Hyp. Let $P(n)$ be the assertion that modBin works $\forall$ inputs $right-left+1=n$

Base case: when $n=1$ then $left=right=m$ and $A[m]=0 \vee 1$ if it's 0 then it trips the first if statement and its nested else returning 0. Otherwise if it's 1 then it trips the first if and the nested if returning $n=1$ which is correct as there would be one 1.

Inductive Step: we assume modBin works as long as $right-left+1 \leq k$. We must prove $right-left+1 \leq k+1$. We have 4 cases where, $left=right$, $A[m]=1 \wedge A[m-1]=0$, $A[m]=1$ and $A[m]=0$.

Case 1, $left=right$:
When $left=right$ we have 2 sub cases firstly where $A[left]=A[right]=1$ here n is returned signifying there are n 1's in A. Second where $A[left]=A[right]=0$ here 0 is returned as no 1's exist.

Case 2, $A[m]=1 \wedge A[m-1]=0$
This is obvious as the function works and will return that there are $n-m+1$ 1's.

Case 3, $A[m]=1$
We know the array must be sorted and that $A[m-1]=1$ so the first occurrence of 1 must lie between $A[left]$ and $A[m-1]$. If the recursive call works correctly so shall this call. So $n=m-1-left$ or $n=\lfloor \frac{left+right}{2} \rfloor -1-left = \frac{right-left}{2}-1$ when $left+right$ is even. When it's odd $n=\frac{right-left-1}{2}-1$. both of these are $< left-right+1=k+1>0$. So the recursive call must be between 1 and n and correct by Hyp.

Case 4, $A[m]=0$ is symmetrical to Case 3 and thus is correct
$\therefore$ as all cases prove the inductive step works the modBin is correct by induction $\square$

TCA: $T(n)=T(\frac{n}{2})+O(1)$; $a=1, b=2, f(n)=O(1)$, watershed $= n^{\log_2 1}=n^0=1$ as $1 \in \Theta(f(n))$ where $k=0$ then $T(n)=\Theta(\lg n)$.