

Hyp. Let $P(n)$ be the assertion that circularBinarySearch works for all inputs $\text{right} - \text{left} + 1 = n$.

Base Case: when $n=1$ then $\text{right} = \text{left} = m$ so t is found in $A[m]$ as the function works correctly.

Inductive Step: We assume circularBinarySearch works as long as $\text{right} - \text{left} + 1 \leq k$ and must prove $\text{right} - \text{left} + 1 \leq k+1$ in 3 cases: $A[m] = t$, $A[m] > t$, $A[m] < t$.

Case 1: $A[m] = t$

As the function works this will return m , the index of t .

Case 2: $A[m] > t$

This halves the array is either sorted or circularly sorted. Thus t must lie to the left of m . If the recursive call works correctly so shall this one. So $n = (m-1) - \text{left} + 1 = \lfloor \frac{\text{right} + \text{left}}{2} \rfloor - \text{left} + 1$. If $\text{right} + \text{left}$ is even then $n = \frac{\text{right} + \text{left}}{2} - \frac{2\text{left}}{2} - 1 + 1 = \frac{\text{right} - \text{left}}{2} \leq \text{right} - \text{left} + 1$. When $\text{right} + \text{left}$ is odd $n = \frac{\text{right} + \text{left} + 1}{2} - \frac{2\text{left}}{2} - 1 + 1 = \frac{\text{right} - \text{left} + 1}{2} \leq \text{right} - \text{left} + 1$. As $\text{right} - \text{left} + 1 \leq k$ then the recursive call is made on interval 0 to k and is correct by Hyp. Thus circularBinarySearch must also work for interval $k+1$.

Case 3: $A[m] < t$

This is symmetrical to the previous case. Thus $n = \text{right} - (m+1) + 1 = \text{right} - (\lfloor \frac{\text{right} + \text{left}}{2} \rfloor + 1) + 1$ so when $\text{right} + \text{left}$ is even $n = \text{right} - \frac{\text{right} + \text{left}}{2} - 1 + 1 = \frac{\text{right} - \text{left}}{2} \leq \text{right} - \text{left} + 1$. When odd $n = \frac{\text{right} - \text{left} + 1}{2} \leq \text{right} - \text{left} + 1$ as well $\frac{\text{right} - \text{left}}{2} \leq \text{right} - \text{left} + 1 \leq k$. So the recursive call is made on interval 0 to k and is correct by Hyp. Thus circularBinarySearch must also work for interval $k+1$.

\therefore as the inductive step works in all cases we can see circularBinarySearch works in all cases.