5.4 Q 1(a)

Modified Euler's Method

$y' = te^{3t} - 2y$, $0 \le t \le 1$ Actual solution: $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$

$y(0) = 0$, $h = 0.5$

Modified Euler's Method

$$\begin{cases} w_0 = \alpha \\ w_{i+1} = w_i + \frac{h}{2}\left(f(t_i, w_i) + f(t_i + h, w_i + h \cdot f(t_i, w_i)\right\} \end{cases}$$

$w_0 = \alpha = 0$

$t_i = t_0 + h \cdot i = 0 + 0.5i = 0.5i$

$$w_1 = w_{0+1} = w_0 + \frac{h}{2}\left(f(t_0, w_0) + f(t_0 + h, w_0 + hf(t_0, w_0)\right)$$

$$= 0 + \frac{0.5}{2}\left[f(0,0) + f(0.5, 0 + 0.5f(0,0)\right]$$

$$= 0.25\left[f(0,0) + f(0.5, 0.5f(0,0))\right]$$

$$= 0.25\left[0 + f(0.5, 0)\right]$$

$$= 0.25\left[0 + \left(0.5e^{3(0.5)} - 2(0)\right)\right]$$

$$= 0.25\left[0 + 2.240844535\right]$$

$$w_1 = 0.5602111338$$

$$w_2 = w_{1+1} = w_1 + \frac{h}{2}\left(f(t_1, w_1) + f(t_1 + h, w_1 + h \cdot f(t_1, w_1)\right]$$

$$= w_1 + \frac{0.5}{2}\left[f(0.5, w_1) + f(1, w_1 + 0.5f(0.5, w_1)\right]$$

$$= 0.5602 + 0.25\left[f(0.5, 0.5602) + f(1, 0.5602 + 0.5f(0.5, 0.5602)\right]$$

$$= 5.30149$$

Actual values: $y(0.5) = 0.283617$

$y(1) = 3.2191$

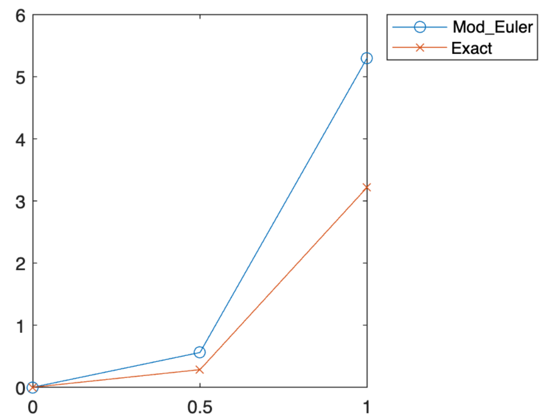
$\text{Error}[i] = |y[t_i] - w_i|$

$$\Rightarrow |y(t_1) - w_1| = |y(0.5) - w_1| = |0.5602... - 0.283617|$$
$$= 0.2765946$$

$$|y(t_2) - w_2| = |y(1) - w_2| = |3.2191 - 5.301491|$$

$$= 2.082391$$

Plots



Q5a) Midpoint Method

$y' = te^{3t} - 2y \quad , \quad 0 \le t \le 1$ Actual solution: $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$

$y(0) = 0 \quad , \quad h = 0.5$

Midpoint Method

$$\begin{cases} w_0 = \alpha \\ w_{i+1} = w_i + h \cdot f\left(t_i + \frac{h}{2},\ w_i + \frac{h}{2} f(t_i, w_i)\right) \end{cases}$$ for $i = 0, 1, 2, \dots, N-1$

$w_0 = 0$

$t_0 = 0$

$h = 0.5$

$f(t, y) = te^{3t} - 2y$

$t_i = t_0 + i \cdot h$

$t_1 = 0 + 0.5 = 0.5$

$w_1 = w_{0+1} = w_0 + hf\left(t_0 + \frac{h}{2},\ w_0 + \frac{h}{2} f(t_0, w_0)\right)$

$= 0 + (0.5) f\left(0 + 0.25,\ 0 + 0.25 f(0, 0)\right)$

$= 0.5 f\left(0.25,\ 0.25 f(0, 0)\right)$

$= 0.5 f\left(0.25,\ 0.25\left(0e^{3 \cdot 0} - 2(0)\right)\right)$

$= 0.5 f(0.25, 0)$

$= 0.5 \left[0.25 e^{(3 \times 0.25)} - 2(0)\right]$

$= 0.2646250021$

$w_2 = w_{1+1} = w_1 + hf\left[t_1 + \frac{h}{2},\ w_1 + \frac{h}{2} f(t_1, w_1)\right]$

$= 0.264625 + 0.5\, f\left(0.5 + 0.25,\ 0.264625 + 0.25 f(0.5, 0.264625)\right)$

$= 0.264625 + 0.5\, f\left(0.75,\ 0.264625 + 0.25 \cdot \left(0.5 e^{(3\times 0.5)} - 2(0.264625)\right)\right)$

$= 3.13000$

Actual values: $y(0.5) = 0.283617$

$y(1) = 3.2191$

Error $[i] = |y[t_i] - w_i|$

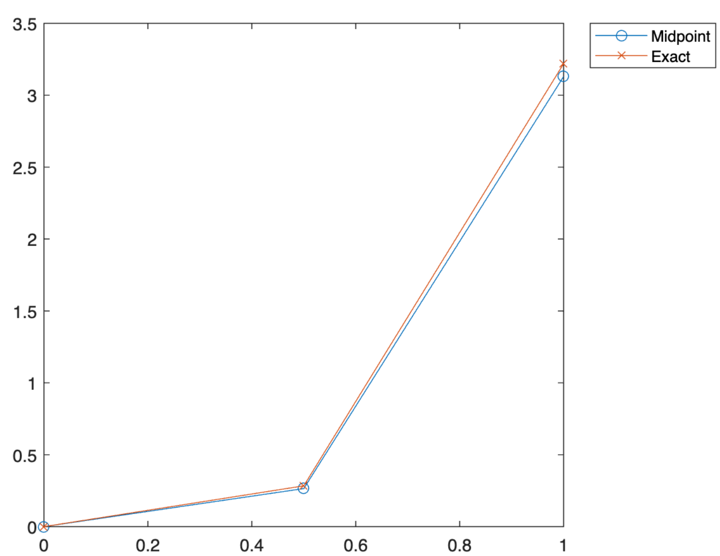
$\Rightarrow |y(t_1) - w_1| = |y(0.5) - w_1| = |0.283617 - 0.2646250021|$

$= 0.0189919979$

$|y(t_2) - w_2| = |y(1) - w_2| = |3.2191 - 3.130001$

$= 0.0891$

Plots:



Q13 a) Runge - Kutta of order 4.

(Done using Matlab)

Actual values: $y(0.5) = 0.283617$

$y(1) = 3.2191$

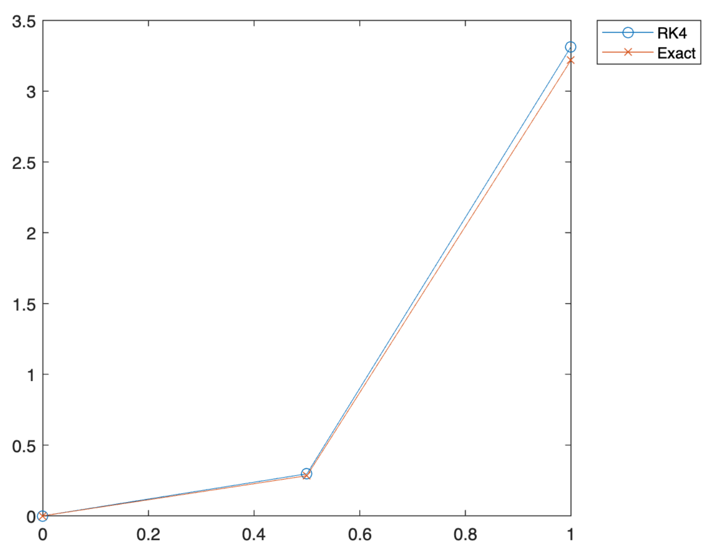
$\text{Error}[i] = |y[t_i] - w_i|$

$$\Rightarrow |y(t_1) - w_1| = |y(0.5) - w_1| = |0.283617 - 0.296997|$$
$$= 0.01338$$

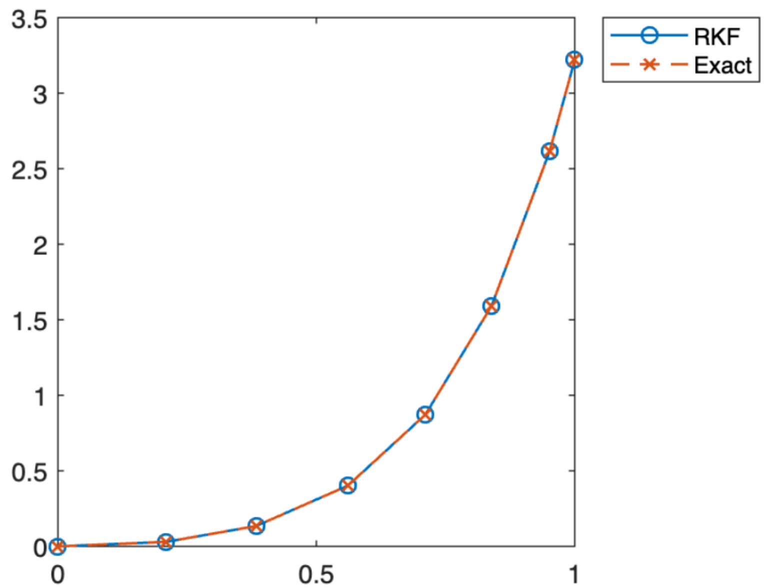
$$|y(t_2) - w_2| = |y(1) - w_2| = |3.2191 - 3.31431|$$
$$= 0.09521$$

Code:

```
t0 = 0; t1 = 1; % Define the interval
h = 0.5;        % Step size
w0 = 0; % Initial condition

syms f(t,y)
f(t,y) = t*exp(3*t) -(2*y);

[t,w] = RK4(t0,t1,h,w0,f);
y = (1/5)*t.*exp(3*t) - (1/25)*exp(3*t) + (1/25)*exp(-2*t)
figure
plot(t,w, '-o',t,y,'-x')
legend('RK4','Exact', "Location","bestoutside")


function [t, w] = RK4(t0,t1,h,w0,f)
t = t0:h:t1;
w = zeros(size(t));
w(1) = w0;
for i = 1:size(t,2)-1
    k_1 = h*f(t(i),w(i));
    k_2 = h*f(t(i)+0.5*h,w(i)+0.5*k_1);
    k_3 = h*f((t(i)+0.5*h),(w(i)+0.5*k_2));
    k_4 = h*f((t(i)+h),(w(i)+k_3));
    w(i+1) = w(i) + (1/6)*(k_1+2*k_2+2*k_3+k_4);
end
end
```



5.5 Q1a)




```
Output:

fivept5_q1
     0    0.2094    0.3833    0.5610    0.7107    0.8388    0.9513    1.0000

     0    0.0298    0.1343    0.4016    0.8708    1.5894    2.6140    3.2190

error between the approximation and the exact solution is: 0 -1.5296e-05
-2.2722e-05 -4.2226e-05 -5.0911e-05 -5.3878e-05 -5.4428e-05  -4.958e-05
```

5.5 Q2a)