Shamir Secret Sharing with no ID numbers

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Shamir Secret Sharing: review

Problem description

You just got the private key of a wallet holding 1000 ETH!

(volcano laptop monster decide october blue now drastic laptop slow effort collect)

For secure long-term storage, you want to:

- Split the key into several 12-word seedphrases, such that two of them are needed for reconstruction.
- You want the secret to be accessible even if one piece gets lost.

You will create 3 pieces. This is called a (2,3) threshold scheme.

Share generation:

volcano laptop monster decide october
blue now drastic laptop slow effort collect

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volcano laptop monster decide october
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11110101101 01111101000
10001111010 00111000110
10011001000 00011000010
10010111001 01000010010
01111101000 11001100001
01000110101 00101101100

(Encoding with BIP-39’s word dictionary)

Share generation:

volcano laptop monster decide october blue now drastic laptop slow effort collect

11110101101 01111101000
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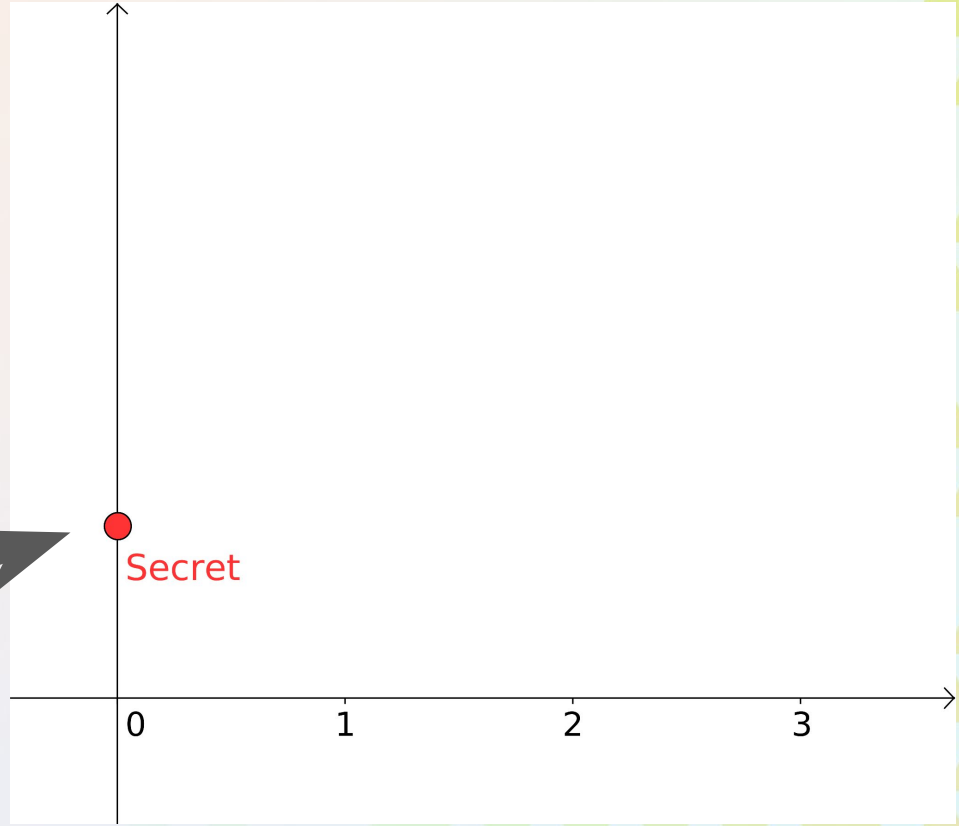
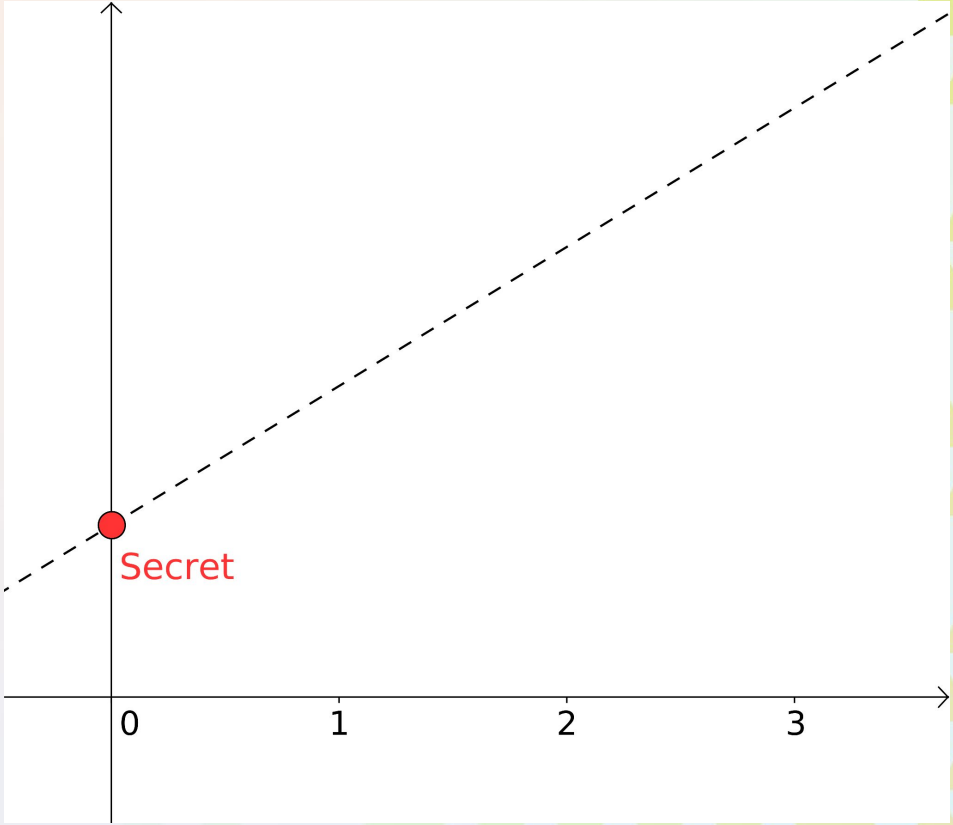


Share generation:

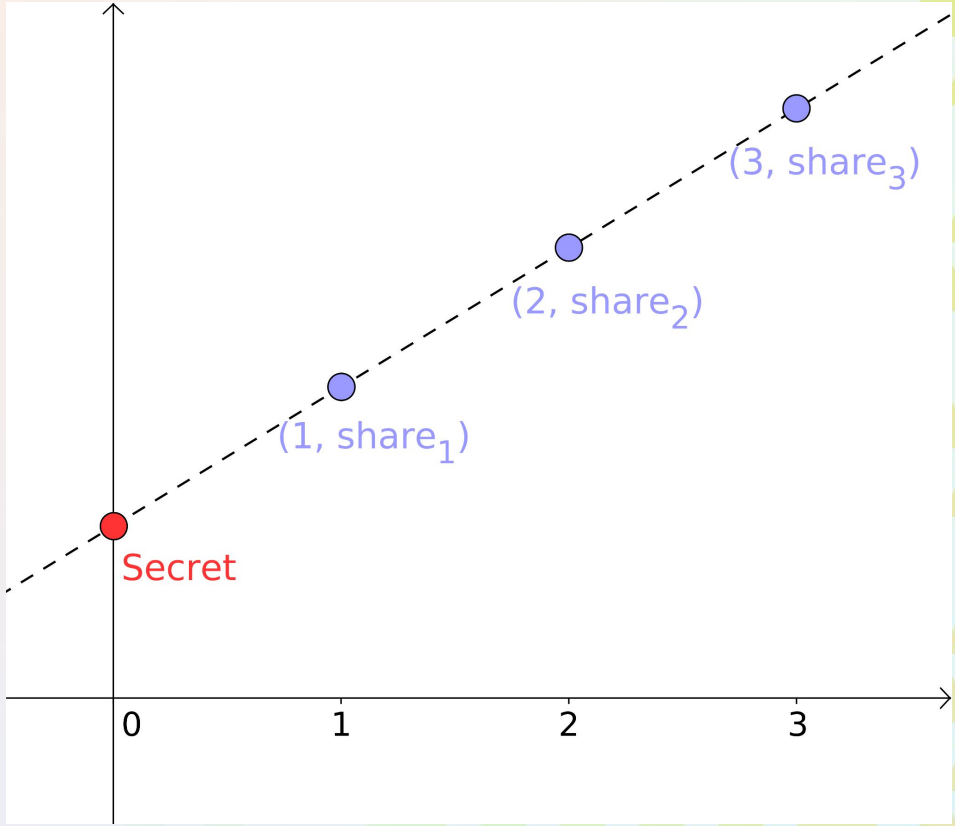
Choose a random straight line passing through the secret.



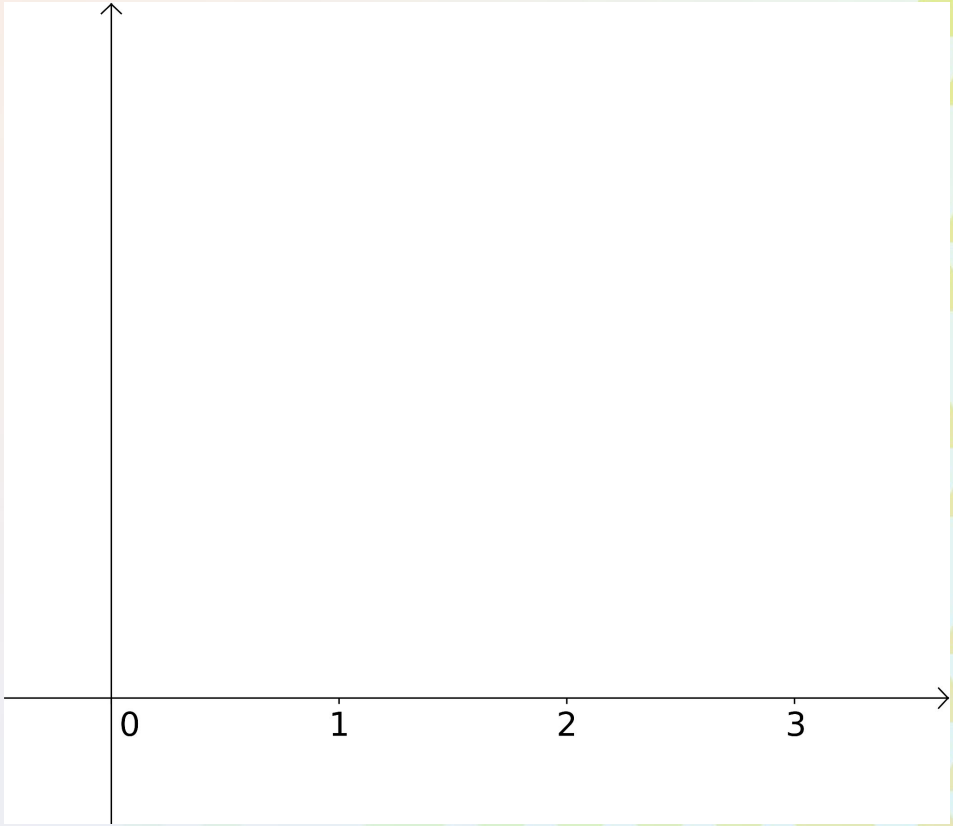
`Share generation:`

Pick three points on the line.

These are your shares.

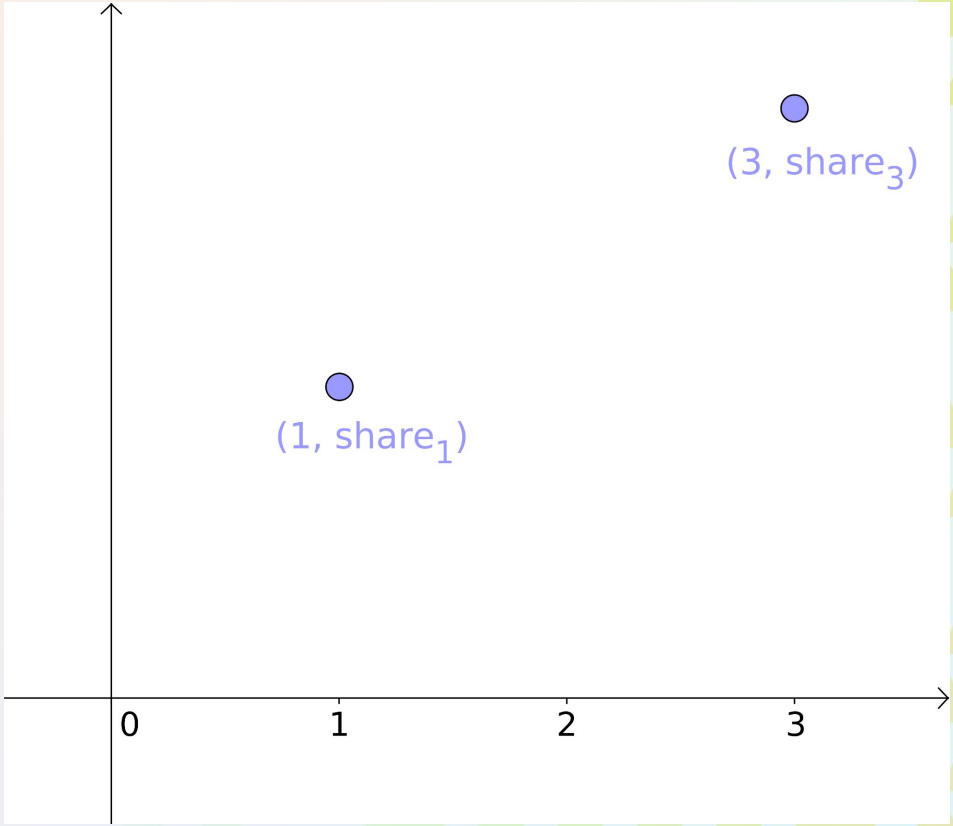


Share generation:



`Share generation:`

Any two shares generate the correct secret.



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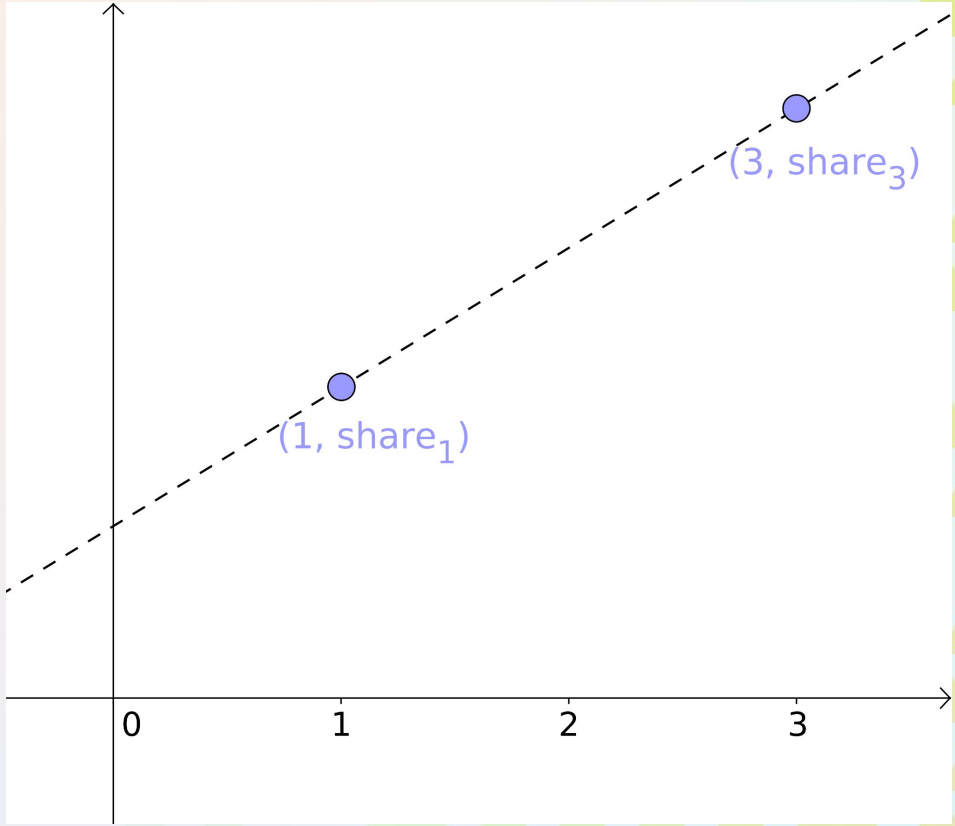

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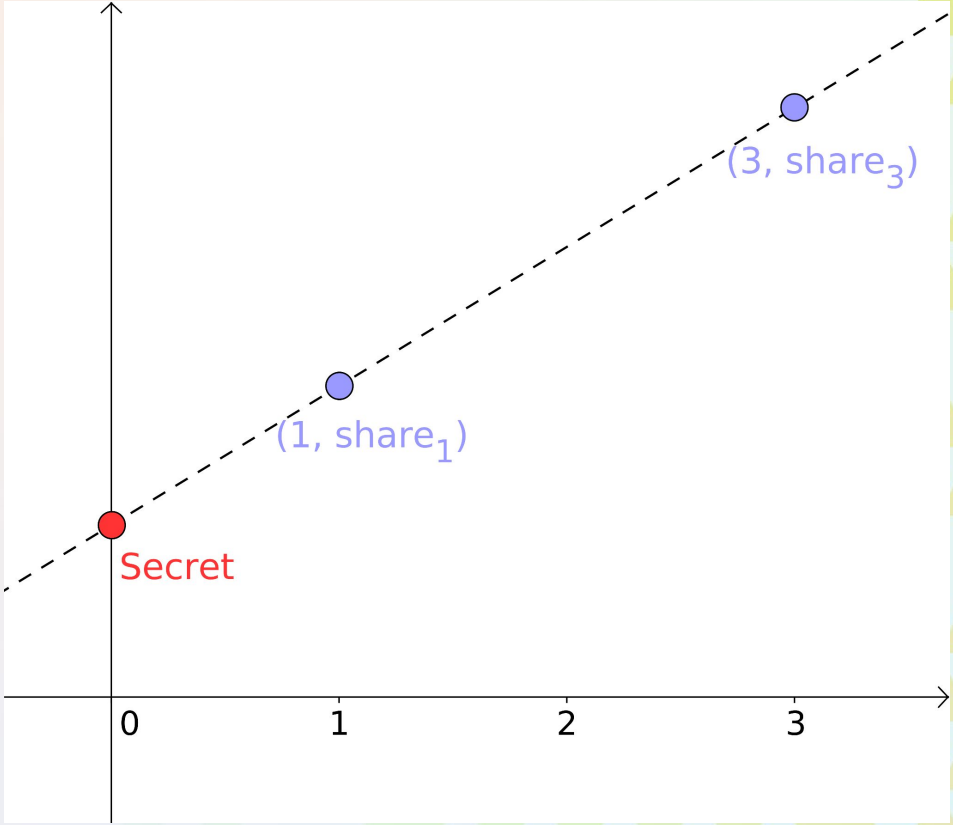
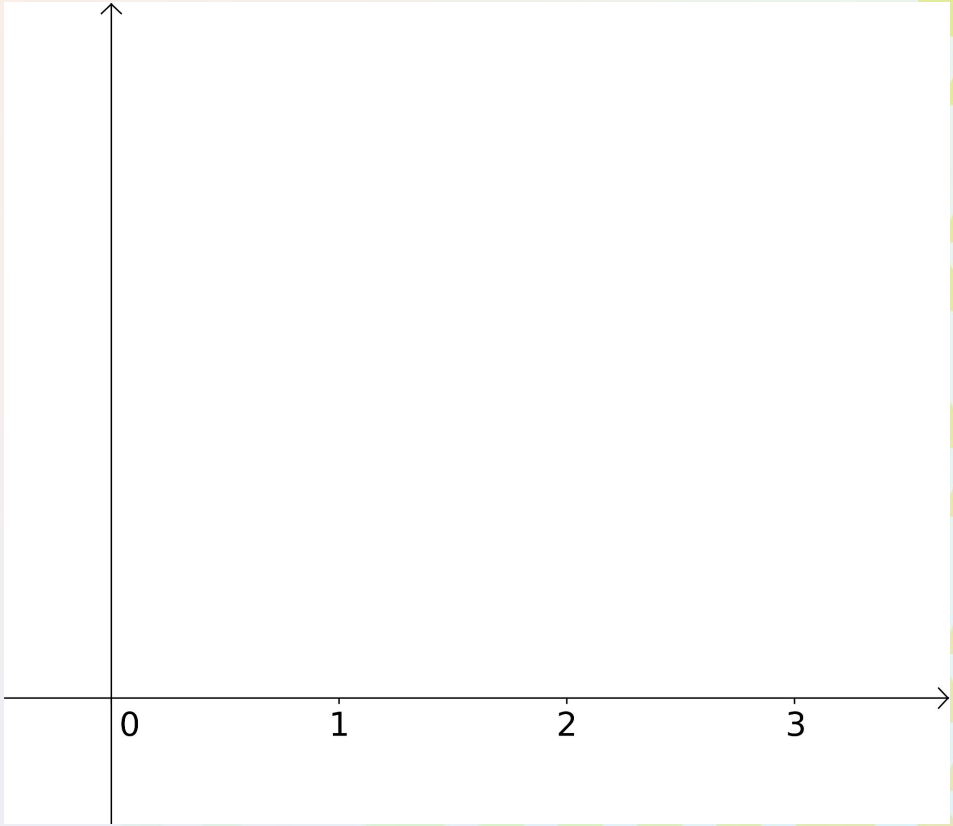


Share generation:

Another example.



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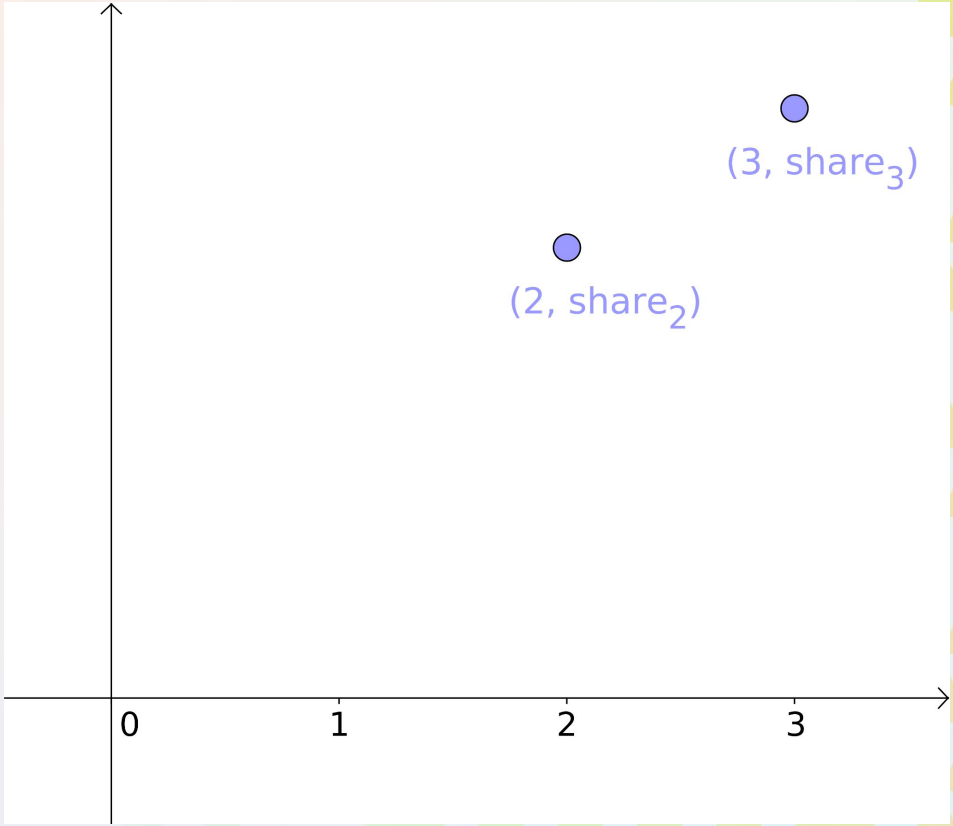

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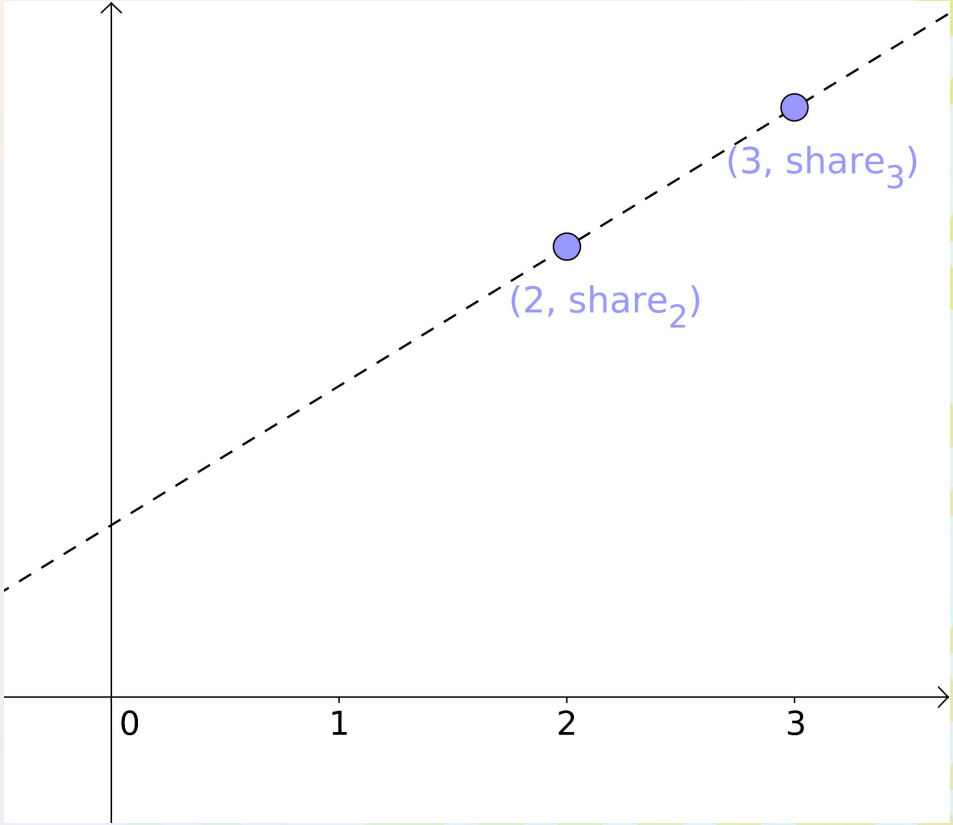

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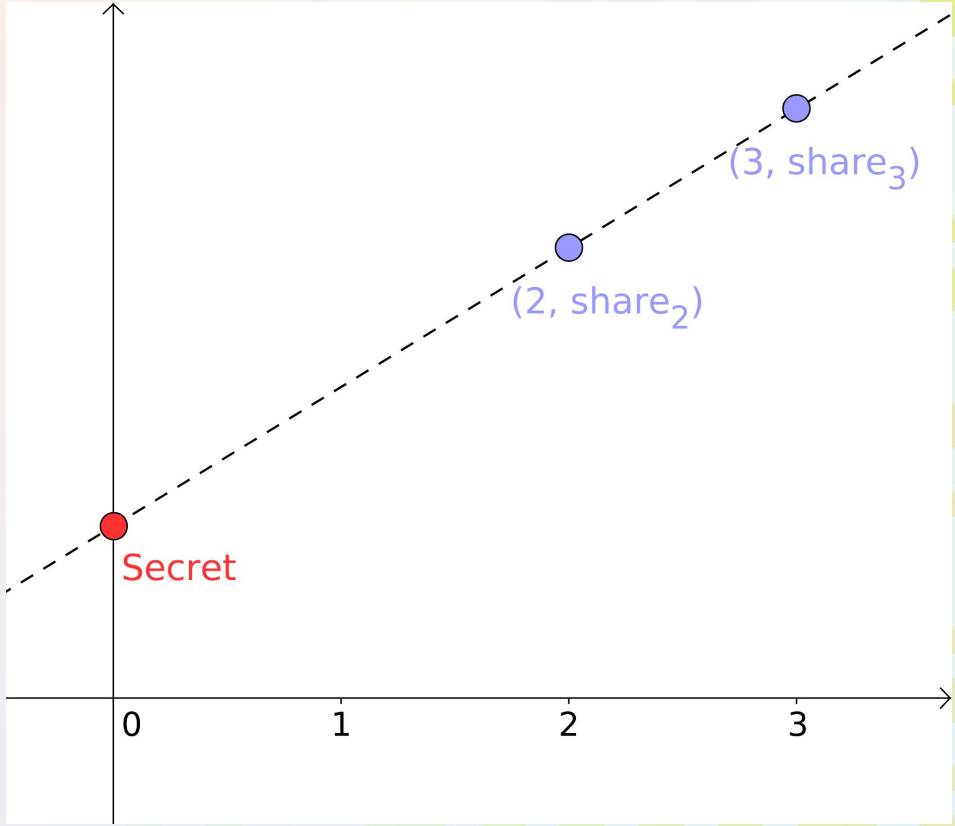


In this example, the shares are

(**1**, 10011000111 11110011100 01001111111 11000110100 10111111110 01101000011 00001101000 11100111001 01010000001 00001001000 10100100101 10011000101)

(**2**, 00101111000 01100000001 00001110001 11000100010 11010100100 11111000001 10100011010 00001000100 00100111011 01000110010 10000010100 01000111110)

(**3**, 01000010010 11101110101 11001110100 00111010000 11110010010 10001000000 00111001011 10101101111 00001010010 10000011011 01100000100 11110010111)

The x-values are called ID numbers.

Encoding the binary into seedphrases, we get:

(**1**, “ocean vicious exit shoot save half artist transfer expand animal pigeon obvious”)

(**2**, “congress gasp athlete session stand wealth person ancient cherry edge little elephant”)

(**3**, “drastic upgrade soldier deliver venture marine defense pupil apart lock gauge very”)

Remark: ID numbers are crucial

What if we miswrote the ID number of:

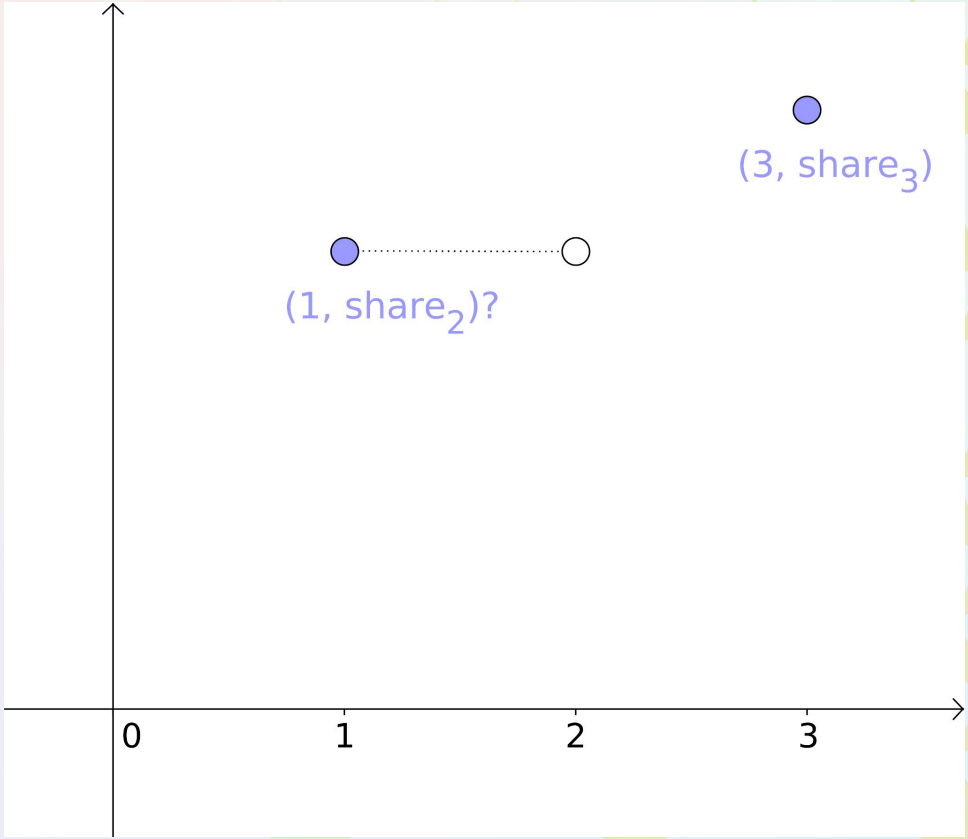
(**2**, “congress gast athlete session stand wealth person ancient cherry edge little elephant”)

Remark: ID numbers are crucial

What if we miswrote the ID number of:

(**1**, “congress gast athlete session stand wealth person ancient cherry edge little elephant”)

…and used a 1 instead?



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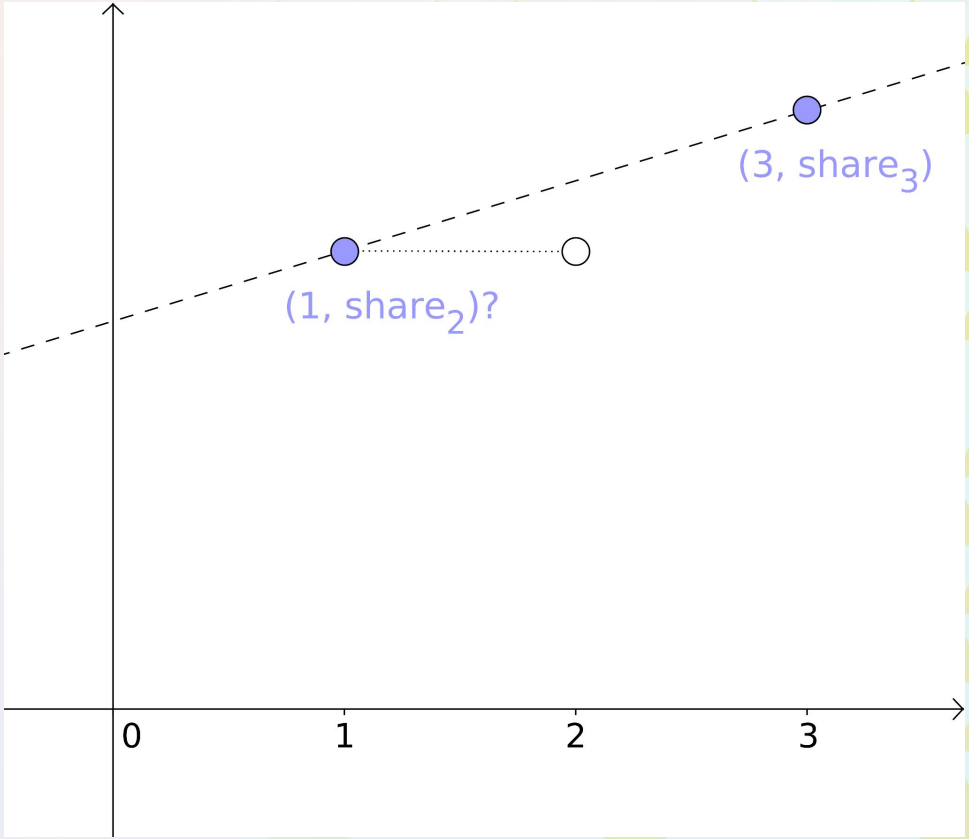

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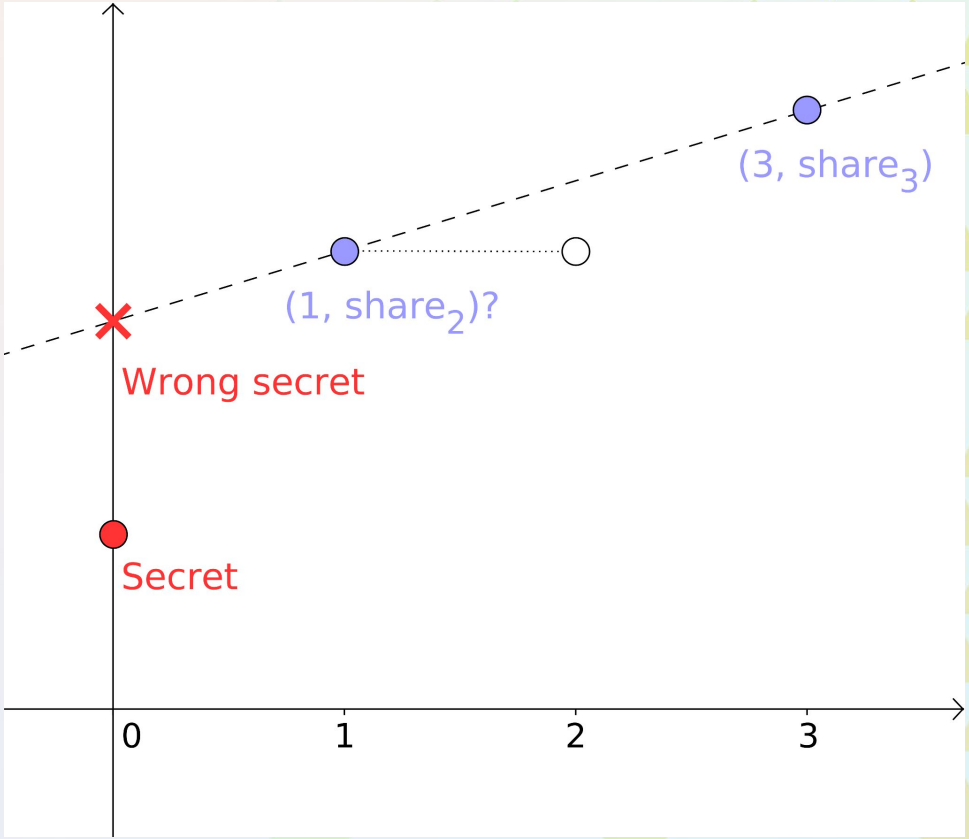

Can we improve this?

(Goal: encode the ID numbers in the seedphrase itself, without adding extra words)

BIP-39 seedphrase standard review

Not all the bits corresponding to the seed phrase carry independent information.

11110101101 01111101000 10001111010 00111000110
10011001000 00011000010 10010111001 01000010010
01111101000 11001100001 01000110101 00101101100

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11110101101 01111101000 10001111010 00111000110
10011001000 00011000010 10010111001 01000010010
01111101000 11001100001 01000110101 0010110**1100**

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11110101101 01111101000 10001111010 00111000110
10011001000 00011000010 10010111001 01000010010
01111101000 11001100001 01000110101 0010110**1100**

sha256(11110101101...) = 1100...

No need to include the checksum bits for share generation/reconstruction

volcano laptop monster decide october blue now drastic laptop slow effort collect

11110101101 01111101000 10001111010 00111000110
10011001000 00011000010 10010111001 01000010010
01111101000 11001100001 01000110101 0010110~~1100~~

Do SSS on the non-checksum bits, get new shares.

(**1**, 00010101010 11110100001 01010010100 10101111001
00100010111 11110111001 10110101001 00000101110
11111111001 00101110111 10111001100 1010100)

(**2**, 00110100010 01101111011 00110110111 00010111001
11101110111 11000110101 11010011001 11001101011
01111011011 00001001100 10111000110 0010011)

(**3**, 01000010010 11101110101 11001110100 00111010000
11110010010 10001000000 00111001011 10101101111
00001010010 10000011011 01100000100 1111001)

Complete the last 4 bits with ID numbers in binary

(**1**, 00010101010 11110100001 01010010100 10101111001
00100010111 11110111001 10110101001 00000101110
11111111001 00101110111 10111001100 1010100**0001**)

(**2**, 00110100010 01101111011 00110110111 00010111001
11101110111 11000110101 11010011001 11001101011
01111011011 00001001100 10111000110 0010011**0010**)

(**3**, 01000010010 11101110101 11001110100 00111010000
11110010010 10001000000 00111001011 10101101111
00001010010 10000011011 01100000100 1111001**0011**)

Convert to seedphrase.

best vintage family quality carry warm release alarm you confirm ridge popular

crowd hunt dad blame upon shop spring sniff kiwi another rhythm chaos

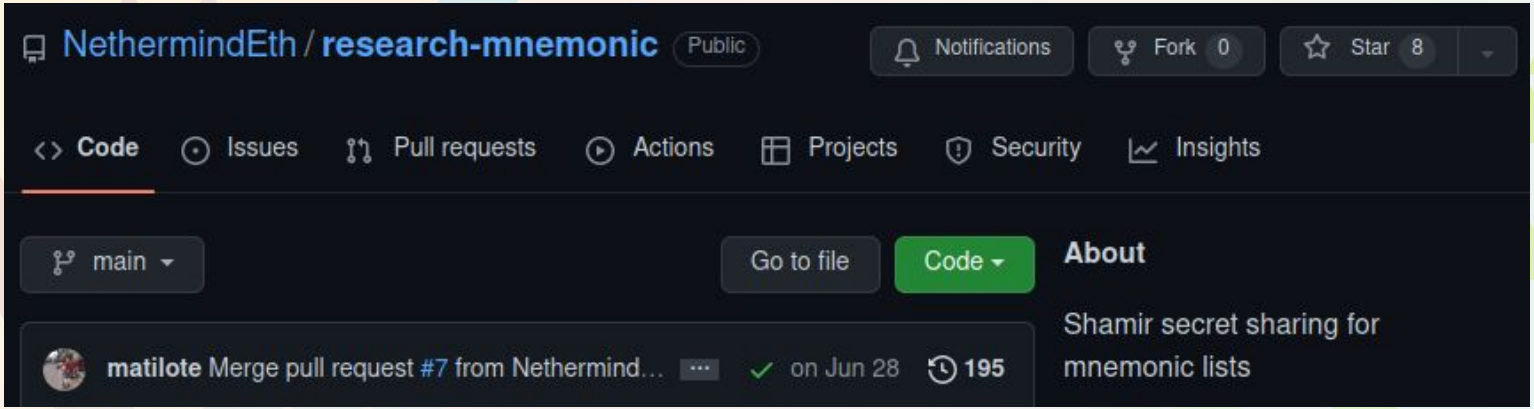
start town twenty liar fence clap van memory welcome twice elevator pen

Last 4 bits in the last word encode the ID number!

Popular = 10101000**0001**

Chaos = 0010011**0010**

Pen = 1111001**0011**



Thank you!

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