

Hyp. Let $P(n)$ be the assertion that circularBinarySearch works for all inputs $\text{right-left+1} \leq n$.

Base Case: when $n=1$ then $\text{right-left}=m$ so t is found in $A[m]$ as the function works correctly.

Inductive Step: We assume circularBinarySearch works as long as $\text{right-left+1} \leq k$ and must prove $\text{right-left+1} \leq k+1$ in 3 cases $A[m]=t$, $A[m] > t$, $A[m] < t$.

Case 1: $A[m]=t$

As the function works this will return m , the index of t .

Case 2: $A[m] > t$

This half of the array is either sorted or circularly sorted. Thus t must lie to the left of m . If the recursive call works correctly for shaft this one. So $n = (m-1)-\text{left}+1 = (\lfloor \frac{\text{right-left}}{2} \rfloor - 1) - \text{left} + 1$ if right-left is even then $n = \frac{\text{right-left}}{2} - \frac{2\text{left}-1}{2} + 1 = \frac{\text{right-left}}{2} \leq \text{right-left-1}$. When right-left is odd $n = \frac{\text{right-left-1}}{2} \cdot \frac{2\text{left}}{2} - \frac{1}{2} + 1 = \frac{\text{right-left-1}}{2} \leq \text{right-left-1}$. As $\text{right-left+1} \leq k$ then the recursive call is made on interval 0 to k and is correct by Hyp. Thus circularBinarySearch must also work for interval $k+1$.

Case 3: $A[m] < t$

This is symmetrical to the previous case. Thus $n = \text{right} - (m+1) + 1 = \text{right} - (\lfloor \frac{\text{right-left}}{2} \rfloor + 1) + 1$ so when right-left is even $n = \text{right} - \frac{\text{right-left}}{2} - \frac{1}{2} + 1 = \frac{2\text{right}-\text{right-left}}{2} - \frac{\text{right-left}}{2} = \frac{\text{right-left}}{2}$. When odd $n = \frac{\text{right}-\text{right-left+1}}{2} - \frac{1}{2} + 1 = \frac{\text{right}-\text{right-left+1}}{2} \leq \text{right-left+1}$ as well $\frac{\text{right-left}}{2} \leq \text{right-left+1} \leq \text{right-left+1} \leq k$. So the recursive call is made on interval 0 to k and is correct by Hyp. Thus circularBinarySearch must also work for interval $k+1$.

∴ as the inductive step works in all cases we can see circularBinarySearch works in all cases.