Improved Image Segmentation Through Energy Minimization Based Subspace Fusion

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Abstract

We discuss here an image segmentation scheme which combines multiple segmentations from different subspaces. A subspace is any image characteristic which can help in segmentation, and in this paper we consider texture and intensity. Energy minimization via graph cuts is used to fuse the intensity segmentation and the texture segmentation into a final segmentation. Previously published work used normalized cuts for the intensity segmentation algorithm and the texture segmentation algorithm. In this paper we investigate alternate algorithms for these steps in order to get better performance. We also propose a method to compute the quality input to the energy minimization algorithm more efficiently for a modified quality definition. The quality computation involves an algorithm developed by the author for computing a measure of the variability within a particular window size throughout an image.

1 Introduction

In image segmentation the goal is to divide the input into separate logical components. In this paper we describe a segmentation scheme based on [4]. Our scheme uses different internal algorithms and is able to get better performance. We also show how it is possible to execute the quality computation part of the algorithm more efficiently for a modified quality definition.

In the next section we discuss the scheme and in the section after we discuss our results and make a few concluding remarks.

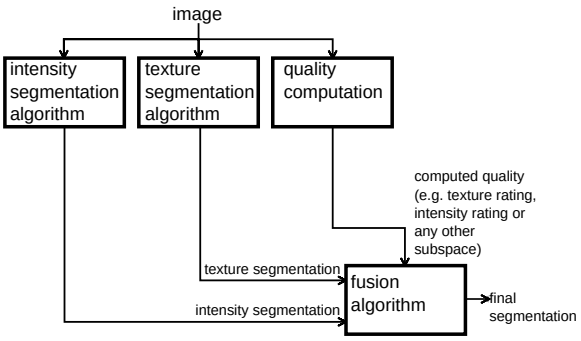


Figure 1: Overview of the scheme

2 Scheme

Fig. 1 shows the basic components of the scheme. The idea is to find segmentations in different subspaces and then to fuse them to form a better total segmentation. A subspace could be any particular characteristic of images that could help in segmentation. In this scheme we consider texture and intensity.

Sometimes an intensity segmentation will be better and sometimes a texture segmentation will be better. For example, if the task is distinguishing between different animal furs a texture segmentation will be more appropriate. On the other hand an intensity segmentation would be more appropriate for distinguishing objects on a desk. By taking information from both the intensity segmentation and the texture

segmentation one can get a better total segmentation.

The quality computation is used to identify whether a particular image region belongs to a particular subspace (e.g. whether it is intensity or texture). The figure only shows a single box for the quality computation, since the quality computation for texture and the quality computation for intensity are computed in the same step, but in principle there could be a separate metric and corresponding computation for each separate subspace. [4] discusses some additional possible subspaces.

In the following sections we describe in more detail the intensity segmentation algorithm [5], the texture segmentation algorithm, the quality computation, and the fusion algorithm [3]. An implementation is available at **FIXFIX**.

2.1 Intensity Segmentation

The first step of the intensity algorithm [5] is to form a graph from the image. Each pixel is a point and is connected to its immediate neighbors (including diagonally). The edge weight is the difference in intensity between the two pixels.

The next step is to sort all the edges from smallest to largest. The algorithm then iterates through the edges and considers for each one whether there is sufficient evidence between the two components to join them. The algorithm takes into account the internal difference within the components as well as the value of the edge weight. Initially, each pixel is a single component. If the internal difference within each component is small and the edge weight is large then it will not join the two components as one would expect, but if the edge weight is small in comparison to the internal difference it will join them. One parameter of the algorithm is referred to as k and sets the scale of observation. A larger k will tend to favor larger components, and a smaller k will tend to favor more numerous smaller components. More details on the exact criteria can be found in [5].

Initially the algorithm will go through the smaller edges and there will be more joins. At the end however it will be going through the larger edges and there will be more non-joins. The algorithm is completed after it has iterated through all the edges.

The implementation of this algorithm was downloaded from `http://people.cs.uchicago.edu/~pff/segment/` (updated link as of summer 2017: `http://cs.brown.edu/~pff/segment/`). The code used is also available in the source in its original form.

2.2 Texture Segmentation

The first step of the texture segmentation algorithm is to find the Fourier transform vectors. One of the parameters passed to the texture segmentation algorithm is a window length. The image to be segmented is divided into squares each containing window length$\times$window length number of pixels, and the two dimensional Fourier transform is taken of each of these windows.

After the Fourier vectors have been found the next step is to cluster them into separate textural segments. The previous segmentation algorithm can be used if the Fourier vectors are formed into a graph. In our implementation each vector is a separate point and the edge weights between different points is the sum of the absolute value of each coordinate difference.

It is possible to form the entire graph with all edges using brute force. However, it is possible to get reasonable results by only forming for each point the edge to its k closest neighbors (in our implementation $k = 5$). Also, one can use a more efficient algorithm [1] than brute force to find the nearest neighbors (as suggested in [5]). The previous segmentation algorithm can then be run on this graph.

We call our implementation of the algorithm described in [1] the cell algorithm. The first step is to partition all of the points into separate cells. The entire set is divided into two parts based on the average value in the dimension of maximum spread ([1] calls this the midpoint split rule). Each subsequent set is recursively divided and the algorithm continues until each cell has *bucketsize* or less points in it.

After the points have been split finding the nearest neighbors involves iterating around the cells close to the query point. Points are successively added to a point priority queue which is prioritized based on how far the point is from the query point. If a certain

criteria is met then the closest point is popped off the point priority queue and placed in a result array which has k slots for the result of the search. The criteria is based on how far the current cell is from the query point versus how far the point on the top of the priority queue is from the query point. If the current cell is distant from the query point then it is likely the closest point(s) have already been encountered as the cells are enumerated based on how far they are from the query point. The parameter epsilon controls how fast versus how precise the algorithm is. Fig. 2 provides further details.

Testing showed that as epsilon decreases accuracy improves but runtime is longer (as expected). Accuracy was measured by looking at the sum of the distances to the 5 close neighbors found by the cell algorithm to the actual 5 closest neighbors found by brute force. In the case of epsilon equal to 50 the percent in which the sum of the distances for the 5 close neighbors was within 10% of the sum for the actual 5 closest neighbors went as 67.24%, 57.33%, 53.83%, and 52.61% for the four images (`car.ppm`, `friends.ppm`, `husky.ppm`, and `olympics2016.ppm`). In the case of epsilon equal to 10 the accuracy improved so that it went as 94.71%, 93.17%, 91.61%, 91.86%; however, it is also the case that the execution time increased going from 3s, 4s, 2s and 11s to 20s, 30s, 11s, and 38s. The default in the file is still set to epislon equal to 50. Comparing the actual segmentations produced it is not clear that the theoretical improvement in accuracy translates to better segmentations for the additional time. It does seem to be the case though that for the `friends.ppm` image and `olympics2016.ppm` there is more consolidation of classifications (larger groups get larger and smaller groups get smaller). This test and the output generated can be found in the source.

2.3 Quality Computation

To compute the quality we follow the same approach as in [4], and use the standard deviation as a measure of texture. That is, given a particular window radius the standard deviation is taken for all pixel values within that radius (or possibly less pixel values if the center pixel is near an edge). This is done for each pixel so that each pixel gets a different value. In the implementation we do not take the square root so it is implemented to be the sum of the squares of the differences from the mean divided by the total number of pixels in the window.

Since this is done for each pixel it is clearly possible to recompute the mean more efficiently than just readding each time. When we move down a row $2R + 1$ points are deleted and $2R + 1$ points are inserted. It is only necessary to keep track of the total sum and to subtract from it the values that are deleted and to add to it the values that are inserted.

Our implementation shows how it is possible to do a similar thing for a modified definition of the variance. The change is to simply consider instead the sum of the absolute value of each difference from the mean.

We now introduce some notation. The `below_count` is the total number of points below the mean. The `below_sum` is the sum of the differences of each below point from the mean. The `above_count` and `above_sum` are defined similarly. It can also be helpful to visualize the points on a one dimensional number line with the mean in the center. The `above_sum` and `below_sum` are always positive so the sum of these two (divided by the total number of points) is the desired quantity. A transition point is a point that changes from being below the mean to above it or from being above the mean to below it when the mean changes.

Clearly if there are no transition points then finding the new state is not difficult. This will happen for example if there are many points clustered close together to the left and many points clustered close together on the right, and the mean simply gets shifted slightly to the right. Then the `below_count` and the `above_count` remain the same and the `below_sum` increases by `below_count * (new_mean - old_mean)` while the `above_sum` decreases by `above_count * (new_mean - old_mean)`.

However, if there are transition points it is necessary to account for them. The discussion now follows the code in fig. 3. The code shows the case for the mean increasing. The case for the mean decreasing is similar. It is also necessary in the implementation to handle points close to the edge.

Maintain a cell priority queue and a point priority queue. The priority being how close the cell or point is to the query point.

Maintain a saved result vector to keep the closest points found so far.

Set the current cell to the cell which contains the query point.

Process the current cell:

> Add all points from the current cell to the point priority queue.
>
> Add all points which satisfy the criteria to the saved result and pop them from the point priority queue. The criteria being whether the distance of the point from the query point is less than $(1+\varepsilon)$ times the distance of the current cell from the query point.
>
> if the length of the saved result vector is greater than or equal to *k* then:
>
> > return the saved result vector
>
> Add all neighbors of the current cell to the cell priority queue (that have not already been added).
>
> Pop the processed cell off the cell priority queue and set the current cell to be the next cell at the top of the queue.

Figure 2: Pseudocode for the cell algorithm

```
dels - points deleted from the set

adds - points added to the set

currpts - points from the previous
pixel that were not deleted

n - total number of points

/* It is assumed here that the new_mean has
already been computed and the state updated
from the deletion of the dels */

transptscnt = 0;
Iterate through currpts:
(iteration variable currpoint)
{
 if ( old_mean <= currpoint <= new_mean )
 {
  below_sum = below_sum +
              (new_mean - currpoint);
  above_sum = above_sum -
              (currpoint - old_mean);
  transptscnt++;
 }
}

above_count = above_count - transptscnt;
above_sum = above_sum - above_count *
           (new_mean - old_mean);
below_sum = below_sum + below_count *
           (new_mean - old_mean);
below_count = below_count + transptscnt;

Iterate through adds and update below_count,
below_sum, above_count, and above_sum;

return ((float)(below_sum + above_sum)) / n;
```

Figure 3: Pseudocode for the quality computation

The first step is to find the transition points. For each transition point that is found we increase the `below_sum` by the difference between the `new_mean` and the current point. We decrease the `above_sum` by the difference between the current point and the `old_mean`. We also keep track of how many transition points were found.

We then decrease the `above_count` by the total number of transition points and the `above_sum` is decreased by `above_count*(new_mean - old_mean)`.

We then increase the `below_sum` by `below_count*(new_mean - old_mean)` and the `below_count` is increased by the total number of transition points found.

After accounting for the additional points added to the set the desired quantity is found by adding together the `below_sum` and the `above_sum` and dividing by the total number of points. The total number of points will increase at edges when going toward the center and will decrease at edges when moving away from the center, but in the middle will remain the same.

It is also possible to do this without saving a lot of state. By moving down each column it is only necessary to save the state at the top of the column in order to transition from one column to the next. It is only necessary then to keep track of two states: the state for the previous pixel that was just processed, and the state for the pixel at the top of the column.

Experimental results indicate a performance im-

provement that increases with the size of the window radius. It is not known whether a larger or a smaller window radius for the quality computation produces better segmentations. The source code for the tests and output fo the testing machine can be found in the source.

2.4 Fusion Algorithm

In this step we have three inputs, the intensity segmentation, the texture segmentation, and the quality computation. The objective is to merge these two segmentations to produce a fused segmentation for the whole image.

We have the quality computation which measures how likely a region is to be intensity or texture. We could then just produce a fused segmentation by using the texture segmentation for each pixel that looks like texture and the intensity segmentation for each pixel that looks like intensity. However, this may not always be the best approach. There might be a few pixels that look only slightly more like texture than intensity within a larger region which looks almost certainly like intensity. Those pixels may just be noise or some object in the scene that will not be relevant to a higher application. Instead of asking the question: "does that pixel look like texture or intensity?" it may be better to ask the question: "what arrangement of labels (texture or intensity) best satisfies some specific criteria?". The criteria taking into account both the local information and how well a particular label assignment fits with other labels around it.

This is a common problem in Computer Vision and the criteria is typically called the energy. The objective is to minimize a function of the following form:

$$E = \sum_{p \in \mathcal{P}} D(p) + \sum_{p \text{ adjacent to } q} V(p, q)$$

$D(N)$ is the data cost and is over each pixel while $V(N)$ is the smoothness cost and is over each pair of adjacent pixels. For example, when doing stereo the data cost will be disparities, and the smoothness cost will be some function which penalizes transitions between different labels. In our case the labels are intensity and texture, but for stereo the labels would be different distance levels.

There are multiple ways to approach this minimization problem (although, in most cases there is no known way to get the exact answer efficiently). This scheme models the problem as a maximum flow minimum cut problem as done before in [4]. The specific graph construction can be found in [3].

In our implementation the smoothness function used is:

$$V(p, q) = \begin{cases} C & \text{if } label(p) = label(q) \\ 0 & \text{if } label(p) \neq label(q) \end{cases}$$

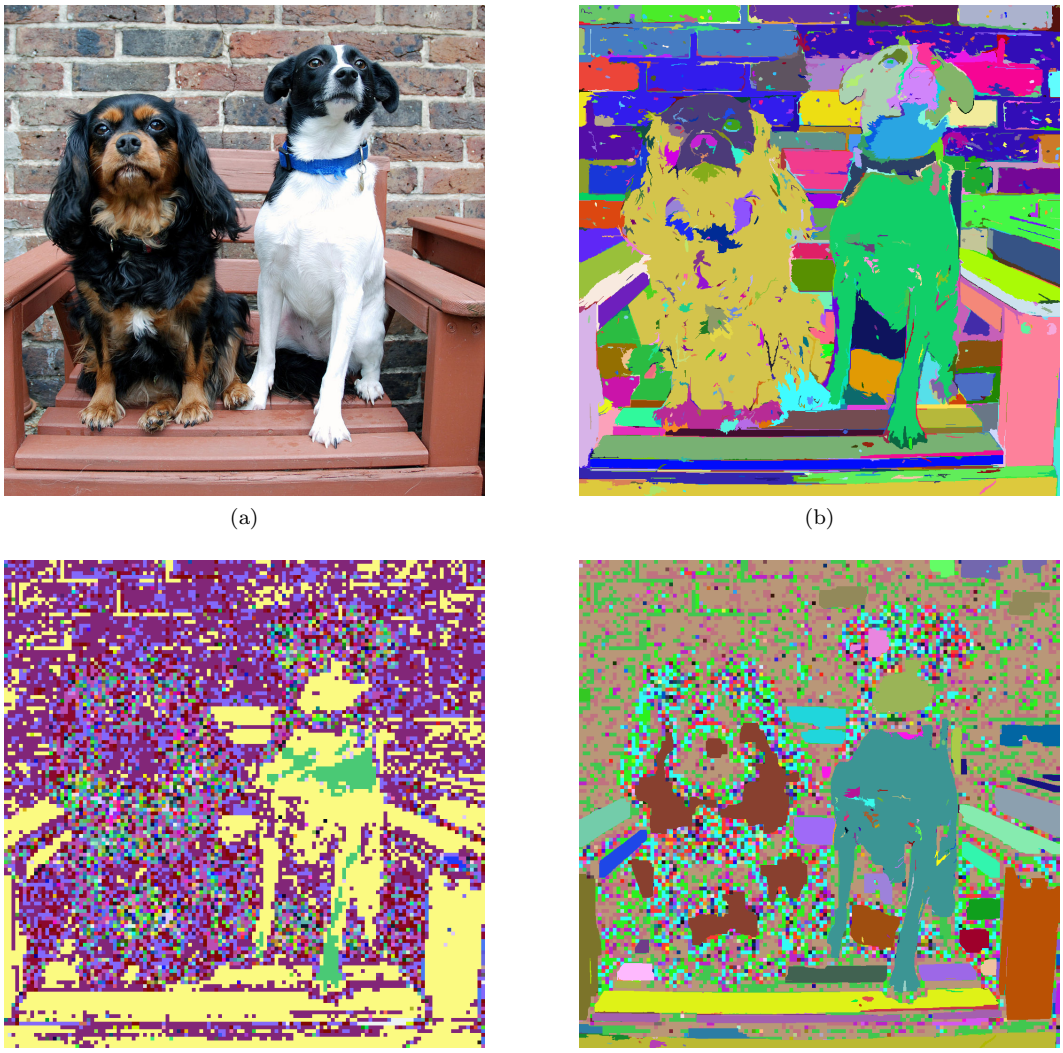
which is referred to in [3] as the Potts model. The quality is computed as described in the previous section. The intensity datacost of a pixel is equal to the quality value at that pixel (lower variance meaning more likely intensity). The texture datacost should be in same way inversely related to the intensity datacost (higher variance meaning more likely texture). In this scheme the additive inverse is used so if the intensity datacost is x the texture datacost is $K - x$. However, setting K to be the highest quality value found could lead to everything be labeled as intensity since the highest quality value found may be a large outlier, so K is set to be the 75th percentile quality value.

All pixels labeled as intensity take the segment of the intensity segmentation and all pixels labeled as texture take the segment of the texture segmentation.

The code for this algorithm was downloaded from `http://www.csd.uwo.ca/~olga/code.html` (updated link as of summer 2017: `http://www.csd.uwo.ca/faculty/olga/code.html`) , and the version used for this implementation is also included in the source.

3 Results and Concluding Remarks

Figs. 4b, 4c, and 4d show the results of the intensity segmentation, texture segmentation, and fused segmentation for the image shown in fig. 4a.



(a) (b) (c) (d)

Figure 4: (a) The original image (b) Intensity segmentation (c) Texture Segmentation (d) Fused segmentation. The original image taken by Allen Watkin and posted on Flickr under a CC BY-SA 2.0 license was downloaded from Wikimedia Commons.

The variation in the bricks is large enough that those are considered texture while the variation in the parts of the chair are small enough that the intensity segmentation is used. The bricks are then composed of one principle group with two smaller groups close in number. The variation in the coat of the left dog is large enough that texture is used for most parts where about one one-fifth of the pixels have a texture classification also found in the bricks. For the dog on the right the body has small enough variation that

intensity is used.

The total amount of time taken to segment this image of size 1024 x 1045 was 22 seconds. Further images and segmentations can be found in the source.

Possible further work includes:

- Improving the texture segmentation algorithm. Previously [4] used Gabor convolution kernels [6] so this could be investigated in more detail.
- Considering alternate metrics for determining what part of an image is texture versus what is not.
- Considering other subspaces as mentioned in [4].

4 Revisions made in 2017

- The algorithm was run on new images for which the license information was readily available. Licensing issues were also addressed for the code, and licensing information was added to the paper and also in the source.
- Consistent with new images the tests were rerun and the paragraphs in the paper reporting on the results of the tests were updated.
- In the course of testing a bug was fixed that would occur in the cell algorithm if all the pixel values were (0, 0, 0) resulting in a division by zero.
- One paragraph related to testing was removed concerning absolute value and branching. The assembly produced for the testing machine did not use branching for absolute value and further investigation indicated that there are a variety of ways to avoid branching when doing absolute value.
- Various other revisions were made such as changes to formatting, etc.

References

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