http://algs4.cs.princeton.edu

5.5 Data Compression

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*

http://algs4.cs.princeton.edu

5.5 Data Compression

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*

Data compression

Compression reduces the size of a file:

- To save space when storing it.
- To save time when transmitting it.
- Most files have lots of redundancy.

Who needs compression?

- Moore's law: # transistors on a chip doubles every 18–24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, …

> *" Everyday, we create 2.5 quintillion bytes of data—so much that 90% of the data in the world today has been created in the last two years alone. "* — *IBM report on big data (2011)*

Basic concepts ancient (1950s), best technology recently developed.

Applications

Generic file compression.

- Files: GZIP, BZIP, 7z.
- Archivers: PKZIP.
- File systems: NTFS, ZFS, HFS+, ReFS, GFS.

Multimedia.

- Images: GIF, JPEG.
- Sound: MP3.
- Video: MPEG, DivX™, HDTV.

Communication.

- ITU-T T4 Group 3 Fax.
- V.42bis modem.
- Skype, Google hangout.

Databases. Google, Facebook, ...

Lossless compression and expansion

Message. Binary data B we want to compress.

Compress. Generates a "compressed" representation $C(B)$.

Expand. Reconstructs original bitstream B.

uses fewer bits
(you hope)

Basic model for data compression

Compression ratio. Bits in $C(B)$ / bits in B.

Ex. 50–75% or better compression ratio for natural language.

Food for thought

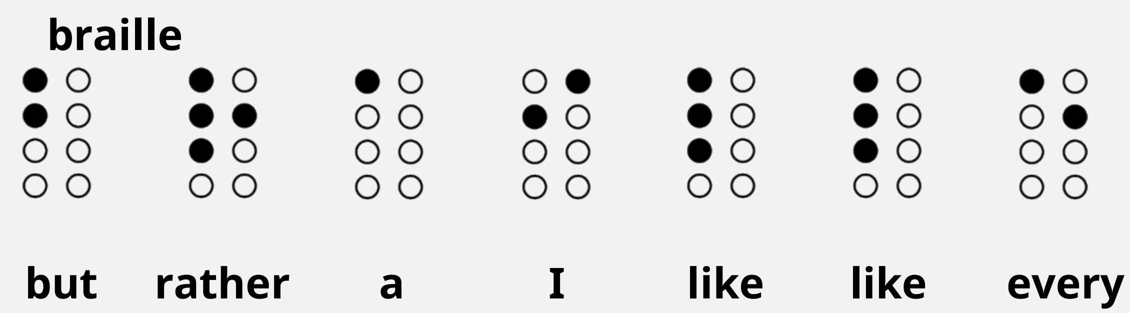
Data compression has been omnipresent since antiquity:

- Number systems.
- Natural languages.
- Mathematical notation.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

has played a central role in communications technology,

- Grade 2 Braille.
- Morse code.
- Telephone system.



and is part of modern life.

- MP3.
- MPEG.

Q. What role will it play in the future?

Data representation: genomic code

Genome. String over the alphabet { A, C, T, G }.

Goal. Encode an N-character genome: ATAGATGCATAG...

Standard ASCII encoding.

- 8 bits per char.
- $8\ N$ bits.

char	hex	binary
A	41	01000001
C	43	01000011
T	54	01010100
G	47	01000111

Two-bit encoding.

- 2 bits per char.
- $2\ N$ bits.

char	binary
A	00
C	01
T	10
G	11

Fixed-length code. k-bit code supports alphabet of size 2^k.

Amazing but true. Some genomic databases in 1990s used ASCII.

Reading and writing binary data

Binary standard input. Read bits from standard input.

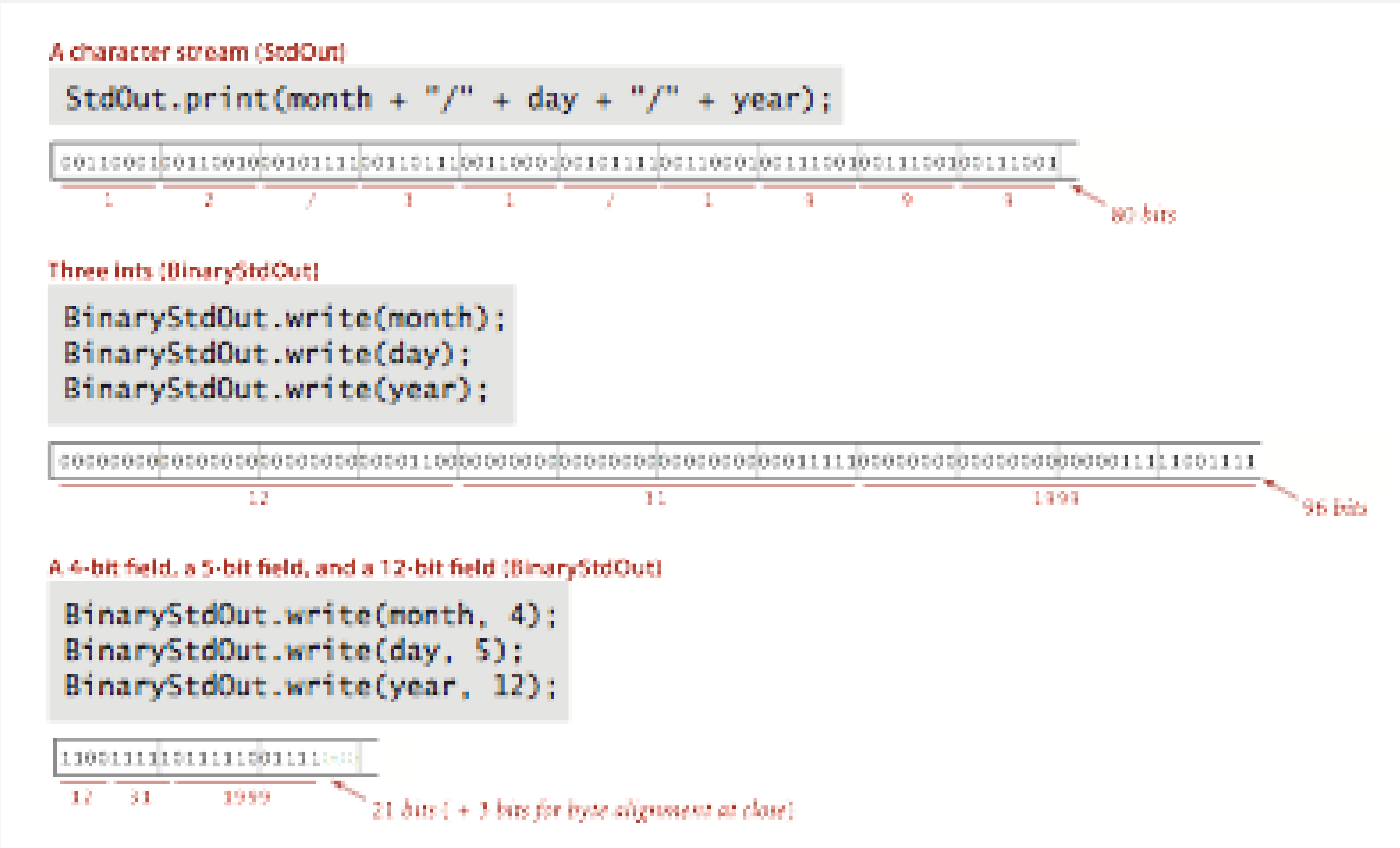
public class BinaryStdIn		
boolean	readBoolean()	*read 1 bit of data and return as a* boolean *value*
char	readChar()	*read 8 bits of data and return as a* char *value*
char	readChar(int r)	*read* r *bits of data and return as a* char *value*
[similar methods for byte *(8 bits);* short *(16 bits);* int *(32 bits);* long *and* double *(64 bits)]*		
boolean	isEmpty()	*is the bitstream empty?*
void	close()	*close the bitstream*

Binary standard output. Write bits to standard output

public class BinaryStdOut		
void	write(boolean b)	*write the specified bit*
void	write(char c)	*write the specified 8-bit* char
void	write(char c, int r)	*write the* r *least significant bits of the specified* char
[similar methods for byte *(8 bits);* short *(16 bits);* int *(32 bits);* long *and* double *(64 bits)]*		
void	close()	*close the bitstream*

Writing binary data

Date representation. Three different ways to represent 12/31/1999.



Binary dumps

Q. How to examine the contents of a bitstream?

Standard character stream

```
% more abra.txt
ABRACADABRA!
```

Bitstream represented as 0 and 1 characters

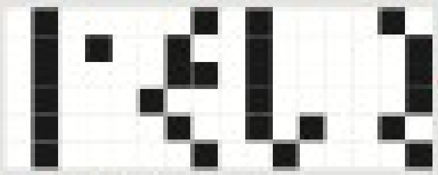
```
% java BinaryDump 16 < abra.txt
0100000101000010
0101001001000001
0100001101000001
0100010001000001
0100001001010010
0100000100100001
96 bits
```

Bitstream represented with hex digits

```
% java HexDump 4 < abra.txt
41 42 52 41
43 41 44 41
42 52 41 21
12 bytes
```

Bitstream represented as pixels in a Picture

```
% java PictureDump 16 6 < abra.txt
```




```
96 bits
```

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

Hexadecimal to ASCII conversion table

Universal data compression

US Patent 5,533,051 on "Methods for Data Compression", which is capable of compression all files.

Slashdot reports of the Zero Space Tuner™ and BinaryAccelerator™.

> *" ZeoSync has announced a breakthrough in data compression that allows for 100:1 lossless compression of random data. If this is true, our bandwidth problems just got a lot smaller. ... "*

Universal data compression

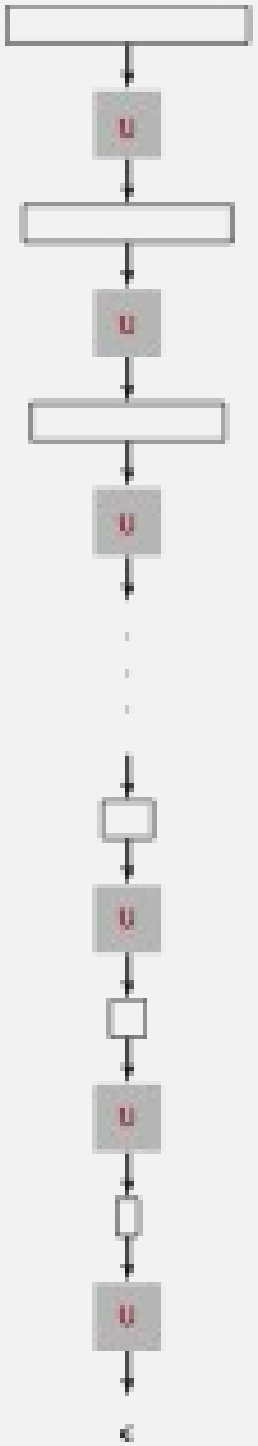
Proposition. No algorithm can compress every bitstring.

Pf 1. [by contradiction]

- Suppose you have a universal data compression algorithm U that can compress every bitstream.
- Given bitstring B_0, compress it to get smaller bitstring B_1.
- Compress B_1 to get a smaller bitstring B_2.
- Continue until reaching bitstring of size 0.
- Implication: all bitstrings can be compressed to 0 bits!

Pf 2. [by counting]

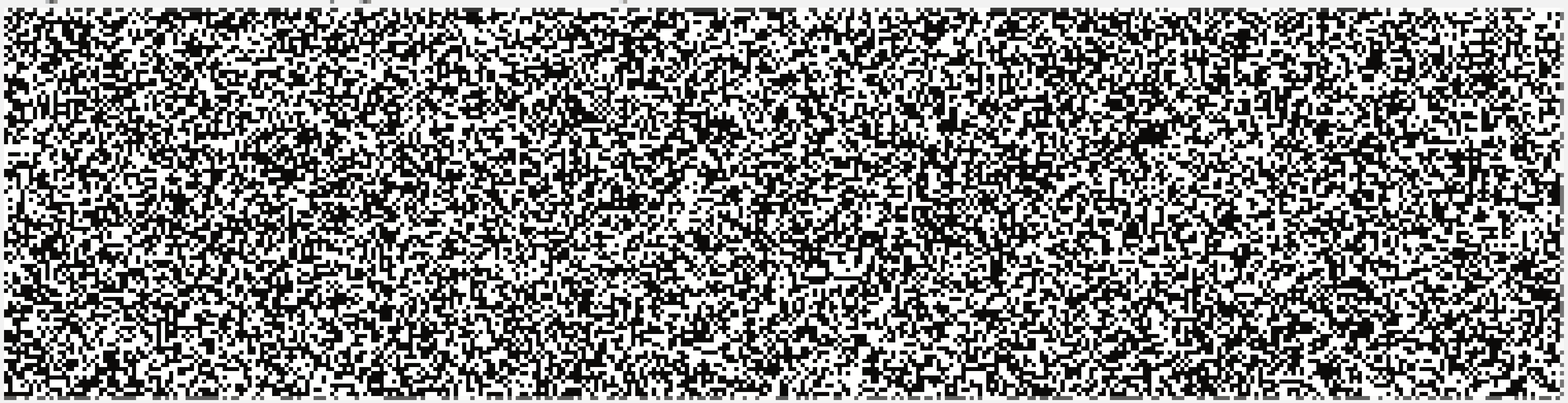
- Suppose your algorithm that can compress all $1{,}000$-bit strings.
- 2^{1000} possible bitstrings with $1{,}000$ bits.
- Only $1 + 2 + 4 + \ldots + 2^{998} + 2^{999}$ can be encoded with ≤ 999 bits.
- Similarly, only 1 in 2^{499} bitstrings can be encoded with ≤ 500 bits!



Universal
data compression?

Undecidability

% java RandomBits | java PictureDump 2000 500



1000000 bits

A difficult file to compress: one million (pseudo-) random bits

```
public class RandomBits
{
   public static void main(String[] args)
   {
      int x = 11111;
      for (int i = 0; i < 1000000; i++)
      {
         x = x * 314159 + 218281;
         BinaryStdOut.write(x > 0);
      }
      BinaryStdOut.close();
   }
```

Rdenudcany in Enlgsih lnagugae

Q. How mcuh rdenudcany is in the Enlgsih lnagugae?

> *" ... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi's work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang. "* — Graham Rawlinson

A. Quite a bit.

http://algs4.cs.princeton.edu

5.5 Data Compression

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*

Run-length encoding

Simple type of redundancy in a bitstream. Long runs of repeated bits.

0000000000000001111111000000011111111111

40 bits

Representation. 4-bit counts to represent alternating runs of 0s and 1s: 15 0s, then 7 1s, then 7 0s, then 11 1s.

1111011101111011

15 7 7 11

16 bits (instead of 40)

Q. How many bits to store the counts?

A. We'll use 8 (but 4 in the example above).

Q. What to do when run length exceeds max count?

A. If longer than 255, intersperse runs of length 0.

Applications. JPEG, ITU-T T4 Group 3 Fax, ...

Run-length encoding: Java implementation

```
public class RunLength
{
   private final static int R   = 256;
   private final static int lgR = 8;

   public static void compress()
   {  /* see textbook */  }

   public static void expand()
   {
      boolean bit = false;
      while (!BinaryStdIn.isEmpty())
      {
         int run = BinaryStdIn.readInt(lgR);
         for (int i = 0; i < run; i++)
            BinaryStdOut.write(bit);
         bit = !bit;
      }
      BinaryStdOut.close();
   }
```

maximum run-length count

number of bits per count

read 8-bit count from standard input

write 1 bit to standard output

pad 0s for byte alignment

http://algs4.cs.princeton.edu

5.5 Data Compression

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*

David Huffman

Variable-length codes

Use different number of bits to encode different chars.

Ex. Morse code: • • • - - - • • •

Issue. Ambiguity.

SOS ?

V7 ?

IAMIE ?

EEWNI ?

In practice. Use a medium gap to separate codewords.

codeword for S is a prefix of codeword for V

Letters		Numbers	
A	•—	1	•————
B	—•••	2	••———
C	—•—•	3	•••——
D	—••	4	••••—
E	•	5	•••••
F	••—•	6	—••••
G	——•	7	——•••
H	••••	8	———••
I	••	9	————•
J	•———	0	—————
K	—•—		
L	•—••		
M	——		
N	—•		
O	———		
P	•——•		
Q	——•—		
R	•—•		
S	•••		
T	—		
U	••—		
V	•••—		
W	•——		
X	—••—		
Y	—•——		
Z	——••		

Variable-length codes

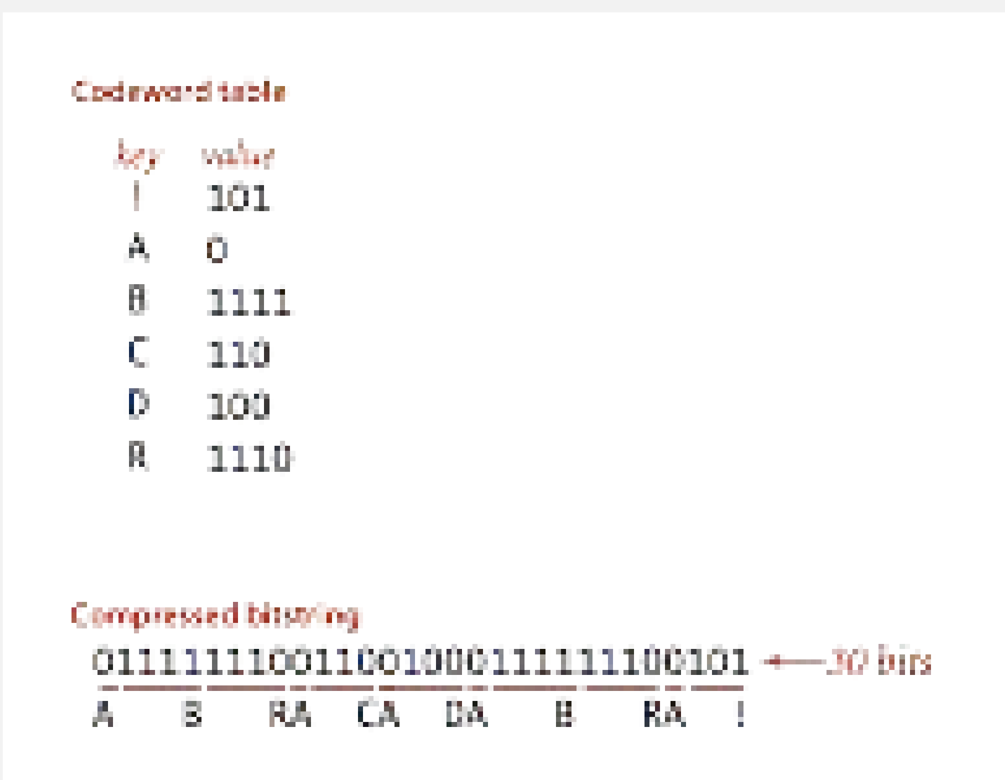
Q. How do we avoid ambiguity?

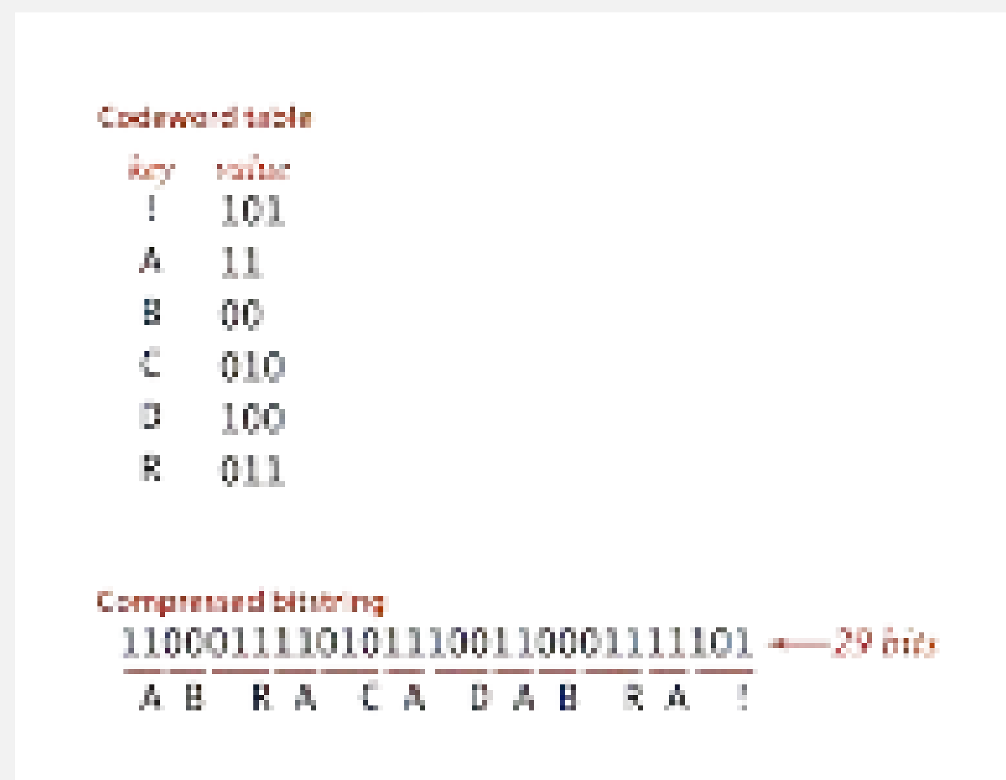
A. Ensure that no codeword is a prefix of another.

Ex 1. Fixed-length code.

Ex 2. Append special stop char to each codeword.

Ex 3. General prefix-free code.



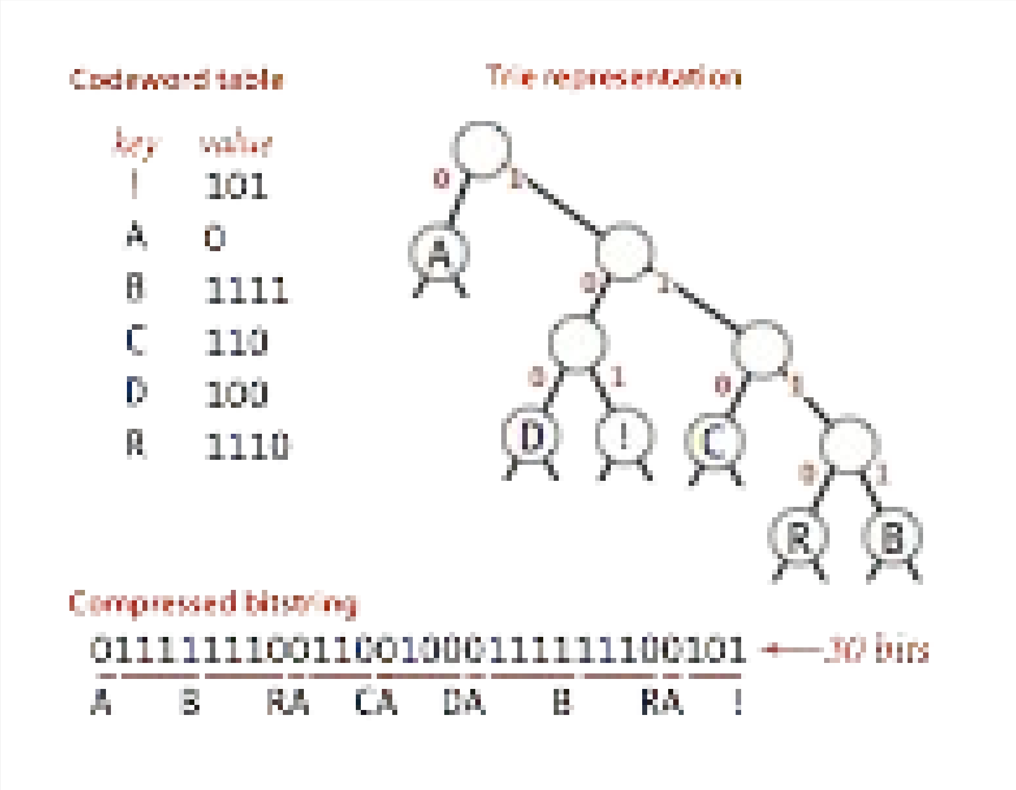


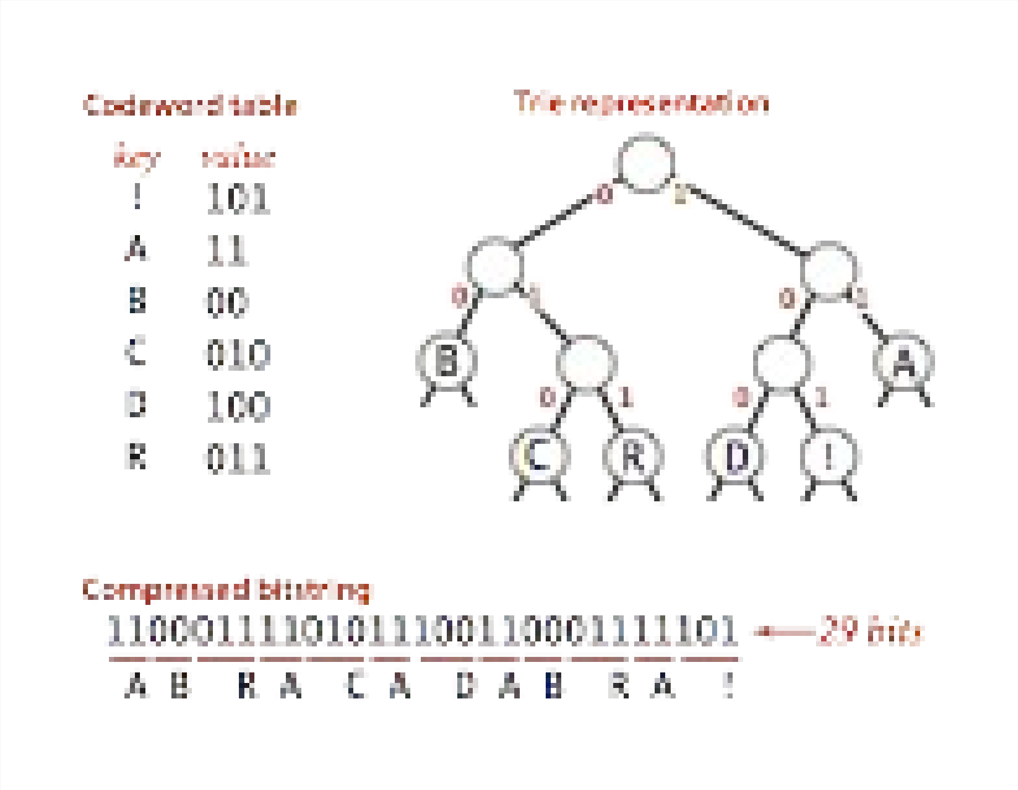
Prefix-free codes: trie representation

Q. How to represent the prefix-free code?

A. A binary trie!

- Chars in leaves.
- Codeword is path from root to leaf.





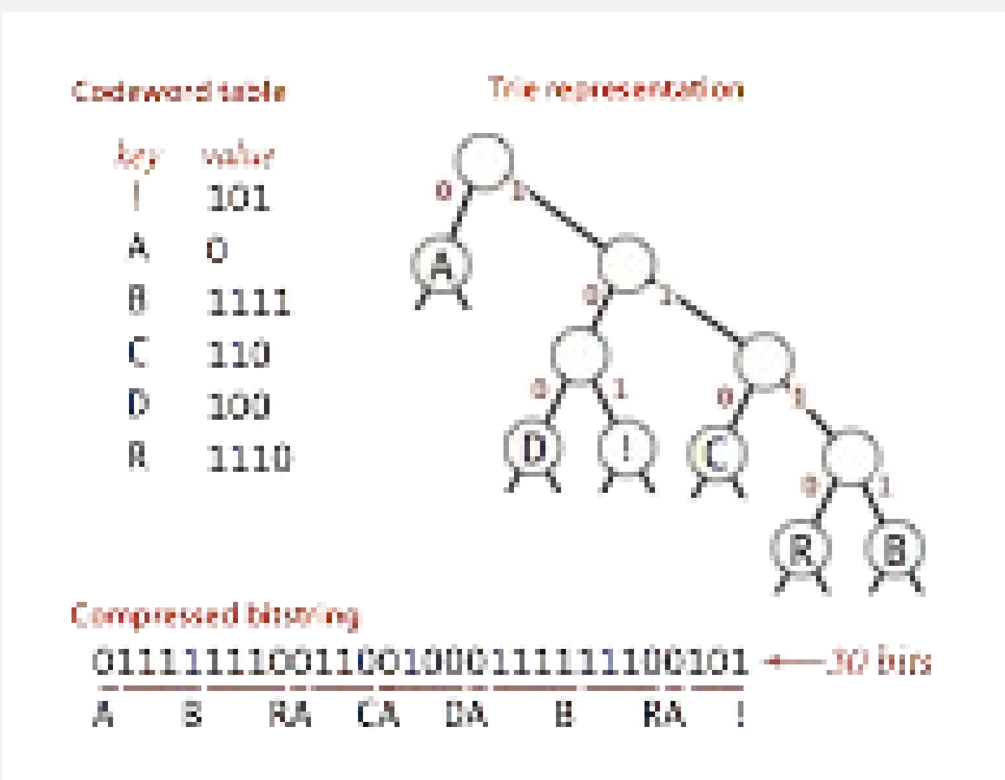
Prefix-free codes: compression and expansion

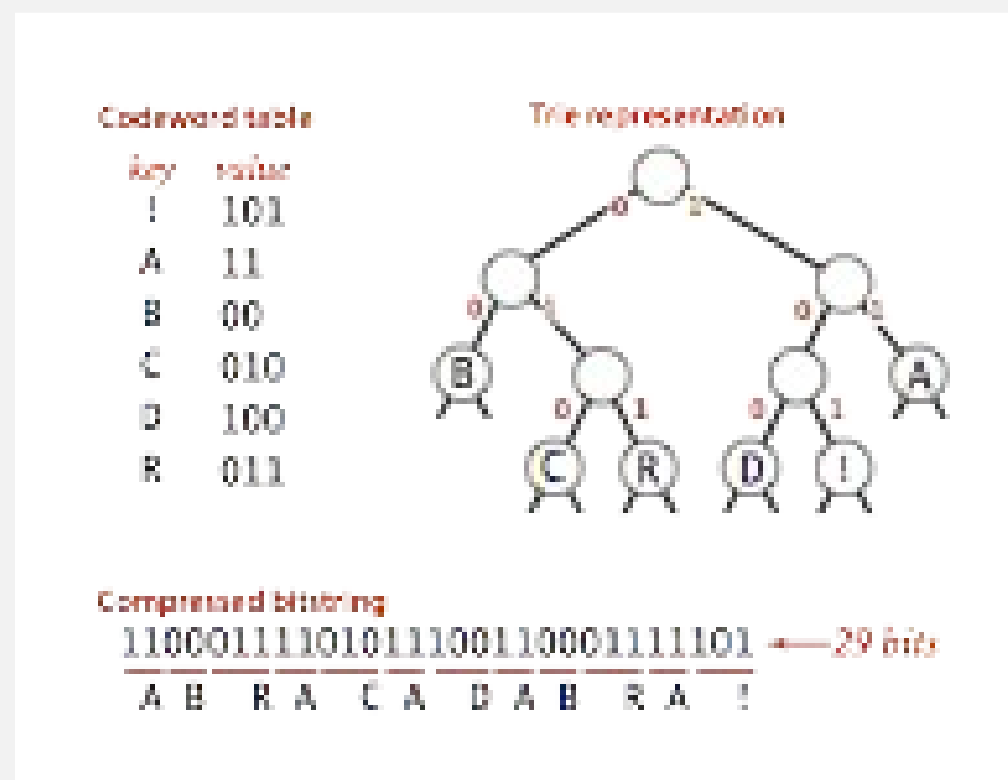
Compression.

- Method 1: start at leaf; follow path up to the root; print bits in reverse.
- Method 2: create ST of key-value pairs.

Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, print char and return to root.





Huffman coding overview

Dynamic model. Use a custom prefix-free code for each message.

Compression.

- Read message.
- Built best prefix-free code for message. How?
- Write prefix-free code (as a trie) to file.
- Compress message using prefix-free code.

Expansion.

- Read prefix-free code (as a trie) from file.
- Read compressed message and expand using trie.

Huffman trie node data type

```
private static class Node implements Comparable<Node>
{
   private final char ch;   // used only for leaf nodes
   private final int freq;  // used only for compress
   private final Node left, right;

   public Node(char ch, int freq, Node left, Node right)
   {
      this.ch    = ch;
      this.freq  = freq;
      this.left  = left;
      this.right = right;
   }

   public boolean isLeaf()
   {  return left == null && right == null; }

   public int compareTo(Node that)
   {  return this.freq - that.freq;  }
}
```

initializing constructor

is Node a leaf?

compare Nodes by frequency

(stay tuned)

Prefix-free codes: expansion

```
public void expand()
{
   Node root = readTrie();
   int N = BinaryStdIn.readInt();

   for (int i = 0; i < N; i++)
   {
      Node x = root;
      while (!x.isLeaf())
      {
         if (!BinaryStdIn.readBoolean())
            x = x.left;
         else
            x = x.right;
      }
      BinaryStdOut.write(x.ch, 8);
   }
   BinaryStdOut.close();
}
```

read in encoding trie

read in number of chars

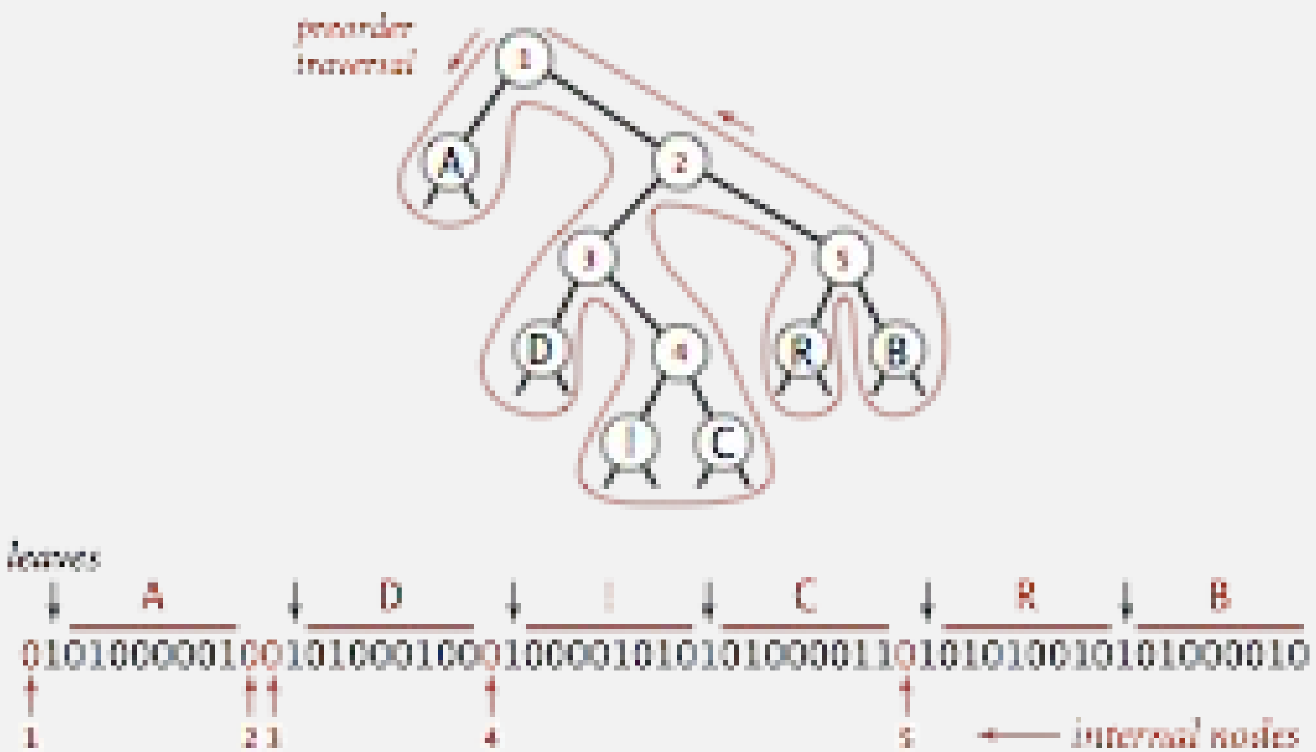
expand codeword for i^{th} char

Running time: Linear in input size N.

Prefix-free codes: how to transmit

Q. How to write the trie?

A. Write preorder traversal of trie; mark leaf and internal nodes with a bit.



Using preorder traversal to encode a trie as a bitstream

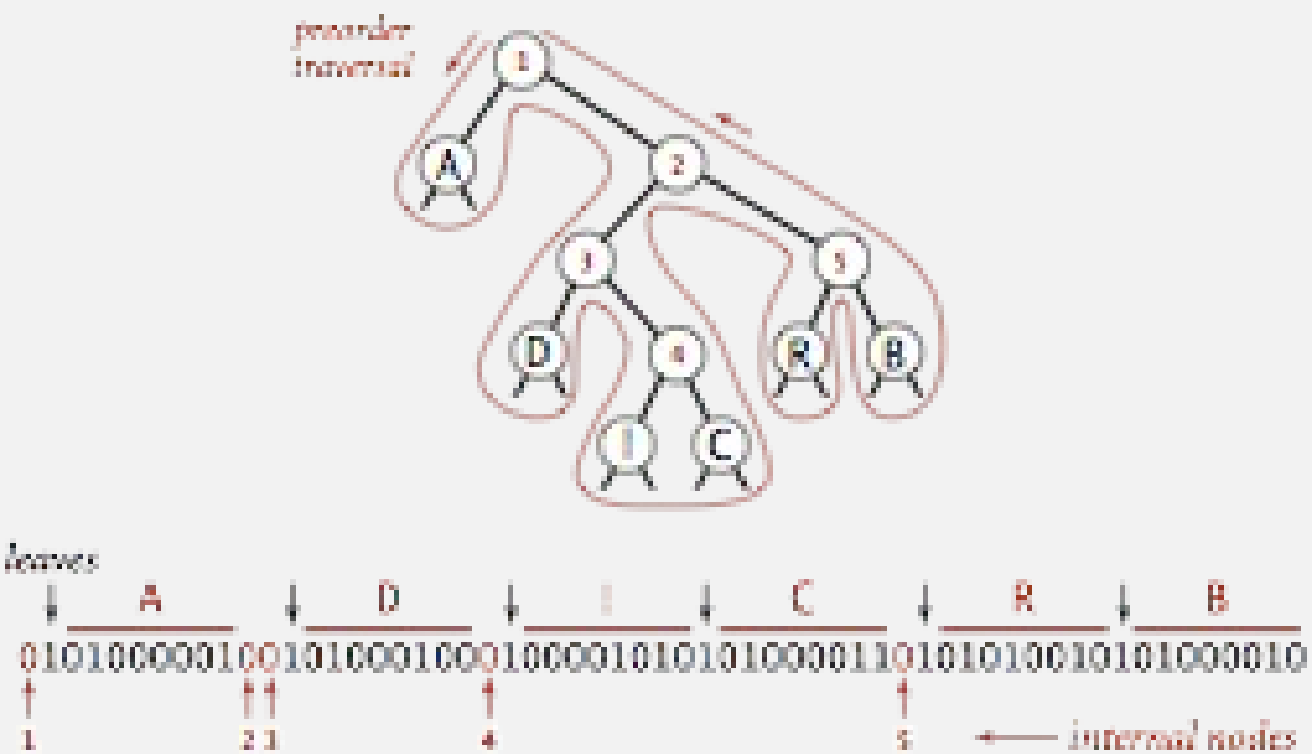
```
private static void writeTrie(Node x)
{
   if (x.isLeaf())
   {
      BinaryStdOut.write(true);
      BinaryStdOut.write(x.ch, 8);
      return;
   }
   BinaryStdOut.write(false);
   writeTrie(x.left);
   writeTrie(x.right);
}
```

Note. If message is long, overhead of transmitting trie is small.

Prefix-free codes: how to transmit

Q. How to read in the trie?

A. Reconstruct from preorder traversal of trie.



Using preorder traversal to encode a trie as a bitstream

```
private static Node readTrie()
{
   if (BinaryStdIn.readBoolean())
   {
      char c = BinaryStdIn.readChar(8);
      return new Node(c, 0, null, null);
   }
   Node x = readTrie();
   Node y = readTrie();
   return new Node('\0', 0, x, y);
}
```

arbitrary value
(value not used with internal nodes)

Shannon-Fano codes

Q. How to find best prefix-free code?

Shannon-Fano algorithm:

- Partition symbols S into two subsets S_0 and S_1 of (roughly) equal freq.
- Codewords for symbols in S_0 start with 0; for symbols in S_1 start with 1.
- Recur in S_0 and S_1.

char	freq	encoding
A	5	0...
C	1	0...

S_0 = codewords starting with 0

char	freq	encoding
B	2	1...
D	1	1...
R	2	1...
!	1	1...

S_1 = codewords starting with 1

Problem 1. How to divide up symbols?

Problem 2. Not optimal!

Huffman algorithm demo

- Count frequency for each character in input.

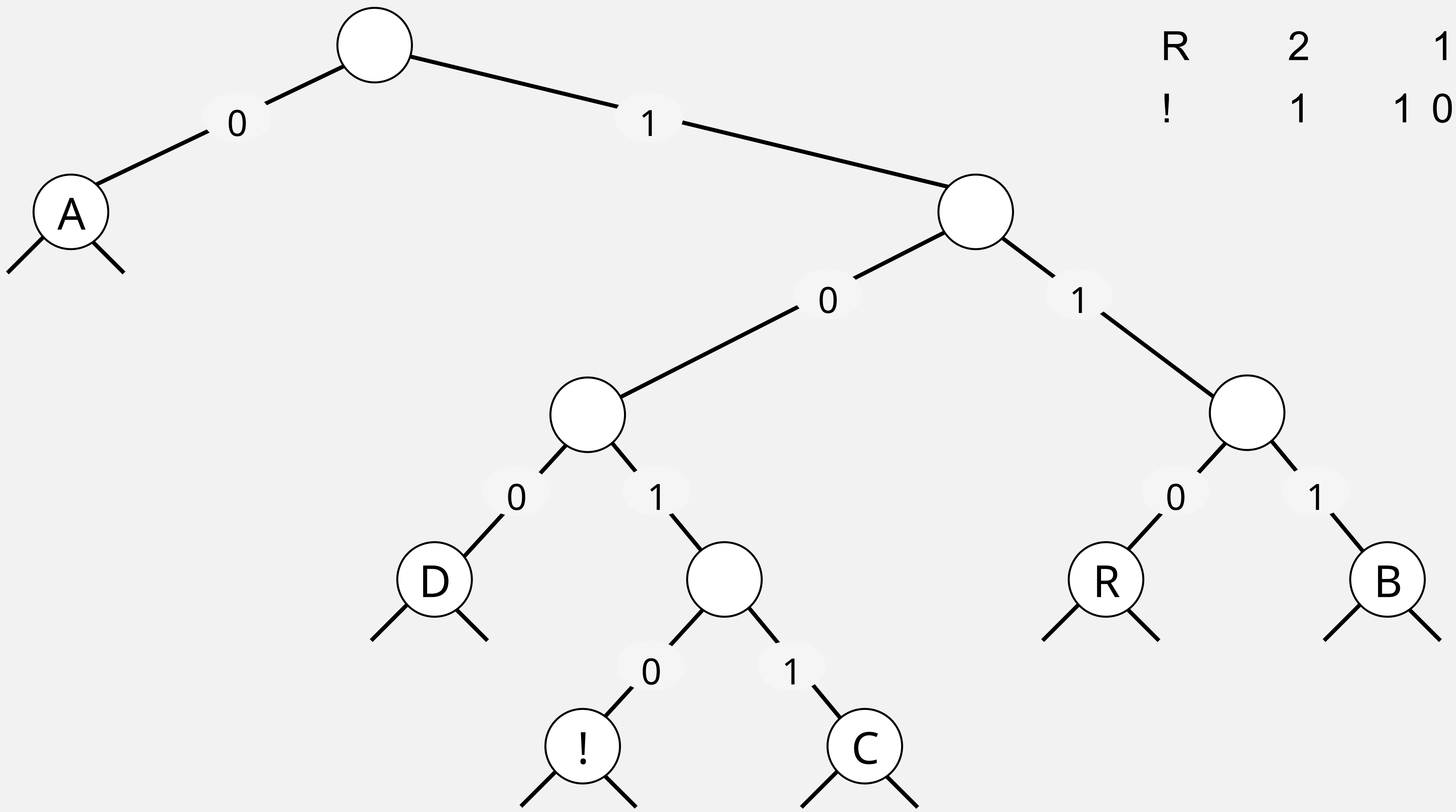
char	freq	encoding
A	5	
B	2	
C	1	
D	1	
R	2	
!	1	

input

A B R A C A D A B R A !

Huffman algorithm demo

char	freq	encoding
A	5	0
B	2	1 1 1
C	1	1 0 1 1
D	1	1 0 0
R	2	1 1 0
!	1	1 0 1 0



Huffman codes

Q. How to find best prefix-free code?

Huffman algorithm:

- Count frequency freq[i] for each char i in input.
- Start with one node corresponding to each char i (with weight freq[i]).
- Repeat until single trie formed:
  - select two tries with min weight freq[i] and freq[j]
  - merge into single trie with weight freq[i] + freq[j]

Applications:

Constructing a Huffman encoding trie: Java implementation

```
private static Node buildTrie(int[] freq)
{
   MinPQ<Node> pq = new MinPQ<Node>();
   for (char i = 0; i < R; i++)
      if (freq[i] > 0)
         pq.insert(new Node(i, freq[i], null, null));

   while (pq.size() > 1)
   {
      Node x = pq.delMin();
      Node y = pq.delMin();
      Node parent = new Node('\0', x.freq + y.freq, x, y);
      pq.insert(parent);
   }

   return pq.delMin();
}
```

initialize PQ with singleton tries

merge two smallest tries

not used for internal nodes

total frequency

two subtries

Huffman encoding summary

Proposition. [Huffman 1950s] Huffman algorithm produces an optimal prefix-free code.

no prefix-free code uses fewer bits

Pf. See textbook.

Implementation.

- Pass 1: tabulate char frequencies and build trie.
- Pass 2: encode file by traversing trie or lookup table.

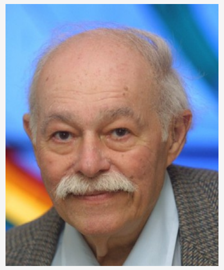
Running time. Using a binary heap $\Rightarrow$ $\underbrace{N}_{\text{input size}} + \underbrace{R}_{\text{alphabet size}} \log R$.

Q. Can we do better? [stay tuned]

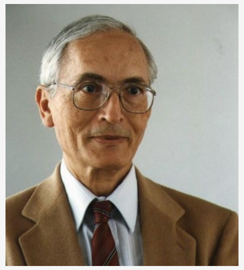
http://algs4.cs.princeton.edu

5.5 Data Compression

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*



Abraham Lempel



Jacob Ziv

Statistical methods

Static model. Same model for all texts.

- Fast.
- Not optimal: different texts have different statistical properties.
- Ex: ASCII, Morse code.

Dynamic model. Generate model based on text.

- Preliminary pass needed to generate model.
- Must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read text.

- More accurate modeling produces better compression.
- Decoding must start from beginning.
- Ex: LZW.

LZW compression demo

input	A	B	R	A	C	A	D	A B	R A	B R	A B R	A	
matches	A	B	R	A	C	A	D	A B	R A	B R	A B R	A	
value	41	42	52	41	43	41	44	81	83	82	88	41	80

LZW compression for A B R A C A D A B R A B R A B R A

key	value	key	value	key	value
⋮	⋮	AB	81	DA	87
A	41	BR	82	ABR	88
B	42	RA	83	RAB	89
C	43	AC	84	BRA	8A
D	44	CA	85	ABRA	8B
⋮	⋮	AD	86		

codeword table

Lempel-Ziv-Welch compression

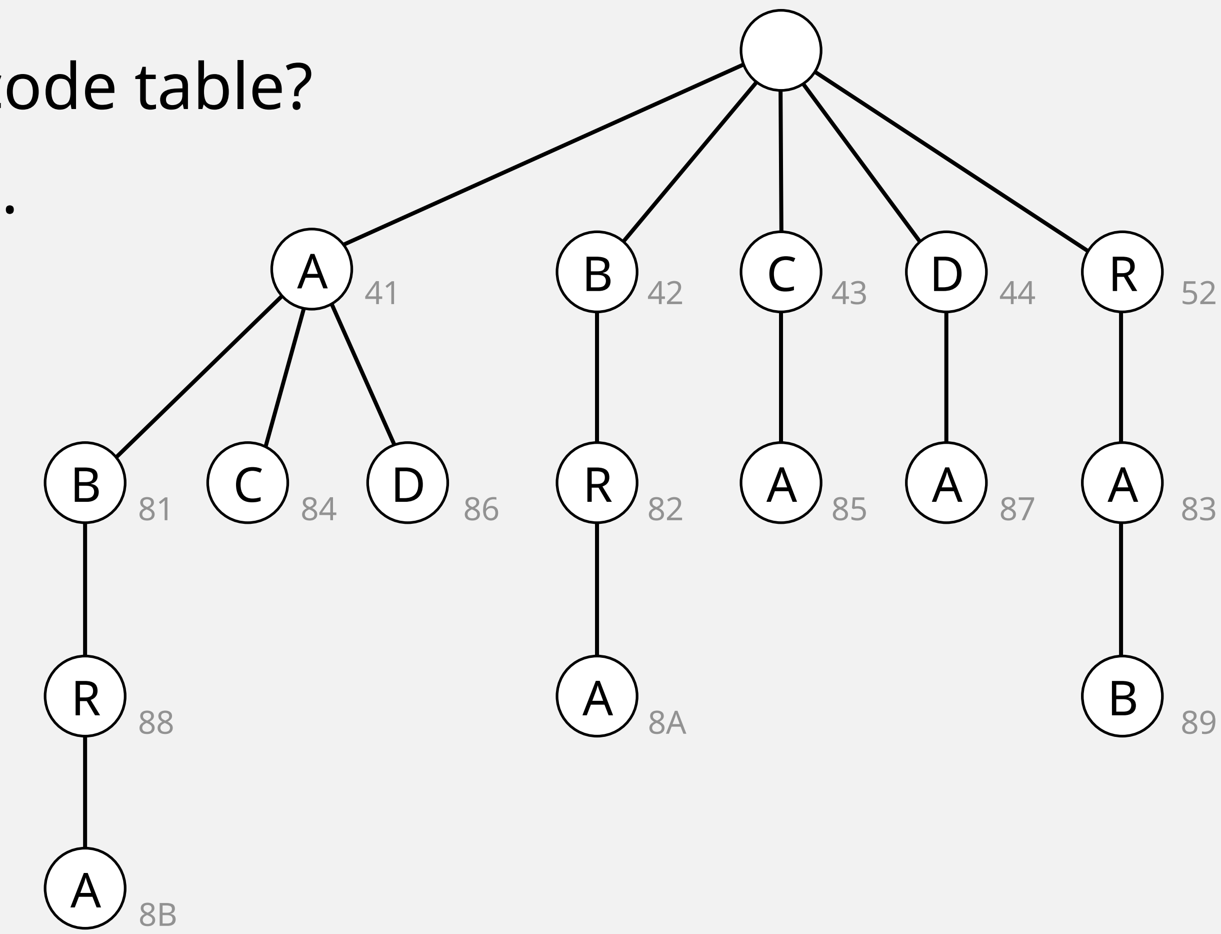
LZW compression.

- Create ST associating W-bit codewords with string keys.
- Initialize ST with codewords for single-char keys.
- Find longest string s in ST that is a prefix of unscanned part of input.
- Write the W-bit codeword associated with s.
- Add $s + c$ to ST, where c is next char in the input.

longest prefix match

Q. How to represent LZW compression code table?

A. A trie to support longest prefix match.



LZW expansion demo

value	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	A B	R A	B R	A B R	A	

LZW expansion for 41 42 52 41 43 41 44 81 83 82 88 41 80

key	value
⋮	⋮
41	A
42	B
43	C
44	D
⋮	⋮

key	value
81	AB
82	BR
83	RA
84	AC
85	CA
86	AD

key	value
87	DA
88	ABR
89	RAB
8A	BRA
8B	ABRA

codeword table

LZW expansion

LZW expansion.

- Create ST associating string values with W-bit keys.
- Initialize ST to contain single-char values.
- Read a W-bit key.
- Find associated string value in ST and write it out.
- Update ST.

Q. How to represent LZW expansion code table?

A. An array of size 2^W.

key	value
⋮	⋮
65	A
66	B
67	C
68	D
⋮	⋮
129	AB
130	BR
131	RA
132	AC
133	CA
134	AD
135	DA
136	ABR
137	RAB
138	BRA
139	ABRA
⋮	⋮

LZW tricky case: compression

input	A	B	A	B	A	B	A	
matches	A	B	A B		A B A			
value	41	42	81		83			80

LZW compression for ABABABA

key	value
⋮	⋮
A	41
B	42
C	43
D	44
⋮	⋮

key	value
AB	81
BA	82
ABA	83

codeword table

LZW tricky case: expansion

value	41	42	81	83	80
output	A	B	A B	A B A	

need to know which key has value 83 before it is in ST!

LZW expansion for 41 42 81 83 80

key	value
⋮	⋮
41	A
42	B
43	C
44	D
⋮	⋮

key	value
81	AB
82	BA
83	ABA

codeword table

LZW implementation details

How big to make ST?

- How long is message?
- Whole message similar model?
- [many other variations]

What to do when ST fills up?

- Throw away and start over. [GIF]
- Throw away when not effective. [Unix compress]
- [many other variations]

Why not put longer substrings in ST?

- [many variations have been developed]

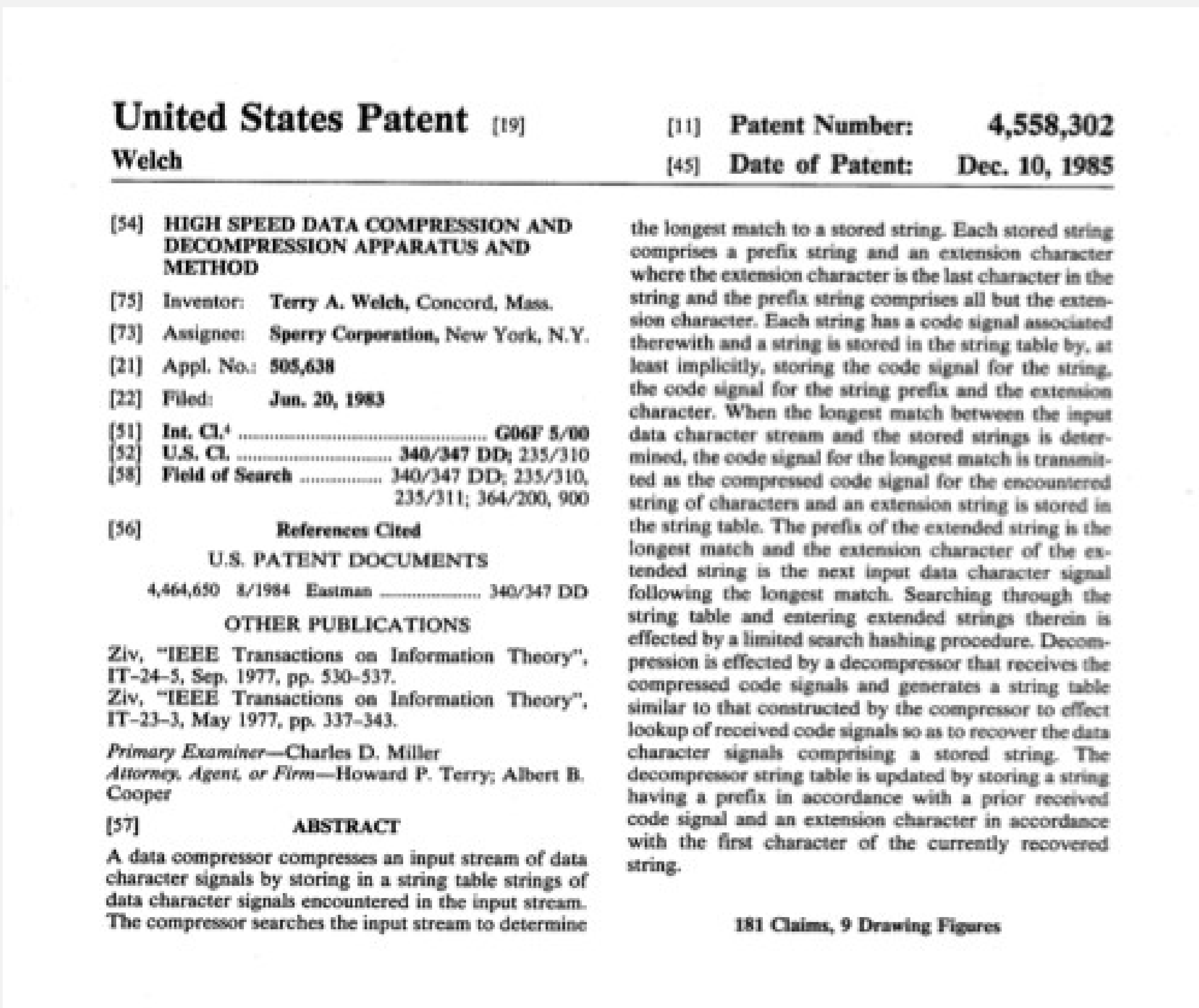
LZW in the real world

Lempel-Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate / zlib = LZ77 variant + Huffman.

LZ77 not patented ⇒ widely used in open source

LZW patent #4,558,302 expired in U.S. on June 20, 2003



United States Patent [19]
Welch

[11] **Patent Number: 4,558,302**
[45] **Date of Patent: Dec. 10, 1985**

[54] **HIGH SPEED DATA COMPRESSION AND DECOMPRESSION APPARATUS AND METHOD**

[75] Inventor: **Terry A. Welch,** Concord, Mass.

[73] Assignee: **Sperry Corporation,** New York, N.Y.

[21] Appl. No.: **505,638**

[22] Filed: **Jun. 20, 1983**

[51] **Int. Cl.**[4] **G06F 5/00**
[52] **U.S. Cl.** .... **340/347 DD;** 235/310
[58] **Field of Search** .... 340/347 DD; 235/310, 235/311; 364/200, 900

[56] **References Cited**

U.S. PATENT DOCUMENTS

4,464,650 8/1984 Eastman .... 340/347 DD

OTHER PUBLICATIONS

Ziv, "IEEE Transactions on Information Theory", IT-24-5, Sep. 1977, pp. 530-537.
Ziv, "IEEE Transactions on Information Theory", IT-23-3, May 1977, pp. 337-343.

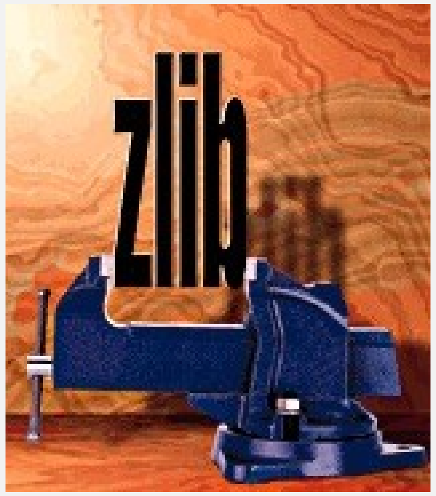
Primary Examiner—Charles D. Miller
Attorney, Agent, or Firm—Howard P. Terry; Albert B. Cooper

[57] **ABSTRACT**

A data compressor compresses an input stream of data character signals by storing in a string table strings of data character signals encountered in the input stream. The compressor searches the input stream to determine the longest match to a stored string. Each stored string comprises a prefix string and an extension character where the extension character is the last character in the string and the prefix string comprises all but the extension character. Each string has a code signal associated therewith and a string is stored in the string table by, at least implicitly, storing the code signal for the string, the code signal for the string prefix and the extension character. When the longest match between the input data character stream and the stored strings is determined, the code signal for the longest match is transmitted as the compressed code signal for the encountered string of characters and an extension string is stored in the string table. The prefix of the extended string is the longest match and the extension character of the extended string is the next input data character signal following the longest match. Searching through the string table and entering extended strings therein is effected by a limited search hashing procedure. Decompression is effected by a decompressor that receives the compressed code signals and generates a string table similar to that constructed by the compressor to effect lookup of received code signals so as to recover the data character signals comprising a stored string. The decompressor string table is updated by storing a string having a prefix in accordance with a prior received code signal and an extension character in accordance with the first character of the currently recovered string.

181 Claims, 9 Drawing Figures

LZW in the real world



Lempel-Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate / zlib = LZ77 variant + Huffman.

Unix compress, GIF, TIFF, V.42bis modem: LZW.

zip, 7zip, gzip, jar, png, pdf: deflate / zlib.

iPhone, Sony Playstation 3, Apache HTTP server: deflate / zlib.

Lossless data compression benchmarks

year	scheme	bits / char
1967	ASCII	7
1950	Huffman	4.7
1977	LZ77	3.94
1984	LZMW	3.32
1987	LZH	3.3
1987	move-to-front	3.24
1987	LZB	3.18
1987	gzip	2.71
1988	PPMC	2.48
1994	SAKDC	2.47
1994	PPM	2.34
1995	Burrows-Wheeler	2.29
1997	BOA	1.99
1999	RK	1.89

next programming assignment

data compression using Calgary corpus

Data compression summary

Lossless compression.

- Represent fixed-length symbols with variable-length codes. [Huffman]
- Represent variable-length symbols with fixed-length codes. [LZW]

Lossy compression. [not covered in this course]

- JPEG, MPEG, MP3, …
- FFT, wavelets, fractals, …

Theoretical limits on compression. Shannon entropy: $H(X) = -\sum_{i}^{n} p(x_i) \lg p(x_i)$

Practical compression. Use extra knowledge whenever possible.