

Case 1:  $t = A[m]$

As the function works this will return  $m$  as  $\text{left} \leq m \leq \text{right}$

Case 2:  $t$  does not exist in  $A[m]$

As the array is sorted when  $A[m]$  should be equal to  $t$  instead it will find  $A[m] < t$  thus the function will be called once more making  $\text{right} > \text{left}$  returning  $-1$

Case 3:  $t < A[m]$

We know, as the array is sorted, that  $t$ , if it exists, must lie between  $A[\text{left}]$  and  $A[m-1]$ . If the recursive call works correctly then so must this one. So  $n = (m-1) - \text{left} + 1$  or  $n = (\frac{\text{left} + \text{right}}{2} - 1) - \text{left} + 1$ . So if  $\text{left} + \text{right}$  is even then  $n = \frac{\text{left} + \text{right}}{2} - \frac{\text{left}}{2} - 1 + 1 = \frac{\text{right} - \text{left}}{2} + 1 - 1$ . When odd  $n = \frac{\text{left} + \text{right} - 1}{2} - \frac{\text{left}}{2} - 1 + 1 = \frac{\text{right} - \text{left}}{2} + 1 - 1$  thus  $\frac{\text{right} - \text{left} - 1}{2} + 1 - 1 \leq \frac{\text{right} - \text{left}}{2} + 1 - 1 \leq \frac{\text{right} - \text{left} + 1}{2}$  so  $\frac{\text{right} - \text{left}}{2} + 1 - 1 \leq k$  thus the binary search works after this recursive call in the range of  $1:k$  and must be correct by Hyp (works on  $k+1$  interval)

Case 4:  $t > A[m]$

We know as the array is sorted that  $t$  must, if it exists, lie between  $A[m+1]$  and  $A[\text{right}]$ . If the recursive call works so must this one. So  $n = \text{right} - (m+1) + 1$ . So if  $\text{left} + \text{right}$  is even then  $n = \frac{\text{right} - \text{left}}{2} + 1 - 1 = \frac{\text{right} - \text{left}}{2}$ . When odd  $n = \frac{\text{right} - \text{left} - 1}{2} + 1 - 1 = \frac{\text{right} - \text{left} + 1}{2}$  as  $\frac{\text{right} - \text{left}}{2} \leq \text{right} - \text{left} + 1$  and  $\frac{\text{right} - \text{left} + 1}{2} \leq \text{right} - \text{left} + 1$  then as both are  $\leq k$  then the recursive call is made on an interval between  $0$  and  $k$  and is correct by Hyp (works on interval  $k+1$ )

Thus as the inductive step works in all cases, we can see exp. search works in all cases.