Cost of Feudalism

Towards a Theory of Maximal Extractable Value (MEV)

Tarun Chitra

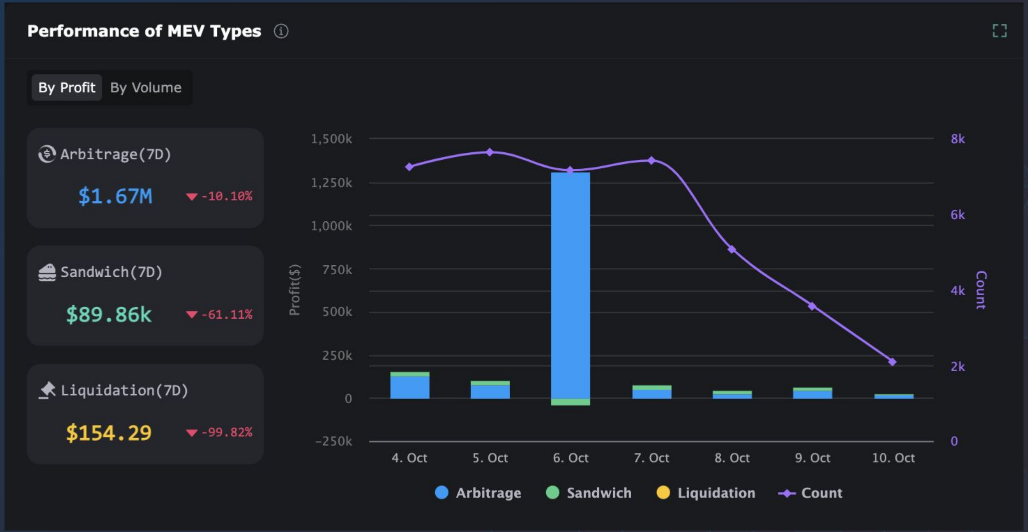
GAUNTLET

Joint work w/ Theo Diamandis (MIT), Kshitij Kulkarni (Berkeley)

MEV: "You know it when you see it"

Maximal extractable value comes in many shapes and sizes

- Sandwich attacks 🥪
- Liquidations 🌊
- Arbitrage 🧙
- NFT mint front-running 👩‍🎨
- Cross-chain ⛓



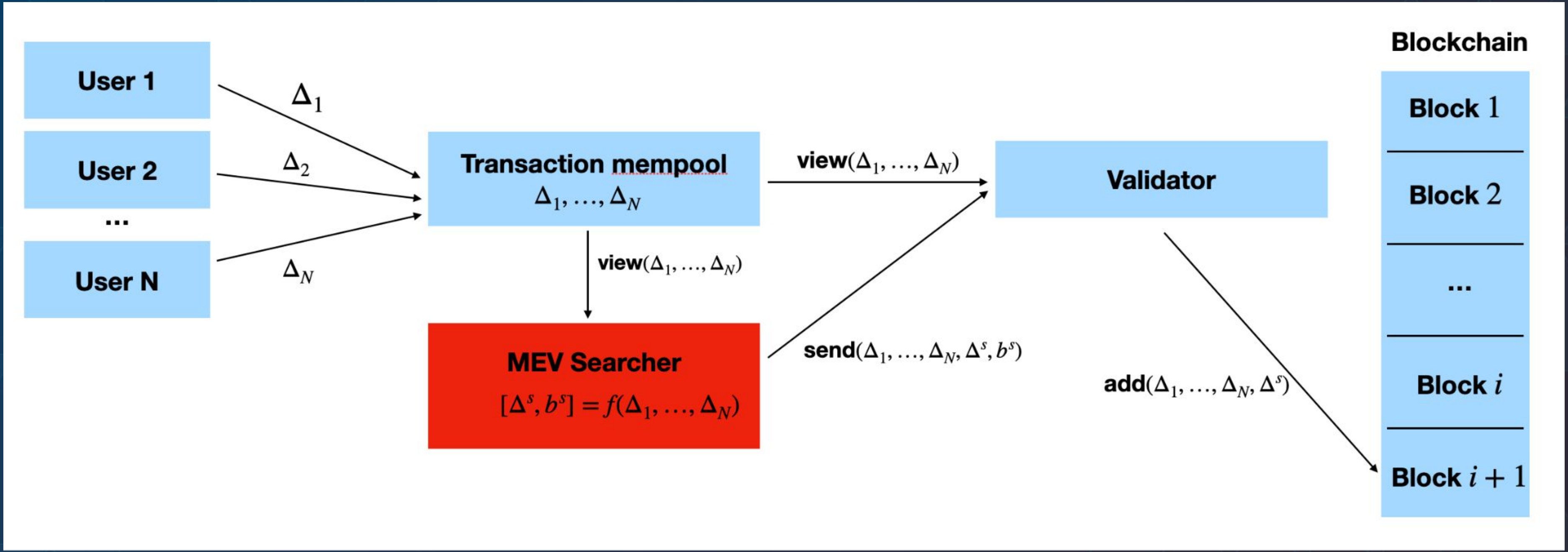
Source: Eigenphi.io

What is MEV?

- Maximal Extractable Value (MEV) is any ***excess value*** captured by validators
    - **Reordering** transactions to adversarial, non-FIFO ordering
    - Strategically **adding** txns before and/or after other users' txns
    - **Eliding** txns from particular bundles

- Currently managed via **off-chain auctions** (e.g. Flashbots 🤖)
    - **Pro**: Ensures low spam from strategic users rendering the network unusable for non-strategic/passive users
    - **Con**: Adds centralization vector since auction isn't {credible, verifiable}

Transaction Flows in Blockchains



But how do we know when value captured via MEV is **excessive**?

Describing Value Flows in MEV is difficult

- Do we do optimize for the **welfare** of users or the **revenue** of validators?
  - User welfare is important for network success
  - Validator revenue is important for **economic security**
    - **Competing goals for any decentralized network**
- What pieces are missing in our description of MEV?
  - **User utilities**: How much users *intrinsically value* a particular txn(s)
  - **User payments:** Set of transaction fees that users are willing to pay
  - **Allocation**: How an auctioneer (Flashbots, Proposer in PBS) allocates block space to users
- Components are **dependent on the applications involved**
  - E.g. utilities for NFT minters and DeFi traders are very different

Formalizing Value in MEV

- **Formalism**:
    - Allocation of block space to users: $x_1, \ldots, x_n \in \{0,1\}$
        - $x_i=1$ if the i^{th} user's transaction makes it in
    - Utilities, payments of users: $u_1, \ldots, u_n, p_1, \ldots, p_n$
        - Note: $p_i = p_i(x_1, \ldots, x_n)$
- **Social Welfare**:

$$\mathbf{SW}(x) = \mathbf{E}[u_1(x_1)x_1 + \cdots + u(x_n)x_n]$$

- **Revenue**:

$$\mathbf{Rev}(p) = \mathbf{E}[p_1 + \cdots + p_n]$$

- **Equilibria**: $(x^*, p^*) \in \mathbf{Eq} = \{(x,y) \in (\mathrm{argmax}_x\, \mathbf{SW}(x), \mathrm{argmax}_p\, \mathbf{Rev}(p))\}$
    - If $p_i^* \neq p_i(x^*)$, then we do not have a consistent equilibria for user welfare maximization and revenue maximization!
        - **Happens all the time in MEV!**

Quantifying MEV at the system level

- Quantifying how good or bad a set of equilibria via approximation ratios, such as the *Price of Anarchy*

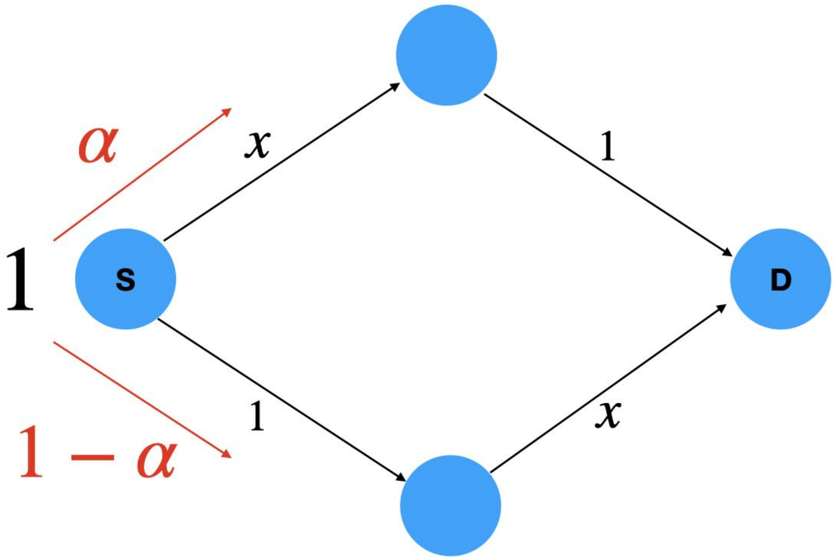
$$\mathrm{POA} = \frac{\sup_{x^*, p^* \in \mathbf{Eq}} \sum_{i=1}^{n} p_i^* - p_i(x^*)}{\inf_{x^*, p^* \in \mathbf{Eq}} \sum_{i=1}^{n} p_i^* - p_i(x^*)}$$

- In words: How many times more is the worst case deviation between optimal price and the user's demanded price relative to the best case?

- **POA** = **O(1)** is **good**, **POA** = **o(n)** is **okay**, **POA** = **Ω(n)** is **bad**
  - This quantity depends deeply on how you define the utilities and prices paid, which is **application-specific** and **not uniquely defined**

Price of Anarchy in examples:

Braess’s Paradox



Equilibrium:

$$1 + \alpha^\star = 1 + 1 - \alpha^\star$$

$$\implies \alpha^\star = \boxed{\frac{1}{2}}$$

Net cost:

$$\frac{1}{2}\left(1 + \frac{1}{2}\right) + \frac{1}{2}\left(1 + \frac{1}{2}\right)$$

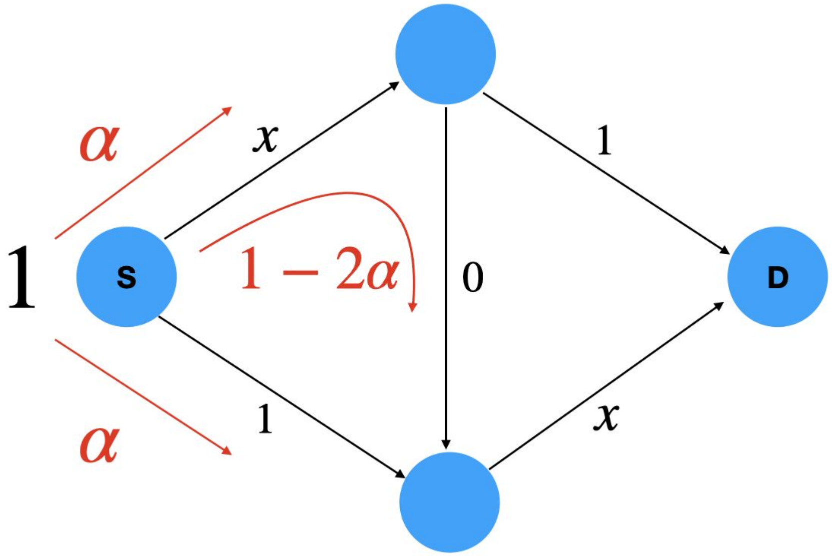
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = \boxed{\frac{3}{2}}$$

Braess's Paradox: A Tale of 4 Cities

- Traffic network with 4 cities, 4 roads
- Two roads have travel times dep. on the percentage of traffic on that road (*x*)
- Assume each driver is myopic and selfish
    - Given that the two paths take 1+x time to travel from S to D, the expected time is 3/2

Adding a road *worsens* congestion

- You might think that adding an extra road will improve congestion — but that's not always the case!
- Adding a road that takes 0 time to traverse between the middle cities, if everyone is greedily optimizing, leads to congestion on the same path!



Equilibrium:

$$\alpha^\star + 1 - 2\alpha^\star + 1 = \alpha^\star + 1 - 2\alpha^\star + 1 - 2\alpha^\star$$
$$\implies \alpha^\star = \boxed{0}$$

Net cost:

$$1 \cdot (1 + 1)$$
$$= \boxed{2}$$

At this point, you might think...

MEV is *always* bad


Opinion

Miners, Front-Running-as-a-Service Is Theft

There's a simple word for projects that seek to advantage miners while systematically exploiting blockchain users, say three researchers.

By Ari Juels, Ittay Eyal, Mahimna Kelkar Layer 2

Application Specificity can give MEV positive externalities

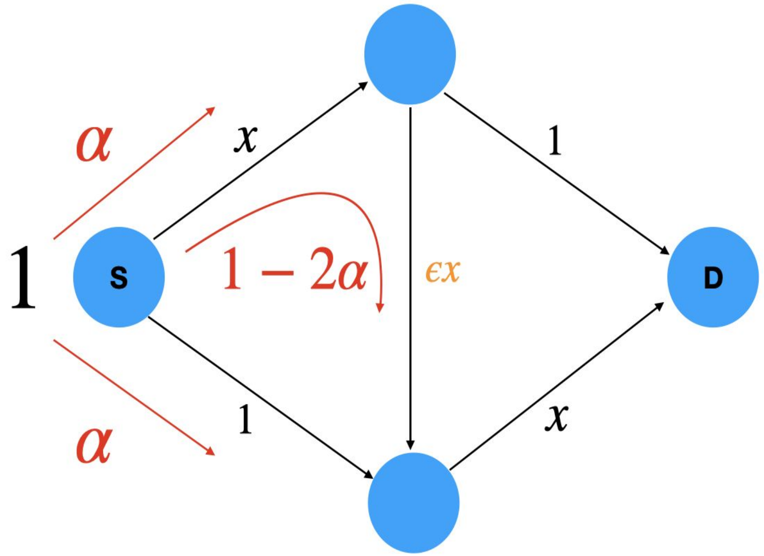
- ***Not all applications are the same***
  - MEV allows for strategic users to make a profit while simultaneously improving the welfare of non-strategic users
- ***Routing across multiple contracts is hard for non-strategic users***
  - Finding optimal routes for trading has been hard for non-strategic users
    - Reliance on 3rd party services like 1inch, Matcha, Gem, etc.
  - Algorithmic Game Theory has studied selfish routing for decades: does any of it apply to MEV?
- **Example:** Braess's Paradox for selfish routing
  - "Sometimes adding more capacity can slow the network down if the incentives aren't tuned correctly"

The *Inverse Braess Paradox*

Now add some small congestion cost on the middle link (ε, due to MEV)

Counterintuitively: these costs actually ***improve*** the overall network flow!

e.g. by *disincentivizing* bad selfish behavior



Equilibrium:

$$\alpha^\star + 1 - 2\alpha^\star + 1 = \alpha^\star + 1 - 2\alpha^\star + (\epsilon + 1)1 - 2\alpha^\star$$

$$\implies \alpha^\star = \frac{\epsilon}{2(\epsilon + 1)} \to \boxed{\frac{1}{2}} \textbf{ as } \epsilon \to \infty$$

Net cost:

$$\to \boxed{\frac{3}{2}} \textbf{ as } \epsilon \to \infty$$

Inverse Braess Paradox for CFMMs (e.g. Uniswap)

Replace 'travel times' in the road network w/ CFMM price impact function

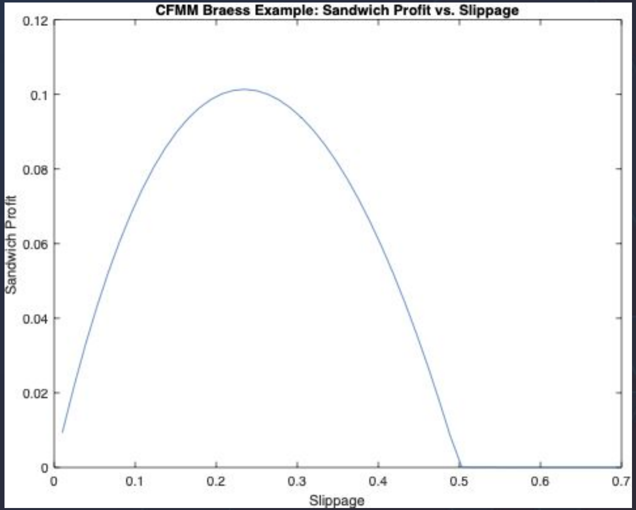
- Travel routes are token trades
  e.g. A→B→C
- Congestion is many users trying to trade on the same link
- Sandwich attacks are equal to adding congestion on a link ε

Shockingly even w/ sandwiches:
PoA = O(1)

Theorem 2. *Suppose that* $f(\kappa, \mu, \eta), g(\kappa, \mu, \eta) \in O((1 + (\alpha\beta\kappa)^{O(1)})^{1/\mathsf{diam}(G)}$. *Then there exists a function* $C(\kappa, \alpha, \beta, \mu, \eta)$ *that is constant in the size of the network graph* G *such that*

$$\mathsf{PoA}(\Delta) \leq C(\kappa, \alpha, \beta, \mu, \eta) \tag{23}$$

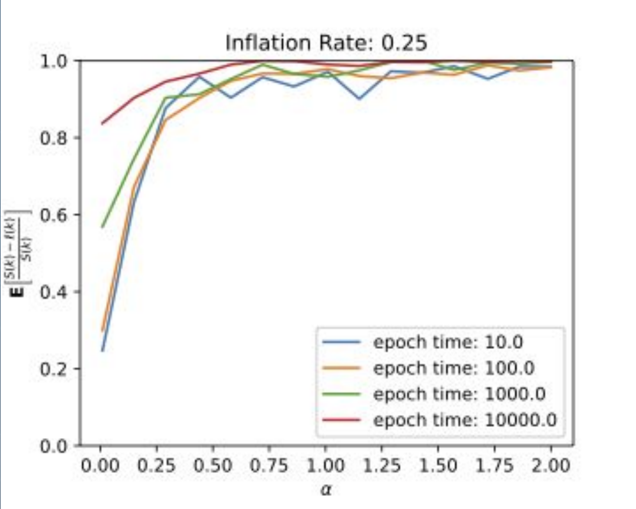
Towards a Theory of Maximal Extractable Value I: CFMMs
Kulkarni, Diamandis, **C**, 2022



If MEV is not so bad, how do we harness it for **good**?

Redistributing MEV to validators can let you lower inflation!

Redistributing MEV (e.g. sharing a percentage of captured MEV pro-rata with validators) increases the stickiness of staked/delegated capital



Improving Proof of Stake Economic Security via MEV Redistribution
Kulkarni, **C**, 2022

Redistribute MEV to validators and governance participants

Multidimensional MEV Auctions can improve allocative efficiency

Now: Ethereum bundles MEV into one auction — Uniswap arbitrage competes with NFT minting

Future: Rollups or apps run their own auctions that get aggregated (*hierarchical PBS*)

α-leak: **C**, Kulkarni, Ferreira (unpublished, 2022) prove that disaggregation can sometimes **improve** auction efficiency

Unbundle **MEV to improve social welfare in MEV auctions**

Theoretical Foundations of MEV are important!

- **Then**: MEV started at an emergent/unstudied phenomena
- **Now**: Design space for redistributing and optimizing MEV for users relies on theoretical understanding of networks
  - Surprising formalized truths evince that **MEV isn't always bad!**
- Without algorithmic game theory and probability theory, it is hard to reason about such truths
  - Our papers are just the beginning!
- Open Problems:
  - **Optimal** auctions
  - Information theoretic **lower bounds**
  - Aggregation vs. Disaggregation effects
    - *"What is the Coase theorem for MEV?"*

Thank you!

AERA

P.S. We launched Aera today!

aera.finance

@tarunchitra
@gauntletnetwork

GAUNTLET