



<http://algs4.cs.princeton.edu>

## 2.3 Partitioning Demos

---

- *Sedgewick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*



<http://algs4.cs.princeton.edu>

## 2.3 Partitioning Demos

---

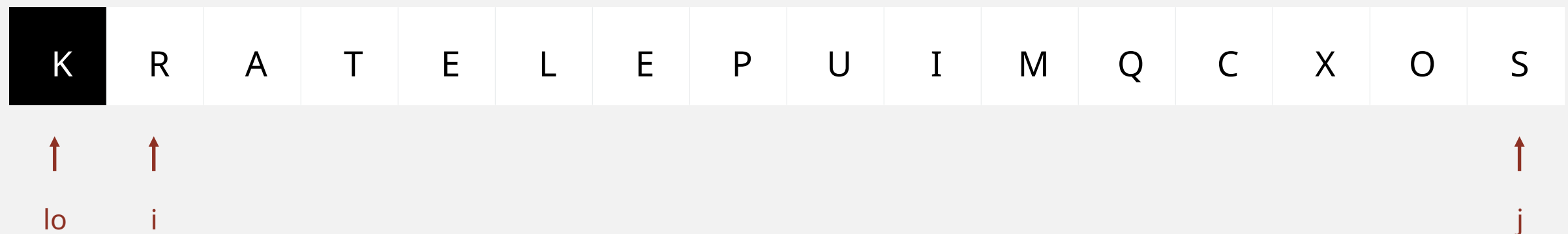
- *Sedgewick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*

# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .

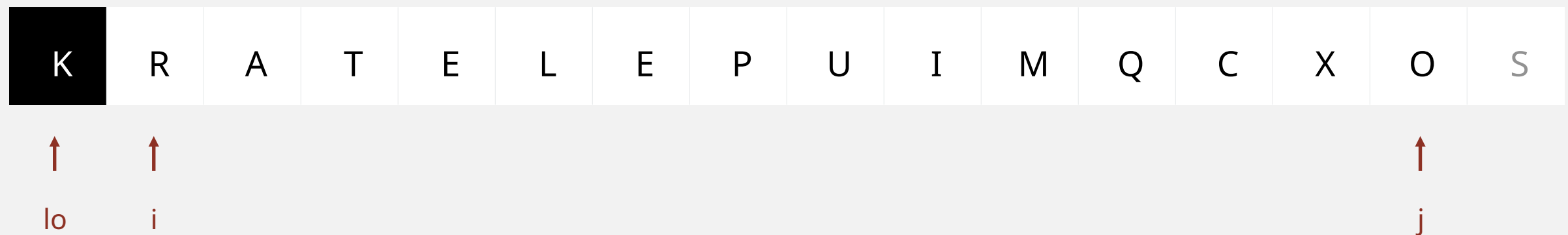


**stop i scan because  $a[i] \geq a[lo]$**

# Quicksort partitioning demo

Repeat until i and j pointers cross.

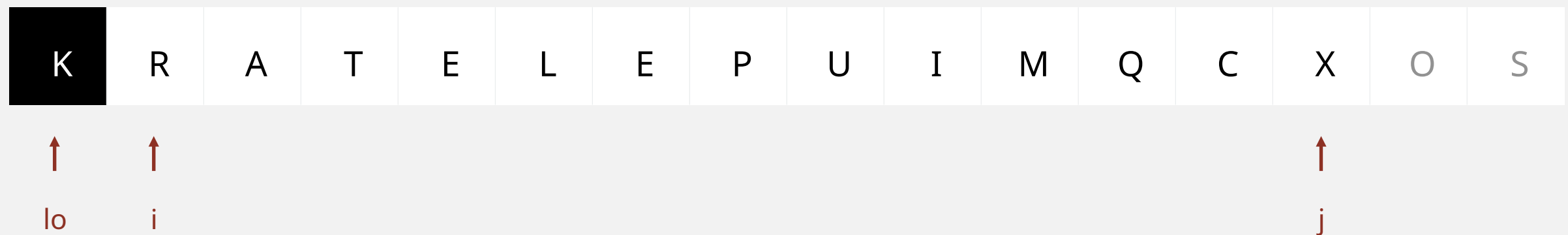
- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .



# Quicksort partitioning demo

Repeat until i and j pointers cross.

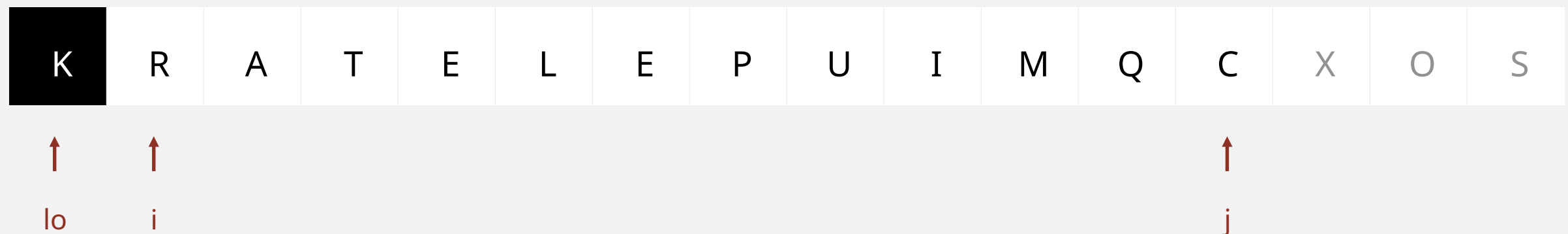
- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .



# Quicksort partitioning demo

Repeat until i and j pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .

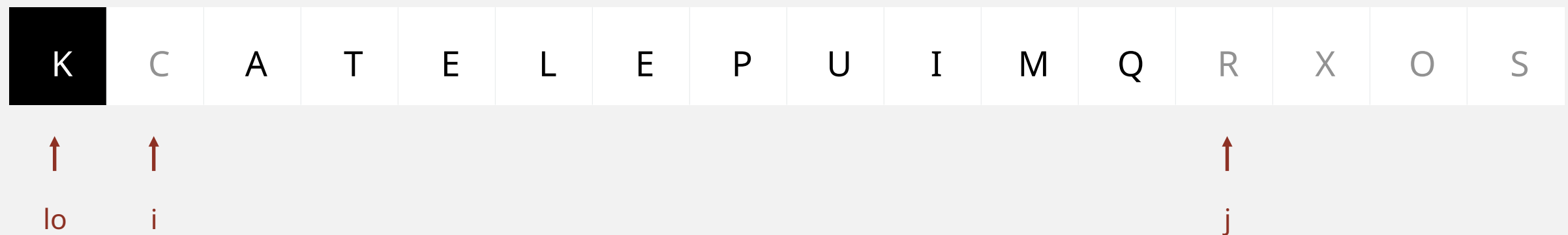


**stop j scan and exchange  $a[i]$  with  $a[j]$**

# Quicksort partitioning demo

Repeat until i and j pointers cross.

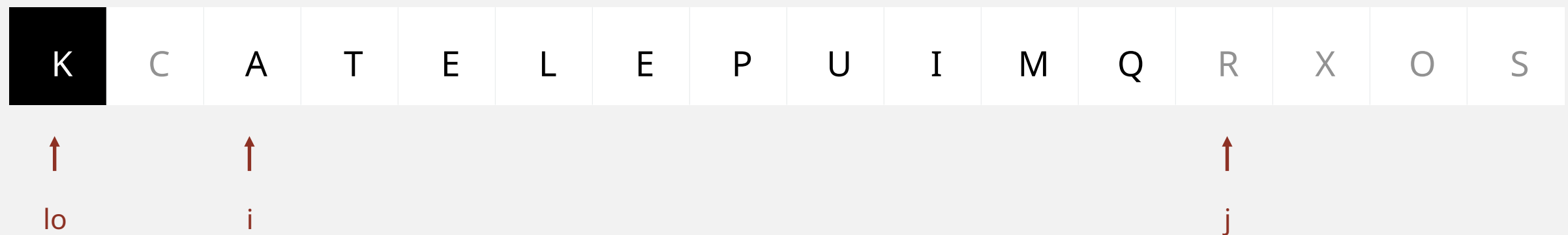
- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .



# Quicksort partitioning demo

Repeat until i and j pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .



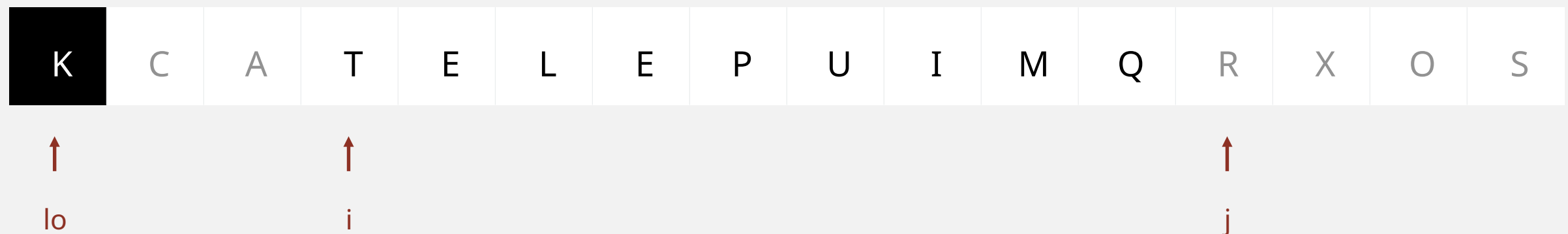


# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .



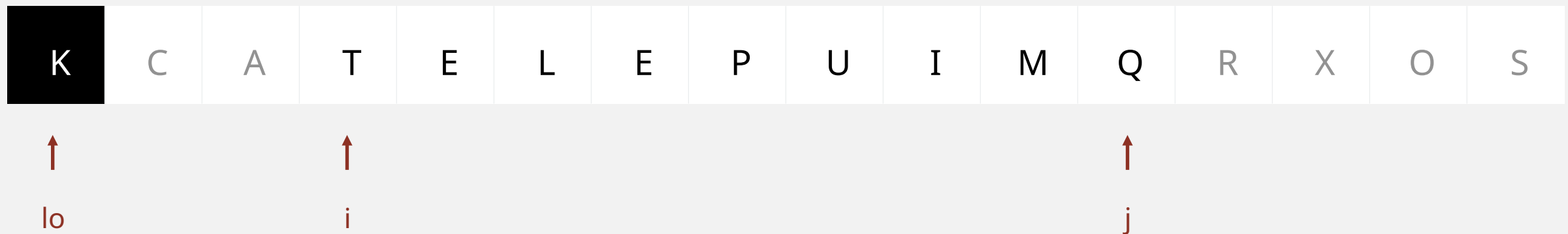
**stop i scan because  $a[i] \geq a[lo]$**

# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .

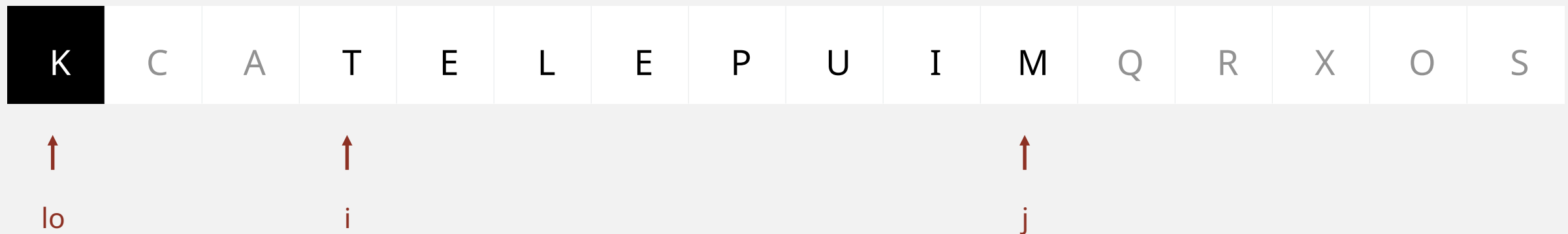


# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .

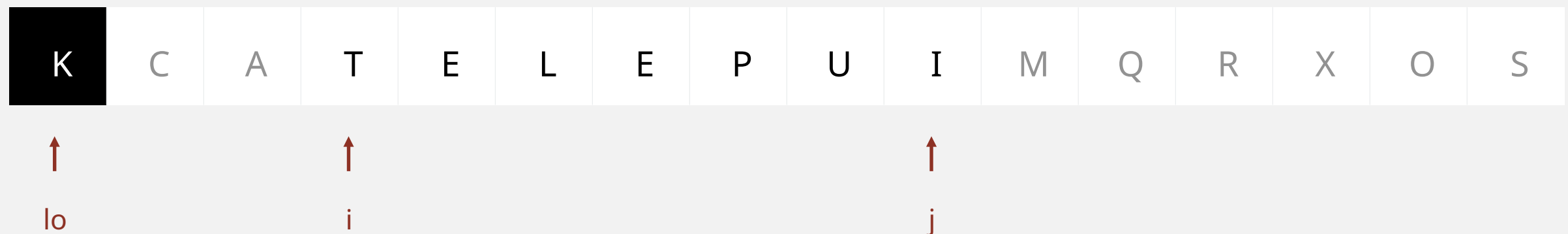


# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .



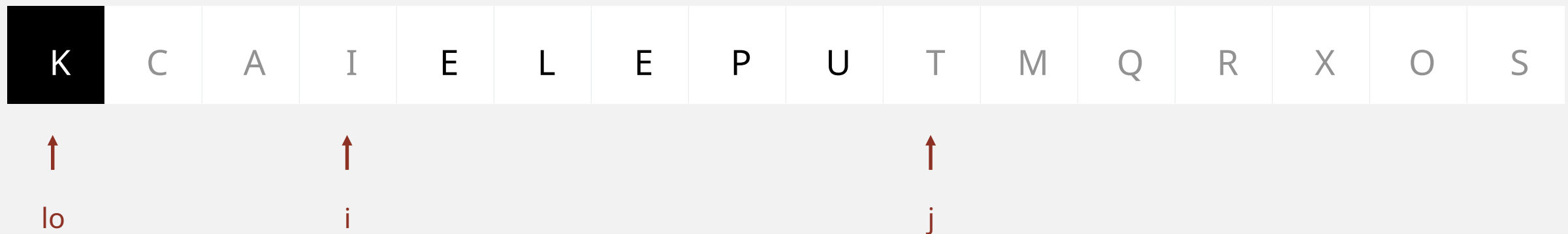
**stop  $j$  scan and exchange  $a[i]$  with  $a[j]$**

# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .

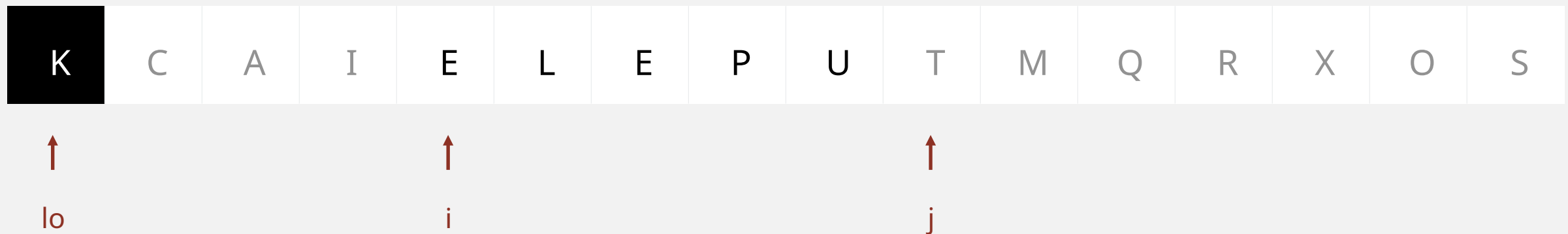


# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

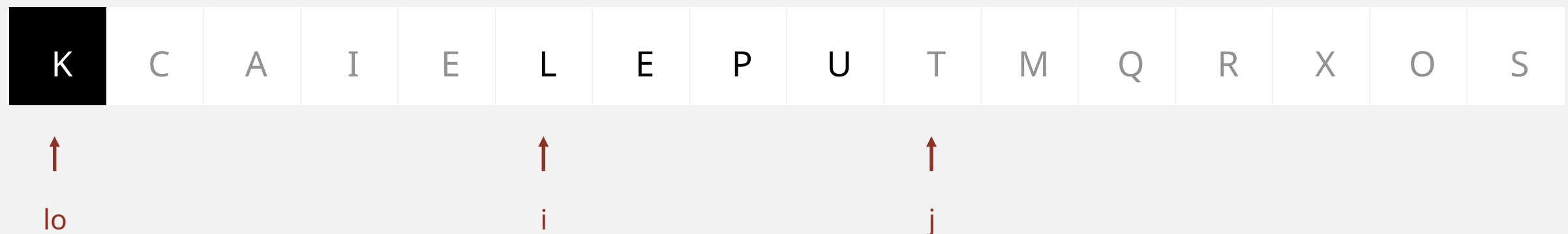
- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .



# Quicksort partitioning demo

Repeat until i and j pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .



stop i scan because  $a[i] \geq a[lo]$

# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .





# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

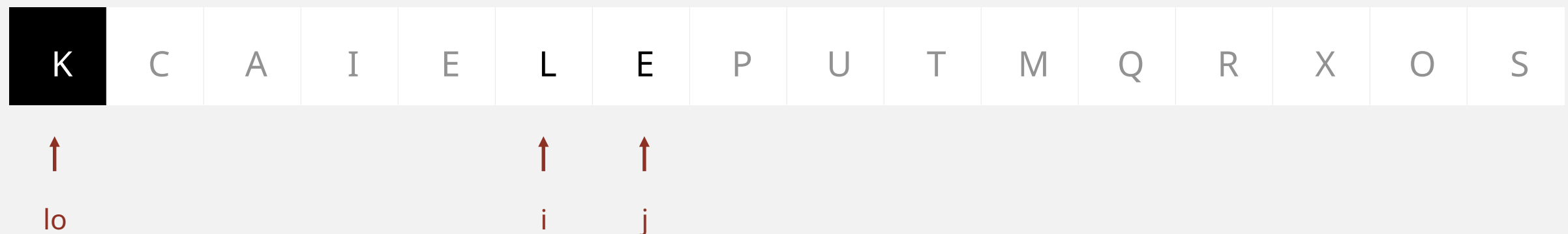
- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .



# Quicksort partitioning demo

Repeat until i and j pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .



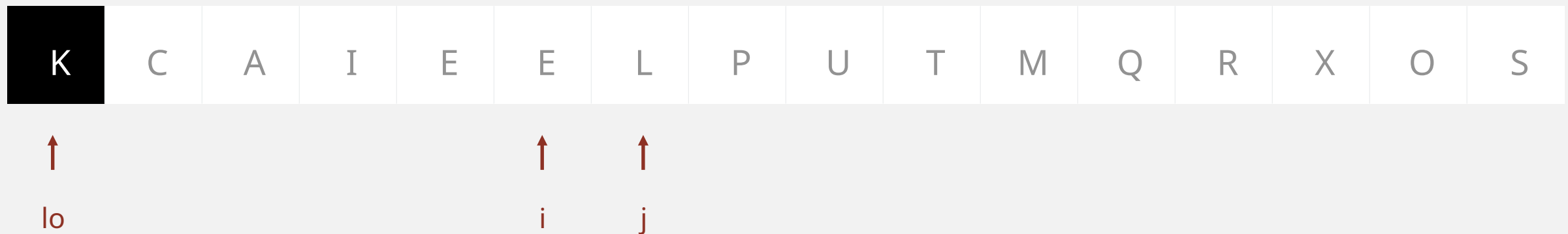
**stop j scan and exchange  $a[i]$  with  $a[j]$**

# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .

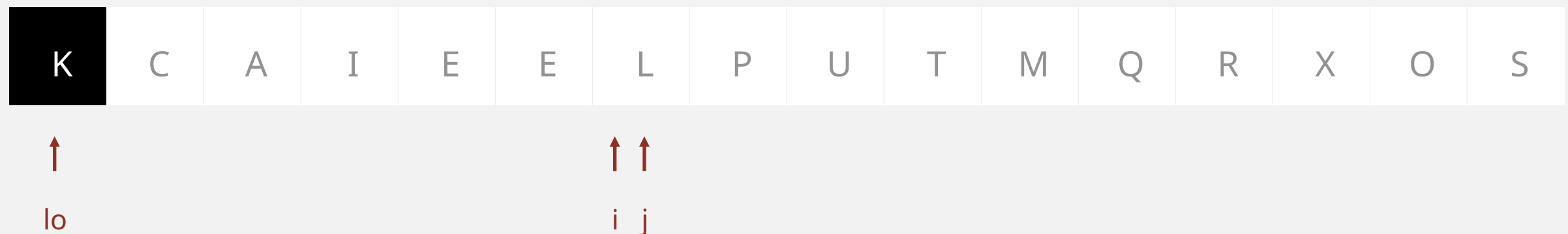


# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .

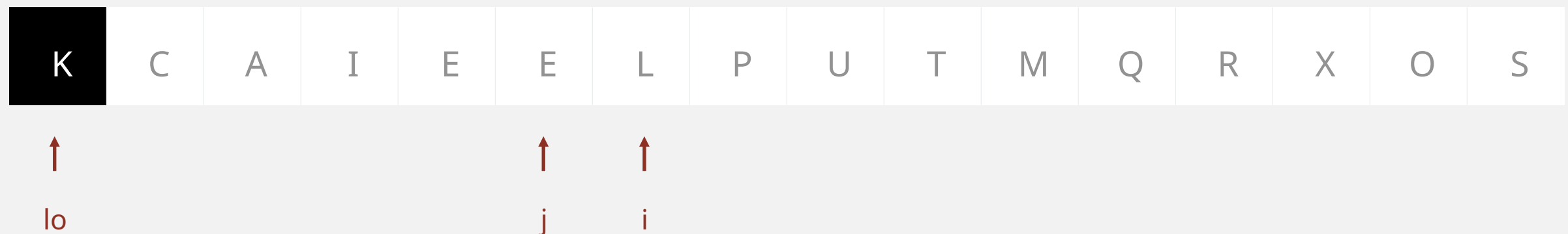


**stop  $i$  scan because  $a[i] \geq a[lo]$**

# Quicksort partitioning demo

Repeat until i and j pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .



**stop j scan because  $a[j] \leq a[lo]$**

# Quicksort partitioning demo

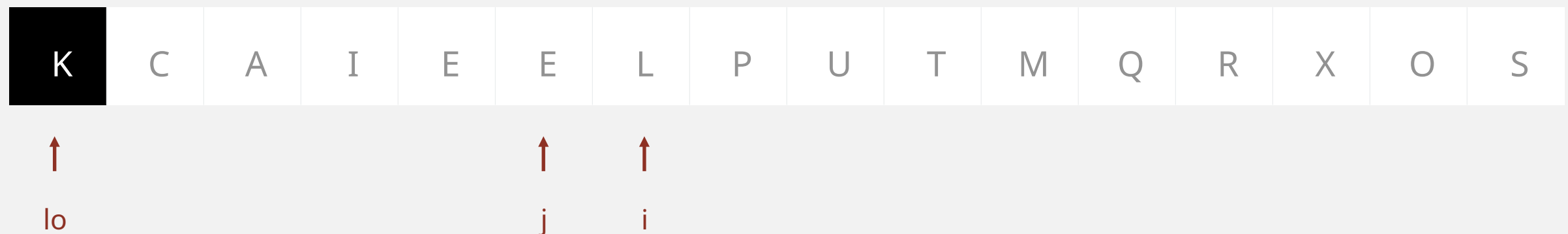
---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .

When pointers cross.

- Exchange  $a[lo]$  with  $a[j]$ .



**pointers cross: exchange  $a[lo]$  with  $a[j]$**

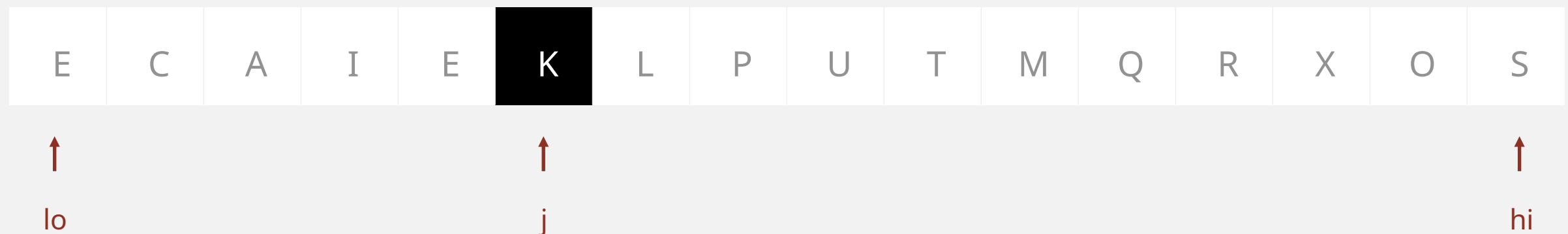
# Quicksort partitioning demo

Repeat until i and j pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[l_o])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[l_o])$ .
- Exchange  $a[i]$  with  $a[j]$ .

# When pointers cross.

- Exchange  $a[\text{lo}]$  with  $a[\text{j}]$ .



## partitioned!



<http://algs4.cs.princeton.edu>

## 2.3 Partitioning Demos

---

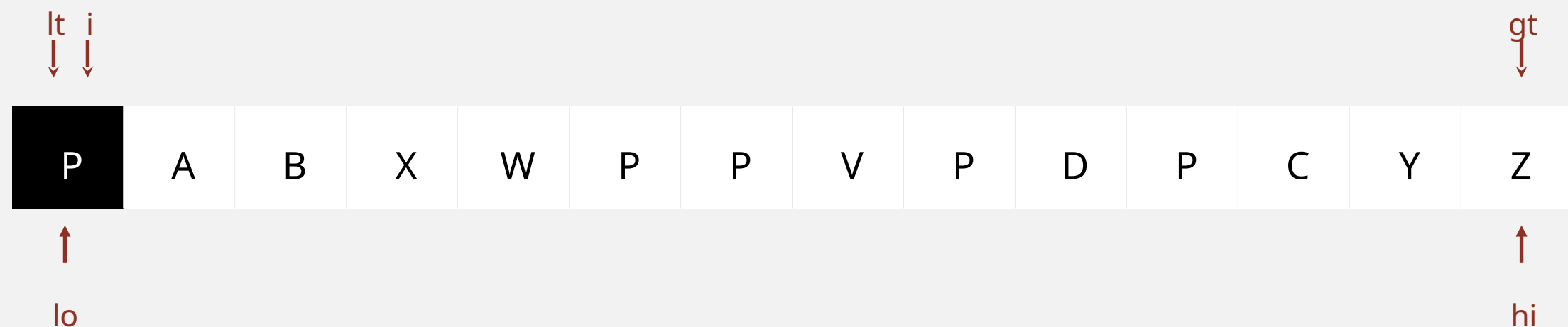
- *Sedgewick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*



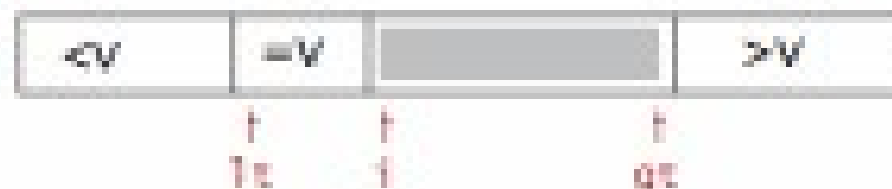
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



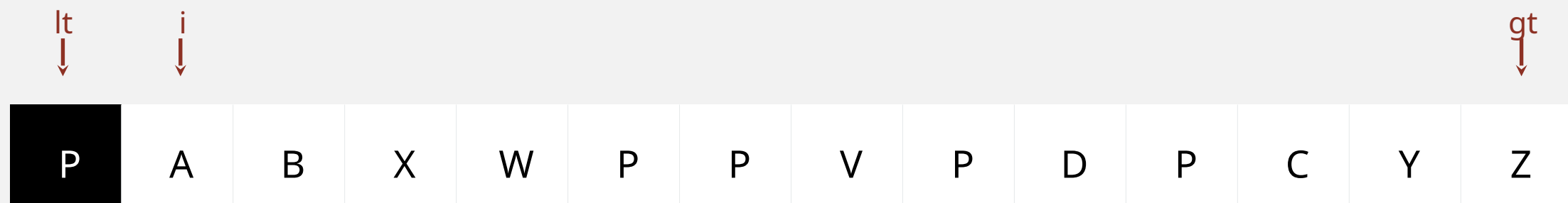
**invariant**



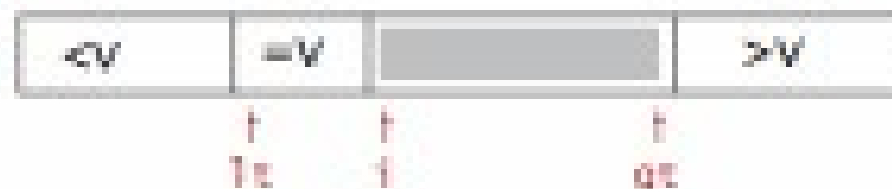
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



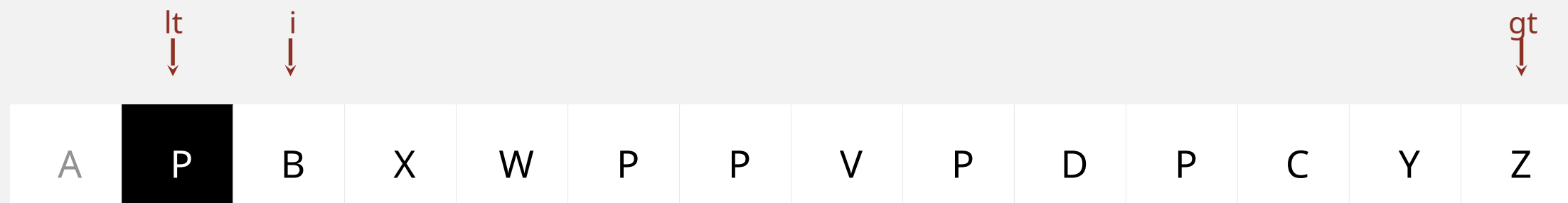
**invariant**



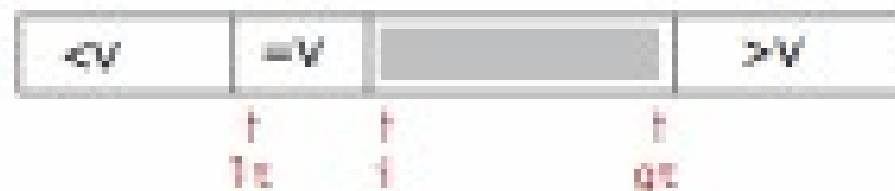
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[l_0]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[l_t]$  with  $a[i]$ ; increment both  $l_t$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[g_t]$  with  $a[i]$ ; decrement  $g_t$
  - ( $a[i] == v$ ): increment  $i$



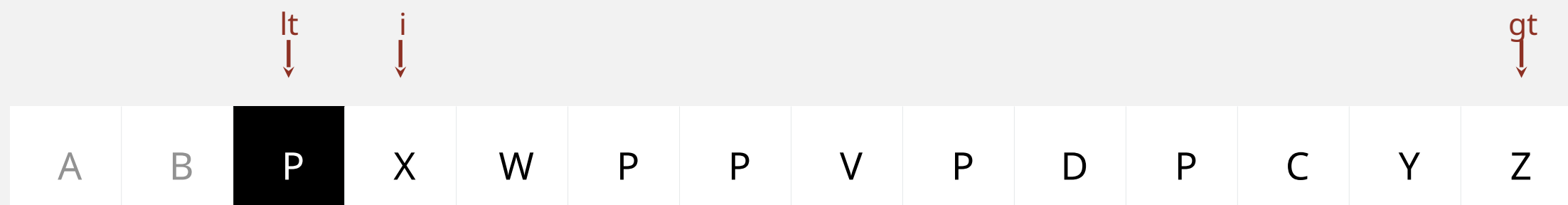
**invariant**



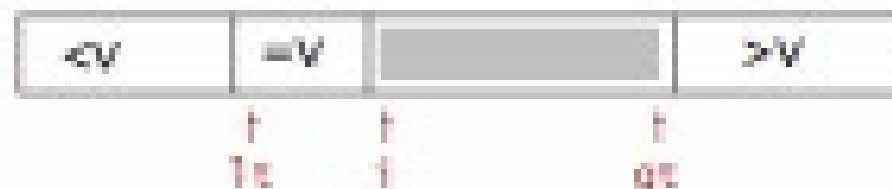
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[l_0]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[l_t]$  with  $a[i]$ ; increment both  $l_t$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[g_t]$  with  $a[i]$ ; decrement  $g_t$
  - ( $a[i] == v$ ): increment  $i$



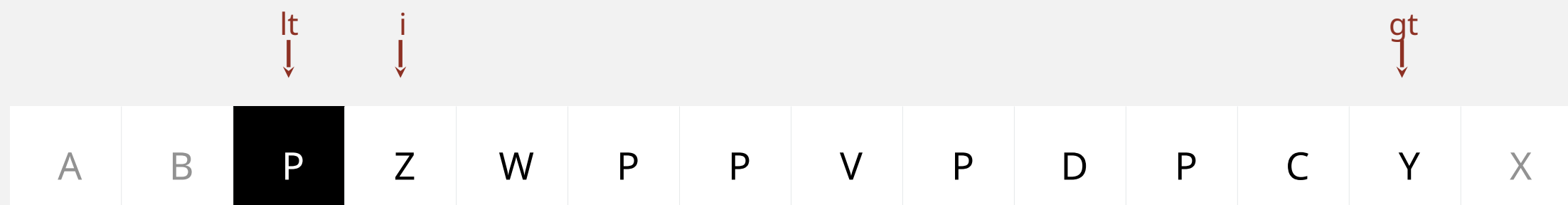
**invariant**



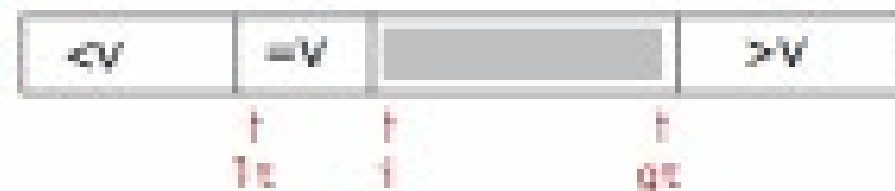
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



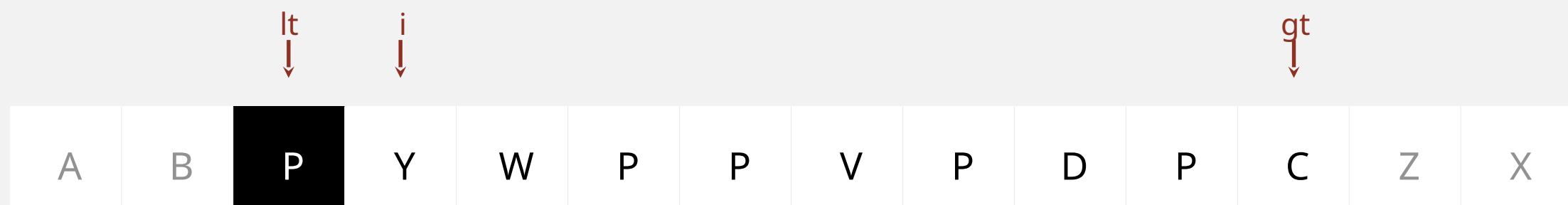
**invariant**



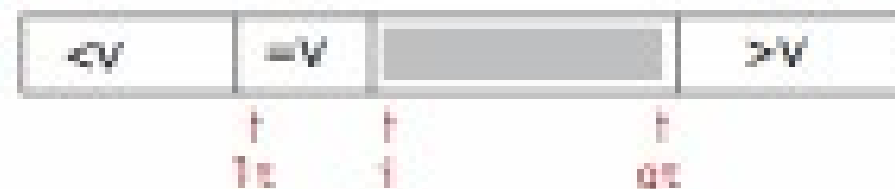
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



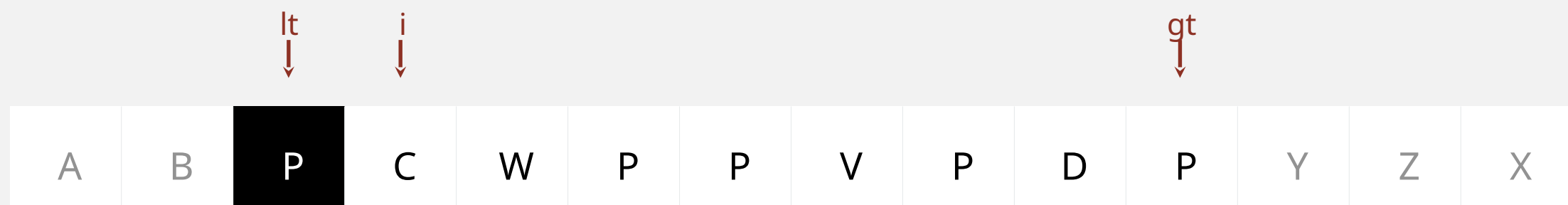
**invariant**



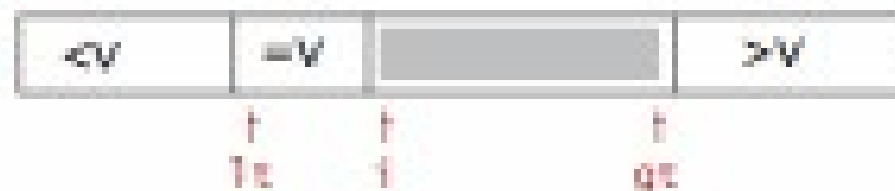
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



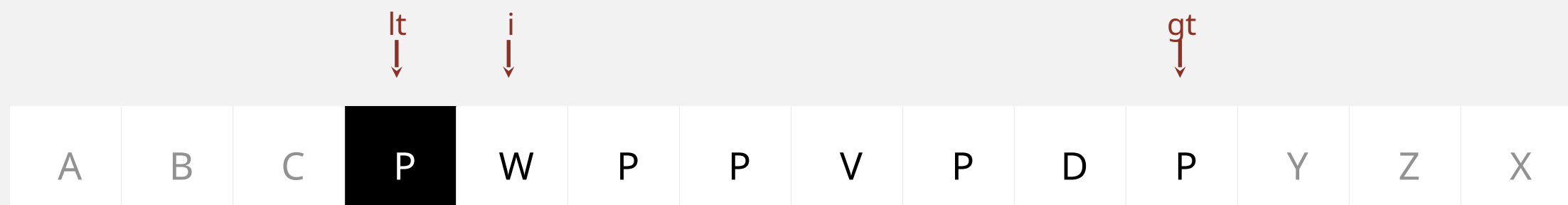
**invariant**



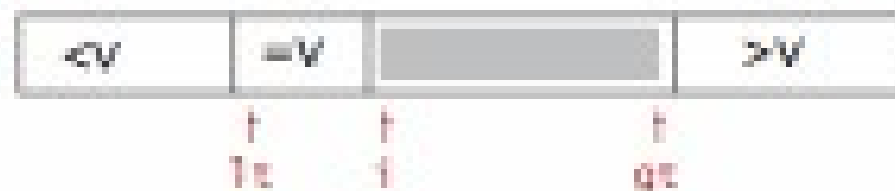
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



**invariant**

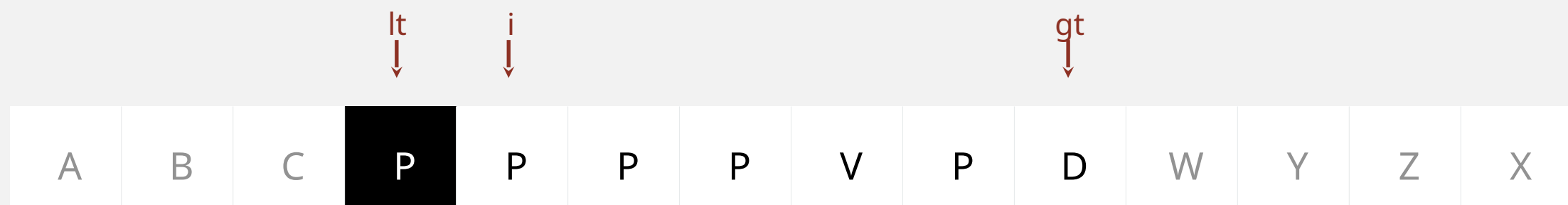




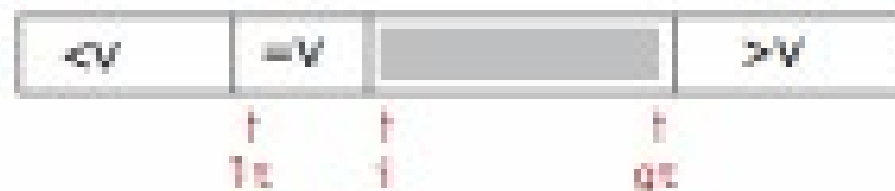
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



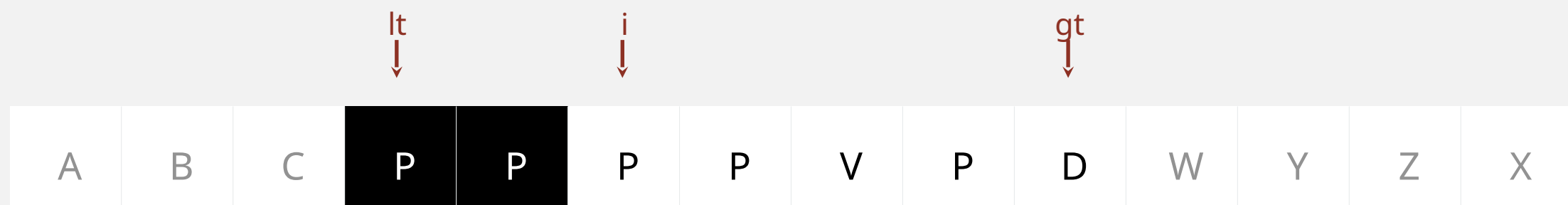
**invariant**



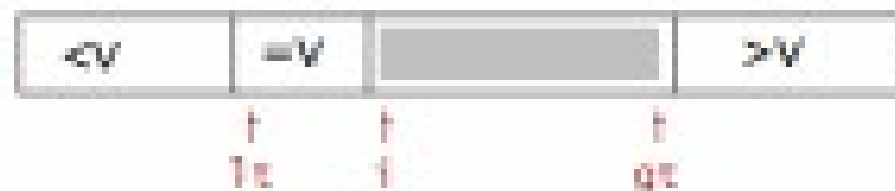
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



**invariant**



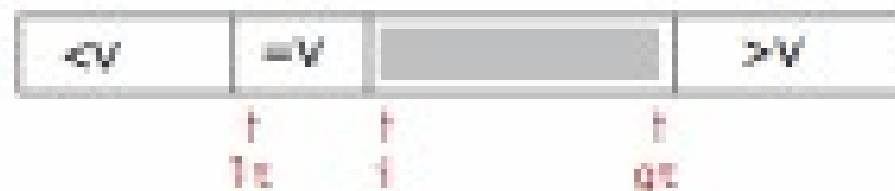
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



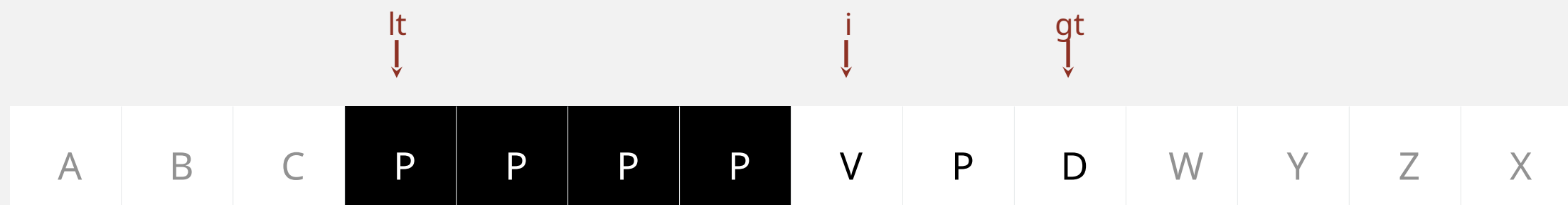
**invariant**



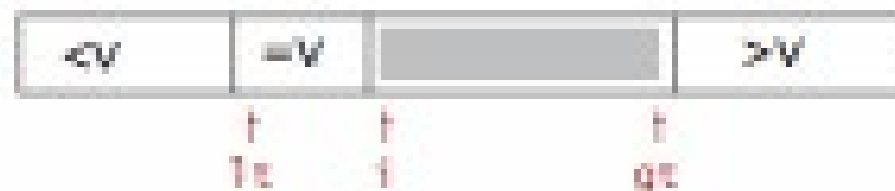
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



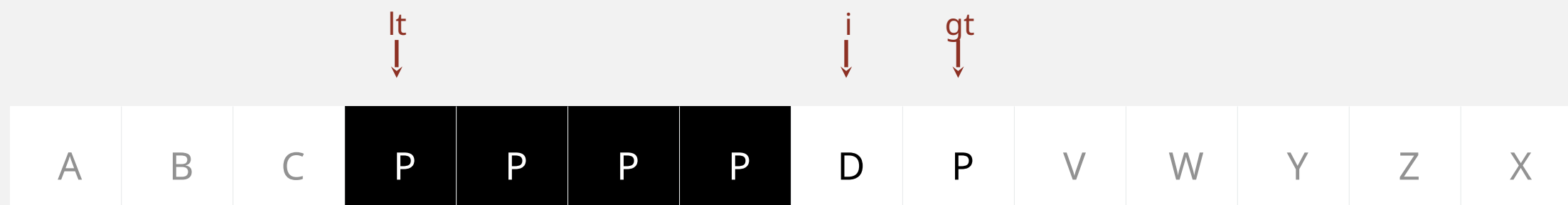
**invariant**



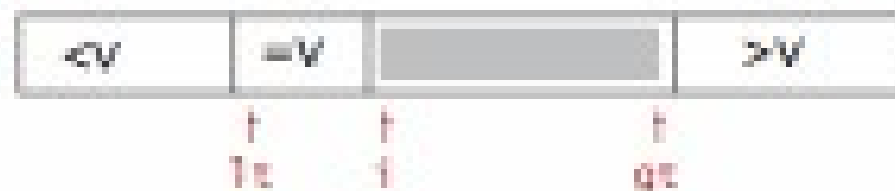
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



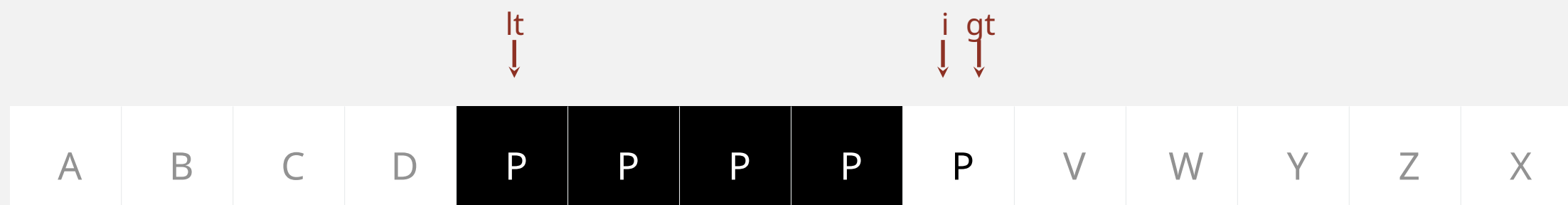
**invariant**



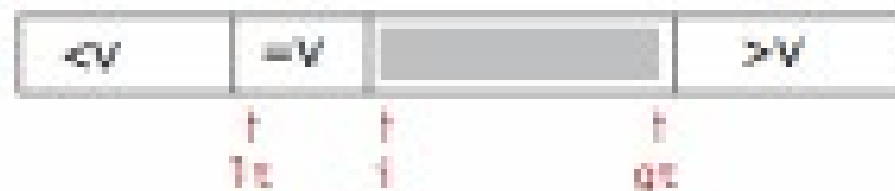
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[l_0]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[l_t]$  with  $a[i]$ ; increment both  $l_t$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[g_t]$  with  $a[i]$ ; decrement  $g_t$
  - ( $a[i] == v$ ): increment  $i$



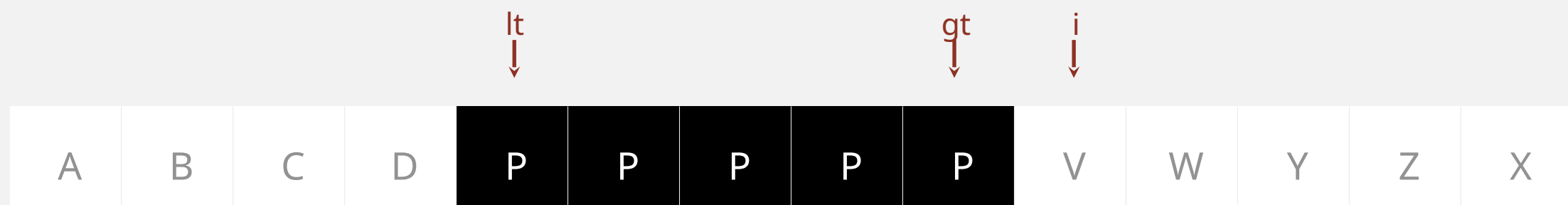
**invariant**



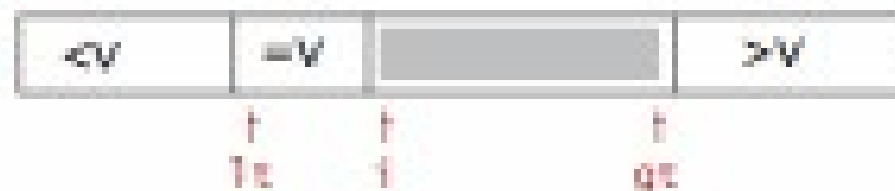
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - $(a[i] < v)$ : exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - $(a[i] > v)$ : exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - $(a[i] == v)$ : increment  $i$



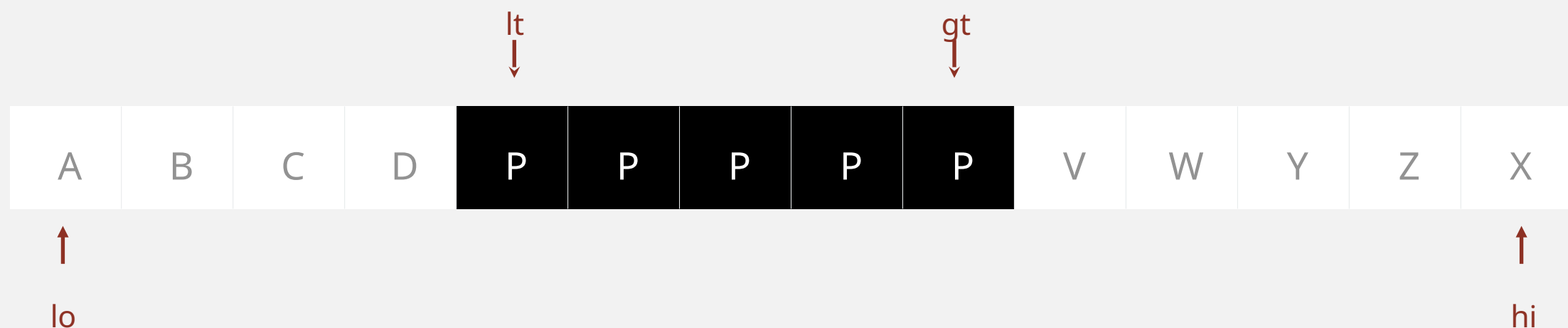
**invariant**



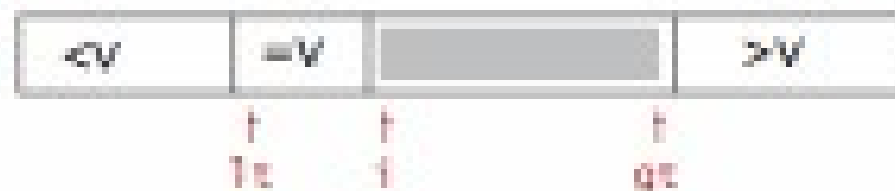
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



**invariant**







<http://algs4.cs.princeton.edu>

## 2.3 Partitioning Demos

---

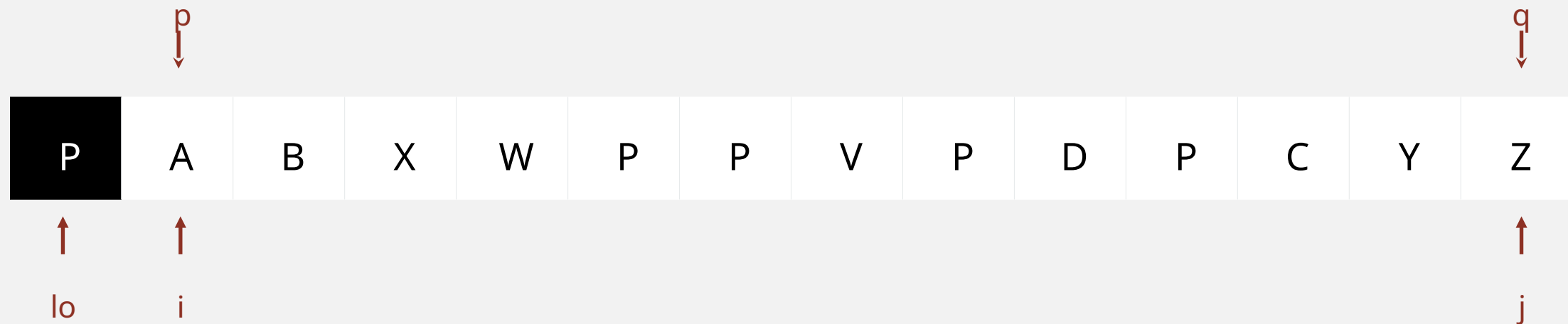
- *Sedgewick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

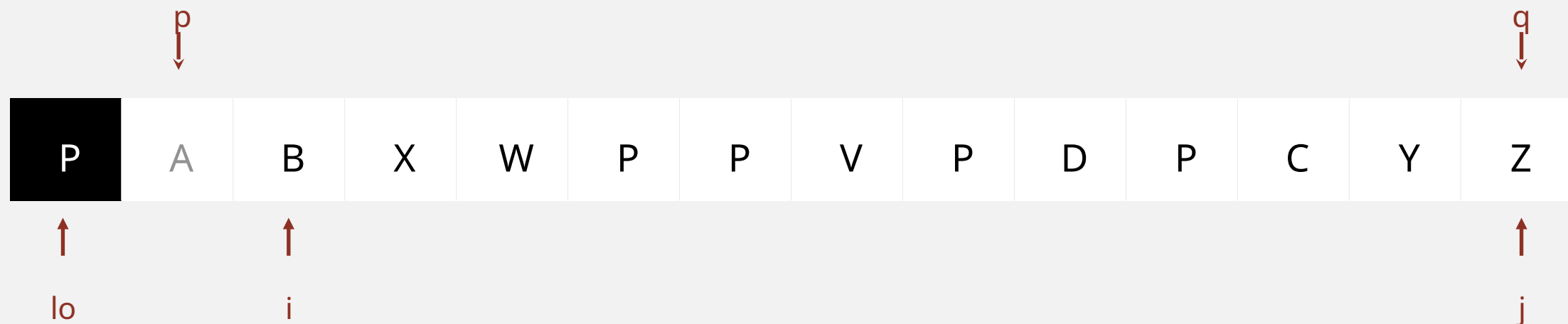


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

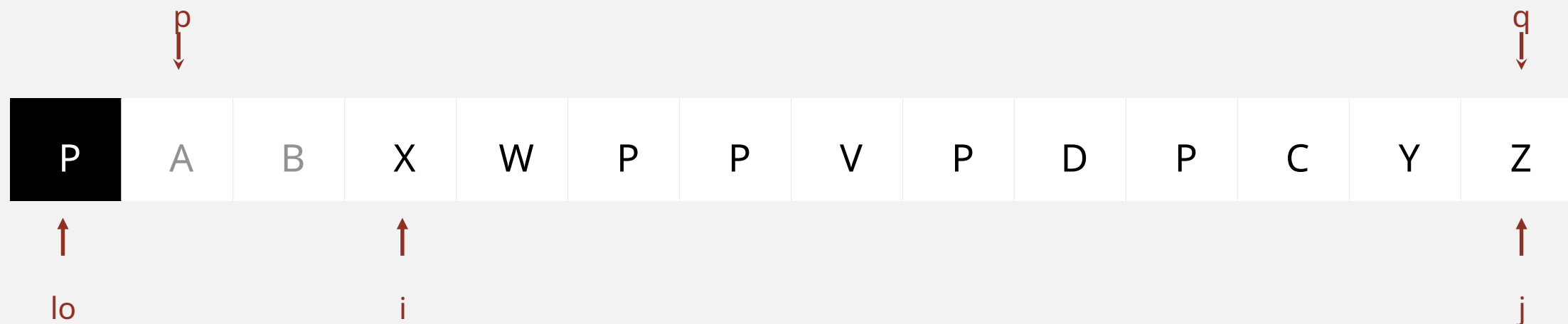


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

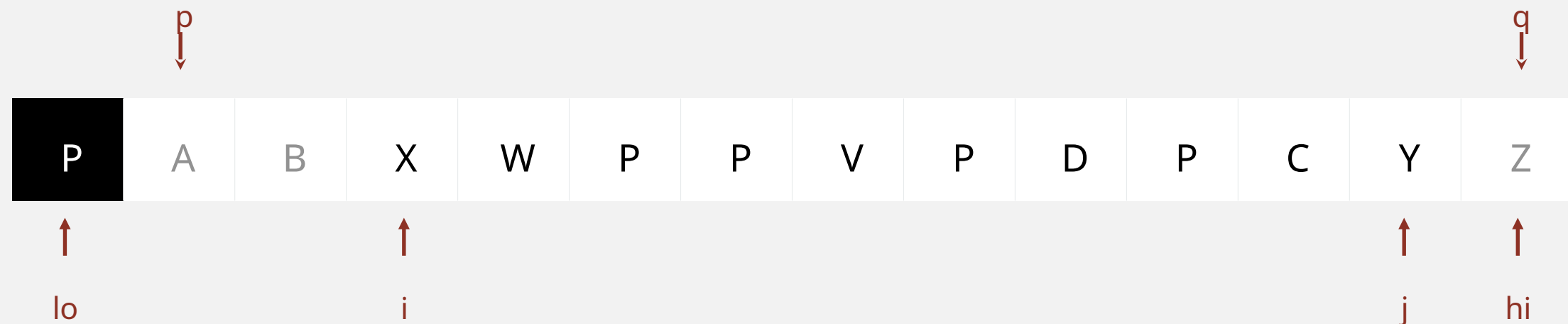


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

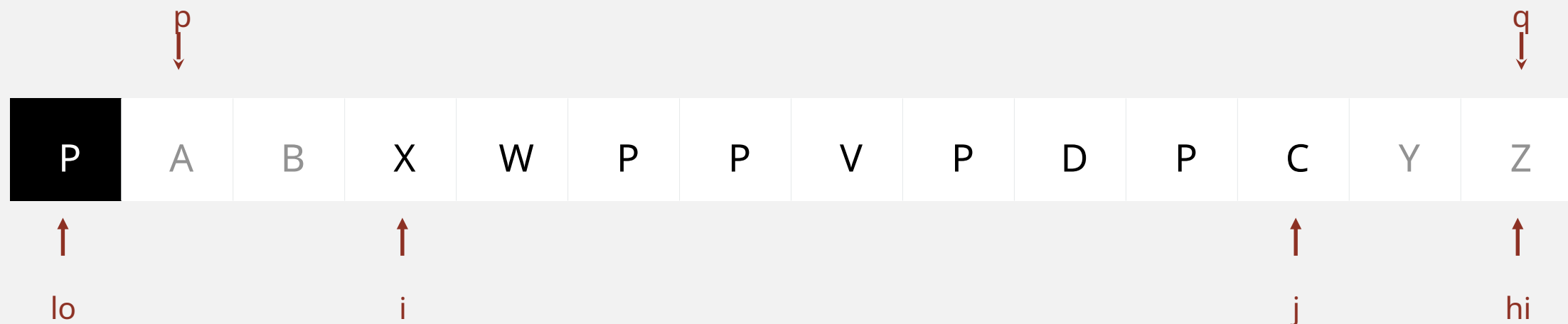


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



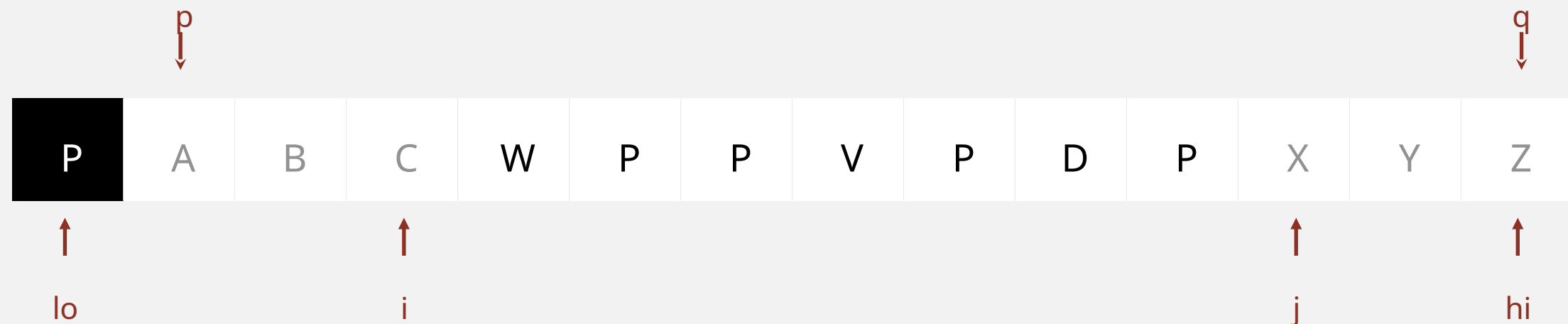
exchange  $a[i]$  with  $a[j]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

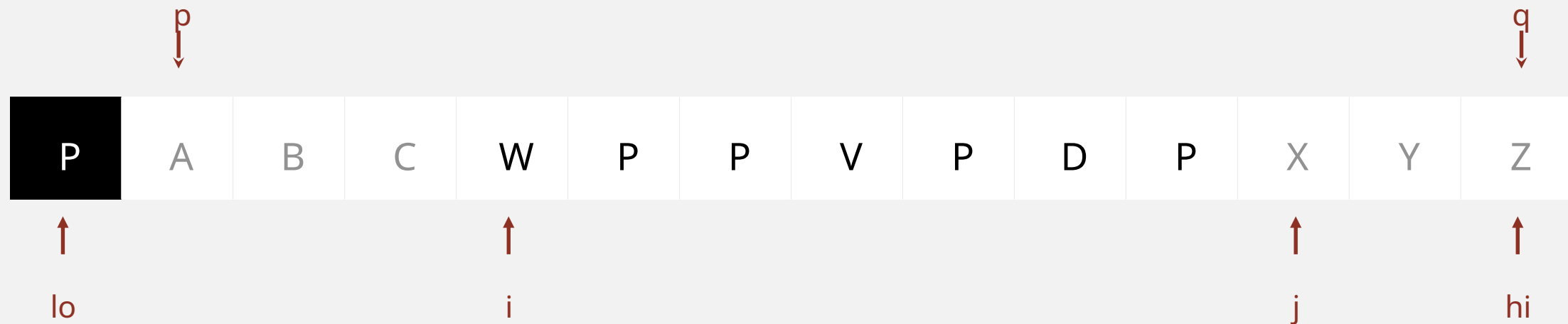


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



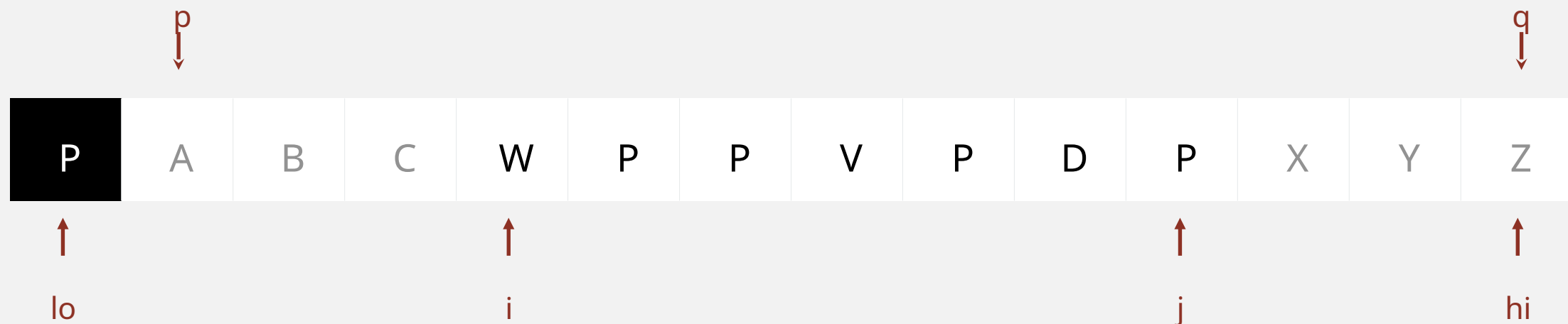


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



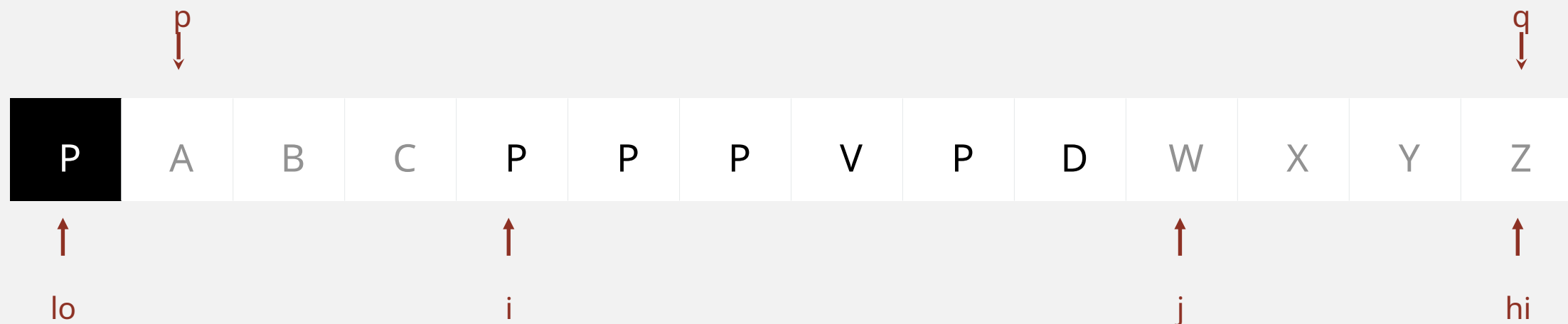
exchange  $a[i]$  with  $a[j]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



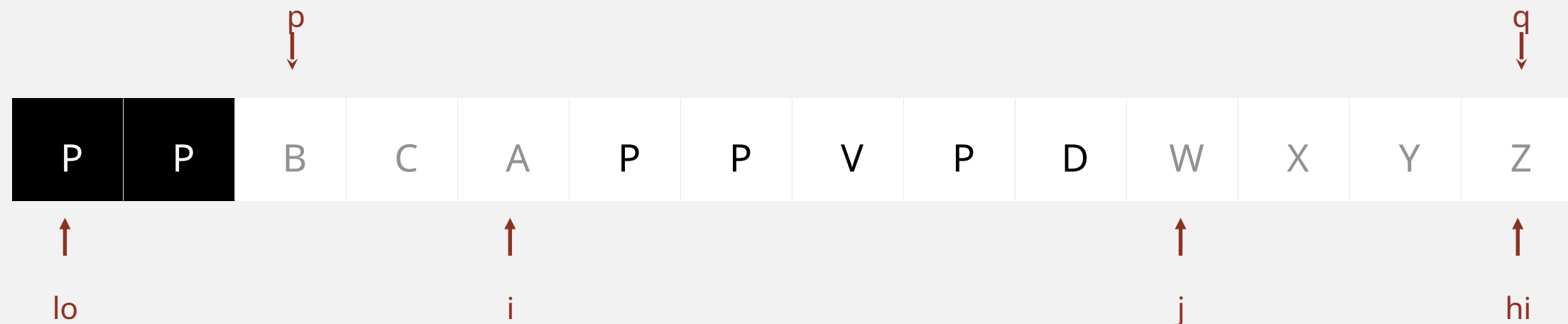
exchange  $a[i]$  with  $a[p]$  and increment  $p$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

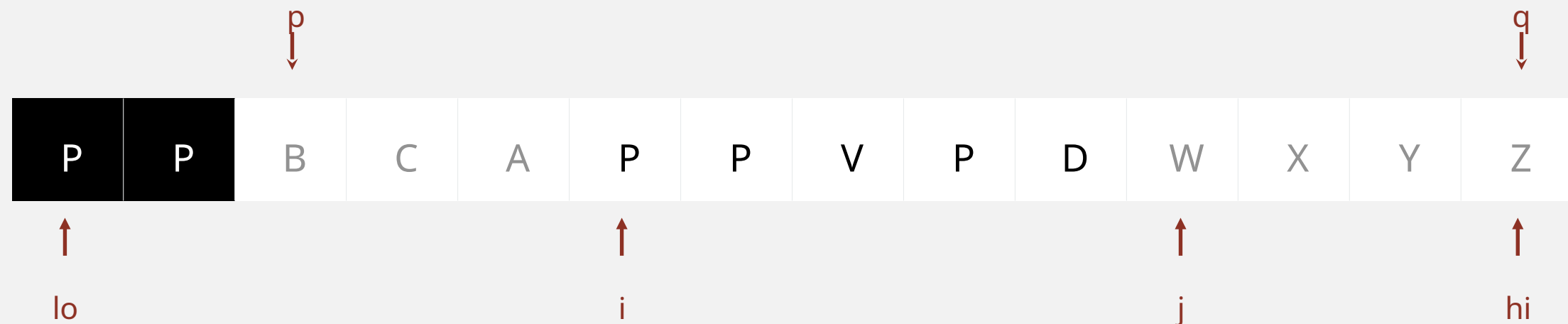


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

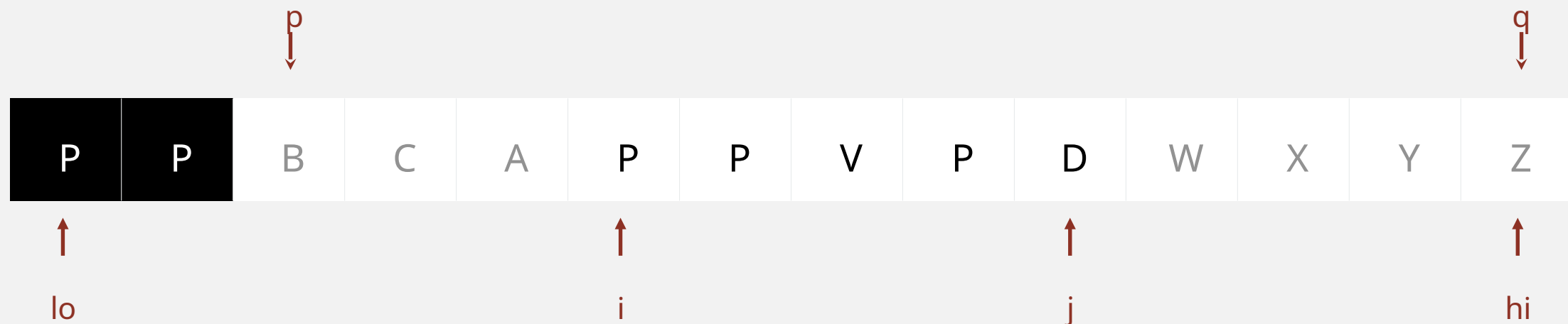


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



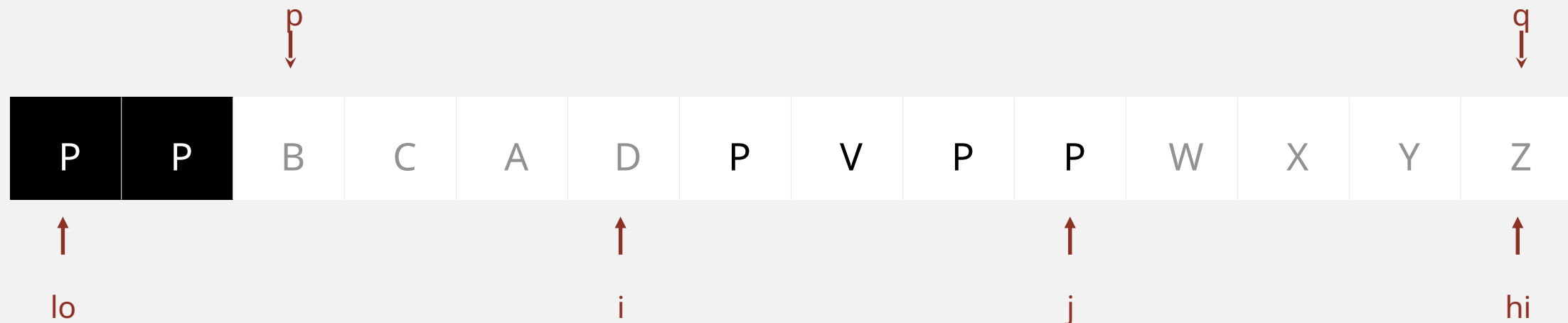
exchange  $a[i]$  with  $a[j]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



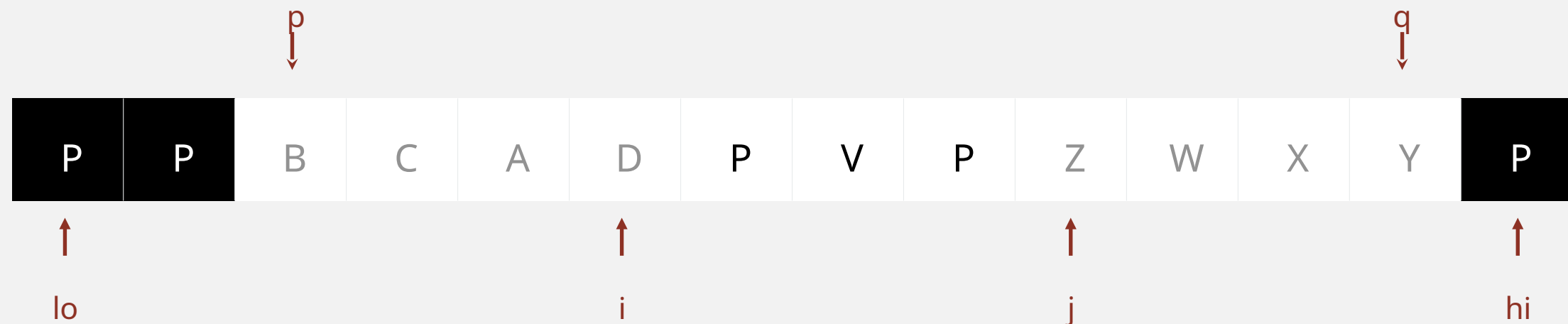
exchange  $a[j]$  with  $a[q]$  and decrement  $q$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

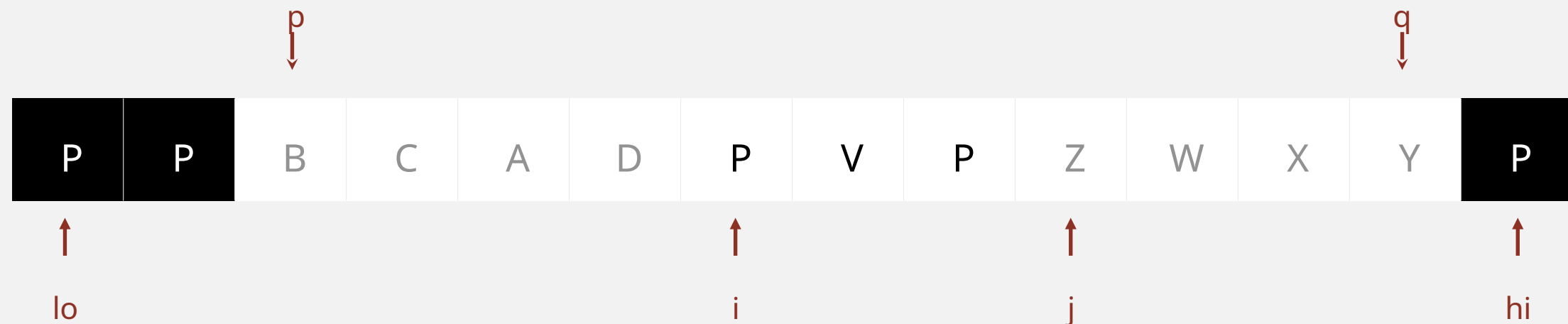


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



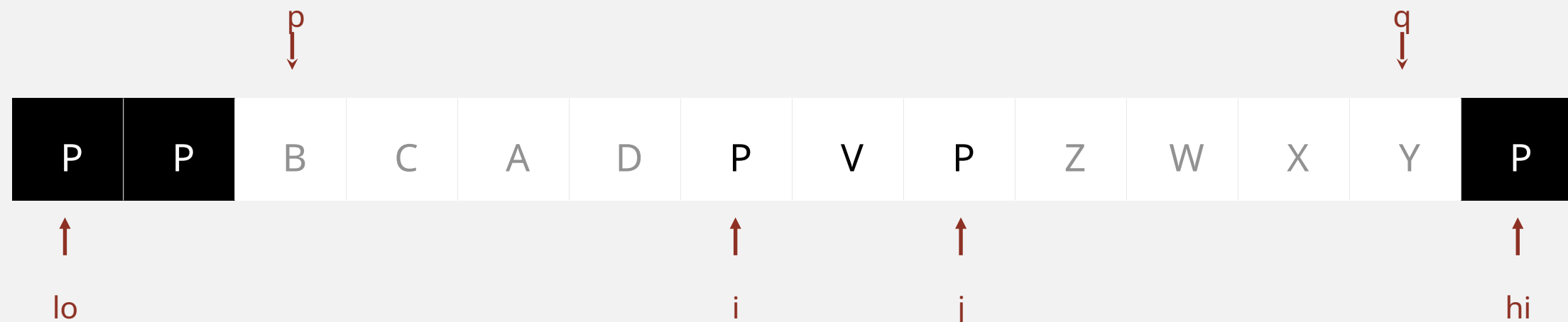


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



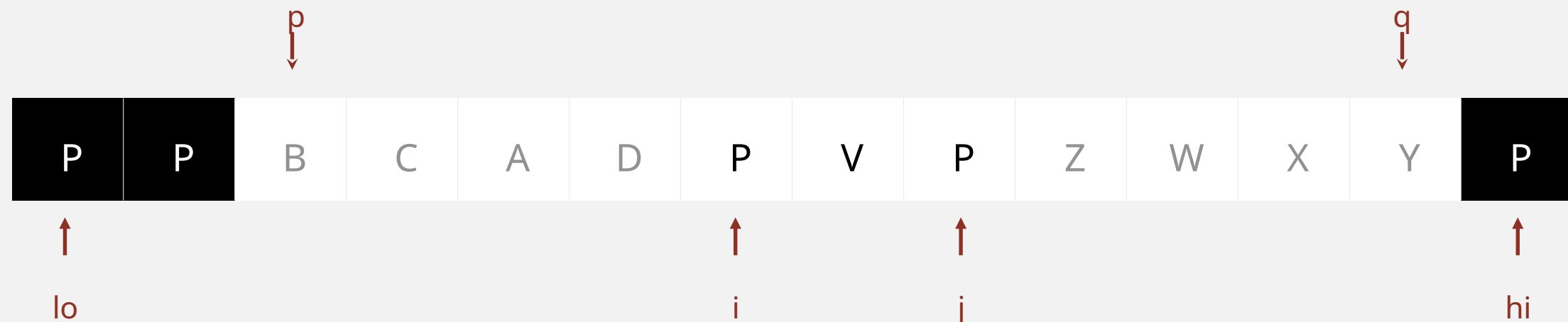
exchange  $a[i]$  with  $a[j]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



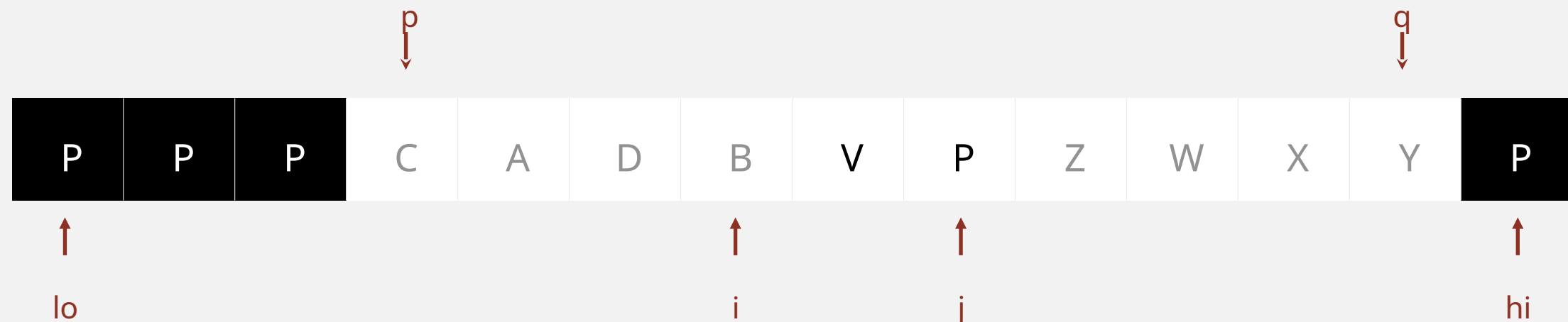
exchange  $a[i]$  with  $a[p]$  and increment  $p$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



exchange  $a[j]$  with  $a[q]$  and decrement  $q$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

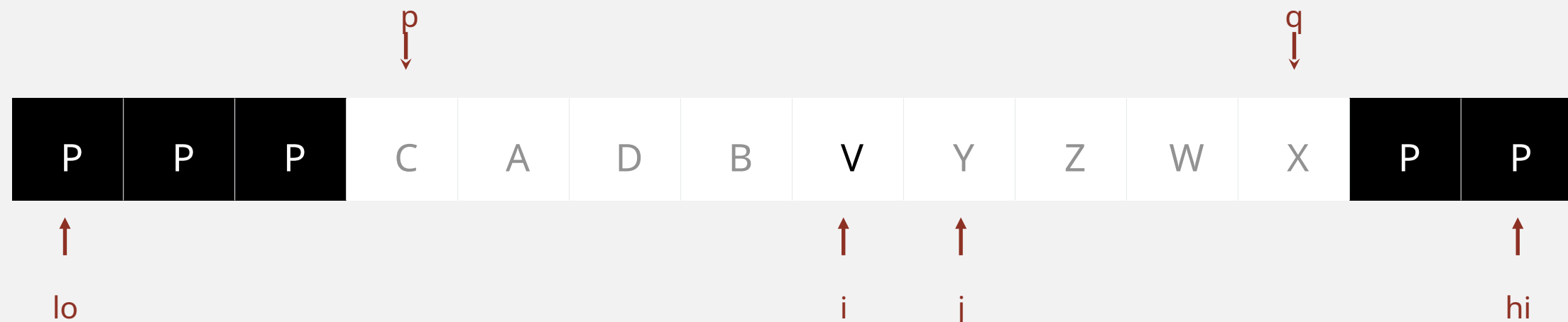


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

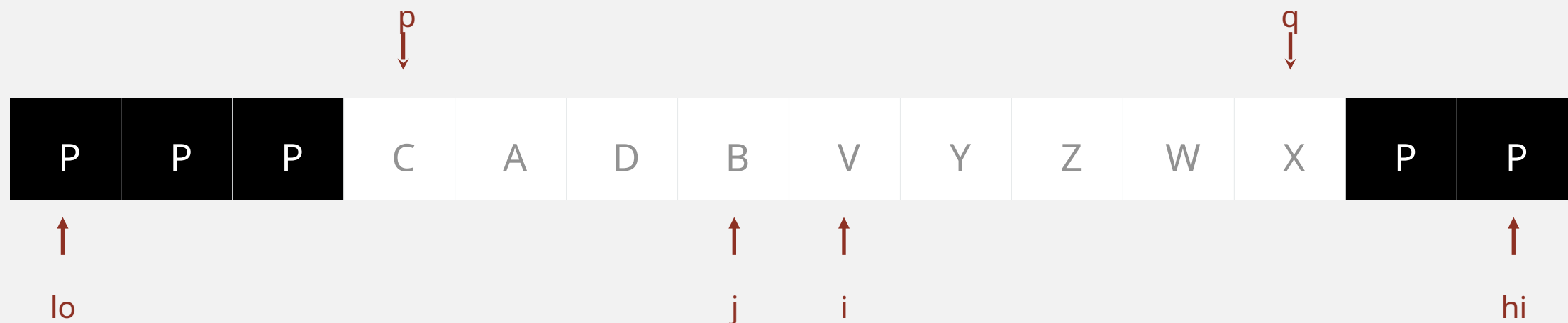


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



pointers cross

# Bentley-McIlroy 3-way partitioning demo

---

Phase II. Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



exchange  $a[j]$  with  $a[p]$

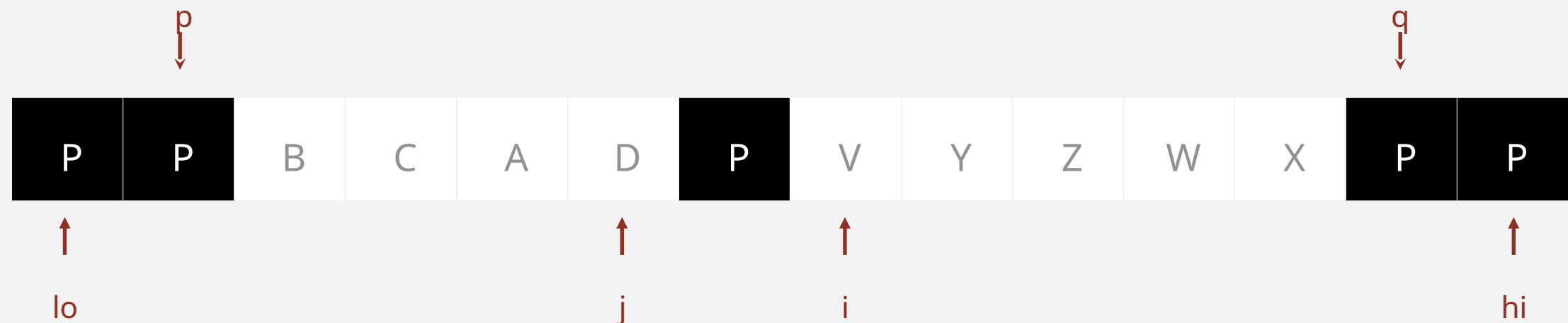


# Bentley-McIlroy 3-way partitioning demo

---

Phase II. Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



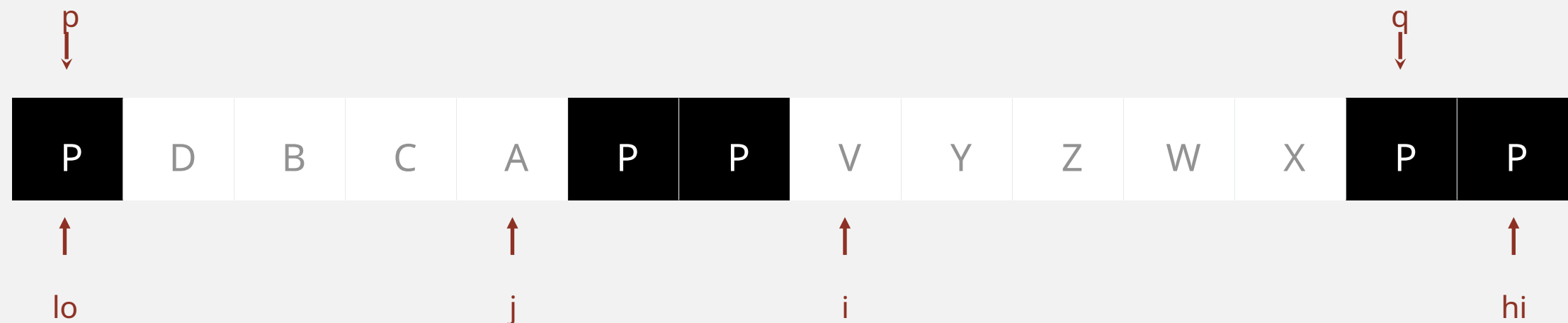
exchange  $a[j]$  with  $a[p]$

# Bentley-McIlroy 3-way partitioning demo

---

Phase II. Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



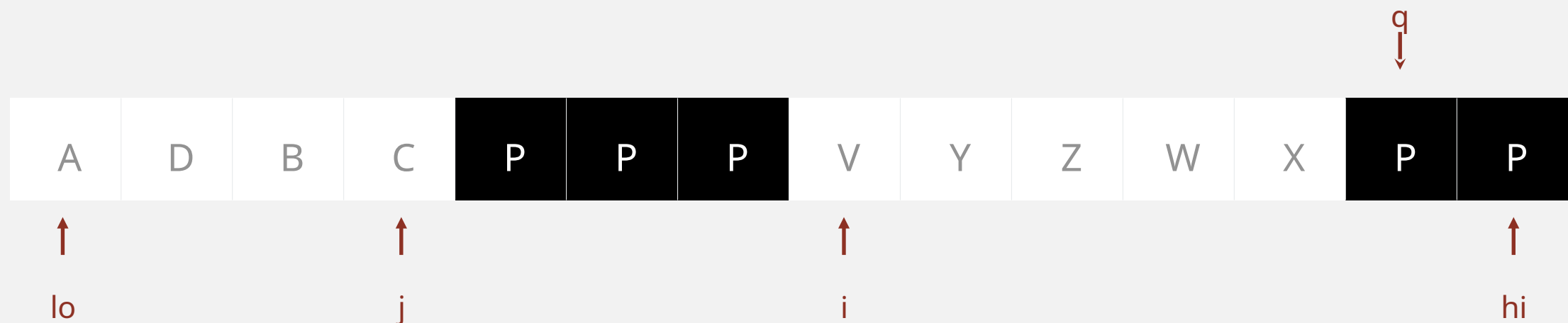
exchange  $a[j]$  with  $a[p]$

# Bentley-McIlroy 3-way partitioning demo

---

Phase II. Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



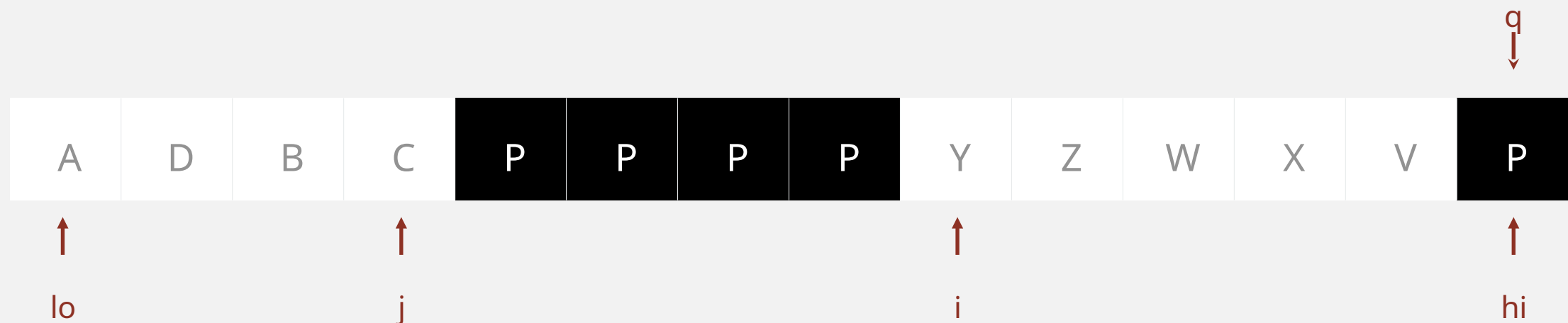
exchange  $a[i]$  with  $a[q]$

# Bentley-McIlroy 3-way partitioning demo

---

Phase II. Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



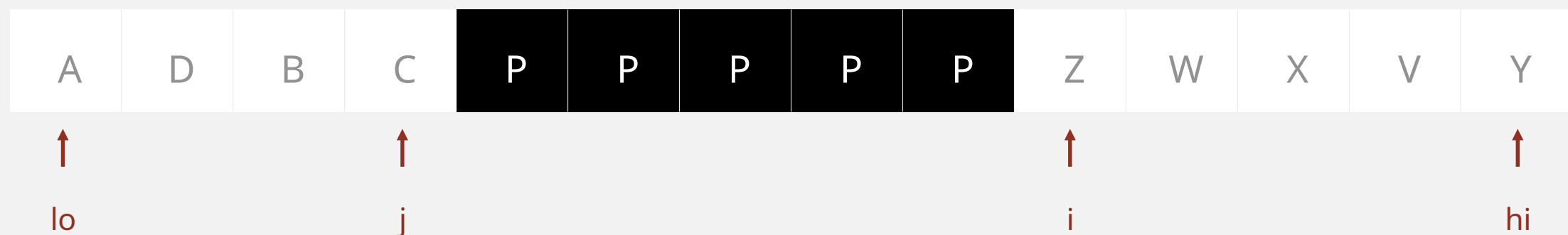
exchange  $a[i]$  with  $a[q]$

# Bentley-McIlroy 3-way partitioning demo

---

Phase II. Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



3-way partitioned



<http://algs4.cs.princeton.edu>

## 2.3 Partitioning Demos

---

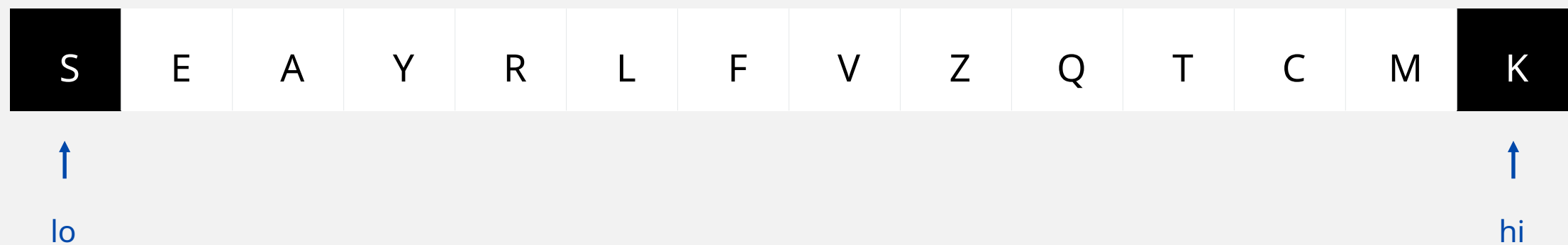
- *Sedgewick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*

# Dual-pivot partitioning demo

---

## Initialization.

- Choose  $a[lo]$  and  $a[hi]$  as partitioning items.
- Exchange if necessary to ensure  $a[lo] \leq a[hi]$ .



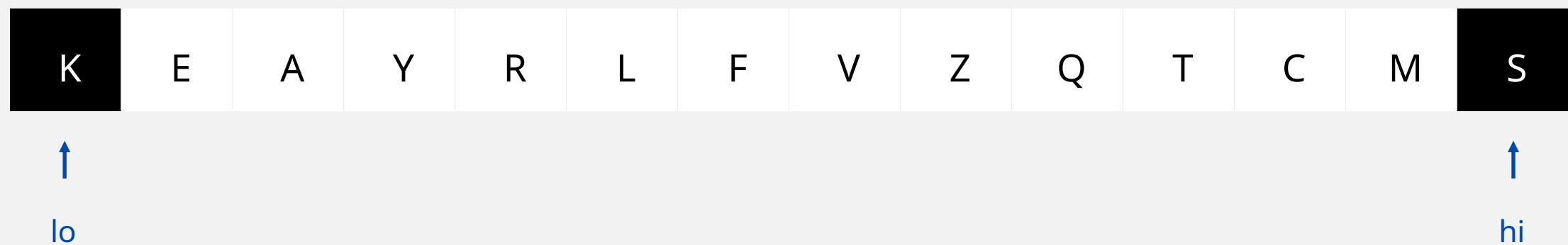
**exchange  $a[lo]$  and  $a[hi]$**

# Dual-pivot partitioning demo

---

## Initialization.

- Choose  $a[lo]$  and  $a[hi]$  as partitioning items.
- Exchange if necessary to ensure  $a[lo] \leq a[hi]$ .

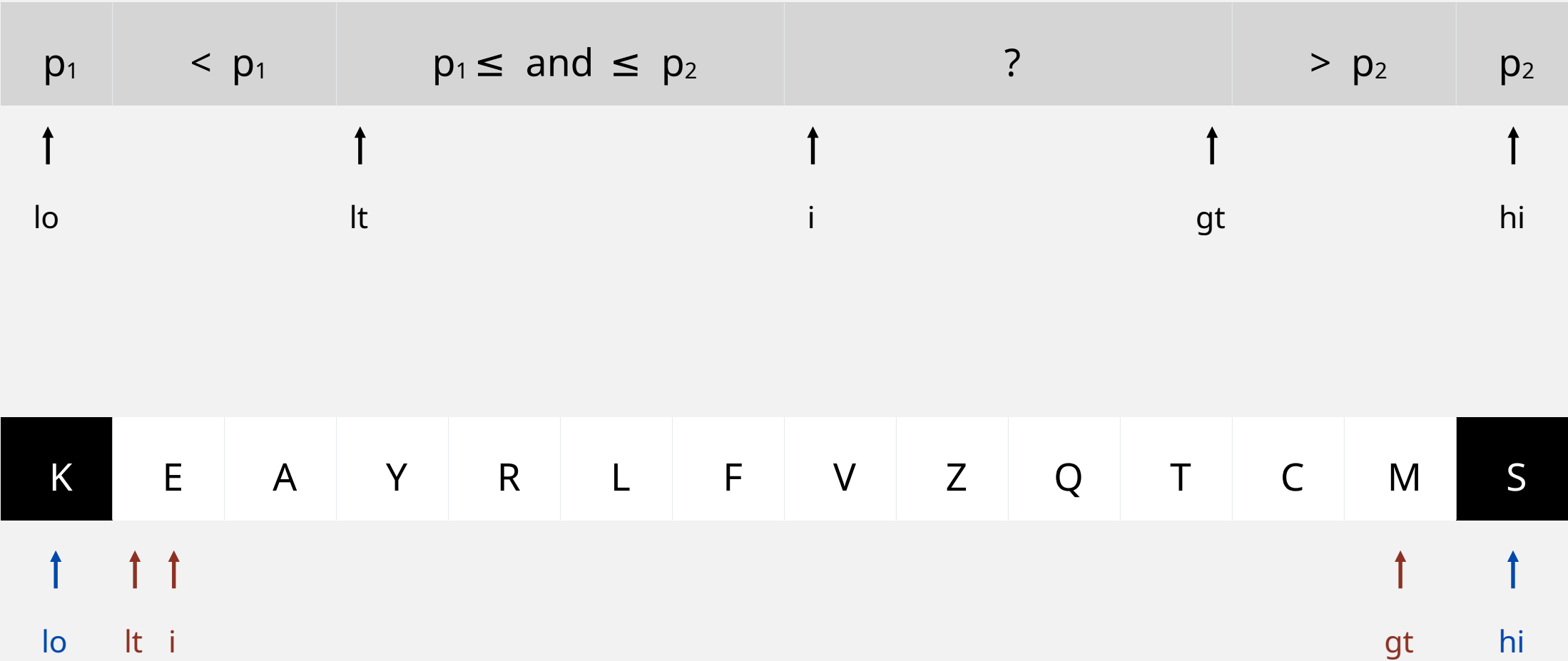




# Dual-pivot partitioning demo

**Main loop.** Repeat until *i* and *gt* pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment *lt* and *i*.
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement *gt*.
- Else, increment *i*.

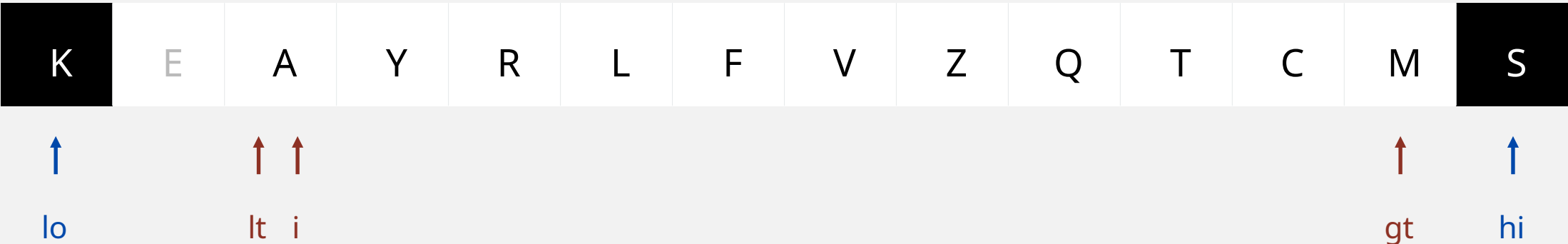
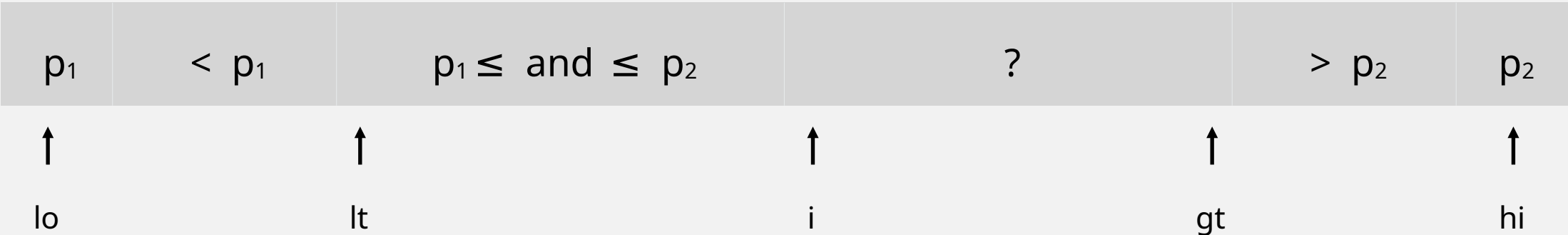


**exchange  $a[i]$  and  $a[lt]$ ; increment *lt* and *i***

# Dual-pivot partitioning demo

**Main loop.** Repeat until i and gt pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .

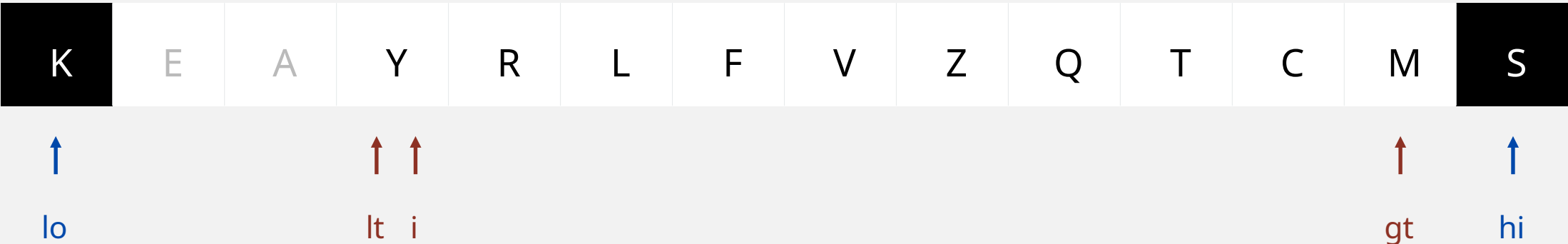
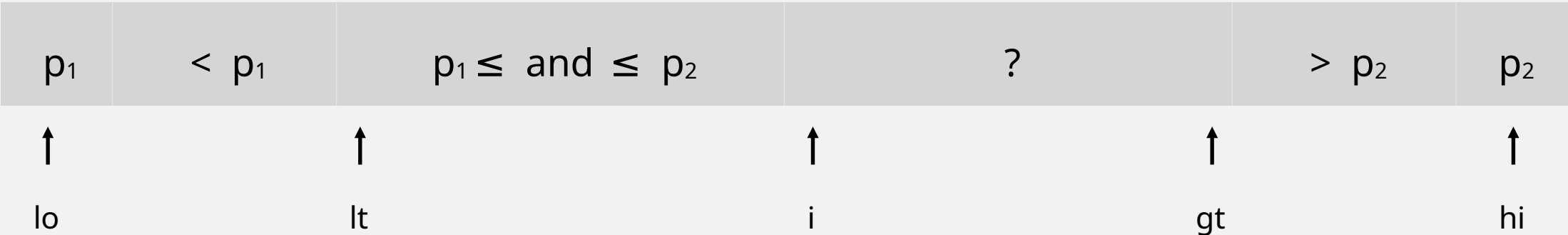


**exchange  $a[i]$  and  $a[lt]$ ; increment  $lt$  and  $i$**

# Dual-pivot partitioning demo

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .

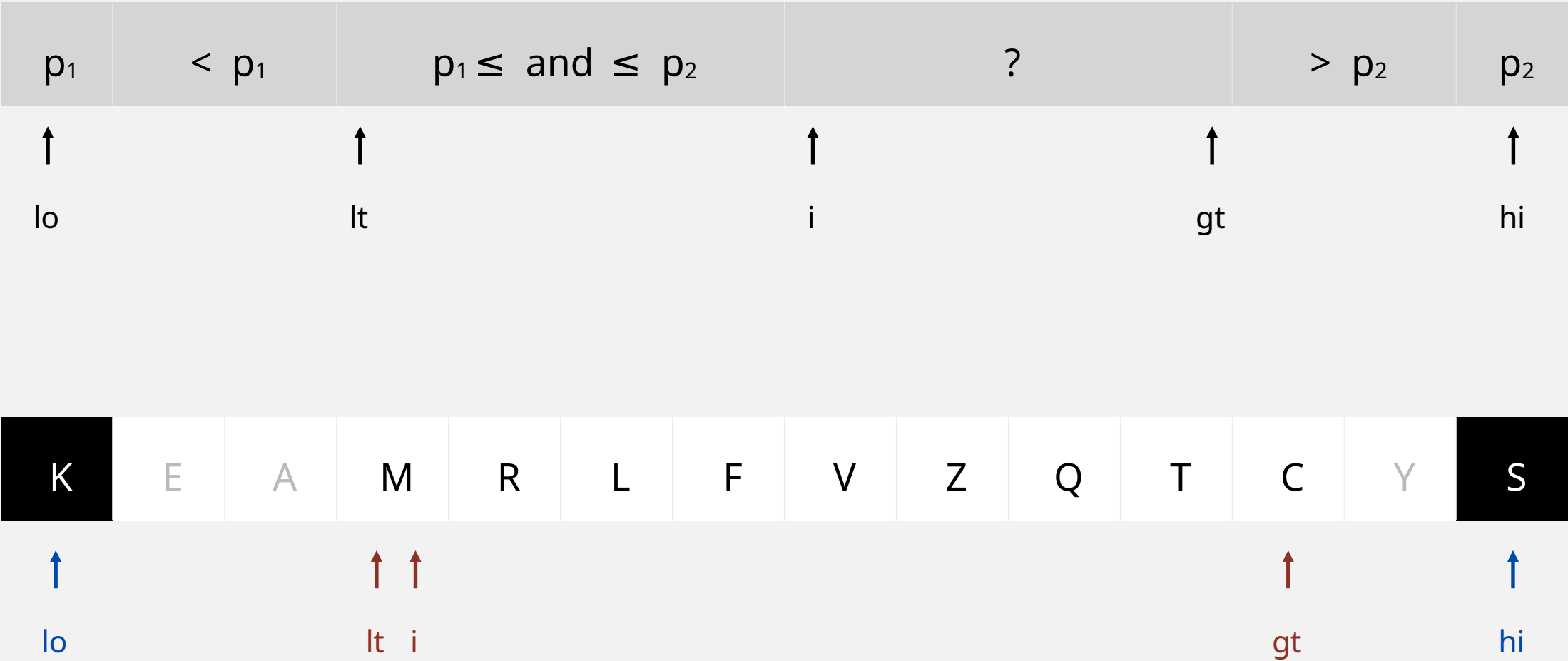


exchange  $a[i]$  and  $a[gt]$ ; decrement  $gt$

# Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .

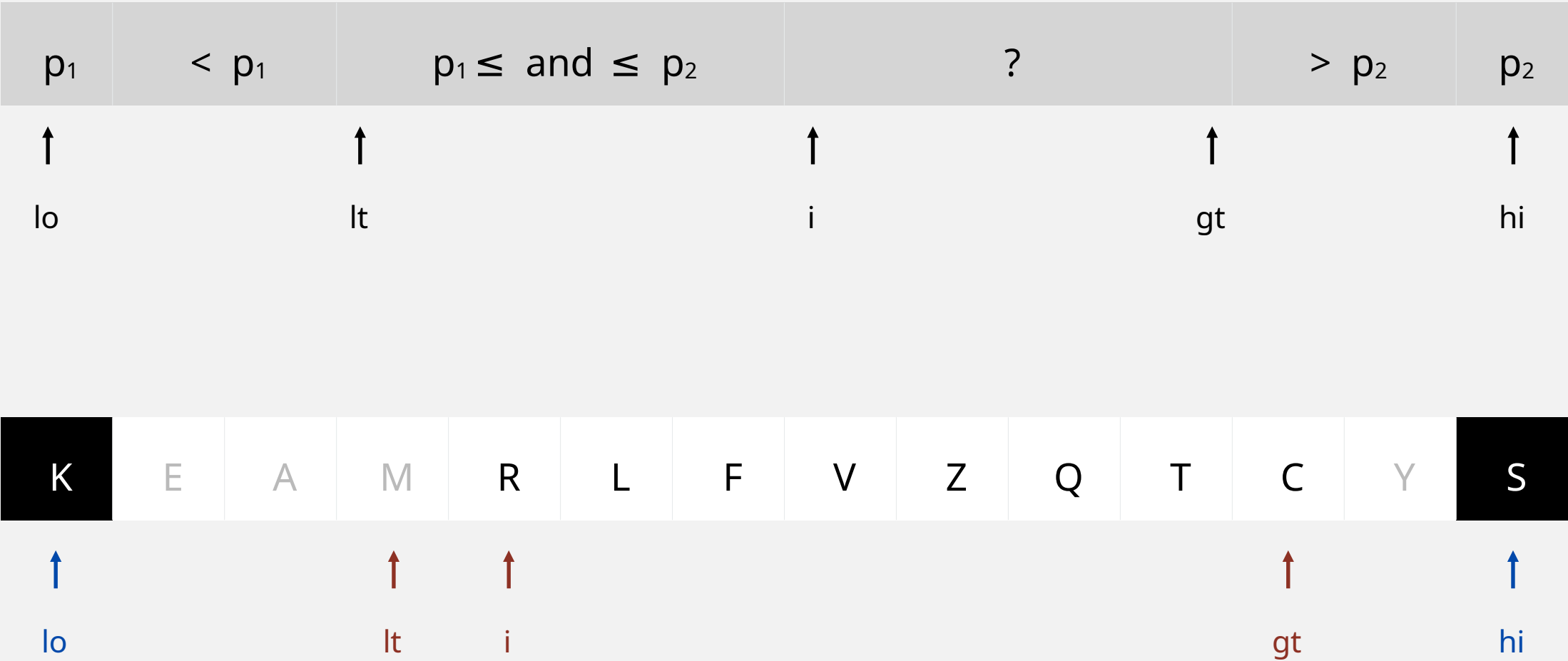


increment i

# Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .

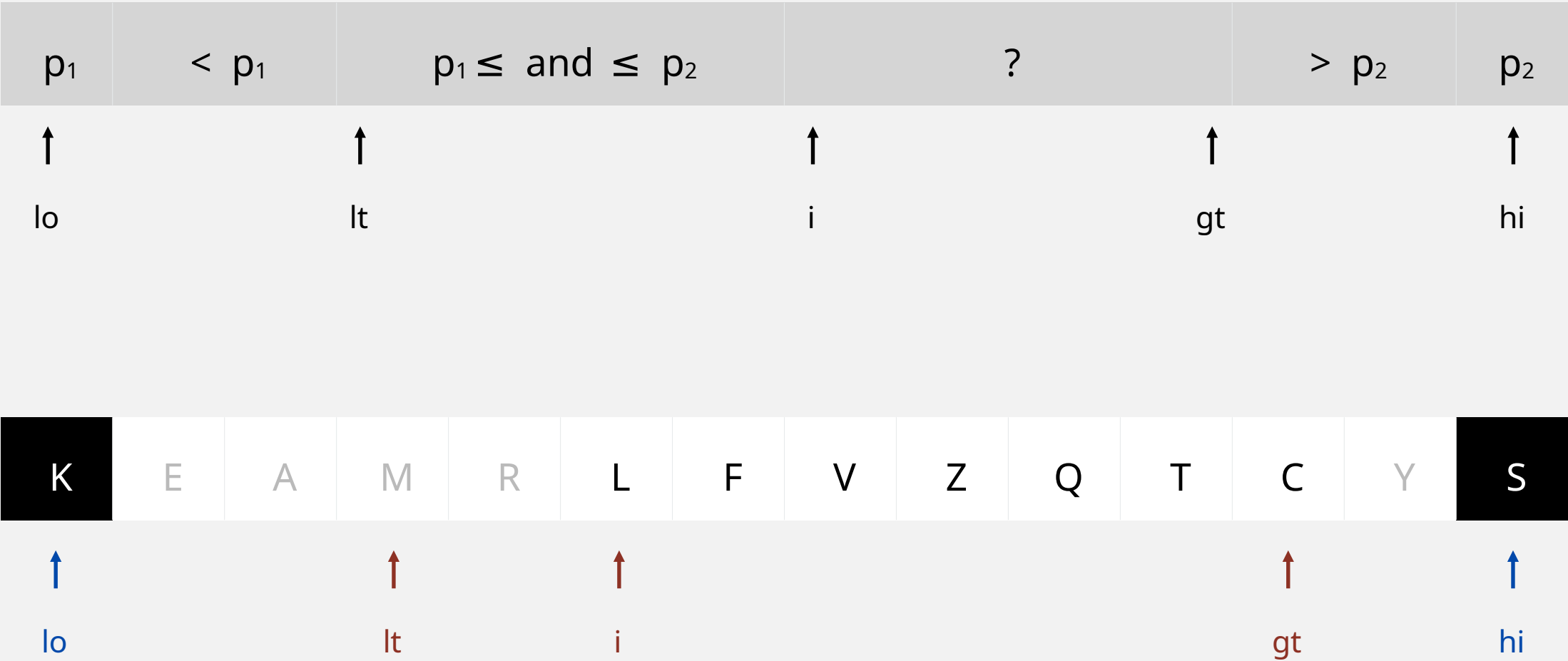


increment i

# Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



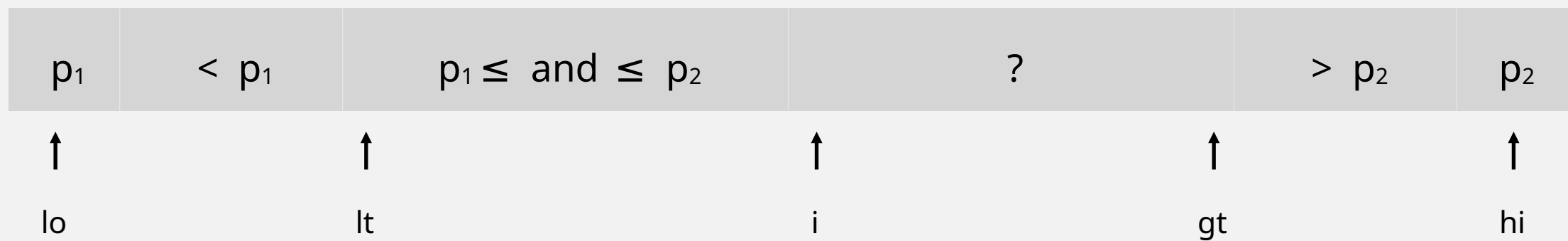
increment i

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .

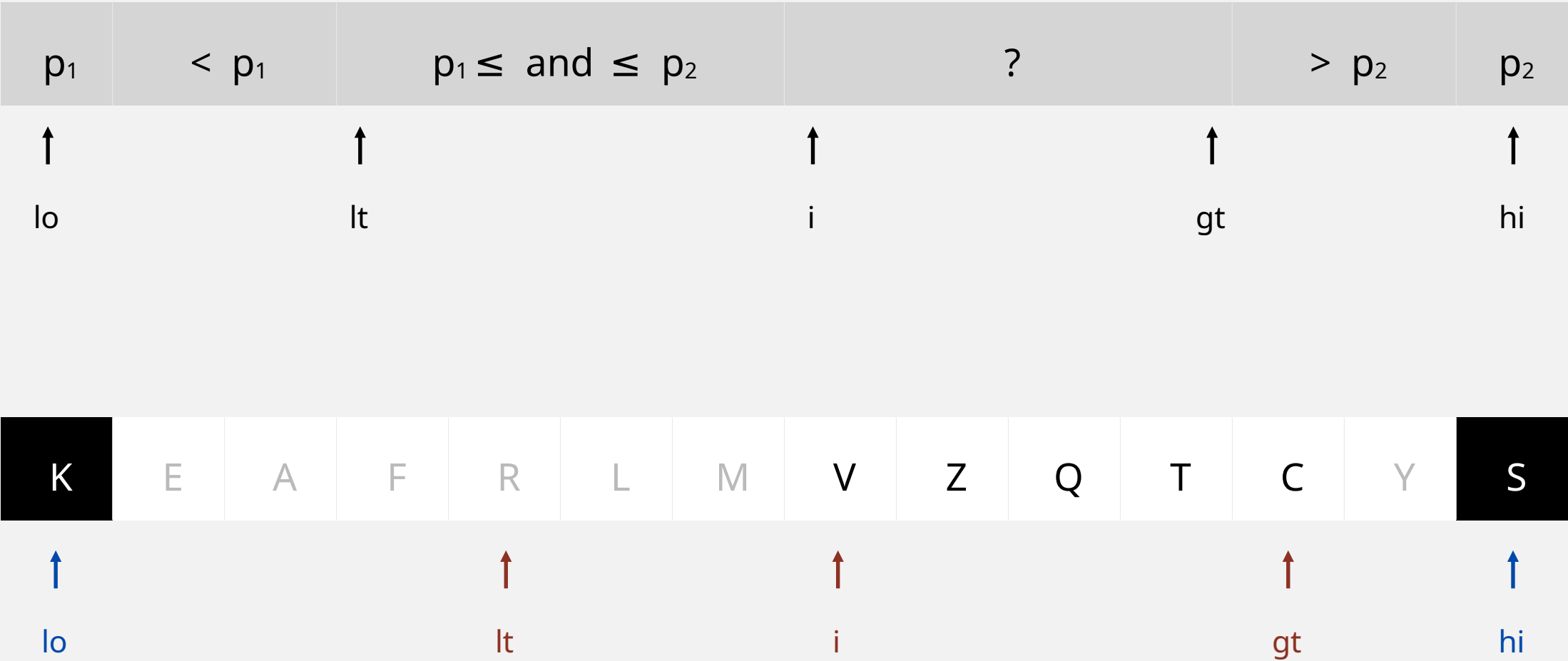


**exchange  $a[i]$  and  $a[lt]$ ; increment  $lt$  and  $i$**

# Dual-pivot partitioning demo

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lr]$  and increment  $lr$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



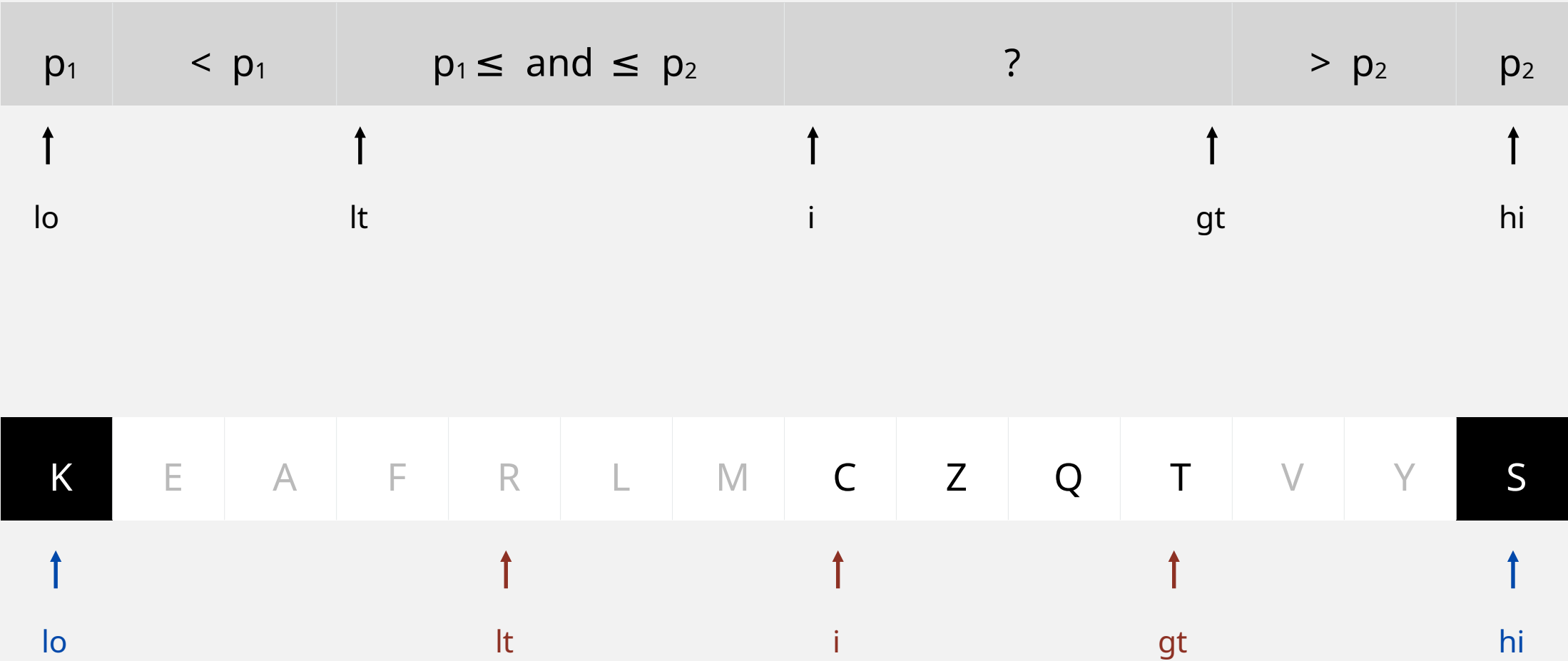
exchange  $a[i]$  and  $a[gt]$ ; decrement  $gt$



# Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



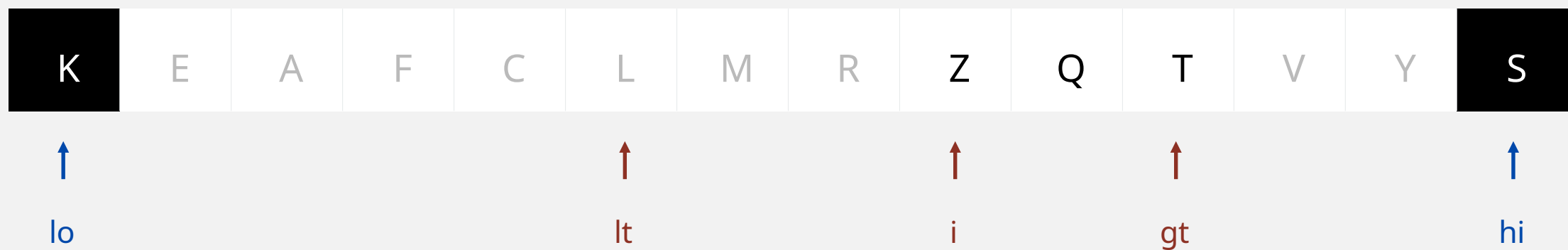
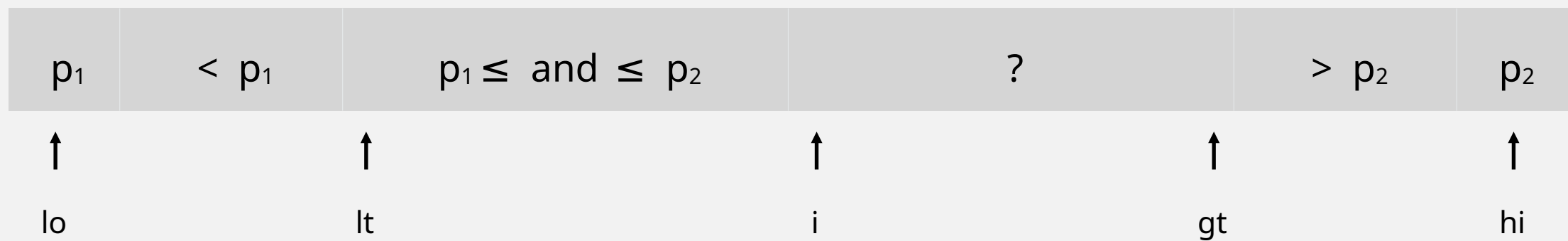
exchange  $a[i]$  and  $a[lt]$ ; increment  $lt$  and  $i$

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .

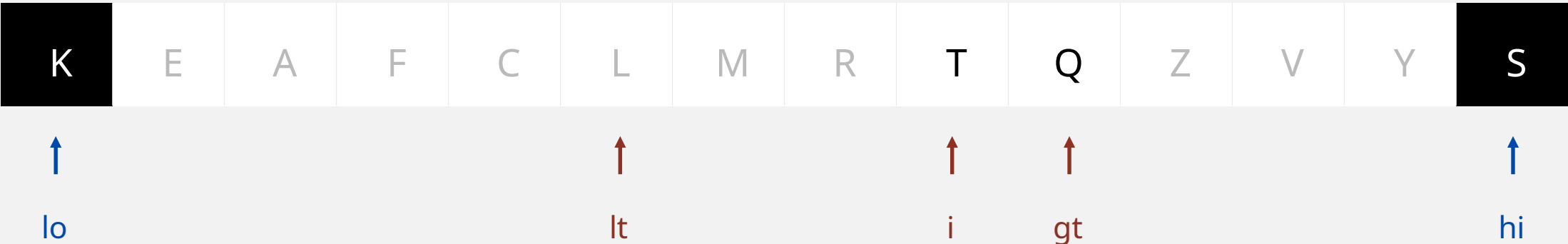
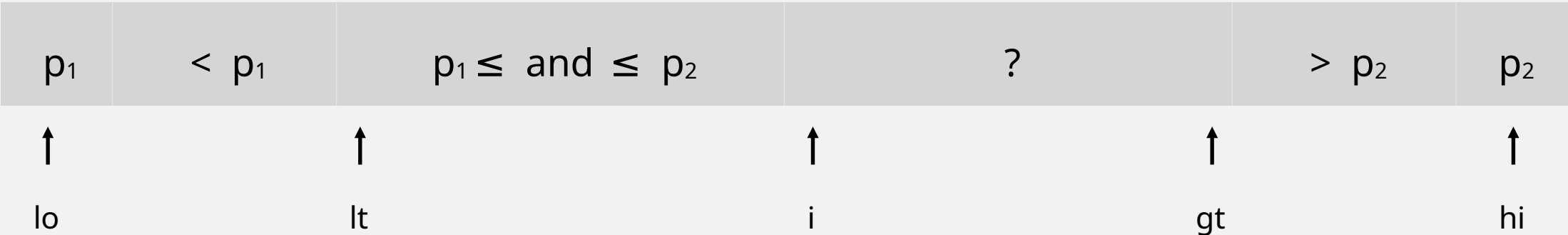


**exchange  $a[i]$  and  $a[gt]$ ; decrement  $gt$**

# Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .

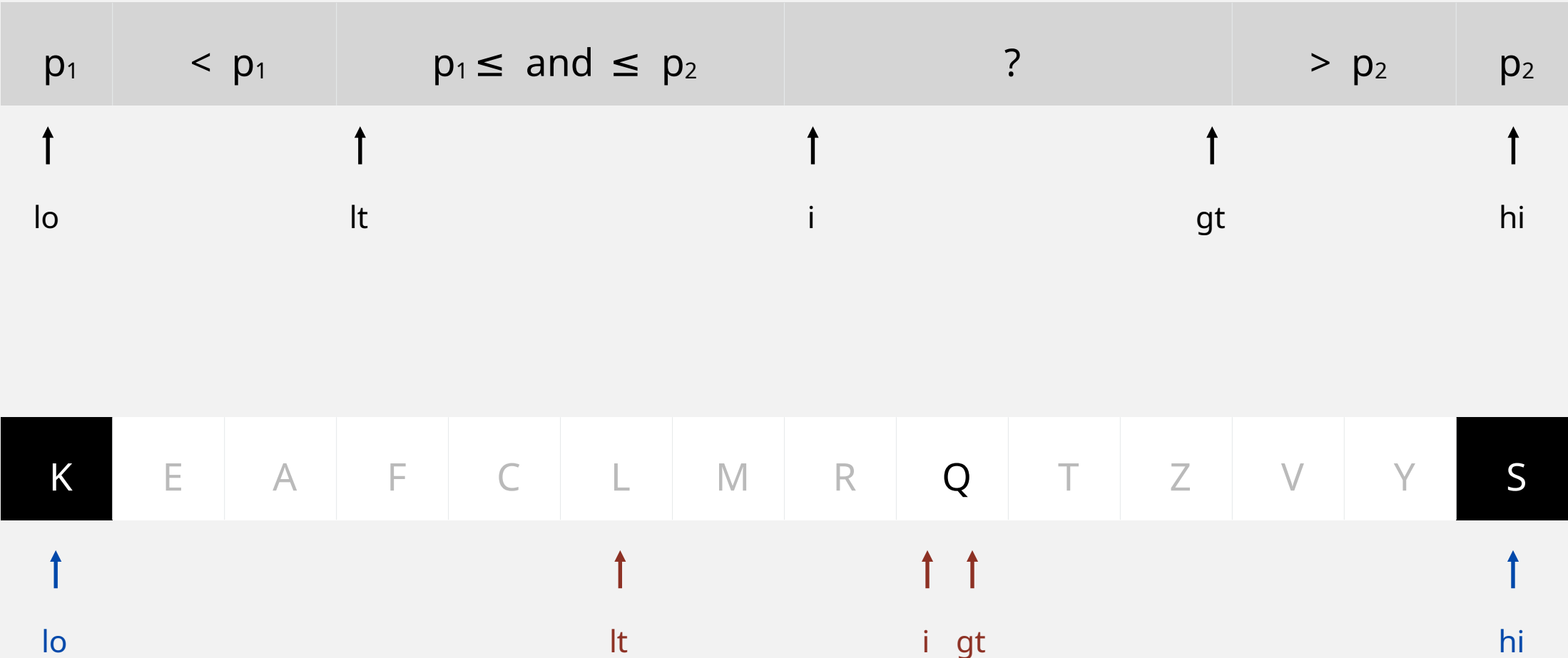


exchange  $a[i]$  and  $a[gt]$ ; decrement  $gt$

# Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .

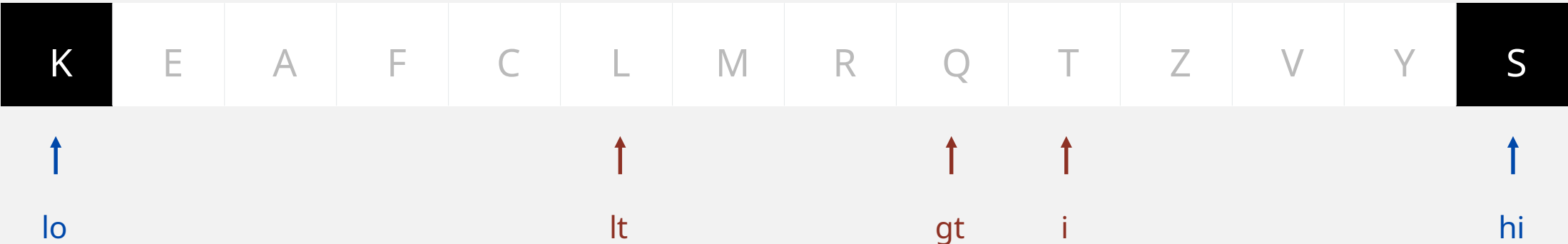
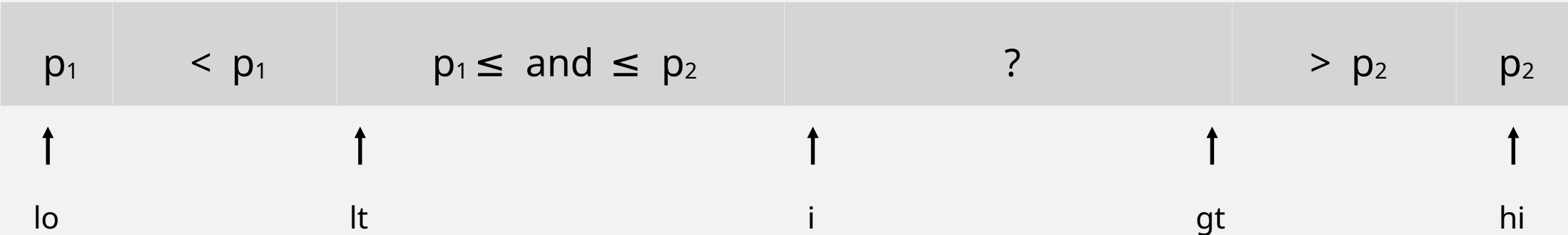


increment i

# Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



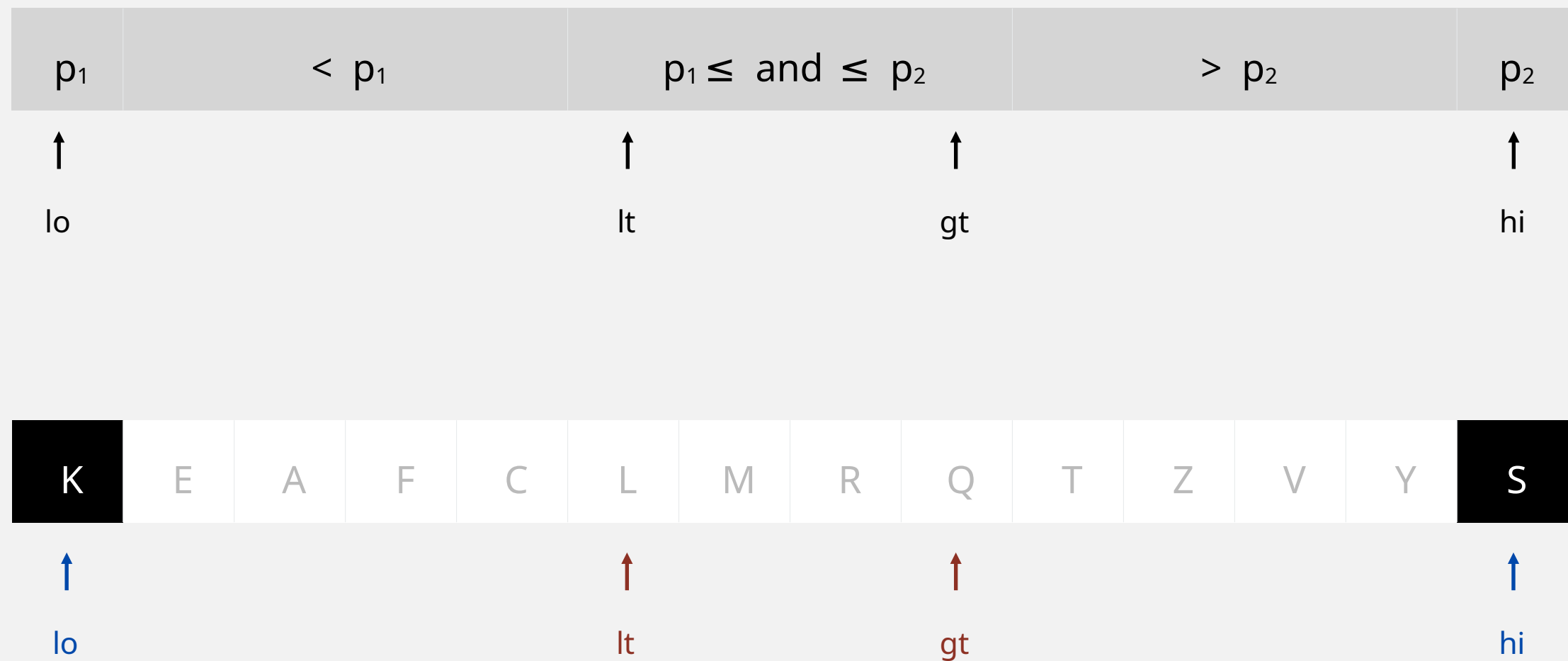
stop when pointers cross

# Dual-pivot partitioning demo

---

## Finalize.

- Exchange  $a[lo]$  with  $a[--lt]$ .
- Exchange  $a[hi]$  with  $a[++gt]$ .

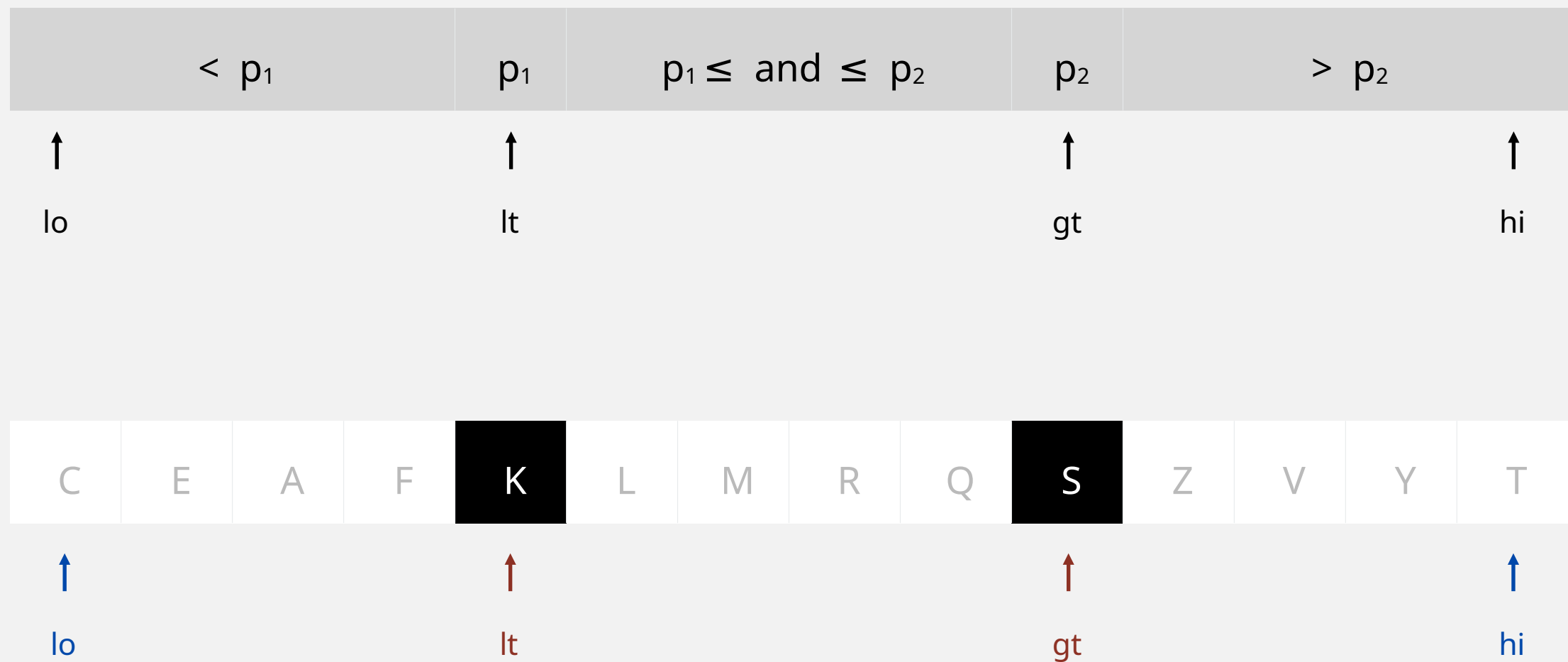


# Dual-pivot partitioning demo

---

## Finalize.

- Exchange  $a[lo]$  with  $a[--lt]$ .
- Exchange  $a[hi]$  with  $a[++gt]$ .



**3-way partitioned**