Hyp: Let $P(n)$ be the assertion that modBin works for all inputs $right - left + 1 = n$.

Base case: when $n = 1$ then $left = right = m$ and $A[m] = 0 \vee 1$. In case $A[m] = 0$ then as the function works 0 is returned. If $A[m] = 1$ then as the function works 1 is returned.

Inductive Step: We assume modBin works as long as $right - left + 1 \leq k$. We must prove $right - left + 1 \leq k + 1$. There are 4 cases. Where $A[left] = A[right]$, $A[m] = 1 \wedge A[m-1] = 0$, $A[m] = 1$, and $A[m] = 0$

Case 1: $A[left] = A[right]$

When $A[left] = A[right]$ there are two sub cases. First $A[left] = A[right] = 1$. Here n is returned as there is n amount of 1's. Second where $A[left] = A[right] = 0$. Here 0 is returned as there are no 1's.

Case 2: $A[m] = 1 \wedge A[m-1] = 0$

As the function work this will return $n - m + 1$ as that is the amount of 1's.

Case 3: $A[m] = 1$

We know the array is sorted and $A[m-1] = 1$ so the first occurence of 1 must be between $A[left]$ and $A[m-1]$. If the recursive call works correctly so shall this call. So $n = (m-1) - left + 1$ or $n = (\lfloor \frac{left + right}{2} \rfloor - 1) - left + 1$. When $left + right$ is even $n = \frac{right + left}{2} - \frac{2left}{2} - 1 + 1 = \frac{right - left}{2}$. When $right + left$ is odd $n = \frac{right + left - 1}{2} - \frac{2left}{2} - 1 + 1 = \frac{right - left - 1}{2} \leq \frac{right - left}{2} \leq right - left + 1$ then $\frac{right - left}{2} \leq k$. Then the recursive call is made on interval 0 to k and is correct by Hyp thus working on $k+1$.

Case 4: $A[m] = 0$

This is symmetrical to the previous case as such $n = right - (m+1) + 1 = right - (\lfloor \frac{left + right}{2} \rfloor + 1) + 1$. When $right + left$ is even then $n = \frac{2right}{2} - \frac{right - left}{2} - 1 + 1 = \frac{right - left}{2}$. When $right + left$ is odd then $n = \frac{2right}{2} - \frac{right - left + 1}{2} - 1 + 1 = \frac{right - left + 1}{2} \leq right - left + 1 \leq k$ and $\frac{right - left}{2} \leq k$. Then the recursive call is made on interval 0 to k and is correct by Hyp thus working on $k+1$.

$\therefore$ As the inductive step works in all cases we can see modBin works in all cases ■