Given a linear map $A : \mathbb{R}^n \to \mathbb{R}^m$ and a vector in its range $b \in \mathbb{R}^m$, form the linear system $Ax = b$.

A basis $\{e_1 ... e_n\}$ in the domain of A now gives many ways to express b as a combination of the image of the basis $\{Ae_i\}_i$:

$$b = \sum_{i \in B} \lambda_i A e_i$$

for many choices of m-sized $B \subset \{1...n\}$ and $\lambda_i, i \in B$.

For each such choice of B, we can rewrite $b = A \sum_{i \in B} \lambda_i e_i$ and denote

$$x_B := \sum_{i \in B} \lambda_i e_i$$

so that $b = Ax_B$.

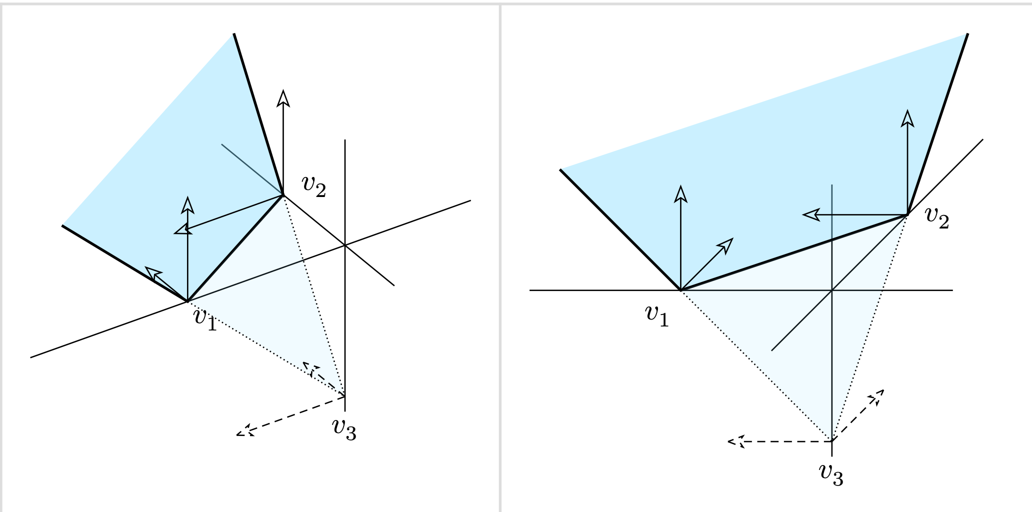
x_B is now well-defined for any choice B of (a subset of) basis vectors in the domain of A whose images form a basis of A's range. Such x_B are *special* solutions to the system $Ax = b$ and are called *basic* solutions.

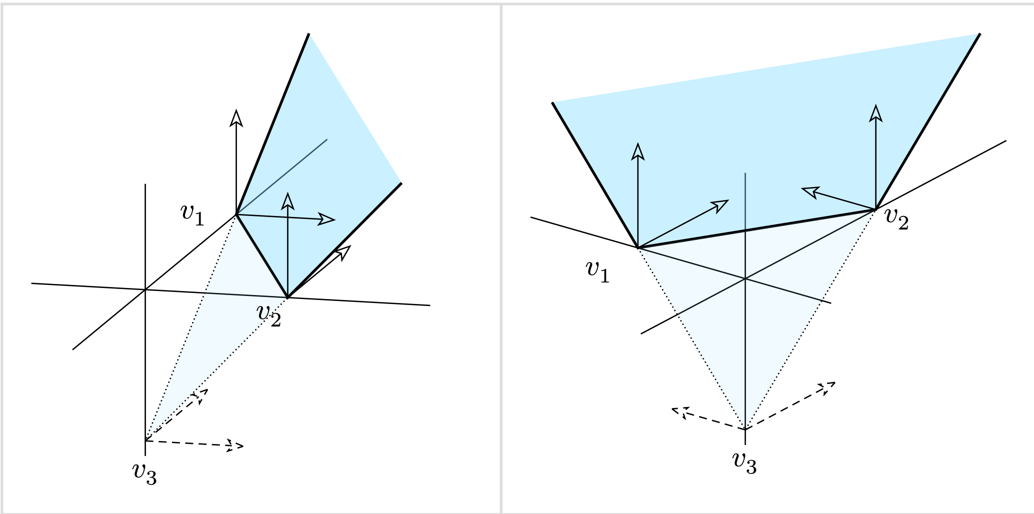
A general solution x can be decomposed as

$$x = x_B + x_N$$

which defines x_N given B. Put $N = \{1...n\} \setminus B$. Since x is a combination of $\{e_1 ... e_n\}$ and x_B is a (unique) combination of the B subset of the basis vectors, x_N is then a (unique) combination of the rest of the basis vectors.

Those e_i that form x_B are called *basic*, and those forming x_N are called *non-basic*.





v_1
v_2
v_3
v_2
v_1
v_3