node:
```

```
        print(node.value)
        node = node.pointer

        # add a node in the first position
        # read will never be changed again while not empty
        def _addFirst(self, value):
            self.length = 1
            node = Node(value)
            self.head = node
            self.tail = node

            # delete a node in the first position, ie
            # when there is no previous node
            def _deleteFirst(self):
                self.length = 0
                self.head = None
                self.tail = None
                print('The list is empty.')

                # add a node in a position different from head,
                # ie, in the end of the list
                def _add(self, value):
                    self.length += 1
                    node = Node(value)
                    if self.tail:
                        self.tail.pointer = node
                    self.tail = node

                    # add nodes in general
                    def addNode(self, value):
                        if not self.head:
                            self._addFirst(value)
                        else:
                            self._add(value)

                        # locate node with some index
                        def _find(self, index):
                            prev = None
                            node = self.head
```

```
i = 0
while node and i < index:
    prev = node
    node = node.pointer
    i += 1
return node, prev, i

# delete nodes in general
def deleteNode(self, index):
    if not self.head or not self.head.pointer:
        self._deleteFirst()
    else:
        node, prev, i = self._find(index)
        if i == index and node:
            self.length -= 1
            if i == 0 or not prev:
                self.head = node.pointer
            else:
                prev.pointer = node.pointer
            if not self.tail == node:
                self.tail = prev
        else:
            print('Node with index {} not found'.format(index))

if __name__ == '__main__':
    ll = LinkedListFIFO()
    for i in range(1, 5):
        ll.addNode(i)
    print('The list is:')
    ll._printList()
    print('The list after deleting node with index 2:')
    ll.deleteNode(2)
    ll._printList()
    print('The list after adding node with value 15')
    ll._add(15)
    ll._printList()
    print("The list after deleting everything...")
    for i in range(ll.length-1, -1, -1):
        ll.deleteNode(i)
```

11._printList()

Linked lists have a dynamic size at runtime and they are good for when you have an unknown number of items to store. Insertion is $\mathcal{O}(1)$ but deletion and searching can be $\mathcal{O}(n)$ because locating an element in a linked list is slow and is it done by a *sequential search*. Traversing a linked list backward or sorting it are even worse, being both $\mathcal{O}(n^2)$. A good trick to obtain deletion of a node i at $\mathcal{O}(1)$ is copying the data from $i + 1$ to i and then to deleting the node $i + 1$.

7.6 Hash Tables

A hash table is used to associate keys to values, so that each key is associated with one or zero values. Each key should be able to compute a **hash** function. The hash table consists of an array of hash buckets. For example, if a hash value is 42, and there are 5 buckets, one use the **mod** function to decide to put the mapping in bucket $42 \bmod 5$, which is 2.

A problem arises when two keys hash to the same bucket, which is called a **collision**. One way to deal with this is to store a linked list of key-value pairs for each bucket.

Insertion, removal, and lookup take $\mathcal{O}(1)$ time, provided that the hash is random. In the worst-case, each key hashes to the same bucket, so each operation takes $\mathcal{O}(n)$ time.

```
[abstract_structures/Hash_Tables/Hash_Table.py]

#!/usr/bin/python

__author__ = "Mari Wahl"
__email__ = "marina.wahl@gmail.com"

''' design a Hash table using chaining to avoid collisions.'''


#import abstract_structures.linked_list.linked_list_fifo
#import abstract_structures.linked_list.node
```

```
from linked_list_fifo import LinkedListFIFO

class HashTableLL(object):
    def __init__(self, size):
        self.size = size
        self.slots = []
        self._createHashTable()

    def _createHashTable(self):
        for i in range(self.size) :
            self.slots.append(LinkedListFIFO())

    def _find(self, item):
        return item%self.size

    def _add(self, item):
        index = self._find(item)
        self.slots[index].addNode(item)

    def _delete(self, item):
        index = self._find(item)
        self.slots[index].deleteNode(item)

    def _print(self):
        for i in range(self.size):
            print('\nSlot {}:{}.'.format(i))
            print(self.slots[i]._printList())

def test_hash_tables():
    H1 = HashTableLL(3)
    for i in range (0, 20):
        H1._add(i)
    H1._print()
    print('\n\nNow deleting:')
    H1._delete(0)
```

```
H1._delete(1)
H1._delete(2)
H1._delete(0)

H1._print()

if __name__ == '__main__':
    test_hash_tables()
```

7.7 Additional Exercises

Stacks

Stacks are very useful when data has to be sorted and retrieved in *reverse order*. In the example below we use our Stack class to reverse a string:

```
[adt/stacks/reverse_string_with_stack.py]

import sys
import stack

def reverse_string_with_stack(str1):
    s = stack.Stack()
    revStr = ''
    for c in str1:
        s.push(c)
    while not s.isEmpty():
        revStr += s.pop()
    return revStr

def test_reverse_string_with_stack():
    str1 = 'Buffy is a Slayer!'
    assert(reverse_string_with_stack(str1) == '!reyalS a si yffuB')
    print('Tests passed!')

if __name__ == '__main__':
    test_reverse_string_with_stack()
```

The following example uses a stack to balance parenthesis in a string:

```
[adt/stacks/balance_parenthesis_str_stack.py]

from stack import Stack

def balance_par_str_with_stack(symbolString):
    s = Stack()
    balanced = True
    index = 0
    while index < len(symbolString) and balanced:
```

```

symbol = symbolString[index]
if symbol == "(":
    s.push(symbol)
else:
    if s.isEmpty():
        balanced = False
    else:
        s.pop()
index = index + 1

if balanced and s.isEmpty():
    return True
else:
    return False

def test_balance_par_str_with_stack(module_name='this module'):
    print(balance_par_str_with_stack('((()))'))
    print(balance_par_str_with_stack('(()'))
    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_balance_par_str_with_stack()

```

The following example uses a stack to transform a decimal number to binary number:

```

[atf/stacks/dec2bin_with_stack.py]

from stack import Stack

def dec2bin_with_stack(decnum):
    s = Stack()
    str_aux = ''
    while decnum > 0:
        dig = decnum % 2
        decnum = decnum//2
        s.push(dig)

```

```

while not s.isEmpty():
    str_aux += str(s.pop())
return str_aux

def test_dec2bin_with_stack(module_name='this module'):
    decnum = 9
    assert(dec2bin_with_stack(decnum) == '1001')
    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_dec2bin_with_stack()

```

The following example implements a stack that has $\mathcal{O}(1)$ minimum lookup:

```
[adt/stacks/stack_with_min.py]

class Stack(list):
    def push(self, value):
        if len(self) > 0:
            last = self[-1]
            minimum = self._find_minimum(value, last)
        else:
            minimum = value
        self.minimum = minimum
        self.append(NodeWithMin(value, minimum))

    def _find_minimum(self, value, last_value):
        if value < last_value.minimum:
            return value
        return last_value.minimum

    def min(self):
        return self.minimum

class NodeWithMin(object):
    def __init__(self, value, minimum):
        self.value = value

```

```

        self.minimum = minimum

    def __repr__(self):
        return str(self.value)

    def min(self):
        return self.minimum

def main():
    stack = Stack()
    stack.push(1)
    stack.push(2)
    stack.push(3)
    node = stack.pop()
    print(node.minimum)
    stack.push(0)
    stack.push(4)
    node = stack.pop()
    print(node.min())
    print(stack.min())
    print(stack)

if __name__ == '__main__':
    main()

```

The following example implements a set of stacks, composed of several stacks. It creates a new stack when the previous stack exceeds capacity. The `push` and `pop` methods are identical to a single stack:

```

[adt/stacks/set_of_stacks.py]

class SetOfStacks(list):
    def __init__(self, capacity=4):
        self.stacks = []
        self.last_stack = []
        self.capacity = capacity
        self.stacks.append(self.last_stack)

    def __repr__(self):

```

```
    return str(self.stacks)

def push(self, value):
    last_stack = self.last_stack
    if len(last_stack) is self.capacity:
        last_stack = []
        self.last_stack = last_stack
        self.stacks.append(last_stack)
    last_stack.append(value)

def pop(self):
    last_stack = self.last_stack
    value = last_stack.pop()
    if len(last_stack) is 0:
        self.stacks.pop()
        self.last_stack = self.stacks[-1]
    return value

def main():
    stack = SetOfStacks()
    stack.push(1)
    stack.push(2)
    stack.push(3)
    stack.push(4)
    stack.push(5)
    stack.push(6)
    print(stack)
    stack.pop()
    stack.pop()
    stack.pop()
    print(stack)

if __name__ == '__main__':
    main()
```

Queues

The example below uses the concepts of a queue to rotate an array from right to left for a given number n .³

```
[adt/queues/rotating_array.py]

def rotating_array(seq, n):
    myqueue = []
    for i in range(n):
        myqueue.append(seq.pop())
    myqueue.reverse()
    return myqueue + seq

def test_rotating_array(module_name='this module'):
    seq = [1, 2, 3, 4, 5, 6, 7]
    n = 4
    assert(rotating_array(seq, N) == [4, 5, 6, 7, 1, 2, 3])

    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_rotating_array()
```

Deques

A nice application for a double-ended queue is verifying whether a string is a palindrome:

```
[adt/queues/palindrome_checker_with_deque.py]

import sys
import string
import collections
```

³We could get the same effect using `collections.deque` with the method `rotate(n)`.

```
def palindrome_checker_with_deque(str1):
    d = collections.deque()
    eq = True
    strip = string.whitespace + string.punctuation + "\\'\\\""
    for s in str1.lower():
        if s not in strip: d.append(s)
    while len(d) > 1 and eq:
        first = d.pop()
        last = d.popleft()
        if first != last:
            eq = False
    return eq

def test_palindrome_checker_with_deque():
    str1 = 'Madam Im Adam'
    str2 = 'Buffy is a Slayer'
    assert(palindrome_checker_with_deque(str1) == True)
    assert(palindrome_checker_with_deque(str2) == False)
    print('Tests passed!')

if __name__ == '__main__':
    test_palindrome_checker_with_deque()
```

Priority Queues and Heaps

The example below uses Python's `heapq` package to find the N largest and smallest items in a sequence:

```
[adt/heap/find_N_largest_smallest_items_seq.py]

import heapq

def find_N_largest_items_seq(seq, N):
    return heapq.nlargest(N, seq)

def find_N_smallest_items_seq(seq, N):
    return heapq.nsmallest(N, seq)
```

```

def find_smallest_items_seq_heap(seq):
    ''' find the smallest items in a sequence using heapify first'''
    ''' heap[0] is always the smallest item '''
    heapq.heapify(seq)
    return heapq.heappop(seq)

def find_smallest_items_seq(seq):
    ''' if it is only one item, min() is faster '''
    return min(seq)

def find_N_smallest_items_seq_sorted(seq, N):
    ''' N ~ len(seq), better sort instead'''
    return sorted(seq) [:N]

def find_N_largest_items_seq_sorted(seq, N):
    ''' N ~ len(seq), better sort instead'''
    return sorted(seq)[len(seq)-N:]

def test_find_N_largest_smallest_items_seq(module_name='this
module'):
    seq = [1, 3, 2, 8, 6, 10, 9]
    N = 3
    assert(find_N_largest_items_seq(seq, N) == [10, 9, 8])
    assert(find_N_largest_items_seq_sorted(seq, N) == [8, 9, 10])
    assert(find_N_smallest_items_seq(seq, N) == [1,2,3])
    assert(find_N_smallest_items_seq_sorted(seq, N) == [1,2,3])
    assert(find_smallest_items_seq(seq) == 1)
    assert(find_smallest_items_seq_heap(seq) == 1)

    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_find_N_largest_smallest_items_seq()

```

The following example uses Python's `heapq` package to merge a two sorted sequences with little overhead.⁴

```
[adt/heap/merge_sorted_seqs.py]

import heapq

def merge_sorted_seqs(seq1, seq2):
    result = []
    for c in heapq.merge(seq1, seq2):
        result.append(c)
    return result

def test_merge_sorted_seq(module_name='this module'):
    seq1 = [1, 2, 3, 8, 9, 10]
    seq2 = [2, 3, 4, 5, 6, 7, 9]
    seq3 = seq1 + seq2
    assert(merge_sorted_seq(seq1, seq2) == sorted(seq3))

    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_merge_sorted_seq()
```

Linked List

Find the kth Element from the End of a Linked List

```
[adt/linked_lists/find_kth_from_the_end.py]

''' Find the mth-to-last element of a linked list.
One option is having two pointers, separated by m. P1 start at
the roots
(p1 = self.root) and p2 is m-behind pointer, which is created
when p1 is at m.
```

⁴Note that the result would not be sorted if we just added both lists.

```

    When p1 reach the end, p2 is the node. '''

from linked_list_fifo import LinkedListFIFO
from node import Node

class LinkedListFIFO_find_kth(LinkedListFIFO):

    def find_kth_to_last(self, k):
        p1, p2 = self.head, self.head
        i = 0
        while p1:
            if i > k:
                try:
                    p2 = p2.pointer
                except:
                    break
            p1 = p1.pointer
            i += 1
        return p2.value

if __name__ == '__main__':
    ll = LinkedListFIFO_find_kth()
    for i in range(1, 11):
        ll.addNode(i)
    print('The Linked List:')
    print(ll._printList())
    k = 3
    k_from_last = ll.find_kth_to_last(k)
    print("The %dth element to the last of the LL of size %d is %d"
          %(k, ll.length, k_from_last))

```

Partitioning a Linked List in an Elements

```

[adt/linked_lists/part_linked_list.py]

''' This function divides a linked list in a value, where
everything smaller than this value
goes to the front, and everything large goes to the back:'''

```

```
from linked_list_fifo import LinkedListFIFO
from node import Node


def partList(ll, n):

    more = LinkedListFIFO()
    less = LinkedListFIFO()

    node = ll.head

    while node:
        item = node.value

        if item < n:
            less.addNode(item)

        elif item > n:
            more.addNode(item)

        node = node.pointer

    less.addNode(n)
    nodemore = more.head

    while nodemore:
        less.addNode(nodemore.value)
        nodemore = nodemore.pointer

    return less


if __name__ == '__main__':
    ll = LinkedListFIFO()
    l = [6, 7, 3, 4, 9, 5, 1, 2, 8]
    for i in l:
        ll.addNode(i)

    print('Before Part')
```

```

ll._printList()

print('After Part')
newll = partList(ll, 6)
newll._printList()

```

A Doubled Linked List FIFO

```

[adt/linked_lists/doubled_linked_list_fifo.py]

''' Implement a double-linked list, which is very simple, we just
    need inherits
from a Linked List Class and add an attribute for previous.'''

from linked_list_fifo import LinkedListFIFO

class dNode(object):
    def __init__(self, value=None, pointer=None, previous=None):
        self.value = value
        self.pointer = pointer
        self.previous = previous

class dLinkedList(LinkedListFIFO):

    # print each node's value, starting from tail
    def printListInverse(self):
        node = self.tail
        while node:
            print(node.value)
            try:
                node = node.previous
            except:
                break

    # add a node in a position different from head,
    # ie, in the end of the list
    def _add(self, value):
        self.length += 1

```

```
node = dNode(value)
if self.tail:
    self.tail.pointer = node
    node.previous = self.tail
self.tail = node

# delete a node in some position
def _delete(self, node):
    self.length -= 1
    node.previous.pointer = node.pointer
    if not node.pointer:
        self.tail = node.previous

# locate node with some index
def _find(self, index):
    node = self.head
    i = 0
    while node and i < index:
        node = node.pointer
        i += 1
    return node, i

# delete nodes in general
def deleteNode(self, index):
    if not self.head or not self.head.pointer:
        self._deleteFirst()
    else:
        node, i = self._find(index)
        if i == index:
            self._delete(node)
        else:
            print('Node with index {} not found'.format(index))

if __name__ == '__main__':
    from collections import Counter

    ll = dLinkedList()
    for i in range(1, 5):
```

```

ll.addNode(i)
print('Printing the list...')
ll._printList()
print('Now, printing the list inversely...')
ll.printListInverse()
print('The list after adding node with value 15')
ll._add(15)
ll._printList()
print("The list after deleting everything...")
for i in range(ll.length-1, -1, -1):
    ll.deleteNode(i)
ll._printList()

```

Check whether a Linked List is a Palindrome

```

[adt/linked_lists/check_pal.py]
''' Given a linked list, check if the nodes form a palindrome '''

from linked_list_fifo import LinkedListFIFO, Node
from node import Node

def isPal(l1):

    if len(l1) < 2:
        return True
    if l1[0] != l1[-1]:
        return False

    return isPal(l1[1:-1])

def checkLlPal(ll):
    node = ll.head
    l = []

    while node:
        l.append(node.value)
        node = node.pointer

```

```
    return isPal(l)

if __name__ == '__main__':
    ll = LinkedListFIFO()
    ll = [1, 2, 3, 2, 1]

    for i in ll:
        ll.addNode(i)

    assert(checkllPal(ll) == True)

    ll.addNode(2)
    ll.addNode(3)

    assert(checkllPal(ll) == False)
```

Summing Digits in in Linked Lists

```
[adt/linked_lists/sum_linked_list.py]

''' Supposing two linked lists representing numbers, such that in
each of their
nodes they carry one digit. This function sums the two numbers
that these
two linked lists represent, returning a third list representing
the sum:'''

from linked_list_fifo import LinkedListFIFO
from node import Node


class LinkedListFIFOYield(LinkedListFIFO):

    # print each node's value, starting from the head
    def _printList(self):
        node = self.head
```

```
        while node:
            yield(node.value)
            node = node.pointer

def sumlls(l1, l2):

    lsum = LinkedListFIFOYield()
    dig1 = l1.head
    dig2 = l2.head
    pointer = 0

    while dig1 and dig2:
        d1 = dig1.value
        d2 = dig2.value
        sum_d = d1 + d2 + pointer
        if sum_d > 9:
            pointer = sum_d//10
            lsum.addNode(sum_d%10)

        else:
            lsum.addNode(sum_d)
            pointer = 0

        dig1 = dig1.pointer
        dig2 = dig2.pointer

    if dig1:
        sum_d = pointer + dig1.value
        if sum_d > 9:
            lsum.addNode(sum_d%10)
        else:
            lsum.addNode(sum_d)
        dig1 = dig1.pointer

    if dig2:
        sum_d = pointer + dig2.value
        if sum_d > 9:
            lsum.addNode(sum_d%10)
        else:
```

```
        lsum.addNode(sum_d)
        dig2 = dig2.pointer

    return lsum

if __name__ == '__main__':
    l1 = LinkedListFIFOYield() # 2671
    l1.addNode(1)
    l1.addNode(7)
    l1.addNode(6)
    l1.addNode(2)

    l2 = LinkedListFIFOYield() # 455
    l2.addNode(5)
    l2.addNode(5)
    l2.addNode(4)

    lsum = sumlls(l1, l2)
    l = list(lsum._printList())
    for i in reversed(l):
        print i
```

Find a Circular Linked List

```
[adt/linked_lists/circular_linked_list.py]

''' implement a function to see whether a linked list is circular.
To implement this, we just need two pointers with different
paces (for example, one goes twice faster)'''

from linked_list_fifo import LinkedListFIFO
from node import Node


class circularLinkedListFIFO(LinkedListFIFO):
    def _add(self, value):
        self.length += 1
```

```
        node = Node(value, self.head)
        if self.tail:
            self.tail.pointer = node
        self.tail = node

def isCircularll(ll):
    p1 = ll.head
    p2 = ll.head

    while p2:
        try:
            p1 = p1.pointer
            p2 = p2.pointer.pointer
        except:
            break

        if p1 == p2:
            return True
    return False

if __name__ == '__main__':
    ll = LinkedListFIFO()
    for i in range(10):
        ll.addNode(i)
    ll._printList()

    print(isCircularll(ll))

    lcirc = circularLinkedListFIFO()
    for i in range(10):
        lcirc.addNode(i)
    print(isCircularll(lcirc))
```

Chapter 8

Asymptotic Analysis

Asymptotic analysis is a method to describe the limiting behavior and the performance of algorithms when applied to very large input datasets. To understand why asymptotic analysis is important, suppose you have to sort a billion of numbers ($n = 10^9$)¹ in a common desktop computer. Suppose that this computer has a CPU clock time of 1 GHz, which roughly means that it executes 10^9 processor cycles (or operations) per second.² Then, for an algorithm that has a runtime of $\mathcal{O}(n^2)$, it would take approximately one billion of seconds to finish the sorting (in the worst case) which means one entire year!

Another way of visualizing the importance of asymptotic analysis is directly looking to the function's behaviour. In the Fig. 8 we have many classes of functions plotted together and it is clear that when n increases, the number of operations for any polynomial or exponential algorithm is infeasible.

8.1 Complexity Classes

A *complexity class* is a set of problems with related complexity. A *reduction* is a transformation of one problem into another problem which is at least

¹Remember that for memory gigabytes means $1024^3 = 2^{30}$ bytes and for storage it means $1000^3 = 10^9$ bytes. Also, integers usually take 2 or 4 bytes each. However, for this example we are simplifying this by saying that a ‘number’ has 1 byte.

²In this exercise we are not considering other factors that would make the processing slower, such as RAM latency, copy cache operations, etc.

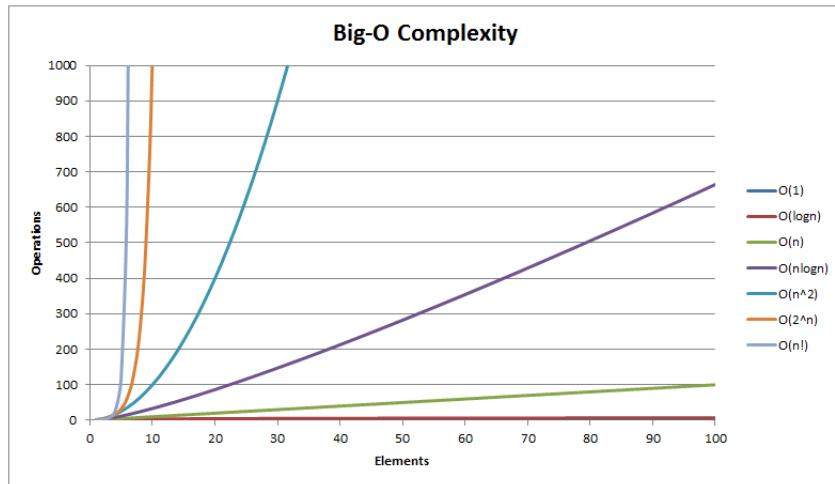


Figure 8.1: Asymptotic behaviour of many classes of functions.

as difficult as the original problem. The most commonly used reduction is a *polynomial-time reduction*, meaning that the reduction process takes polynomial time. A problem is *hard* for a class of problems if every problem in it can be reduced to the original problem.

P

The complexity class of *decision problems* that can be solved on a *deterministic Turing machine in polynomial time* (in the worst case). If we can turn a problem into a decision problem, the result would belong to **P**.

NP

The complexity class of *decision problems* that can be solved on a *non-deterministic Turing machine (NTM) in polynomial time*. In other words, it includes all decision problems whose *yes instances* can be solved in polynomial time with the NTM. A problem is called *complete* if all problems in the class are reduced to it. Therefore, the subclass called *NP-complete* (**NPC**) contains the hardest problems in all of **NP**.

Any problem that is at least as hard (determined by polynomial-time reduction) as any problem in **NP**, but that need not itself be in **NP**, is called **NP-hard**. For example, finding the shortest route through a graph, which is called the *Travelling Salesman (or Salesrep) problem* (TSP).

P=NP?

The class **co-NP** is the class of the *complements* of **NP** problems. For every “yes” answer, we have the “no”, and vice versa. If **NP** is truly asymmetric, then these two classes are different. Although there is overlap between them because all of **P** lies in their intersection: both the yes and no instances in **P** can be solved in polynomial time with an NTM.

What would happen if a **NPC** was found in a intersection of **N** and **co-NP**? First, it would mean that all of **NP** would be inside **co-NP**, so we would show **NP = co-NP** and the asymmetry would disappear. Second, since all of **P** is in this intersection, **P = NP**. If **P = NP**, we could solve any (decision) problem that had a practical (verifiable) solution.

However, it is (strongly) believed that **NP** and **co-NP** are different. For instance, no polynomial solution to the problem of factoring numbers was found, and this problem is in both **NP** and **co-NP**.

8.2 Recursion

The *three laws of recursion* are:

1. A recursive algorithm must have a *base case*.
2. A recursive algorithm must change its state and move toward the base case.
3. A recursive algorithm must call itself, recursively.

For every recursive call, the recursive function has to allocate memory on the *stack* for arguments, return address, and local variables, costing time to push and pop these data onto the stack. Recursive algorithms take at least $\mathcal{O}(n)$ space where n is the depth of the recursive call.

Recursion is very costly when there are duplicated calculations and/or there are overlap among subproblems. In some cases this can cause the stack

to overflow. For this reason, where subproblems overlap, iterative solutions might be a better approach. For example, in the case of the Fibonacci series, the iterative solution runs on $\mathcal{O}(n)$ while the recursive solution runs on exponential runtime.

Recursive Relations

To describe the running time of recursive functions, we use recursive relations:

$$T(n) = a \cdot T(g(n)) + f(n),$$

where a represents the number of recursive calls, $g(n)$ is the size of each subproblem to be solved recursively, and $f(n)$ is any extra work done in the function. The following table shows examples of recursive relations:

$T(n) = T(n - 1) + 1$	$\mathcal{O}(n)$	Processing a sequence
$T(n) = T(n - 1) + n$	$\mathcal{O}(n^2)$	Handshake problem
$T(n) = 2T(n - 1) + 1$	$\mathcal{O}(2^n)$	Towers of Hanoi
$T(n) = T(n/2) + 1$	$\mathcal{O}(\ln n)$	Binary search
$T(n) = T(n/2) + n$	$\mathcal{O}(n)$	Randomized select
$T(n) = 2T(n/2) + 1$	$\mathcal{O}(n)$	Tree transversal
$T(n) = 2T(n/2) + n$	$\mathcal{O}(n \ln n)$	Sort by divide and conquer

Divide and Conquer Algorithms

Recurrences for the *divide and conquer algorithms* have the form:

$$T(n) = a \cdot T(n/b) + f(n),$$

where we have a recursive calls, each with a percentage $1/b$ of the dataset. Summing to this, the algorithm does $f(n)$ of work. To reach the problem of $T(1) = 1$ in the final instance (leaf, as we will learn when we study trees), the *height* is defined as $h = \ln_b n$, Fig. 8.2.

8.3 Runtime in Functions

We are now ready to estimate algorithm runtimes. First of all, if the algorithm does not have any recursive calling, we only need to analyse its data

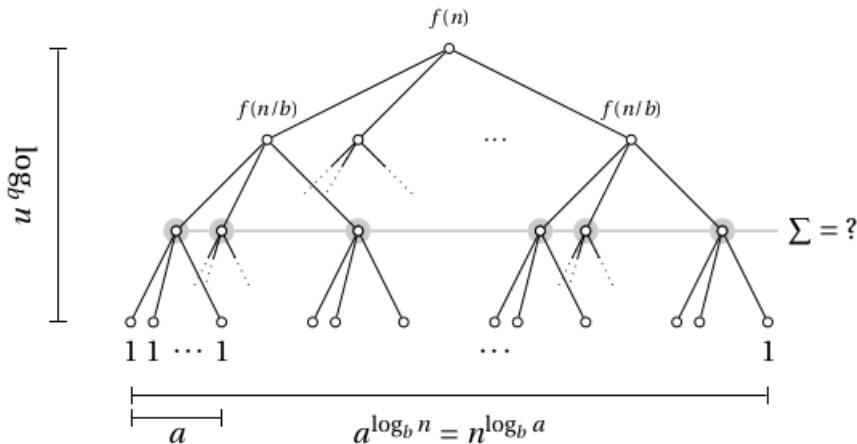


Figure 8.2: Tree illustrating divide and conquer recurrences.

structures and flow blocks. In this case, complexities of code blocks executed one after the other are just added and complexities of nested loops are multiplied.

If the algorithm has recursive calls, we can use the recursive functions from the previous section to find the runtime. When we write a recurrence relation for a function, we must write two equations, one for the general case and one for the base case (that should be $\mathcal{O}(1)$, so that $T(1) = 1$). Keeping this in mind, let us take a look at the example of the algorithm to find the n^{th} element in a Fibonacci sequence, which is known as to be exponential:

```
[general_problems/numbers/find_fibonacci_seq.py]

def find_fibonacci_seq_rec(n):
    if n < 2: return n
    return find_fibonacci_seq_rec(n - 1) +
           find_fibonacci_seq_rec(n - 2)
```

Here, $g(n)$ s are $n - 2$ and $n - 1$, a is 2, and $f(n)$ is 1, so the recursive relation in this case is

$$T(n) = 2T(n - 1) + 1.$$

Now let us open this equation for each next recursion:

$$T(n) = 2^2T(n - 2) + 2 \rightarrow 2^kT(n - k) + k\dots$$

We need to make sure that the function have $\mathcal{O}(1)$ in the base case, where it is $T(1) = 1$, this means that $n - k = 1$ or $k = n - 1$. So plugging back into the equation, we have:

$$T(n) = 2^{n-1} + n - 1 \sim 2^n. \quad (8.3.1)$$

We have indeed proved that **this algorithm is exponential!** The same process can be done for each recursive relation and the following table shows the runtime results for many algorithm:

$\mathcal{O}(n^2)$	quadratic	insertion, selection sort
$\mathcal{O}(n \ln n)$	loglinear	algorithms breaking problem into smaller chunks per invocation, and then sticking the results together, quick and merge sort
$\mathcal{O}(n)$	linear	iteration over a list
$\mathcal{O}(\ln n)$	log	algorithms breaking problem into smaller chunks per invocation, searching a binary search tree
$\mathcal{O}(1)$	constant	hash table lookup/modification
$\mathcal{O}(n^k)$	polynomial	k -nested for loops over n
$\mathcal{O}(k^n)$	exponential	producing every subset of n items
$\mathcal{O}(n!)$	factorial	producing every ordering of n values

Chapter 9

Sorting

The simplest way of *sorting* a group of items is to start by removing the smallest item from the group, and putting it first. Then removing the next smallest, and putting it next and so on. This is clearly an $\mathcal{O}(n^2)$ algorithm, so we need to find a better solution. In this chapter we will look at many examples of sorting algorithms and analyse their characteristics and runtimes.

An *in-place sort* does not use any additional memory to do the sorting (for example, swapping elements in an array). A *stable sort* preserves the relative order of otherwise identical data elements (for example, if two data elements have identical values, the one that was ahead of the other stays ahead). In any *comparison sort* problem, a *key* is the value (or values) that determines the sorting order. A comparison sort requires only that there is a way to determine if a key is less than, equal to, or greater than another key. Most sorting algorithms are comparison sorts where the worst-case running time for such sorts can be no better than $\mathcal{O}(n \ln n)$.

9.1 Quadratic Sort

Insertion Sort

Insertion sort is a simple sorting algorithm with best runtime case runtime of $\mathcal{O}(n)$ and average and worst runtime cases of $\mathcal{O}(n^2)$. It sorts by repeatedly inserting the next unsorted element in an initial sorted segment of the array. For small data sets, it can be preferable to more advanced algorithms such as merge sort or quicksort if the list is already sorted (it is a good way to

add new elements to a presorted list):

```
[sorting/insertion_sort.py]

def insertion_sort(seq):
    for i in range(1, len(seq)):
        j = i
        while j > 0 and seq[j-1] > seq[j]:
            seq[j-1], seq[j] = seq[j], seq[j-1]
            j -= 1
    return seq

def insertion_sort_rec(seq, i = None):
    if i == None: i = len(seq) -1
    if i == 0: return i
    insertion_sort_rec(seq, i-1)
    j = i
    while j > 0 and seq[j-i] > seq[j]:
        seq[j-1], seq[j] = seq[j], seq[j-1]
        j -= 1
    return seq

def test_insertion_sort():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2, 5, 4, 1, 5, 3]
    assert(insertion_sort(seq) == sorted(seq))
    assert(insertion_sort_rec(seq) == sorted(seq))
    print('Tests passed!')

if __name__ == '__main__':
    test_insertion_sort()
```

Selection Sort

Selection sort is based on finding the smallest or largest element in a list and exchanging it to the first, then finding the second, etc, until the end is reached. Even when the list is sorted, it is $\mathcal{O}(n^2)$ (and not stable):

```
[sorting/selection_sort.py]
```

```

def selection_sort(seq):
    for i in range(len(seq) -1, 0, -1):
        max_j = i
        for j in range(max_j):
            if seq[j] > seq[max_j]:
                max_j = j
        seq[i], seq[max_j] = seq[max_j], seq[i]
    return seq

def test_selection_sort():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2]
    assert(selection_sort(seq) == sorted(seq))
    print('Tests passed!')

if __name__ == '__main__':
    test_selection_sort()

```

A more sophisticated version of this algorithm is presented in the next chapter and it is called **quick select** and it is used to find the kth element of an array, and its median.

Gnome Sort

Gnome sort works by moving forward to find a misplaced value and then moving backward to place it in the right position:

```

[sorting/gnome_sort.py]

def gnome_sort(seq):
    i = 0
    while i < len(seq):
        if i == 0 or seq[i-1] <= seq[i]:
            i += 1
        else:
            seq[i], seq[i-1] = seq[i-1], seq[i]
            i -= 1
    return seq

```

```
def test_gnome_sort():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2, 5, 4, 1, 5, 3]
    assert(gnome_sort(seq) == sorted(seq))
    print('Tests passed!')

if __name__ == '__main__':
    test_gnome_sort()
```

9.2 Linear Sort

Count Sort

Count sort sorts integers with a small value range, counting occurrences and using the cumulative counts to directly place the numbers in the result, updating the counts as it goes.

There is a loglinear limit on how fast you can sort if all you know about your data is that they are greater or less than each other. However, if you can also count events, sort becomes linear in time, $\mathcal{O}(n + k)$:

```
[sorting/count_sort.py]

from collections import defaultdict

def count_sort_dict(a):
    b, c = [], defaultdict(list)
    for x in a:
        c[x].append(x)
    for k in range(min(c), max(c) + 1):
        b.extend(c[k])
    return b

def test_count_sort():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2, 5, 4, 1, 5, 3]
    assert(count_sort_dict(seq) == sorted(seq))
    print('Tests passed!')

if __name__ == '__main__':
    test_count_sort()
```

```
test_count_sort()
```

If several values have the same key, they will have the original order with respect with each other, so the algorithm is *stable*.

9.3 Loglinear Sort

The `sort()` and `sorted()` Methods

In Python, we would normally sort a list by using the `list.sort()` (in-place) method or any other iterable item by the `sorted()` function. They are both a very efficient implementation of the Python's `timsort` algorithm¹. With the `sorted()` function, the original object is not changed.

The `sorted()` function can also be customized though optional arguments, for example: `reverse=True`; `key=len`; `str.lower` (treating uppercase and lowercase the same); or even with a custom sorting function.

Merge Sort

Merge sort divides the list in half to create two unsorted lists. These two unsorted lists are sorted and merged by continually calling the merge-sort algorithm, until you get a list of size 1. The algorithm is stable, as well as fast for large data sets. However, since it is not in-place, it requires much more memory than many other algorithms. The space complexity is $\mathcal{O}(n)$ for arrays and $\mathcal{O}(\ln n)$ for linked lists². The best, average, and worst case times are all $\mathcal{O}(n \ln n)$.

Merge sort is a good choice when the data set is too large to fit into the memory. The subsets can be written to disk in separate files until they are small enough to be sorted in memory. The merging is easy, and involves just reading single elements at a time from each file and writing them to the final file in the correct order:

```
[sorting/merge_sort.py]
```

¹Timsort is a hybrid sorting algorithm, derived from merge sort and insertion sort, and invented by Tim Peters for Python.

²Never ever consider to sort a linked list tough.

```
''' Some examples of how to implement Merge Sort in Python.  
--> RUNTIME: WORST/BEST/AVERAGE Is O(nlogn)  
--> space complexity is O(n) for arrays  
--> in general not in place, good for large arrays  
--> In the case of two arrays: we can merge two arrays using  
    the merge function from the merge sort  
--> we can do this for files too, merging each two  
  
1) If we can modify the arrays (pop) we can use:  
def merge(left, right):  
    if not left or not right: return left or right #  
        nothing to be merged  
    result = []  
    while left and right:  
        if left[-1] >= right[-1]:  
            result.append(left.pop())  
        else:  
            result.append(right.pop())  
    result.reverse()  
    return (left or right) + result  
  
2) If we can't modify or we want to in place, we need two  
    pointers:  
>>> l1 = [1, 2, 3, 4, 5, 6, 7]  
>>> l2 = [2, 4, 5, 8]  
>>> merge(l1, l2)  
[1, 2, 2, 3, 4, 4, 5, 5, 6, 7, 8]  
  
3) For example, in the case we have a big array filled 0s  
    in the end, and another array with the size of the  
    number of 0s:  
>>> l1 = [1, 2, 3, 4, 5, 6, 7, 0, 0, 0, 0]  
>>> l2 = [2, 4, 5, 8]  
>>> merge_two_arrays_inplace(l1, l2)  
[1, 2, 2, 3, 4, 4, 5, 5, 6, 7, 8]
```

```
    4) If we want to merge sorted files (and we have plenty of  
        RAM to load all files):  
        >>> list_files = ['1.dat', '2.dat', '3.dat']  
        >>> merge_files(list_files)  
        [1, 1, 2, 3, 3, 3, 4, 5, 5, 5, 6, 7, 8]  
        ,,  
  
        """  
        The typical example...  
        """  
  
def merge_sort(seq):  
    if len(seq) < 2:  
        return seq  
    mid = len(seq)//2  
    lft, rgt = seq[:mid], seq[mid:]  
    if len(lft)>1:  
        lft = merge_sort(lft)  
    if len(rgt)>1:  
        rgt = merge_sort(rgt)  
  
    res = []  
    while lft and rgt:  
        if lft[-1]>= rgt[-1]:  
            res.append(lft.pop())  
        else:  
            res.append(rgt.pop())  
    res.reverse()  
    return(lft or rgt) + res  
  
    ,,  
    We could also divide this sort into two parts, separating  
    the merge part in another function  
    ,,  
  
def merge_sort_sep(seq):  
    if len(seq) < 2 :
```

```
        return seq # base case
    mid = len(seq)//2
    left = merge_sort(seq[:mid])
    right = merge_sort(seq[mid:]) # notice that mid is included!
    return merge(left, right) # merge iteratively

def merge(left, right):
    if not left or not right:
        return left or right # nothing to be merged
    result = []
    i, j = 0, 0
    while i < len(left) and j < len(right):
        if left[i] <= right[j]:
            result.append(left[i])
            i += 1
        else:
            result.append(right[j])
            j += 1
    if left[i:] : result.extend(left[i:]) # REMEMBER TO EXTEND, NOT
        APPEND
    if right[j:] : result.extend(right[j:])
    return result

''' The following merge functions is O(2n) and
illustrate many features in Python that '''
def merge_2n(left, right):
    if not left or not right: return left or right # nothing to be
        merged
    result = []
    while left and right:
        if left[-1] >= right[-1]:
            result.append(left.pop())
        else:
            result.append(right.pop())
    result.reverse()
    return (left or right) + result
```

```
''' Merge two arrays in place '''
def merge_two_arrays_inplace(l1, l2):
    if not l1 or not l2: return l1 or l2 # nothing to be merged
    p2 = len(l2) - 1
    p1 = len(l1) - len(l2) - 1
    p12 = len(l1) - 1
    while p2 >= 0 and p1 >= 0:
        item_to_be_merged = l2[p2]
        item_bigger_array = l1[p1]
        if item_to_be_merged < item_bigger_array:
            l1[p12] = item_bigger_array
            p1 -= 1
        else:
            l1[p12] = item_to_be_merged
            p2 -= 1
        p12 -= 1
    return l1

''' Merge sort for files '''
def merge_files(list_files):
    result = []
    final = []
    for filename in list_files:
        aux = []
        with open(filename, 'r') as file:
            for line in file:
                aux.append(int(line))
        result.append(aux)
    final.extend(result.pop())
    for l in result:
        final = merge(l, final)
    return final

def test_merge_sort():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2]
    seq_sorted = sorted(seq)
    assert(merge_sort(seq) == seq_sorted)
    assert(merge_sort_sep(seq) == seq_sorted)
```

```

    print('Tests passed!')


if __name__ == '__main__':
    test_merge_sort()

```

Quick Sort

Quick sort works by choosing a *pivot* and partitioning the array so that the elements that are smaller than the pivot goes to the left. Then, it recursively sorts the left and right parts.

The choice of the pivot value is a key to the performance. It can be shown that always choosing the value in the middle of the set is the best choice for already-sorted data and no worse than most other choices for random unsorted data.

The worst case is $\mathcal{O}(n^2)$ in the rare cases when partitioning keeps producing a region of $n - 1$ elements (when the pivot is the minimum or the maximum value). The best case produces two $n/2$ -sized lists. This and the average case are both $\mathcal{O}(n \ln n)$. The algorithm is not stable.

```

[sorting/quick_sort.py]

def quick_sort(seq):
    if len(seq) < 2: return seq
    ipivot = len(seq)//2
    pivot = seq[ipivot]
    before = [x for i,x in enumerate(seq) if x <= pivot and i != ipivot]
    after = [x for i,x in enumerate(seq) if x > pivot and i != ipivot]
    return qs(before) + [pivot] + qs(after)

"""
we can also divide them into two functions """
def partition(seq):
    pi,seq = seq[0],seq[1:]
    lo = [x for x in seq if x <= pi]
    hi = [x for x in seq if x > pi]

```

```
    return lo, pi, hi

def quick_sort_divided(seq):
    if len(seq) < 2: return seq
    lo, pi, hi = partition(seq)
    return quick_sort_divided(lo) + [pi] + quick_sort_divided(hi)

''' quick_sort in place, not very efficient '''
def quick_sort_in(seq):
    if len(seq) < 2 : return seq
    if len(seq) == 2 and seq[0] > seq[1]:
        seq[0], seq[1] = seq[1], seq[0] # problems when only 2
            elements because of swap
    pivot = seq[0] # start at the ends because we don't know how
        many elements
    p1, p2 = 1, len(seq) -1 # set pointers at both ends
    while p1 < p2: # must be < or out of range
        if seq[p1] <= pivot: # must be <= because of pivot swap
            seq[p1], seq[p2] = seq[p2], seq[p1]
            p2 -= 1
        else:
            p1 += 1
    seq[0], seq[p1] = seq[p1], pivot
    return quick_sort_in(seq[p1+1:]) + [seq[p1]] +
        quick_sort_in(seq[:p1])

def test_quick_sort():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2]
    assert(quick_sort(seq) == sorted(seq))
    assert(quick_sort_divided(seq) == sorted(seq))
    print('Tests passed!')

if __name__ == '__main__':
    test_quick_sort()
```

Heap Sort

Heap sort is similar to a selection sort, except that the unsorted region is a heap, so finding the largest element n times gives a loglinear runtime.

In a heap, for every node other than the root, the value of the node is at least (at most) the value of its parent. Thus, the smallest (largest) element is stored at the root and the subtrees rooted at a node contain larger (smaller) values than does the node itself.

Although the insertion is only $\mathcal{O}(1)$, the performance of validating (the heap order) is $\mathcal{O}(\ln n)$. Searching (traversing) is $\mathcal{O}(n)$. In Python, a heap sort can be implemented by pushing all values onto a heap and then popping off the smallest values one at a time:

```
[sorting/heap_sort1.py]

import heapq

def heap_sort1(seq):
    ''' heap sort with Python's heapq '''
    h = []
    for value in seq:
        heapq.heappush(h, value)
    return [heapq.heappop(h) for i in range(len(h))]

def test_heap_sort1():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2]
    assert(heap_sort1(seq) == sorted(seq))
    print('Tests passed!')

if __name__ == '__main__':
    test_heap_sort1()
```

If we decide to use the heap class that we have from the last chapters, we can write a heap sort simply by:

```
[sorting/heap_sort2.py]

from heap import Heap
```

```

def heap_sort2(seq):
    heap = Heap(seq)
    res = []
    for i in range(len(seq)):
        res.insert(0, heap.extract_max())
    return res

def test_heap_sort2():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2]
    assert(heap_sort2(seq) == sorted(seq))
    print('Tests passed!')

if __name__ == '__main__':
    test_heap_sort2()

```

Finally, we can also write our heap sort explicitly:

```

[sorting/heap_sort3.py]

def heap_sort3(seq):
    for start in range((len(seq)-2)//2, -1, -1):
        siftdown(seq, start, len(seq)-1)
    for end in range(len(seq)-1, 0, -1):
        seq[end], seq[0] = seq[0], seq[end]
        siftdown(seq, 0, end - 1)
    return seq

def siftdown(seq, start, end):
    root = start
    while True:
        child = root * 2 + 1
        if child > end: break
        if child + 1 <= end and seq[child] < seq[child + 1]:
            child += 1
        if seq[root] < seq[child]:
            seq[root], seq[child] = seq[child], seq[root]
            root = child
        else:
            break

```

```
def test_heap_sort():
    seq = [3, 5, 2, 6, 8, 1, 0, 3, 5, 6, 2]
    assert(heap_sort3(seq) == sorted(seq))
    print('Tests passed!')

if __name__ == '__main__':
    test_heap_sort3()
```

9.4 Comparison Between Sorting Methods

Algorithm	Data Structure	Time Complexity			Worst Case Auxiliary Space Complexity
		Best	Average	Worst	
Quicksort	Array	$\Theta(n \log(n))$	$\Theta(n \log(n))$	$\Theta(n^2)$	$\Theta(n)$
Mergesort	Array	$\Theta(n \log(n))$	$\Theta(n \log(n))$	$\Theta(n \log(n))$	$\Theta(n)$
Heapsort	Array	$\Theta(n \log(n))$	$\Theta(n \log(n))$	$\Theta(n \log(n))$	$\Theta(1)$
Bubble Sort	Array	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(1)$
Insertion Sort	Array	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(1)$
Select Sort	Array	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(1)$
Bucket Sort	Array	$\Theta(n+k)$	$\Theta(n+k)$	$\Theta(n^2)$	$\Theta(nk)$
Radix Sort	Array	$\Theta(nk)$	$\Theta(nk)$	$\Theta(nk)$	$\Theta(n+k)$

9.5 Additional Exercises

Quadratic Sort

The following program implements a *bubble sort*, a very inefficient sorting algorithm:

```
[searching/bubble_sort.py]

def bubble_sort(seq):
    size = len(seq) -1
    for num in range(size, 0, -1):
        for i in range(num):
            if seq[i] > seq[i+1]:
                temp = seq[i]
                seq[i] = seq[i+1]
                seq[i+1] = temp
    return seq

def test_bubble_sort(module_name='this module'):
    seq = [4, 5, 2, 1, 6, 2, 7, 10, 13, 8]
    assert(bubble_sort(seq) == sorted(seq))
    s = 'Tests in {} have {}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_bubble_sort()
```

Linear Sort

The example bellow shows a simple count sort for people ages:

```
def counting_sort_age(A):
    oldestAge = 100
    timesOfAge = [0]*oldestAge
    ageCountSet = set()
    B = []
```

```

for i in A:
    timesOfAge[i] += 1
    ageCountSet.add(i)
for j in ageCountSet:
    count = timesOfAge[j]
    while count > 0:
        B.append(j)
        count -= 1
return B

```

The example below uses *quick sort* to find the k largest elements in a sequence:

```
[sorting/find_k_largest_seq_quicksort.py]

import random

def swap(A, x, y):
    tmp = A[x]
    A[x] = A[y]
    A[y] = tmp

def qselect(A, k, left=None, right=None):
    left = left or 0
    right = right or len(A) - 1
    pivot = random.randint(left, right)
    pivotVal = A[pivot]

    swap(A, pivot, right)
    swapIndex, i = left, left
    while i <= right - 1:
        if A[i] < pivotVal:
            swap(A, i, swapIndex)
            swapIndex += 1
        i += 1

    swap(A, right, swapIndex)
    rank = len(A) - swapIndex
    if k == rank:
```

```
        return A[swapIndex]
    elif k < rank:
        return qselect(A, k, left=swapIndex+1, right=right)
    else:
        return qselect(A, k, left=left, right=swapIndex-1)

def find_k_largest_seq_quickselect(seq, k):
    kth_largest = qselect(seq, k)
    result = []
    for item in seq:
        if item >= kth_largest:
            result.append(item)
    return result

def test_find_k_largest_seq_quickselect():
    seq = [3, 10, 4, 5, 1, 8, 9, 11, 5]
    k = 2
    assert(find_k_largest_seq_quickselect(seq,k) == [10, 11])

if __name__ == '__main__':
    test_find_k_largest_seq_quickselect()
```

Chapter 10

Searching

The most common *searching* algorithms are the *sequential search* and the *binary search*. If an input array is not *sorted*, or the input elements are accommodated by dynamic containers (such as linked lists), the search is usually sequential. If the input is a sorted array, the binary search algorithm is the best choice. If we are allowed to use auxiliary memory, a hash table might help the search, with which a value can be located in $\mathcal{O}(1)$ time with a key.

10.1 Unsorted Arrays

10.1.1 Sequential Search

In the following example we illustrate the runtime of a sequential search for items stored in a collection. If the item is present, the best case is $\mathcal{O}(1)$, the average case is $\mathcal{O}(n/2)$, and the worst case is $\mathcal{O}(n)$. However, if the item is not present, all three cases will be $\mathcal{O}(n)$:

```
[searching/sequential_search.py]

def sequential_search(seq, n):
    for item in seq:
        if item == n: return True
    return False
```

```
def test_sequential_search(module_name='this module'):
    seq = [1, 5, 6, 8, 3]
    n1 = 5
    n2 = 7
    assert(sequential_search(seq, n1) == True)
    assert(sequential_search(seq, n2) == False)

    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_sequential_search()
```

Now, if we sort the sequence first, we can improve the sequential search in the case when the item is not present to have the same runtimes as when the item is present:

```
[searching/ordered_sequential_search.py]

def ordered_sequential_search(seq, n):
    item = 0
    for item in seq:
        if item > n: return False
        if item == n: return True
    return False

def test_ordered_sequential_search(module_name='this module'):
    seq = [1, 2, 4, 5, 6, 8, 10]
    n1 = 10
    n2 = 7
    assert(ordered_sequential_search(seq, n1) == True)
    assert(ordered_sequential_search(seq, n2) == False)
    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_ordered_sequential_search()
```

10.1.2 Quick Select and Order Statistics

We can use an adapted version of **quick sort** for finding the **kth smallest number** in a list. Such a number is called the **kth order statistic**. This includes the cases of finding the minimum, maximum, and median elements. This algorithm is $\mathcal{O}(n)$ in the worst case because we only look to one side of the array in each iteration: $\mathcal{O}(n) = \mathcal{O}(n) + \mathcal{O}(n/2) + \mathcal{O}(n/4)\dots$ Sublinear performance is possible for structured data: we can achieve $\mathcal{O}(1)$ for an array of sorted data.

```
[searching/quick_select.py]
import random

''' The simplest way...'''
def quickSelect(seq, k):
    # this part is the same as quick sort
    len_seq = len(seq)
    if len_seq < 2: return seq

    # we could use a random choice here doing
    #pivot = random.choice(seq)
    ipivot = len_seq // 2
    pivot = seq[ipivot]

    # O(n)
    smallerList = [x for i,x in enumerate(seq) if x <= pivot and i
                   != ipivot]
    largerList = [x for i,x in enumerate(seq) if x > pivot and i !=
                  ipivot]

    # here starts the different part
    m = len(smallerList)
    if k == m:
        return pivot
    elif k < m:
        return quickSelect(smallerList, k)
    else:
        return quickSelect(largerList, k-m-1)
```

```
''' If you don't want to use pythons feature at all and
also select pivot randomly'''

def swap(seq, x, y):
    tmp = seq[x]
    seq[x] = seq[y]
    seq[y] = tmp


def quickSelectHard(seq, k, left=None, right=None):
    left = left or 0
    right = right or len(seq) - 1
    #ipivot = random.randint(left, right)
    ipivot = len(seq)//2
    pivot = seq[ipivot]

    # Move pivot out of the sorting range
    swap(seq, ipivot, right)
    swapIndex, i = left, left
    while i < right:
        if seq[i] < pivot:
            swap(seq, i, swapIndex)
            swapIndex += 1
        i += 1

    # Move pivot to final position
    swap(seq, right, swapIndex)

    # Check if pivot matches, else recurse on the correct half
    rank = len(seq) - swapIndex

    if k == rank:
        return seq[swapIndex]
    elif k < rank:
        return quickSelectHard(seq, k, swapIndex+1, right)
    else:
        return quickSelectHard(seq, k, left, swapIndex-1)
```

```

if __name__ == '__main__':
    # Checking the Answer
    seq = [10, 60, 100, 50, 60, 75, 31, 50, 30, 20, 120, 170, 200]
    #seq = [3, 7, 2, 1, 4, 6, 5, 10, 9, 11]

    # we want the middle element
    k = len(seq) // 2

    # Note that this only work for odd arrays, since median in
    # even arrays is the mean of the two middle elements
    print(quickSelect(seq, k))
    print(quickSelectHard(seq, k))
    import numpy
    print(numpy.median(seq))

```

In general, we can define the median as the *the value that is bigger than half of the array*. This algorithm is important in the context of larger problems such as finding the **nearest neighbor** or the **shortest path**.

10.2 Sorted Arrays

10.2.1 Binary Search

A binary search finds the position of a specified input value (the key) within a sorted array. In each step, the algorithm compares the search key value with the key value of the middle element of the array. If the keys match the item's index, (position) is returned. Otherwise, if the search key is less than the middle element's key, the algorithm repeats the process in the left subarray; or if the search key is greater, on the right subarray. The algorithm runs on $\mathcal{O}(\ln n)$:

```

[searching/binary_search.py]

def binary_search_rec(seq,key): # Recursive

```

```

if not seq : return False
mid = len(seq)//2
if key == seq[mid] : return True
elif key < seq[mid] : return binary_search_rec(seq[:mid],key)
else : return binary_search_rec(seq[mid+1:],key)

def binary_search_iter(seq,key): # Iterative
    hi, lo = len(seq), 0
    while lo < hi :
        mid = (hi + lo)//2
        if key == seq[mid] : return True
        elif key < seq[mid] : hi = mid
        else : lo = mid + 1
    return False

def test_binary_search():
    seq = [1,2,5,6,7,10,12,12,14,15]
    key = 6
    assert(binary_search_iter(seq, key) == 3)
    assert(binary_search_rec(seq, key) == 3)
    print('Tests passed!')

if __name__ == '__main__':
    test_binary_search()

```

The bisect Module

Python's built-in `bisect()` module is available for binary search in a sorted sequence:

```

>>> from bisect import bisect
>>> list = [0, 3, 4, 5]
>>> bisect(list, 5)
4

```

Note that the module returns the index *after* the key, which is *where you should place the new value*. Other available functions are `bisect_right` and `bisect_left`.

10.3 Additional Exercises

Searching in a Matrix

The following module searches an entry in a matrix where the rows and columns are sorted. In this case, every row is increasingly sorted from left to right, and every column is increasingly sorted from top to bottom. The runtime is linear on $\mathcal{O}(m + n)$:

```
[general_problems/numbers/search_entry_matrix.py]

def find_elem_matrix_bool(m1, value):
    found = False
    row = 0
    col = len(m1[0]) - 1
    while row < len(m1) and col >= 0:
        if m1[row][col] == value:
            found = True
            break
        elif m1[row][col] > value:
            col-=1
        else:
            row+=1
    return found

def test_find_elem_matrix_bool(module_name='this module'):
    m1 = [[1,2,8,9], [2,4,9,12], [4,7,10,13], [6,8,11,15]]
    assert(find_elem_matrix_bool(m1,8) == True)
    assert(find_elem_matrix_bool(m1,3) == False)
    m2 = [[0]]
    assert(find_elem_matrix_bool(m2,0) == True)

    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_find_elem_matrix_bool()
```

The following problem searches an element in a matrix where in every row, the values increasing from left to right, but the last number in a row is

smaller than the first number in the next row. The naive brute force solution scans all numbers and costs $\mathcal{O}(mn)$. However, since the numbers are already sorted, the matrix can be viewed as a 1D sorted array and we can use the binary search algorithm with efficiency $\mathcal{O}(\log nm)$:

```
[searching/searching_in_a_matrix.py]

import numpy

def searching_in_a_matrix(m1, value):
    rows = len(m1)
    cols = len(m1[0])
    lo = 0
    hi = rows*cols
    while lo < hi:
        mid = (lo + hi)//2
        row = mid//cols
        col = mid%cols
        v = m1[row][col]
        if v == value: return True
        elif v > value: hi = mid
        else: lo = mid+1
    return False

def test_searching_in_a_matrix():
    a = [[1,3,5],[7,9,11],[13,15,17]]
    b = numpy.array([(1,2),(3,4)])
    assert(searching_in_a_matrix(a, 13) == True)
    assert(searching_in_a_matrix(a, 14) == False)
    assert(searching_in_a_matrix(b, 3) == True)
    assert(searching_in_a_matrix(b, 5) == False)
    print('Tests passed!')

if __name__ == '__main__':
    test_searching_in_a_matrix()
```

Unimodal Arrays

An array is *unimodal* if it consists of an increasing sequence followed by a decreasing sequence. The example below shows how to find the “locally maximum” of an array using binary search:

```
[searching/find_max_unimodal_array.py]

def find_max_unimodal_array(A):
    if len(A) <= 2 : return None
    left = 0
    right = len(A)-1
    while right > left +1:
        mid = (left + right)//2
        if A[mid] > A[mid-1] and A[mid] > A[mid+1]:
            return A[mid]
        elif A[mid] > A[mid-1] and A[mid] < A[mid+1]:
            left = mid
        else:
            right = mid
    return None

def test_find_max_unimodal_array():
    seq = [1, 2, 5, 6, 7, 10, 12, 9, 8, 7, 6]
    assert(find_max_unimodal_array(seq) == 12)
    print('Tests passed!')

if __name__ == '__main__':
    test_find_max_unimodal_array()
```

Calculating Square Roots

The example below implements the square root of a number using binary search:

```
[searching/find_sqrt_bin_search.py]

def find_sqrt_bin_search(n, error=0.001):
    lower = n < 1 and n or 1
```

```

upper = n < 1 and 1 or n
mid = lower + (upper - lower) / 2.0
square = mid * mid
while abs(square - n) > error:
    if square < n:
        lower = mid
    else:
        upper = mid
    mid = lower + (upper - lower) / 2.0
    square = mid * mid
return mid

def test_ind_sqrt_bin_search():
    number = 9
    assert(find_sqrt_bin_search(number) == 3)
    print('Tests passed!')

if __name__ == '__main__':
    test_ind_sqrt_bin_search()

```

Counting Frequency of Elements

The following example finds how many times a k element appears in a sorted list. Since the array is sorted, binary search gives a $\mathcal{O}(log n)$ runtime:

```

[searching/find_time_occurrence_list.py]

from binary_search import binary_search

def find_time_occurrence_list(seq, k):
    index_some_k = binary_serch(seq, k)
    count = 1
    sizet = len(seq)
    for i in range(index_some_k+1, sizet): # go up
        if seq[i] == k: count +=1
        else: break
    for i in range(index_some_k-1, -1, -1): # go down
        if seq[i] == k: count +=1

```

```
        else: break
    return count

def test_find_time_occurrence_list():
    seq = [1,2,2,2,2,2,2,5,6,6,7,8,9]
    k = 2
    assert(find_time_occurrence_list(seq, k) == 6)
    print('Tests passed!')

if __name__ == '__main__':
    test_find_time_occurrence_list()
```

Intersection of Arrays

The snippet below shows three ways to perform the intersection of two sorted arrays. The simplest way is to use sets, however this will not preserve the ordering. The second example uses an adaptation of the merge sort. The third example is suitable when one of the arrays is much larger than other. In this case, binary search is the best option:

```
[searching/intersection_two_arrays.py]

def intersection_two_arrays_sets(seq1, seq2):
    ''' find the intersection of two arrays using set properties
    '''
    set1 = set(seq1)
    set2 = set(seq2)
    return set1.intersection(set2) #same as list(set1 & set2)

def intersection_two_arrays_ms(seq1, seq2):
    ''' find the intersection of two arrays using merge sort '''
    res = []
    while seq1 and seq2:
        if seq1[-1] == seq2[-1]:
            res.append(seq1.pop())
            seq2.pop()
        elif seq1[-1] > seq2[-1]:
```

```
        seq1.pop()
    else:
        seq2.pop()
    res.reverse()
    return res

from binary_search import binary_search
def intersection_two_arrays_bs(seq1, seq2):
    ''' using binary search '''
    if len(seq1) > len(seq2): seq, key = seq1, seq2
    else: seq, key = seq2, seq1
    intersec = []
    for item in key:
        if binary_search(seq, item):
            intersec.append(item)
    return intersec

def test_intersection_two_arrays(module_name='this module'):
    seq1 = [1,2,3,5,7,8]
    seq2 = [3,5,6]
    assert(set(intersection_two_arrays_sets(seq1,seq2)) ==
           set([3,5]))
    assert(intersection_two_arrays_bs(seq1,seq2) == [3,5])
    assert(intersection_two_arrays_ms(seq1,seq2) == [3,5])
    s = 'Tests in {name} have {con}!'
    print(s.format(name=module_name, con='passed'))

if __name__ == '__main__':
    test_intersection_two_arrays()
```

Chapter 11

Dynamic Programming

Dynamic programming is used to simplify a complicated problem by breaking it down into simpler subproblems by means of recursion. If a problem has an *optimal substructure* and *overlapping subproblems*, it may be solved by dynamic programming.

Optimal substructure means that the solution to a given optimization problem can be obtained by a combination of optimal solutions to its subproblems. The first step to utilize dynamic programming is to check whether the problem exhibits such optimal substructure. The second step is having *overlapping problems* by solving subproblems once and then storing the solution to be retrieved. A choice is made at each step in dynamic programming, and the choice depends on the solutions to subproblems, bottom-up manner, from smaller subproblems to larger subproblems.

11.1 Memoization

Dynamically Solving the Fibonacci Series

High-level languages such as Python can implement the recursive formulation directly, *caching* return values. *Memoization* is a method where if a call is made more than once with the same arguments, and the result is returned directly from the *cache*.

For example, we can dynamically solve the exponential Fibonacci series by using a `memo` function designed as an algorithm that uses nested scopes

to give the wrapped function memory:

```
[dynamic_programming/memo.py]

from functools import wraps

def memo(func):
    cache = {}
    @wraps(func)
    def wrap(*args):
        if args not in cache:
            cache[args] = func(*args)
        return cache[args]
    return wrap

>>> fibonacci = memo(fibonacci)
>>> fibonacci(130)
1066340417491710595814572169
```

We could even use the decorator directly in the function:

```
@memo
def fib(i):
    if i < 2: return 1
    return fib(i-1) + fib(i-2)
>>> fibonacci(130)
1066340417491710595814572169
```

11.2 Additional Exercises

Longest Increasing Subsequence

Another interesting application of memoization is to the problem of finding the **longest increasing subsequence**¹ of a given sequence. A naive recursive solution gives a $\mathcal{O}(2^n)$ runtime. However, the iterative solution can be solved in loglinear time using dynamic programming.

In the example below we benchmark these functions with an array with 40 elements to find that the memoized version takes less than one second to run, while without dynamic programming, the function can take over 120 seconds to run:

```
[dynamic_programming/memoized_longest_inc_subseq.py]

from itertools import combinations
from bisect import bisect
from memo import memo
from do_benchmark import benchmark

def naive_longest_inc_subseq(seq):
    ''' exponential solution to the longest increasing subsequence
        problem '''
    for length in range(len(seq), 0, -1):
        for sub in combinations(seq, length):
            if list(sub) == sorted(sub):
                return len(sub)

def longest_inc_subseq1(seq):
    ''' iterative solution for the longest increasing subsequence
        problem '''
    end = []
    for val in seq:
        idx = bisect(end, val)
        if idx == len(end): end.append(val)
        else: end[idx] = val
    return len(end)
```

¹See other versions of this problem in the end of the chapter about lists in Python.

```
def longest_inc_subseq2(seq):
    ''' another iterative algorithm for the longest increasing
    subsequence problem '''
    L = [1] * len(seq)
    for cur, val in enumerate(seq):
        for pre in range(cur):
            if seq[pre] <= val:
                L[cur] = max(L[cur], 1 + L[pre])
    return max(L)

def memoized_longest_inc_subseq(seq):
    ''' memoized recursive solution to the longest increasing
    subsequence problem '''
    @memo
    def L(cur):
        res = 1
        for pre in range(cur):
            if seq[pre] <= seq[cur]:
                res = max(res, 1 + L(pre))
        return res
    return max(L(i) for i in range(len(seq)))

@benchmark
def test_naive_longest_inc_subseq():
    print(naive_longest_inc_subseq(s1))

benchmark
def test_longest_inc_subseq1():
    print(longest_inc_subseq1(s1))

@benchmark
def test_longest_inc_subseq2():
    print(longest_inc_subseq2(s1))

@benchmark
def test_memoized_longest_inc_subseq():
```

```
print(memoized_longest_inc_subseq(s1))

if __name__ == '__main__':
    from random import randrange
    s1 = [randrange(100) for i in range(40)]
    print(s1)
    test_naive_longest_inc_subseq()
    test_longest_inc_subseq1()
    test_longest_inc_subseq2()
    test_memoized_longest_inc_subseq()
```


Part III

Climbing is so last week! I
would rather fly, wouldn't you?

Time to start our engines to
reach the most fun objects in
the algorithm world. Speed up
to beautiful Graphs and Trees!

Chapter 12

Introduction to Graphs

12.1 Basic Definitions

A *graph* is an abstract network, consisting of *nodes* (or vertices, V) connected by *edges* (or arcs, E). A graph can be defined as a pair of sets, $G = (V, E)$, where the node set V is any finite set, and the edge set E is a set of node pairs. For example, some graph can be simply represented by the node set $V = \{a, b, c, d\}$ and the edge set $E = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, a\}\}$.

Direction of a Graph

If a graph has no direction, it is referred as *undirected*. In this case, nodes with an edge between them are *adjacent* and adjacent nodes are *neighbors*.

If the edges have a direction, the graph is *directed* (digraph). In this case, the graph has *leaves*. The edges are no longer unordered: an edge between nodes u and v is now either an edge (u, v) from u to v , or an edge (v, u) from v to u . We can say that in a digraph G , the function $E(G)$ is a relation over $V(G)$.

Subgraphs

A *subgraph* of G consists of a subset of V and E . A *spanning subgraph* contains all the nodes of the original graph.

Completeness of a Graph

If all the nodes in a graph are pairwise adjacent, the graph is called *complete*.

Degree in a Node

The number of undirected edges incident on a node is called *degree*. Zero-degree graphs are called *isolated*. For directed graphs, we can split this number into *in-degree* (incoming edges/parents) and *out-degree/children* (outgoing edges).

Paths, Walks, and Cycle

A *path* in G is a subgraph where the edges connect the nodes in a *sequence*, without revisiting any node. In a directed graph, a path has to follow the directions of the edges.

A *walk* is an alternating sequence of nodes and edges that allows nodes and edges to be visited multiple times.

A *cycle* is like a path except that the last edge links the last node to the first.

Length of a Path

The *length* of a path or walk is the value given by its edge count.

Weight of an Edge

Associating *weights* with each edge in G gives us a *weighted graph*. The weight of a path or cycle is the sum of its edge weights. So, for unweighted graphs, it is simply the number of edges.

Planar Graphs

A graph that can be drawn on the plane without crossing edges is called *planar*. This graph has *regions*, which are areas bounded by the edges. The *Euler's formula* for connected planar graphs says that $V - E + F = 2$, where V, E, F are the number of nodes, edges, and regions, respectively.

Graph Traversal

A *traversal* is a walk through all the connected components of a graph. The main difference between graph traversals is the ordering of the *to-do* list among the unvisited nodes that have been discovered.

Connected and Strongly Connected Components

An undirected graph is *connected* if there is a path from every node to every other node. A directed graph is *connected* if its underlying undirected graph is connected.

A *connected component* is a maximal subgraph that is connected. Connected components can be found using *traversal algorithms* such as *depth-first searching* (DFS) or *breadth-first searching* (BFS), as we will see in following chapters.

If there is a path from every node to every other node in a directed graph, the graph is called *strongly connected*. A *strongly connected component* (SCC) is a maximal subgraph that is strongly connected.

Trees and Forests

A *forest* is a *cycle-free* graph. A *tree* is an acyclic, connected, and directed graph. A forest consists of one or more trees.

In a tree, any two nodes are connected by exactly one path. Adding any new edge to it creates a cycle and removing any edge yields unconnected components.

12.2 The Neighborhood Function

A graph's *neighborhood function*, $N(V)$, is a container (or iterable object) of all neighbors of V . The most well-known data structures used to represent them are *adjacent lists* and *adjacent matrices*.

Adjacent Lists

For each node in an *adjacent list*, we have access to a list (or set or container or iterable) of its neighbor. Supposing we have n nodes, each adjacent (or

neighbor) list is just a list of such numbers. We place the lists into a main list of size n , indexable by the node numbers, where the order is usually arbitrary.

Using Sets as Adjacent Lists:

We can use Python's set type to implement adjacent lists:

```
>>> a,b,c,d,e,f = range(6) # nodes
>>> N = [{b,c,d,f}, {a,d,f}, {a,b,d,e}, {a,e}, {a,b,c}, {b,c,d,e}]
>>> b in N[a] # membership
True
>>> b in N[b] # membership
False
>>> len(N[f]) # degree
4
```

Using Lists as Adjacent Lists:

We can also use Python's lists to implement adjacent lists, which let you efficiently iterate $N(V)$ over any node V . Replacing sets with lists makes membership checking to be $\mathcal{O}(n)$. If all that your algorithm does is *iterating over neighbors*, using list may be preferential. However if the graph is dense (many edges), adjacent sets are a better solution:

```
>>> a,b,c,d,e,f = range(6) # nodes
>>> N = [[b,c,d,f], [a,d,f], [a,b,d,e], [a,e], [a,b,c], [b,c,d,e]]
>>> b in N[a] # membership
True
>>> b in N[b] # membership
False
>>> len(N[f]) # degree
4
```

Deleting objects from the middle of a Python list is $\mathcal{O}(n)$, but deleting from the end is only $\mathcal{O}(1)$. If the order of neighbors is not important, you can delete an arbitrary neighbor in $\mathcal{O}(1)$ time by swapping it in to the last item in the list and then calling `pop()`.

Using Dictionaries as Adjacent Lists:

Finally, we can use dictionaries as adjacent lists. In this case, the neighbors would be the keys, and we are able to associate each of them with some extra value, such as an edge weight:

```
>>> a,b,c,d,e,f = range(6) # nodes
>>> N = [{b:2,c:1,d:4,f:1}, {a:4,d:1,f:4}, {a:1,b:1,d:2,e:4},
    {a:3,e:2}, {a:3,b:4,c:1}, {b:1,c:2,d:4,e:3}]
>>> b in N[a] # membership
True
>>> len(N[f]) # degree
4
>>> N[a][b] # edge weight for (a,b)
2
```

A more flexible approach for node labels is to use dictionaries as a main structure only. For instance, we can use a dictionary with adjacency sets:

```
>>> a,b,c,d,e,f = range(6) # nodes
>>> N = { 'a':set('bcdf'), 'b':set('adf'), 'c':set('abde'),
    'd':set('ae'), 'e':set('abc'), 'f':set('bcde')}
>>> 'b' in N['a'] # membership
True
>>> len(N['f']) # degree
4
```

Adjacent Matrices

In *adjacent matrices*, instead of listing all the neighbors for each node, we have one row with one position for each possible neighbor, filled with `True` and `False` values. The simplest implementation of adjacent matrices is given by nested lists. Note that the diagonal is always `False`:

```
>>> a,b,c,d,e,f = range(6) # nodes
>>> N = [[0,1,1,1,0,1], [1,0,0,1,0,1], [1,1,0,1,1,0],
    [1,0,0,0,1,0], [1,1,1,0,0,0], [0,1,1,1,1,0]]
>>> N[a][b] # membership
1
>>> N[a][e]
```

```
0
>>> sum(N[f]) # degree
4
```

An adjacent matrix for an *undirected graph will always be symmetric*. To add weight to adjacent matrices, we can replace `True` and `False` by values. In this case, non-existent edges can be represented by infinite weights (`float('inf')`, or `None`, `-1`, or very large values):

```
>>> _ = float('inf') # nodes
>>> N = [[_,2,1,4,_,1], [4,_,_,1,_,4], [1,1,_,2,4,_],
      [3,_,_,_,2,_], [3,4,1,_,_,_], [1,2,_,4,3,_]]
>>> N[a][b] <= _ # membership
True
>>> sum(1 for w in N[f] if w <= _) # degree
4
```

Looking up an edge in an adjacent matrix is $\mathcal{O}(1)$ while iterating over a node's neighbor is $\mathcal{O}(n)$.

12.3 Introduction to Trees

While in a graph there may be multiple references to any given node; in a *tree* each node (data element) is referenced only by at most one other node, the *parent node*. The *root node* is the node that has no parent. The nodes referenced by a parent node are called *children*. A tree is said to be *full* and *complete* if all of its leaves are at the bottom and all of the non-leaf nodes have exactly two children.

Height or Depth of a Tree

The *height* (or depth) of a tree is the length of the path from the root to the deepest node in the tree. It is equal to the maximum level of any node in the tree. The depth of the root is zero. If the height of a tree is represented as the log of the number of leaves, the integer number from the log may be also called depth.

Level or Depth of a Node

The *level* (or depth) of a node is the length of path from the root to this node. The set of all nodes at a given depth in a tree is also called the level of the tree.

Representing Trees

The simplest way of representing a tree is by a nested lists:

```
>>> T = ['a', ['b', ['d', 'f'], ['c', ['e', 'g']]]
>>> T[0]
'a'
>>> T[1][0]
'b'
>>> T[1][1][0]
'd'
>>> T[1][1][1]
'f'
>>> T[2][0]
'c'
>>> T[2][1][1]
'g'
```

However, this becomes very hard to handle if we simply add a couple more branches. The only good way to work with trees is representing them as a collection of nodes. Let us start with a simple example, where we define a simple tree class with an attribute for value, another for a children (or ‘next’), and a method for printing the result:

```
[trees/trees/simple_tree/tree.py]

class SimpleTree(object):
    def __init__(self, value=None, children=None):
        self.children = children or []
        self.value = value

    def __repr__(self, level=0):
        ret = "\t"*level+repr(self.value)+"\n"
        for child in self.children:
            ret += child.__repr__(level+1)
        return ret
```

```

        for child in self.children:
            ret += child.__repr__(level+1)
        return ret

def main():
    """
    'a'
    'b'
    'd'
    'e'
    'c'
    'h'
    'g'
    """
    st = SimpleTree('a', [SimpleTree('b', [SimpleTree('d'),
                                             SimpleTree('e')]), SimpleTree('c', [SimpleTree('h'),
                                             SimpleTree('g')])])
    print(st)

if __name__ == '__main__':
    main()

```

In the next chapter we will learn how to improve this class, including many features and methods that a tree can hold. For now, it is useful to keep in mind that when we are prototyping data structures such as trees, we should always be able to come up with a flexible class to specify arbitrary attributes in the constructor. The following program implements what is referred to as *a bunch class*, a generic tool that is a specialization of the Python's `dict` class and that let you create and set arbitrary attributes on the fly:

```

[trees/simple_trees/bunchclass.py]

class BunchClass(dict):
    def __init__(self, *args, **kwds):
        super(BunchClass, self).__init__(*args, **kwds)
        self.__dict__ = self

def main():

```

```
''' {'right': {'right': 'Xander', 'left': 'Willow'}, 'left': {'right': 'Angel', 'left': 'Buffy'}}'''
bc = BunchClass # notice the absence of ()
tree = bc(left = bc(left="Buffy", right="Angel"), right =
           bc(left="Willow", right="Xander"))
print(tree)

if __name__ == '__main__':
    main()
```

In the example above, the function's arguments `*args` and `**kwds` can hold an arbitrary number of arguments and an arbitrary number of keywords arguments, respectively.

Chapter 13

Binary Trees

13.1 Basic Concepts

Binary trees are tree data structures where each node has at most two child nodes: the *left* and the *right*. Child nodes may contain references to their parents. The *root* of a tree (the ancestor of all nodes) can exist either inside or outside the tree.

Binary trees can be seen as a way of *passing an initial number n of tokens down*, meaning that at any point in the tree the sum of all the horizontal nodes will be n . The *degree* of every node is maximum two. Supposing that an arbitrary rooted tree has m internal nodes and each internal node has exactly two children, if the tree has n leaves, the degree of the tree is $n - 1$:

$$2m = n + m - 1 \rightarrow m = n - 1,$$

i.e a tree with n nodes has exactly $n - 1$ branches or degree.

13.2 Representing Binary Trees

The simplest (and silliest) way to represent a binary tree is using Python's lists. The following module constructs a list with a root and two empty sublists for the children. To add a left subtree to the root of a tree, we insert a new list into the second position of the root list. Note that this algorithm is not very efficient due to the restrictions that Python's lists have on inserting and popping in the middle::

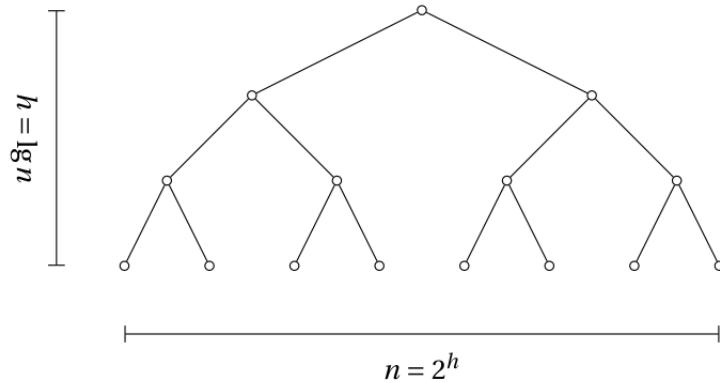


Figure 13.1: The height (h) and width (number of leaves) of a (*perfectly balanced*) binary tree.

```
[trees/binary_trees/BT_lists.py]

def BinaryTreeList(r):
    return [r, [], []]

def insertLeft(root, newBranch):
    t = root.pop(1)
    if len(t) > 1:
        root.insert(1,[newBranch,t,[]])
    else:
        root.insert(1,[newBranch, [], []])
    return root

def insertRight(root, newBranch):
    t = root.pop(2)
    if len(t) > 1:
        root.insert(2,[newBranch,[],t])
    else:
        root.insert(2,[newBranch,[],[]])
    return root

def getRootVal(root):
```

```

    return root[0]

def setRootVal(root, newVal):
    root[0] = newVal

def getLeftChild(root):
    return root[1]

def getRightChild(root):
    return root[2]

def main():
    ...
    3
    [5, [4, [], []], []]
    [7, [], [6, [], []]]
    ...

    r = BinaryTreeList(3)
    insertLeft(r,4)
    insertLeft(r,5)
    insertRight(r,6)
    insertRight(r,7)
    print(getRootVal(r))
    print(getLeftChild(r))
    print(getRightChild(r))

if __name__ == '__main__':
    main()

```

However this method is not very practical when we have many branches (or at least it needs many improvements, for example, how it manages the creation of new lists and how it displays or searches for new elements).

A more natural way to handle binary trees is (again) by representing it as a collection of nodes. A simple node in a binary tree should carry attributes for value and for left and right children, and it can have a method to identify leaves:

[trees/binary_trees/BT.py]

```
class BT(object):
    def __init__(self, value):
        self.value = value
        self.left = None
        self.right = None

    def is_leaf(self):
        return self.left is None and self.right is None

    def insert_left(self, new_node):
        if not self.left:
            self.left = BT(new_node)
        else:
            t = BT(self.left)
            t.left = new_node
            self.left = t

    def insert_right(self, new_node):
        if not self.right:
            self.right = BT(new_node)
        else:
            t = BT(self.right)
            t.right = new_node
            self.right = t

    def __repr__(self):
        return '{}'.format(self.value)

def tests_BT():
    """
        1
      2   3
    4   5   6   7
    """
    tree = BT(1)
    tree.insert_left(2)
    tree.insert_right(3)
    tree.left().insert_left(4)
    tree.left().insert_right(5)
    tree.right().insert_left(6)
```

```

tree.right().insert_right(7)
print(tree.right().right())
tree.right().right().value(8)
print(tree.right().right())
assert(tree.right().is_leaf() == False)
assert(tree.right().right().is_leaf() == True)
print("Tests Passed!")

if __name__ == '__main__':
    tests_BT()

```

13.3 Binary Search Trees

A *binary search tree* (BST) is a node-based binary tree data structure which has the following properties:

1. The left subtree of a node contains only nodes with keys less than the node's key.
2. The right subtree of a node contains only nodes with keys greater than the node's key.
3. Both the left and right subtrees must also be a binary search tree.
4. There must be no duplicate nodes.

If the binary search tree is balanced, the following operations are $\mathcal{O}(\ln n)$: (i) finding a node with a given value (lookup), (ii) finding a node with maximum or minimum value, and (iii) insertion or deletion of a node.

Representing Binary Search Trees

The following code implements a class for a binary search tree using our previous binary tree class as a *superclass*. The main difference now is that we can only insert a new node under the binary search tree conditions, which naturally gives us a method for finding an element in the tree. To be able to achieve this, we use polymorphism on the superclass's `insert_right` and `insert_left` methods, and we unify them to the proper `insert` method:

```
[trees/binary_trees/BST.py]

from BT import BT

class BST(BT):
    def __init__(self, value=None):
        self.value = value
        self.left = None
        self.right = None

    def insert_left(self, new_node):
        self.insert(value)

    def insert_right(self, new_node):
        self.insert(value)

    def insert(self, value):
        if self.value == None:
            self.value = value
        else:
            if value > self.value:
                self.right = self.right and self.right.insert(value)
                or BST(value)
            else:
                self.left = self.left and self.left.insert(value)
                or BST(value)
        return self

    def find(self, value):
        if value == self.value:
            return self
        elif value > self.value and self.right:
            return self.right.find(value)
        elif self.left:
            return self.left.find(value)
        return None
```

```

def main():
    """
        4
       2   6
      1 3 5 7
    """
    tree = BST()
    tree.insert(4)
    tree.insert(2)
    tree.insert(6)
    tree.insert(1)
    tree.insert(3)
    tree.insert(7)
    tree.insert(5)
    print(tree.get_right())
    print(tree.get_right().get_left())
    print(tree.get_right().get_right())
    print(tree.get_left())
    print(tree.get_left().get_left())
    print(tree.get_left().get_right())
    assert(tree.find(30) == None)

if __name__ == '__main__':
    main()

```

There are many other ways that a tree can be created. We could, for instance, think of two classes, one simply for nodes, and a second one that controls these nodes. This is not much different from the previous example (and in the end of this chapter we will see a third hybrid example of these two):

```

[trees/binary_trees/BST_with_Nodes.py]

class Node(object):
    def __init__(self, value):
        self.value = value
        self.left = None
        self.right = None

```

```
def __repr__(self):
    return '{}'.format(self.value)

class BSTwithNodes(object):
    def __init__(self):
        self.root = None

    def insert(self, value):
        if not self.root:
            self.root = Node(value)
        else:
            current = self.root
            while True:
                if value < current.value:
                    if current.left:
                        current = current.left
                    else:
                        current.left = Node(value)
                        break;
                elif value > current.value:
                    if current.right:
                        current = current.right
                    else:
                        current.right = Node(value)
                        break;
                else:
                    break

    def main():
        """
        BST
            4
            2       6
            1   3 5   7
        """
        tree = BSTwithNodes()
        l1 = [4, 2, 6, 1, 3, 7, 5]
        for i in l1: tree.insert(i)
        print(tree.root)
```

```

print(tree.root.right)
print(tree.root.right.left)
print(tree.root.right.right)
print(tree.root.left)
print(tree.root.left.left)
print(tree.root.left.right)

if __name__ == '__main__':
    main()

```

13.4 Self-Balancing BST

A *balanced* tree is a tree where the differences of the height of every subtree is at most equal to 1. A *self-balancing* binary search tree is any node-based binary search tree that automatically keeps itself balanced. By applying a balance condition we ensure that the worst case runtime of common tree operations will be at most $\mathcal{O}(\ln n)$.

Balancing Factor of a Tree

A *balancing factor* can be attributed to each internal node in a tree, being the difference between the heights of the left and right subtrees. There are many balancing methods for trees, but they are usually based on two operations:

- ★ *Node splitting (and merging)*: nodes are not allowed to have more than two children, so when a node become overfull it splits into two subnodes.
- ★ *Node rotations*: process of switching edges. If x is the parent of y , we make y the parent of x and x must take over one of the children of y .

AVL Trees

An *AVL tree* is a binary search tree with a *self-balancing condition* where the difference between the height of the left and right subtrees cannot be more than one.

To implement an AVL tree, we can start by adding a self-balancing method to our BST classes, called every time we add a new node to the

tree. The method works by continuously checking the height of the tree, which is added as a new attribute:

```
def height(node):
    if node is None:
        return -1
    else:
        return node.height

def update_height(node):
    node.height = max(height(node.left), height(node.right)) + 1
```

Now we can go ahead and implement the rebalancing method for our tree. The method will check whether the difference between the new heights of the right and left subtrees are up to 1. If this is not true, the method will perform the rotations:

```
def rebalance(self, node):
    while node is not None:
        update_height(node)

        if height(node.left) >= 2 + height(node.right):
            if height(node.left.left) >= height(node.left.right):
                self.right_rotate(node)
            else:
                self.left_rotate(node.left)
                self.right_rotate(node)

        elif height(node.right) >= 2 + height(node.left):
            if height(node.right.right) >=
                height(node.right.left):
                self.left_rotate(node)
            else:
                self.right_rotate(node.right)
                self.left_rotate(node)

    node = node.value
```

The rotation methods are straightforward: it takes a node and swaps it to the right or to the left children node:

```

def left_rotate(self, x):
    y = x.right
    y.value = x.value
    if y.value is None:
        self.root = y
    else:
        if y.value.left is x:
            y.value.left = y
        elif y.value.right is x:
            y.value.right = y
    x.right = y.left
    if x.right is not None:
        x.right.value = x
    y.left = x
    x.value = y

def right_rotate(self, x):
    y = x.left
    y.value = x.value
    if y.value is None:
        self.root = y
    else:
        if y.value.left is x:
            y.value.left = y
        elif y.value.right is x:
            y.value.right = y
    x.left = y.right
    if x.left is not None:
        x.left.value = x
    y.right = x
    x.value = y

```

We are now ready to write the entire AVL tree class! In the following code we have used our old BST class as a superclass, together with the methods we have described above. In addition, two methods for *traversals* were used, and we will explain them better in the next chapter. For now, it is good to keep the example in mind and that this AVL tree indeed supports insert, find, and delete-min operations at $\mathcal{O}(\ln n)$ time:

```
[trees/binary_trees/avl.py]

from BST_with_Nodes import BSTwithNodes, Node

class AVL(BSTwithNodes):
    def __init__(self):
        self.root = None

    def left_rotate(self, x):
        y = x.right
        y.value = x.value
        if y.value is None:
            self.root = y
        else:
            if y.value.left is x:
                y.value.left = y
            elif y.value.right is x:
                y.value.right = y
        x.right = y.left
        if x.right is not None:
            x.right.value = x
        y.left = x
        x.value = y
        update_height(x)
        update_height(y)

    def right_rotate(self, x):
        y = x.left
        y.value = x.value
        if y.value is None:
            self.root = y
        else:
            if y.value.left is x:
                y.value.left = y
            elif y.value.right is x:
                y.value.right = y
        x.left = y.right
        if x.left is not None:
            x.left.value = x
        y.right = x
```

```
x.value = y
update_height(x)
update_height(y)

def insert_item(self, value):
    if self.root == None:
        self.root = Node(value)
    else:
        current = self.root
        while True:
            if value < current.value:
                if current.left:
                    current = current.left
                else:
                    current.left = Node(value)
                    break;
            elif value > current.value:
                if current.right:
                    current = current.right
                else:
                    current.right = Node(value)
                    break;
            else:
                break

    def insert(self, value):
        node = self.insert_item(value)
        self.rebalance(node)

    def rebalance(self, node):
        while node is not None:
            update_height(node)
            if height(node.left) >= 2 + height(node.right):
                if height(node.left.left) >= height(node.left.right):
                    self.right_rotate(node)
                else:
                    self.left_rotate(node.left)
                    self.right_rotate(node)
```

```
        elif height(node.right) >= 2 + height(node.left):
            if height(node.right.right) >=
                height(node.right.left):
                    self.left_rotate(node)
            else:
                self.right_rotate(node.right)
                self.left_rotate(node)
            node = node.value

    def inorder(self, node):
        if node is not None:
            self.inorder(node.left)
            print(node.value)
            self.inorder(node.right)

    def preorder(self, node):
        if node is not None:
            print(node.value)
            self.preorder(node.left)
            self.preorder(node.right)

    def postorder(self, node):
        if node is not None:
            self.postorder(node.left)
            self.postorder(node.right)
            print(node.value)

    def height(node):
        if node is None: return -1
        else: return node.height

    def update_height(node):
        node.height = max(height(node.left), height(node.right)) + 1

    def main():
        tree = AVL()
        tree.insert(4)
        tree.insert(2)
        tree.insert(6)
        tree.insert(1)
```

```

tree.insert(3)
tree.insert(7)
tree.insert(5)
print('Inorder Traversal:')
tree.inorder(tree.root)

if __name__ == '__main__':
    main()

```

Red-black Trees

Red-black trees are an evolution of a binary search trees that aim to keep the tree balanced without affecting the complexity of the primitive operations. This is done by coloring each node in the tree with either red or black and preserving a set of properties that guarantees that the deepest path in the tree is not longer than twice the shortest one.

Red-black trees have the following properties:

- ★ Every node is colored with either red or black.
- ★ All leaf (nil) nodes are colored with black; if a node's child is missing then we will assume that it has a nil child in that place and this nil child is always colored black.
- ★ Both children of a red node must be black nodes.
- ★ Every path from a node n to a descendent leaf has the same number of black nodes (not counting node n). We call this number the *black height* of n .

Binary Heaps

Binary heaps are *complete* balanced binary trees. The heap property makes it easier to maintain the structure, *i.e.*, the balance of the tree. There is no need to modify a structure of the tree by splitting or rotating nodes in a heap: the only operation will be swapping parent and child nodes.

In a binary heap, the root (the smallest or largest element) is always found in $h[0]$. Considering a node at index i :

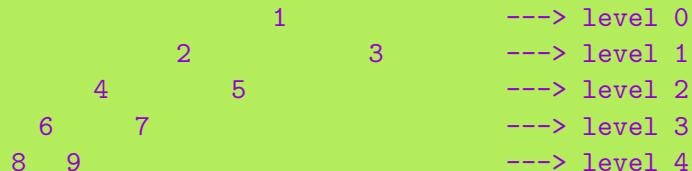
- ★ the parent index is $\frac{i-1}{2}$,
- ★ the left child index is $2i + 1$,
- ★ the right child index is $2i + 2$.

13.5 Additional Exercises

Another (Complete Example) of a Binary Tree

```
[trees/binary_trees/binary_tree.py]
```

`''' Implementation of a binary tree and its properties. For example, the following bt:`



`has the following properties:`

- SIZE OR NUMBER OF NODES: $n = 9$
- NUMBER OF BRANCHES OR INTERNAL NODES: $b = n-1 = 8$
- VALUE OF ROOT = 1
- MAX_DEPTH OR HEIGHT: $h = 4$
- IS BALANCED? NO
- IS BST? NO
- INORDER DFT: 8, 6, 9, 4, 7, 2, 5, 1, 3
- POSTORDER DFT: 8, 9, 6, 7, 4, 5, 2, 3, 1
- PREORDER DFT: 1, 2, 4, 6, 8, 9, 7, 5, 3
- BFT: 1, 2, 3, 4, 5, 6, 7, 8, 9

`,,`

```

from collections import deque
class NodeBT(object):
    def __init__(self, item=None, level=0):
        ''' Construtor for a Node in the Tree '''
        self.item = item
        self.level = level
        self.left = None
        self.right = None
        self.traversal = []
  
```

```
'''  
METHODS TO MODIFY NODES  
'''  
  
def _addNextNode(self, value, level_here=1):  
    ''' Aux for self.addNode(value)'''  
    self.traversal = []  
    new_node = NodeBT(value, level_here)  
    if not self.item:  
        self.item = new_node  
    elif not self.left:  
        self.left = new_node  
    elif not self.right:  
        self.right = new_node  
    else:  
        self.left = self.left._addNextNode(value, level_here+1)  
    return self  
  
'''  
METHODS TO PRINT/SHOW NODES ATTRIBUTES  
'''  
  
def __repr__(self):  
    ''' Private method for this class string representation'''  
    return '{}'.format(self.item)  
  
def _getDFTpreOrder(self, node):  
    ''' Traversal Pre-Order, O(n)'''  
    if node:  
        if node.item: self.traversal.append(node.item)  
        self._getDFTpreOrder(node.left)  
        self._getDFTpreOrder(node.right)  
    return self  
  
def _printDFTpreOrder(self, noderoot):  
    ''' Fill the pre-order traversal array '''  
    self.traversal = []
```

```
    self._getDFTpreOrder(noderoot)
    return self.traversal

def _getDFTinOrder(self, node):
    ''' Traversal in-Order, O(n)'''
    if node:
        self._getDFTinOrder(node.left)
        if node.item: self.traversal.append(node.item)
        self._getDFTinOrder(node.right)
    return self

def _printDFTinOrder(self, noderoot):
    ''' Fill the in-order traversal array '''
    self.traversal = []
    self._getDFTinOrder(noderoot)
    return self.traversal

def _getDFTpostOrder(self, node):
    ''' Traversal post-Order, O(n)'''
    if node:
        self._getDFTpostOrder(node.left)
        self._getDFTpostOrder(node.right)
        if node.item: self.traversal.append(node.item)
    return self

def _getBFT(self, node):
    ''' Traversal bft, O(n)'''
    if node:
        queue = deque()
        queue.append(node)
        while len(queue) > 0:
            current = queue.popleft()
            if current.item: self.traversal.append(current)
            if current.left: queue.append(current.left)
            if current.right: queue.append(current.right)
    return self

def _printBFT(self, noderoot):
    ''' Fill the in-order traversal array '''
    self.traversal = []
```

```
        self._getBFT(noderoot)
        return self.traversal

    def _printDFTpostOrder(self, noderoot):
        ''' Fill the post-order traversal array '''
        self.traversal = []
        self._getDFTpostOrder(noderoot)
        return self.traversal

    def _searchForNode(self, value):
        ''' Traverse the tree looking for the node'''
        if self.item == value: return self
        else:
            found = None
            if self.left: found = self.left._searchForNode(value)
            if self.right: found = found or
                self.right._searchForNode(value)
            return found

    def _findNode(self, value):
        ''' Find whether a node is in the tree.
            if the traversal was calculated, it is just a membership
            checking, which is O(1), otherwise it is necessary to
            traverse
            the binary tree, so best case is O(1) and worst is O(n).
        '''
        if self.traversal: return value in self.traversal
        else: return self._searchForNode(value)

    def _isLeaf(self):
        ''' Return True if the node is a leaf '''
        return not self.right and not self.left

    def _getMaxHeight(self, level=0):
        ''' Get the max height at the node, O(n)'''
        levelr, levell = -1, -1
        if self.right:
            levelr = self.right._getMaxHeight(level +1)
        if self.left:
```

```
        levell = self.left._getMaxHeight(level +1)
    return max(levelr, levell) + 1

def _getMinHeight(self, level=0):
    ''' Get the min height at the node, O(n)'''
    levelr, levell = -1, -1
    if self.right:
        levelr = self.right._getMinHeight(level +1)
    if self.left:
        levell = self.left._getMinHeight(level +1)
    return min(levelr, levell) + 1

def _isBalanced(self):
    ''' Find whether the tree is balanced, by calculating
        heights first, O(n2) '''
    if self._getMaxHeight() - self._getMinHeight() < 2:
        return True
    else:
        if self._isLeaf():
            return True
        elif self.left and self.right:
            return self.left._isBalanced() and
                   self.right._isBalanced()
        elif not self.left and self.right:
            return self.right._isBalanced()
        elif not self.right and self.left:
            return self.left._isBalanced()

def _isBST(self):
    ''' Find whether the tree is a BST, inorder '''
    if self.item:
        if self._isLeaf(): return True
        elif self.left:
            if self.left.item < self.item: return
                self.left._isBST()
            else: return False
        elif self.right:
            if self.right.item > self.item: return
                self.right._isBST()
            else: return False
```

```
        else:
            raise Exception('Tree is empty')

class BinaryTree(object):
    """
    >>> bt = BinaryTree()
    >>> for i in range(1, 10): bt.addNode(i)
    >>> bt.hasNode(7)
    True
    >>> bt.hasNode(12)
    False
    >>> bt.printTree()
    [1, 2, 4, 6, 8, 9, 7, 5, 3]
    >>> bt.printTree('pre')
    [1, 2, 4, 6, 8, 9, 7, 5, 3]
    >>> bt.printTree('bft')
    [1, 2, 3, 4, 5, 6, 7, 8, 9]
    >>> bt.printTree('post')
    [8, 9, 6, 7, 4, 5, 2, 3, 1]
    >>> bt.printTree('in')
    [8, 6, 9, 4, 7, 2, 5, 1, 3]
    >>> bt.hasNode(9)
    True
    >>> bt.hasNode(11)
    False
    >>> bt.isLeaf(8)
    True
    >>> bt.getNodeLevel(1)
    0
    >>> bt.getNodeLevel(8)
    4
    >>> bt.getSizeTree()
    9
    >>> bt.isRoot(10)
    False
    >>> bt.isRoot(1)
    True
```

```
>>> bt.getHeight()
4
>>> bt.isBST()
False
>>> bt.isBalanced()
False
>>> bt.isBalanced(2)
False
>>> bt.getAncestor(8, 5)
2
>>> bt.getAncestor(8, 5, 'pre-post')
2
>>> bt.getAncestor(8, 5, 'post-in')
2
...
...
def __init__(self):
    ''' Construtor for the Binary Tree, which is a container of
    Nodes'''
    self.root = None

...
METHODS TO MODIFY THE TREE
...

def addNode(self, value):
    ''' Add new node to the tree, by the left first, O(n) '''
    if not self.root: self.root = NodeBT(value)
    else: self.root._addNextNode(value)

...
METHODS TO PRINT/SHOW TREES ATTRIBUTES
...

def __repr__(self):
    ''' Private method for this class string representation'''
    return '{}'.format(self.item)
```

```
def printTree(self, order = 'pre'):
    ''' Print Tree in the chosen order '''
    if self.root:
        if order == 'pre': return
            self.root._printDFTpreOrder(self.root)
        elif order == 'in': return
            self.root._printDFTinOrder(self.root)
        elif order == 'post': return
            self.root._printDFTpostOrder(self.root)
        elif order == 'bft': return
            self.root._printBFT(self.root)
        else: raise Exception('Tree is empty')

def hasNode(self, value):
    ''' Verify whether the node is in the Tree '''
    return bool(self.root._findNode(value))

def isLeaf(self, value):
    ''' Return True if the node is a Leaf '''
    node = self.root._searchForNode(value)
    return node._isLeaf()

def getNodeLevel(self, item):
    ''' Return the level of the node, best O(1), worst O(n) '''
    node = self.root._searchForNode(item)
    if node: return node.level
    else: raise Exception('Node not found')

def getSizeTree(self):
    ''' Return how many nodes in the tree, O(n) '''
    return len(self.root._printDFTpreOrder(self.root))

def isRoot(self, value):
    '''Return the root of the tree '''
    return self.root.item == value

def getHeight(self):
    ''' Returns the height/depth of the tree, best/worst O(n)
        '''
```

```
        return self.root._getMaxHeight()

    def isBalanced(self, method=1):
        ''' Return True if the tree is balanced'''
        if method == 1:
            ''' O(n^2)'''
            return self.root._isBalanced()
        else:
            ''' O(n)'''
            return self.root._isBalancedImproved()

    def isBST(self, method=1):
        ''' Return True if the tree is BST'''
        return self.root._isBST()

    def _getAncestorPreIn(self, preorder, inorder, value1, value2):
        ''' Return the ancestor of two nodes with pre and in'''
        root = preorder[0]
        preorder = preorder[1:]
        i = 0
        item = inorder[0]
        value1left, value2left = False, False
        while item != root and i < len(inorder):
            if item == value1: value1left = True
            elif item == value2: value2left = True
            i += 1
            item = inorder[i]
        if (value1left and not value2left) or (value2left and not
            value1left):
            return root
        else:
            return self._getAncestorPreIn(preorder, inorder[:i] +
                inorder[i+1:], value1, value2)

    def _getAncestorPrePost(self, preorder, postorder, value1,
        value2):
        ''' Return the ancestor of two nodes with pre and post'''
        root = preorder[0]
        preorder = preorder[1:]
        postorder = postorder[:-1]
```

```

        value1right, value2right = False, False
        i = len(postorder)-1
        itempre = preorder[0]
        itempos = postorder[i]
        while itempre != itempos and i > 0:
            if itempos == value1: value1right = True
            elif itempos == value2: value2right = True
            i -= 1
            itempos = postorder[i]

            if (value1right and not value2right) or (value2right and
                not value1right):
                return root
            else:
                return self._getAncestorPrePost(preorder, postorder[:i]
                    + postorder[i+1:], value1, value2)

    def _getAncestorInPost(self, inorder, postorder, value1,
        value2):
        ''' Return the ancestor of two nodes with in and post'''
        root = postorder[-1]
        postorder = postorder[:-1]
        value1left, value2left = False, False
        i = 0
        item = inorder[i]
        while item != root and i < len(inorder):
            if item == value1: value1left = True
            elif item == value2: value2left = True
            i += 1
            item = inorder[i]

            if (value1left and not value2left) or (value2left and not
                value1left):
                return root
            else:
                return self._getAncestorInPost(postorder, inorder[:i] +
                    inorder[i+1:], value1, value2)

    def _getAncestorBST(self, preorder, value1, value2):
        ''' Return the ancestor of two nodes if it is a bst'''
```

```
while preorder:
    current = preorder[0]
    if current < value1:
        try: preorder = preorder[2:]
        except: return current
    elif current > value2:
        try: preorder = preorder[1:]
        except: return current
    elif value1 <= current <= value2:
        return current
    return None

def getAncestor(self, value1, value2, method='pre-in'):
    ''' Return the common ancestor for two nodes'''
    if method == 'pre-in':
        ''' Using pre and inorder, best/worst O(n)'''
        preorder = self.root._printDFTpreOrder(self.root)
        inorder = self.root._printDFTinOrder(self.root)
        return self._getAncestorPreIn(preorder, inorder, value1,
                                       value2)
    if method == 'pre-post':
        ''' Using pre and postorder, best/worst O(n)'''
        preorder = self.root._printDFTpreOrder(self.root)
        postorder = self.root._printDFTpostOrder(self.root)
        return self._getAncestorPrePost(preorder, postorder,
                                         value1, value2)
    if method == 'post-in':
        ''' Using in and postorder, best/worst O(n)'''
        inorder = self.root._printDFTinOrder(self.root)
        postorder = self.root._printDFTpostOrder(self.root)
        return self._getAncestorInPost(inorder, postorder,
                                       value1, value2)
    if method == 'bst':
        if self.isBST():
            preorder = self.root._printDFTpreOrder(self.root)
            return self._getAncestorBST(preorder, value1, value2)
        else:
            return Exception('The tree is not a BST')
```

```
if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

Another (Complete Example) of a Binary Search Tree

```
[trees/binary_trees/binary_search_tree.py]

from binary_tree import BinaryTree, NodeBT

''' Implementation of a binary search tree and its properties.
    We use the Binary Tree class and its Node class as
        superclasses, and we modify the methods
    that are needed to create a BST (polymorphism). For example,
        the following bst:

                    7
                  ---> level 0
                  4                 9
                ---> level 1
                2         5           8       10
              ---> level 2
              1         6
              ---> level 3

    has the following properties:

    - SIZE OR NUMBER OF NODES: n = 10
    - NUMBER OF BRANCHES OR INTERNAL NODES: b = n-1 = 9
    - VALUE OF ROOT = 7
    - MAX_DEPTH OR HEIGHT: h = 3
    - IS BALANCED? YES
    - IS BST? YES
    - INORDER DFT: 1, 2, 4, 5, 6, 7, 8, 9, 10
    - POSTORDER DFT: 1, 2, 6, 5, 4, 8, 10, 9, 7
    - PREORDER DFT: 7, 4, 2, 1, 5, 6, 9, 8, 10
    - BFT: 7, 4, 9, 2, 5, 8, 10, 1, 6
    ,,,,
```



```
class NodeBST(NodeBT):
```

```
def _addNextNode(self, value, level_here=1):
    ''' Aux for self.addNode(value): for BST, best O(1), worst
        O(log n) '''
    self.traversal = []
    new_node = NodeBST(value, level_here)
    if not self.item:
        self.item = new_node
    elif value < self.item:
        self.left = self.left and self.left._addNextNode(value,
            level_here+1) or new_node
    else:
        self.right = self.right and
            self.right._addNextNode(value, level_here+1) or
            new_node
    return self

def _searchForNode(self, value):
    ''' Traverse the tree looking for the node. For BST it is
        O(logn) '''
    if self.item == value: return self
    elif value > self.item and self.right: return
        self.right._findNode(value)
    elif value < self.item and self.left: return
        self.left._findNode(value)
    return None

class BinarySearchTree(BinaryTree):
    ...
    >>> bst = BinarySearchTree()
    >>> l1 = [7, 4, 5, 9, 2, 8, 1, 6, 10]
    >>> for i in l1: bst.addNode(i)
    >>> bst.hasNode(3)
    False
    >>> bst.hasNode(10)
    True
    >>> bst.printTree('pre')
    [7, 4, 2, 1, 5, 6, 9, 8, 10]
    >>> bst.printTree('post')
```

```
[1, 2, 6, 5, 4, 8, 10, 9, 7]
>>> bst.printTree('in')
[1, 2, 4, 5, 6, 7, 8, 9, 10]
>>> bst.printTree('bft')
[7, 4, 9, 2, 5, 8, 10, 1, 6]
>>> bst.getHeight()
3
>>> bst.isBST()
True
>>> bst.isBalanced()
False
>>> bst.isBalanced(2)
False
>>> bst.getAncestor(2, 9)
7
>>> bst.getAncestor(2, 9, 'bst')
7
>>> bst.getAncestor(2, 9, 'pre-post')
7
>>> bst.getAncestor(2, 9, 'post-in')
7
, , ,  
  
def addNode(self, value):
    ''' Add new node to the tree, by the left first, O(n).
        The only difference from the Binary Tree class is the
        node class is
        NodeBST and not NodeBT '''
    if not self.root: self.root = NodeBST(value)
    else: self.root._addNextNode(value)  
  
if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

Chapter 14

Traversals and Problems on Graphs and Trees

Traversals are algorithms used to visit the objects (nodes) in some connected structure, such as a tree or a graph. Traversal problems can be either visiting *every node* or visiting only *some specific nodes*.

14.1 Depth-First Search

Depth-first traversal, or *depth-first search* (DFS), are algorithms that searches *deeper* first in a graph or a tree. Their difference when in graphs or trees is that in case of graphs, it is necessary to mark nodes as visited (otherwise we might be stuck in a loop).

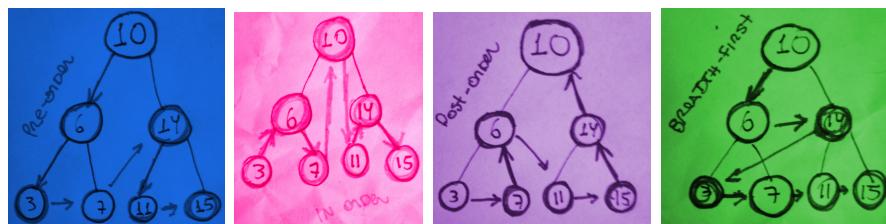


Figure 14.1: Binary tree traversals: preorder, inorder, postorder, and breath-first search.

DFS algorithms are called *once for every node* that is reachable from the start node, looking at its successors. The runtime is $\mathcal{O}(\text{number of reachable nodes} + \text{total number of outgoing edges from these nodes}) = \mathcal{O}(V + E)$. DFSs are usually implemented using LIFO structure such as stacks to keep track of the discovered nodes, and they can be divided in three different strategies:

Preorder: Visit a node before traversing subtrees (root → left → right):

```
def preorder(root):
    if root != 0:
        yield root.value
        preorder(root.left)
        preorder(root.right)
```

Postorder: Visit a node after traversing all subtrees (left → right → root):

```
def postorder(root):
    if root != 0:
        postorder(root.left)
        postorder(root.right)
        yield root.value
```

Inorder: Visit a node after traversing its left subtree but before the right subtree (left → root → right):

```
def inorder(root):
    if root != 0:
        inorder(root.left)
        yield root.value
        inorder(root.right)
```

14.2 Breadth-First Search

Breadth-first traversal, or *breadth-first search* (BFS), are algorithms that yields the values of all nodes of a particular depth *before* going to any deeper node.

Problems that use BFS usually ask to find the fewest number of steps (or the shortest path) needed to reach a certain end point from the starting point. Traditionally, BFSs are implemented using a list to store the values of the visited nodes and then a FIFO *queue* to store those nodes that have yet to be visited. The total runtime is also $\mathcal{O}(V + E)$.

14.3 Representing Tree Traversals

There are many ways we could write traversals. In the following code we use the “BST with nodes” class, defined in the last chapter, to implement each of the traversal algorithms. For the DFS cases, we have also tested two different methods:

```
[trees/traversals/BST_with_Nodes_traversal.py]

from BST_with_Nodes import BSTwithNodes, Node

class BSTTraversal(BSTwithNodes):
    def __init__(self):
        self.root = None
        self.nodes_BFS = []
        self.nodes_DFS_pre = []
        self.nodes_DFS_post = []
        self.nodes_DFS_in = []

    def BFS(self):
        self.root.level = 0
        queue = [self.root]
        current_level = self.root.level

        while len(queue) > 0:
            current_node = queue.pop(0)
            if current_node.level > current_level:
                current_level += 1
```

```

        self.nodes_BFS.append(current_node.value)

        if current_node.left:
            current_node.left.level = current_level + 1
            queue.append(current_node.left)

        if current_node.right:
            current_node.right.level = current_level + 1
            queue.append(current_node.right)

    return self.nodes_BFS

def DFS_inorder(self, node):
    if node is not None:
        self.DFS_inorder(node.left)
        self.nodes_DFS_in.append(node.value)
        self.DFS_inorder(node.right)
    return self.nodes_DFS_in

def DFS_preorder(self, node):
    if node is not None:
        self.nodes_DFS_pre.append(node.value)
        self.DFS_preorder(node.left)
        self.DFS_preorder(node.right)
    return self.nodes_DFS_pre

def DFS_postorder(self, node):
    if node is not None:
        self.DFS_postorder(node.left)
        self.DFS_postorder(node.right)
        self.nodes_DFS_post.append(node.value)
    return self.nodes_DFS_post

def main():
    tree = BSTTraversal()
    l1 = [10, 5, 15, 1, 6, 11, 50]
    for i in l1: tree.insert(i)

    print('Breadth-First Traversal: ', tree.BFS())
    print('Inorder Traversal: ', tree.DFS_inorder(tree.root))

```

```
print('Preorder Traversal: ', tree.DFS_preorder(tree.root))
print('Postorder Traversal: ', tree.DFS_postorder(tree.root))

if __name__ == '__main__':
    main()
```

14.4 Additional Exercises

More Traversals in a BST

As an alternative to the example shown in this chapter, in the following program we implement traversals for our old BST class (without nodes):

```
[traversals/BST_traversal.py]

from BST import BST

class TranversalBST(object):
    def __init__(self):
        self.bst = BST(None)
        self.nodes = []

    def insert(self, value):
        if not self.bst.value:
            self.bst.value = value
        else:
            self.bst.insert(value)

    def contains(self, value):
        return bool(self.bst.find(value))

    def get(self, index):
        for i, value in enumerate(self.inorder()):
            if i == index:
                return value

    def inorder(self):
        current = self.bst
        self.nodes = []
        stack = []
        while len(stack) > 0 or current is not None:
            if current is not None:
                stack.append(current)
                current = current.left
            else:
                current = stack.pop()
                self.nodes.append(current.value)
```

```
        current = current.right
    return self.nodes

def preorder(self):
    self.nodes = []
    stack = [self.bst]
    while len(stack) > 0:
        curr = stack.pop()
        if curr is not None:
            self.nodes.append(curr.value)
            stack.append(curr.right)
            stack.append(curr.left)
    return self.nodes

def preorder2(self):
    self.nodes = []
    current = self.bst
    stack = []
    while len(stack) > 0 or current is not None:
        if current is not None:
            self.nodes.append(current.value)
            stack.append(current)
            current = current.left
        else:
            current = stack.pop()
            current = current.right
    return self.nodes

def main():
    """
        10
      5     15
     1   6   11   50
    """
    t = TranversalBST()
    t.insert(10)
    t.insert(5)
    t.insert(15)
```

```

t.insert(1)
t.insert(6)
t.insert(11)
t.insert(50)
print(t.preorder())
print(t.preorder2())
print(t.inorder())

if __name__ == '__main__':
    main()

```

Balance and Depth in a BST

In the following example we use the class in the previous example with some methods to find the (maximum and minimum) depths, to check whether the tree is balanced and to find a key in preorder and inorder traversals:

```

[trees/traversals/BST_extra_methods.py]

from BST_traversal import TranversalBST
from BST import BST

class BSTwithExtra(TranversalBST):
    def __init__(self):
        self.bst = BST(None)
        self.nodes = []

    def get_inorder(self, k):
        for i, value in enumerate(self.inorder()):
            if value == k:
                return i

    def get_preorder(self, k):
        for i, value in enumerate(self.preorder()):
            if value == k:
                return i

    def is_balanced(self, threshold=1):

```

```
maxd = self.get_max_depth()
mind = self.get_min_depth()
print('Max depth: ' + str(maxd))
print('Min depth: ' + str(mind))
return maxd - mind

def get_min_depth(self, node=None, initial=1):
    if node is None and initial == 1:
        node = self.bst
    if node.left and node.right:
        return 1 + min(self.get_min_depth(node.left, 0),
                        self.get_min_depth(node.right, 0))
    else:
        if node.left:
            return 1 + self.get_max_depth(node.left, 0)
        elif node.right:
            return 1 + self.get_max_depth(node.right, 0)
        else:
            return 0

def get_max_depth(self, node=None, initial=1):
    if node is None and initial == 1:
        node = self.bst
    if node.left and node.right:
        return 1 + max(self.get_max_depth(node.left, 0),
                        self.get_max_depth(node.right, 0))
    else:
        if node.left:
            return 1 + self.get_max_depth(node.left, 0)
        elif node.right:
            return 1 + self.get_max_depth(node.right, 0)
        else:
            return 0

def main():
    """
        10
       5      15
      1   6    11   50
                           60
    """
```

```

    70
    80
"""
t = BSTwithExtra()
l1 = [10, 5, 15, 1, 6, 11, 50, 60, 70, 80]
for i in l1: t.insert(i)
print(t.inorder())
print(t.preorder())
assert(t.get_max_depth() == 5)
assert(t.get_min_depth() == 2)
assert(t.is_balanced() == 3)
assert(t.get_inorder(10) == 3)
assert(t.get_preorder(10) == 0)

"""
      1
      2      3
    4   5   6   7
"""
t2 = BSTwithExtra()
l2 = [1, 2, 3, 4, 5, 6, 7, 8]
for i in l2: t2.insert(i)
print(t2.inorder())
print(t2.preorder())
assert(t2.is_balanced() == 0)

print("Tests Passed!")

if __name__ == '__main__':
    main()

```

Ancestor in a BST

The example below finds the lowest level common ancestor of two nodes in a binary search tree:

```
[trees/traversals/BST_ancestor.py]
```

```
from BST_traversal import TranversalBST

def find_ancestor(path, low_value, high_value):
    ''' find the lowest ancestor in a BST '''
    while path:
        current_value = path[0]
        if current_value < low_value:
            try:
                path = path[2:]
            except:
                return current_value
        elif current_value > high_value:
            try:
                path = path[1:]
            except:
                return current_value
        elif low_value <= current_value <= high_value:
            return current_value
        return None

def test_find_ancestor():
    """
        10
      5     15
     1   6   11   50
    """
    t = TranversalBST()
    l1 = [10, 5, 15, 1, 6, 11, 50]
    for i in l1: t.insert(i)
    path = t.preorder()
    assert(find_ancestor(path, 1, 6) == 5)
    assert(find_ancestor(path, 1, 11) == 10)
    assert(find_ancestor(path, 11, 50) == 15)
    assert(find_ancestor(path, 5, 15) == 10)

    print("Tests passed!")

if __name__ == '__main__':
    test_find_ancestor()
```

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