Compressing Multisets with Large Alphabets

Daniel Severo[1,2,3] James Townsend[4]
Ashish Khisti[2] Alireza Makhzani[2,3] Karen Ullrich[1]

[1]Meta AI [2]University of Toronto
[3]Vector Institute for AI [4]University of Amsterdam

Data Compression Conference, 2022

Outline

Problem setting

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we want to losslessly compress the multiset

$$\mathcal{M} = f(X^n) = \{X_1, \ldots, X_n\}$$

at rate $H(\mathcal{M}) \leq H(X^n)$.

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Would like efficient, rate-optimal method for any $\mathcal{A}, n$.

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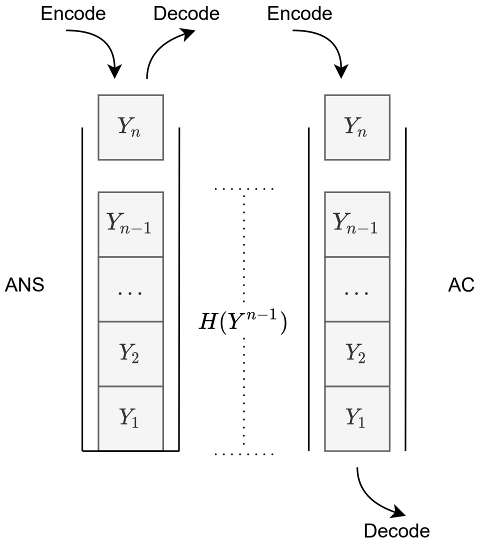
order	queue-like	stack-like

Background: Asymmetric Numeral Systems (ANS)

Key difference: ANS decodes in reverse order



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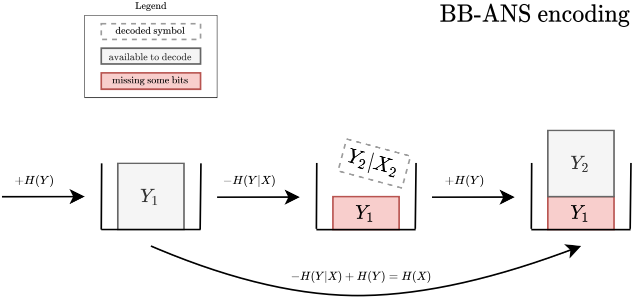

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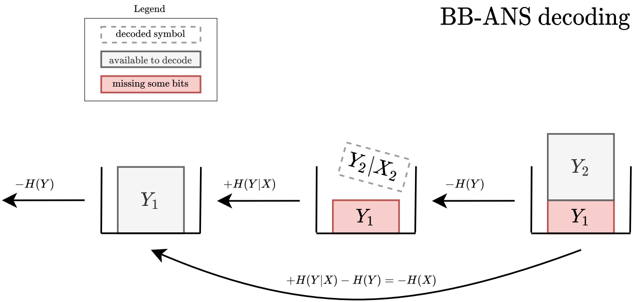

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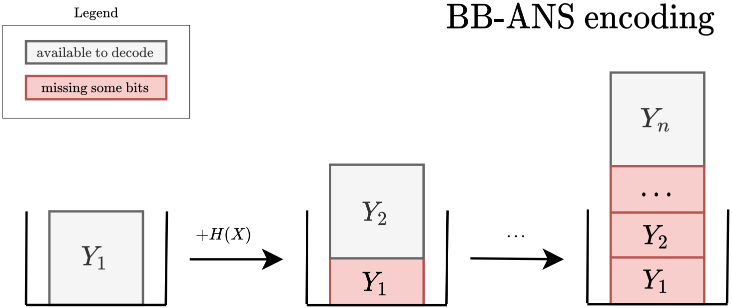
The full picture



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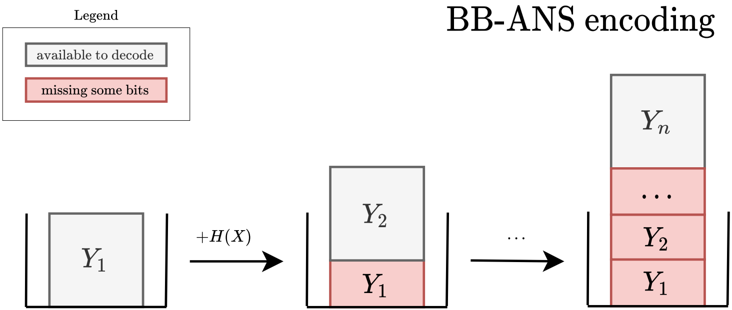
The full picture, with one-time overhead of $+\frac{1}{n}H(Y|X)$



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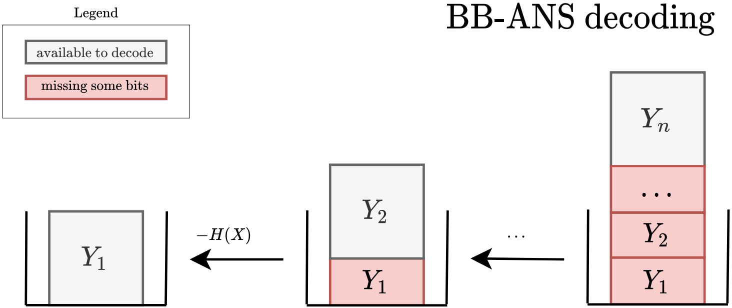

Background: Bits-back with ANS (BB-ANS)

Take-away: BB-ANS gives an operational meaning to the identity

$$H(X) = H(Y) - H(Y \mid X) = I(X;Y),$$

where $X = f(Y)$.

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$H(X^n \,|\, \mathcal{M})$ bits are needed to order symbols in $\mathcal{M}$ to create X^n

It is often called the "order information"

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Can we achieve $H(\mathcal{M})$ on a single multiset $\mathcal{M} = f(X^n)$?

In other words, can we compress $\mathcal{M}$ to $-\log P_{\mathcal{M}}(\mathcal{M})$ bits?

Method: compressing $\mathcal{M}$ to $-\log P_{\mathcal{M}}(\mathcal{M})$ bits

Construct order information $H(X^n \mid \mathcal{M})$ iteratively by "sampling without replacement" from $\mathcal{M}$. Alternate:

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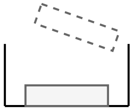

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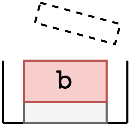

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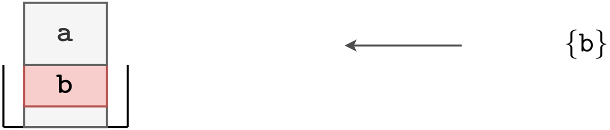

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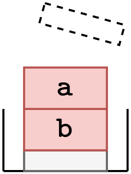

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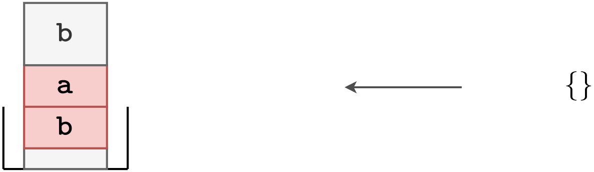

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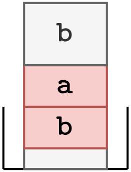

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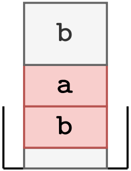

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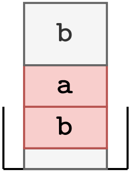

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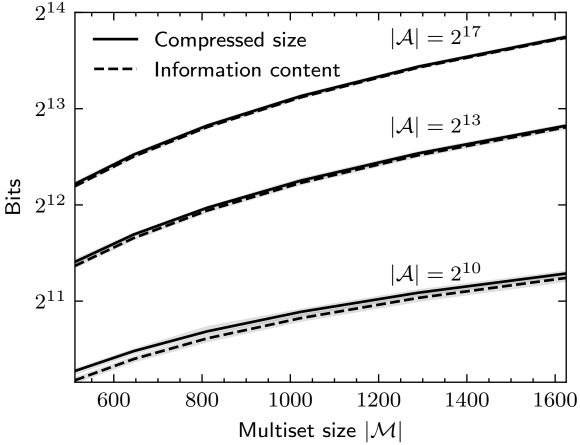
Experiments

Experiments: Synthetic multisets (rate)

Achieves $H(\mathcal{M}) = \mathbb{E}[-\log P_{\mathcal{M}}(\mathcal{M})]$ on single $\mathcal{M}$



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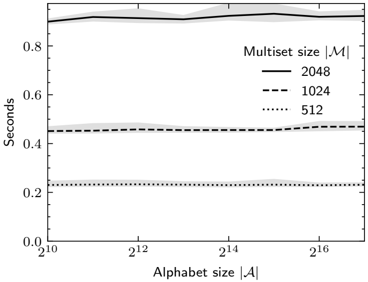

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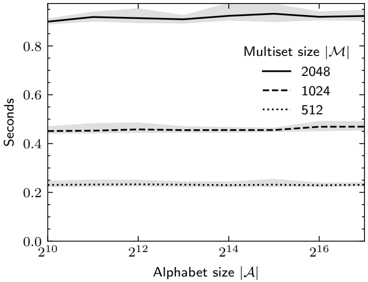

Compute time doesn't scale with $|\mathcal{A}|$, if m is fixed

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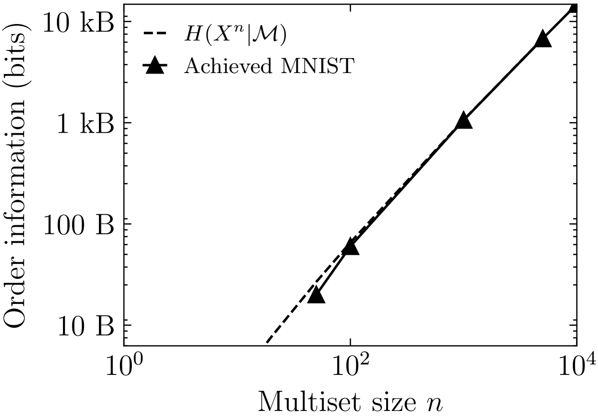

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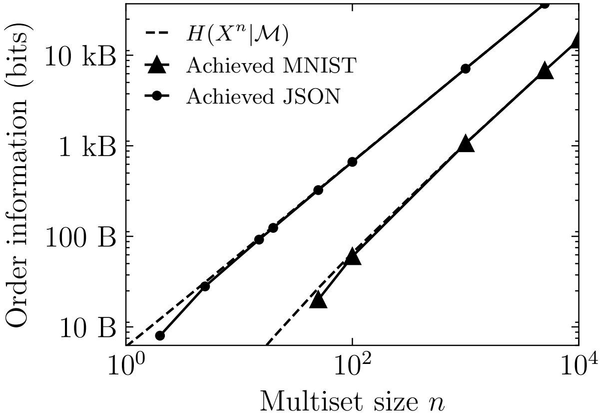

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- Symbols can be anything (e.g. images, text, multisets)

Thank you!

Meta

Daniel Severo

James Townsend

UCL

UNIVERSITY OF TORONTO

Ashish Khisti

Alireza Makhzani

Karen Ullrich

VECTOR INSTITUTE

Presented by: dsevero.com and j-towns.github.io

Code: github.com/facebookresearch/multiset-compression

Bonus: dynamic multiset data structure

We use something like a *Fenwick tree* for the multiset.

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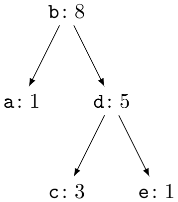

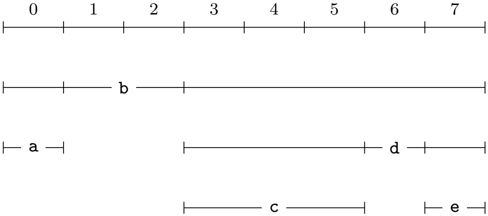

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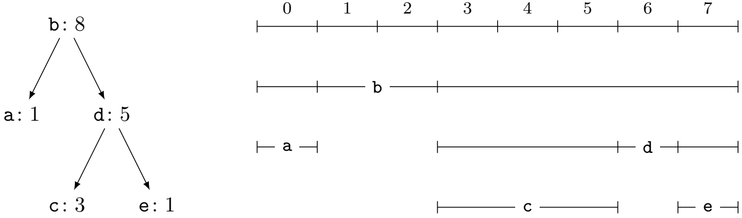

Has $O(\log n)$ insertion, deletion and F_X, P_X lookup :-).