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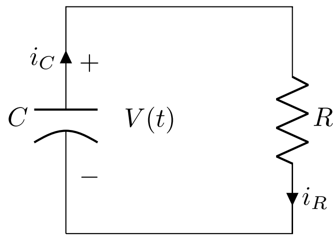
Electrical Sciences

2021-04-28

- 1^{st} **order circuits:**

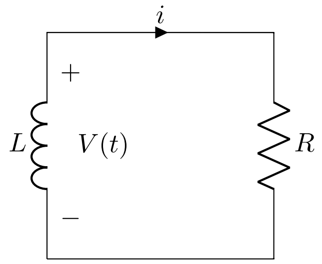
  1. RC circuits:



Using KCL gives:

$$i_C + i_R = 0$$
$$\implies C\left(\frac{dV}{dt}\right) + \frac{V}{R} = 0$$
$$\therefore V(t) = V_0 e^{-\frac{t}{RC}}$$

  2. RL circuits:

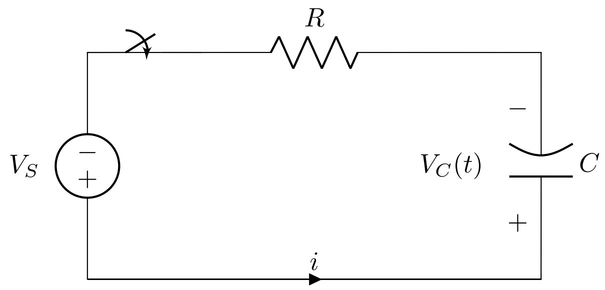


Using KVL gives:

$$Ri + L\left(\frac{di}{dt}\right) = 0$$
$$\therefore i(t) = i_0 e^{-\frac{t}{\tau}}$$

Where τ is the time constant of the circuit given by $\frac{L}{R}$

  3. Complete response:
Consider the following RC circuit with a source voltage V_S



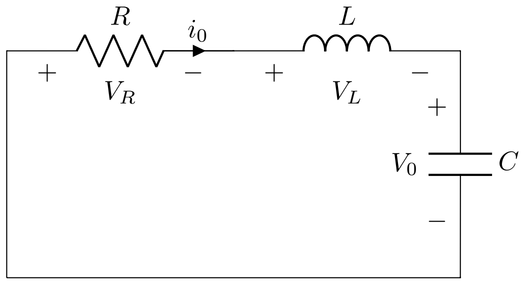
Using KVL in the closed loop gives us the following:

$$V_s = iR + V_C$$
$$\implies V_S = RC\left(\frac{dV_c}{dt}\right) + V_C$$
$$\frac{dV_C}{dt} + \frac{V_C}{RC} = \frac{V_S}{RC}$$
$$\therefore V_C(t) = Ae^{-\frac{t}{\tau}} + V_S$$

The exponential term corresponds to the **natural response**, whereas the constant term V_S corresponds to the **forced response**. The value of the coeffecient A is computed using the boundary conditions of the differential equation: $\boxed{A = V_C(0) - V_S}$

- 2^{nd} **order circuits**:

1. Natural response:



Applying KVL in the closed loop gives

$$V_R + V_L + V_0 = 0$$
$$\implies iR + L\left(\frac{di}{dt}\right) + V_0 = 0$$
$$\therefore L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

Thus, we have the following characteristic differential equation: $\boxed{s^2 + 2\alpha s + \omega_n^2 = 0}$. The solutions of this equaiton take the form

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2}$$

Here, $\alpha = \frac{R}{2L}$ and $\omega_n = \frac{1}{\sqrt{LC}}$ From this, we have 3 different cases that arise on account of the nature of the roots s_1, s_2:

(a) **Overdamped**: s_1, s_2 are **real and distinct**. Thus, $\alpha > \omega_n$. The natural solution takes the form $\boxed{V_0(t) = A_1e^{s_1t} + A_2e^{s_2t}}$

(b) **Critically damped**: s_1, s_2 are **real**. Thus, $\alpha = \omega_n$. The natural solution takes the form $\boxed{V_0(t) = A_1e^{st} + A_2te^{st}}$

(c) **Under damped**: s_1, s_2 are **complex**. Thus $\alpha < \omega_n$. The roots $s_1, s_2 = -\alpha + j\omega_d$ where j is the quantity $\sqrt{-1}$ and $\omega_d = \sqrt{\omega_n^2 - \alpha^2}$. Hence, the natural solution of the the system is of the form $\boxed{V_0(t) = B_1e^{-\alpha t}\cos\omega_d t + B_2e^{-\alpha t}\sin\omega_d t}$

2. Forced response: If we add a voltage source V_S to the previous circuit, it is wasy to observe that by the superposition theorem, the quantity V_S simply gets added to the natural response solutions of the characteristic differential equations.

- **AC analysis:**

1. Impedance and admittance:
All the results (in partiucular, the solutions of 2^{nd} order circuits) of DC circuit analysis carry forward here.
The **impedance** (Z) of the circuit elements is the AC analog resistance.

$$Z = \frac{V}{i} = R \text{ or } j\omega \text{ or } \frac{1}{j\omega C}$$

Similarly, the AC analog of conductance is called **admittance** (Y)

$$Y = \frac{i}{V} = \frac{1}{R} \text{ or } \frac{1}{j\omega L} \text{ or } j\omega C$$

In both of these, j denotes the complex square root ($\sqrt{-1}$) and ω is the angular freqeuncy of the voltage source.

2. Power:
   Consider an AC voltage source with a time varying current $i(t)$ and a time varying voltage $V(t)$. The power due to this source is

$$\begin{aligned} p(t) = v(t)i(t) &= V_m \cos(\omega t + \varphi_1) I_m \sin(\omega t + \varphi_2) \\ &= \frac{1}{2} V_m I_M [\cos(\omega t + \varphi_1 + \omega t + \varphi_2) + \cos(\varphi_1 - \varphi_2)] \end{aligned}$$

   Here, V_m and I_M denote the peak voltage and current respectively. From here, we calculate the average power to be

$$\langle p(t) \rangle = \frac{1}{2} V_M I_M \cos(\varphi_1 - \varphi_2)$$

   Here, we define another quantity, the **RMS voltage** $(V_{\text{RMS}}) = \frac{V_M}{\sqrt{2}}$. Using this, the average power is written as

$$\langle p(t) \rangle = V_{\text{RMS}} I_{\text{RMS}} \cos\theta$$

   θ being the phase difference $(\varphi_1 - \varphi_2)$. Note that φ_1 denotes the phase angle of voltage and φ_2 denotes the phase angle of current.
   Also, the RMS voltage (and current) is computed by performing the following integral:

$$V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0 + t} v^2(t) dt}$$