

Case  $t = A[m]$  is obvious as the function works

Case  $t < A[m]$

We know, since the array is sorted, that  $t$  must lie if it exists between  $A[\text{left}]$  and  $A[m-1]$ . So if the recursive call works correctly this call will as well. Therefore this is  $n = m-1-\text{left}$  or  $n = \frac{\text{left}+\text{right}}{2}-1-\text{left} < \text{right}-\text{left}$ . Case  $t$  does not exist, since the array is sorted when  $A[m]$  should be  $t$  it will not find an  $A[m] = t$  but instead  $A[m] < t$  this will cause the function to be called again where  $\text{left} > \text{right}$  thus returning 1.

Case  $t > A[m]$ , We know, as the array is sorted, that  $t$ , if it exists, must lie between  $A[\text{left}]$  and  $A[m-1]$ . If the recursive call works correctly, so will this call, so  $n = m-1-\text{left}$  or  $n = \frac{\text{left}+\text{right}}{2}-1-\text{left}$ . So if  $\text{left}+\text{right}$  is even then  $n = \frac{\text{left}+\text{right}}{2}-1-\text{left}$  is  $\frac{\text{left}+\text{right}-2\text{left}}{2}-1 = \frac{\text{right}-\text{left}}{2}-1$  which is smaller than  $\text{right}-\text{left}+1$ . If  $\text{right}-\text{left}$  is odd then  $n = \frac{\text{left}+\text{right}-1}{2}-1-\text{left} = \frac{\text{right}-\text{left}-1}{2}-1$  which is also smaller than  $\text{right}-\text{left}+1$ . So the recursive call must be between 1 and  $n$  and must be correct by Hyp. Case  $t > A[m]$  this is symmetrical to case  $t < A[m]$ .

∴ as all cases the induction step works we can conclude expSearch works  $\square$

$$\text{TCA: } T(n) = T\left(\frac{n}{2}\right) + O(1); a=1, b=2, f(n)=O(1); n^{\log_2^1} = n^0 = 1$$

as  $O(1) \in \Theta(1)$  where  $k=0$  then by case 2 of masters theorem

$$T(n) = \Theta(1 \lg n) \text{ or } T(n) = \Theta(\lg n) \quad \square$$