Hyp: Let $P(n)$ be the assertion that misBin works for all inputs $right - left + 1 = n$

Base Case: when $n=1$ then ~~A~~ $right = left = m$ and $A[m] = t \vee \neq t$. if $A[m] = t$ the function works and returns m if $A[m] \neq t$ then the next call is the ~~doe~~ making $right > left$ returning -1.

Inductive Step: We assume misBin works as long as ~~...~~ $right - left + 1 \leq k$ and must prove $right - left + 1 \leq k+1$ to do this we must prove 4 cases. t doesn't exist in A, $A[m-1] \vee A[m] \vee A[m+1] = t$, $A[m] > t$ and $A[m] < t$.

Case 1: t doesn't exist in A

When $A[m]$ should equal t $A[m]$ will be found to be $A[m] < t$ forcing a recursive call where after $left > right$ returning -1

Case 2: $A[m-1] \vee A[m] \vee A[m+1] = t$

as the function works if $A[m-1] = t$ then $m-1$ is returned, if $A[m+1] = t$; $m+1$ returned. If $A[m] = t$, m is returned.

Case 3: $A[m] > t$

We know, as the array is sorted and $A[m \pm 1] \neq t$ that t must lie between $A[left]$ and $A[m-1]$. If the recursive call works correctly so shall this one. So $n = (m-1) - left + 1 = (\lfloor \frac{right+left}{2} \rfloor - 1) - left + 1$. When $right + left$ is even $n = \frac{right+left}{2} - 1 - \frac{2left}{2} + 1 = \frac{right-left}{2}$. When $right + left$ is odd then $\frac{right+left-1}{2} - 1 - \frac{2left}{2} + 1 = \frac{right-left-1}{2} \leq \frac{right-left}{2} \leq right - left + 1 \leq k$. Thus the recursive call is made on an interval from 0 to k and is correct by Hyp. Thus misBin must also work for interval $k+1$.

Case 4: $A[m] < t$

This is symmetrical to the previous case. Thus $n = right - (m+1) + 1 = right - (\lfloor \frac{right+left}{2} \rfloor + 1) + 1$ when $right + left$ is even then $n = \frac{2right}{2} - \frac{right-left}{2} - 1 + 1 = \frac{right-left}{2}$. When $right + left$ is odd then $n = \frac{2right}{2} - \frac{right-left+1}{2} - 1 + 1 = \frac{right-left+1}{2} \leq right - left + 1 \wedge \frac{right-left}{2} \leq k$. Thus the recursive call is made on an interval from 0 to k and is correct by Hyp. Thus misBin must also work for interval $k+1$.

$\therefore$ as the inductive step works in all cases we can see misBin works in all cases.