

recursive proofs:

applications and affordances

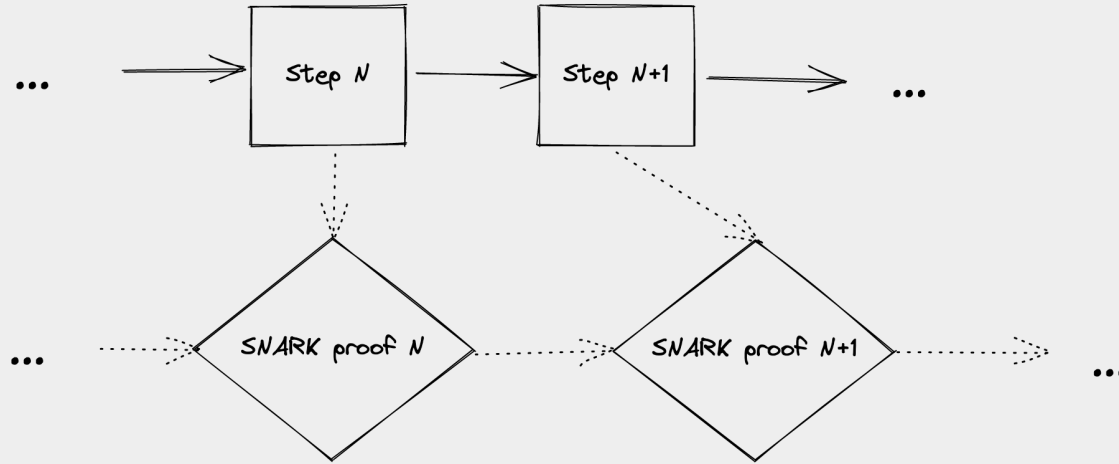
Ying Tong

Core Engineer, Electric Coin Company

Nalin

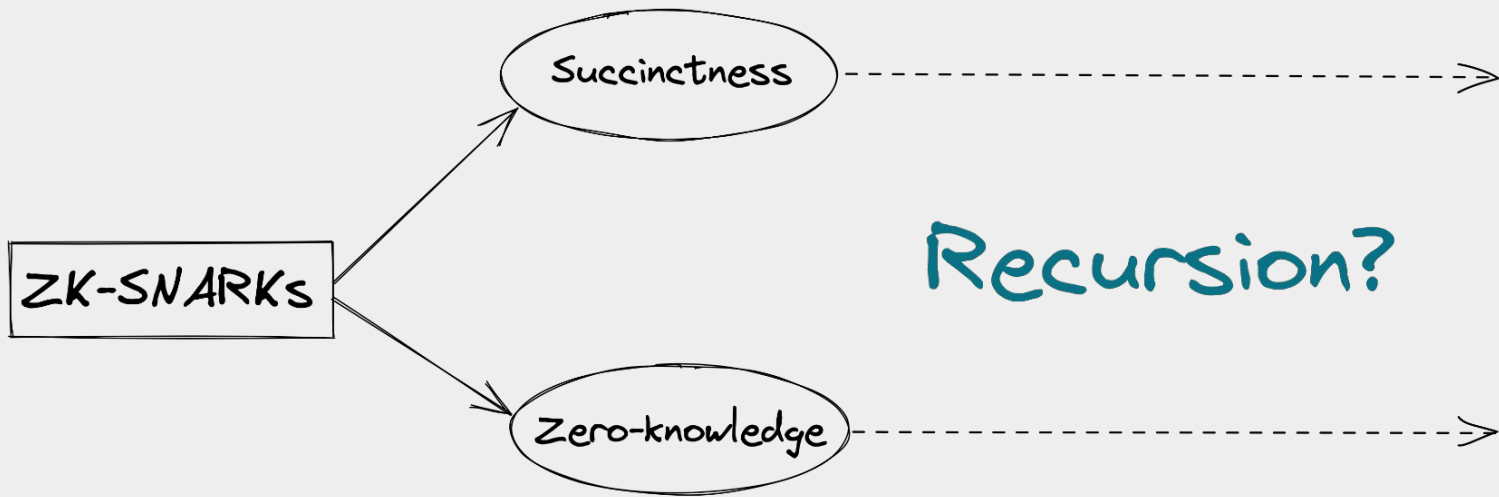
Person, 0xPARC

what are recursive proofs?

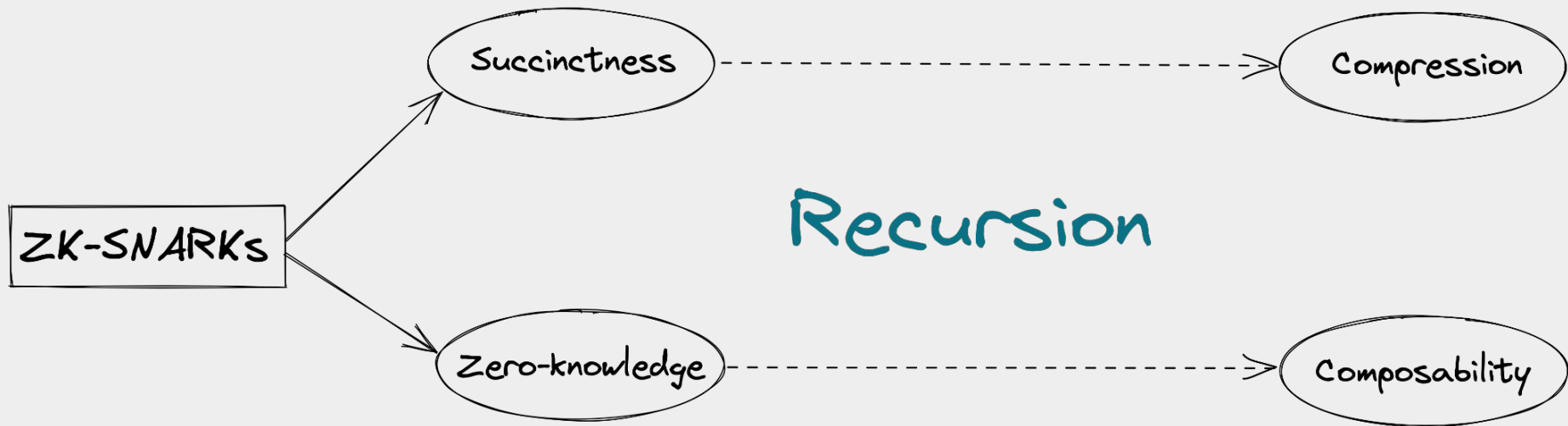


Verify SNARK proof N and Step $N+1$
(instead of verifying all N steps again)

unlocks from recursion



unlocks from recursion





Section 1

compression (supercharging succinctness)



compression: supercharging succinctness

for a low verifier cost, a prover can show:

"i know \ll pieces of knowledge"

compression: supercharging succinctness

for a low verifier cost, a prover can show

"i know \mathbb{N} pieces of knowledge"

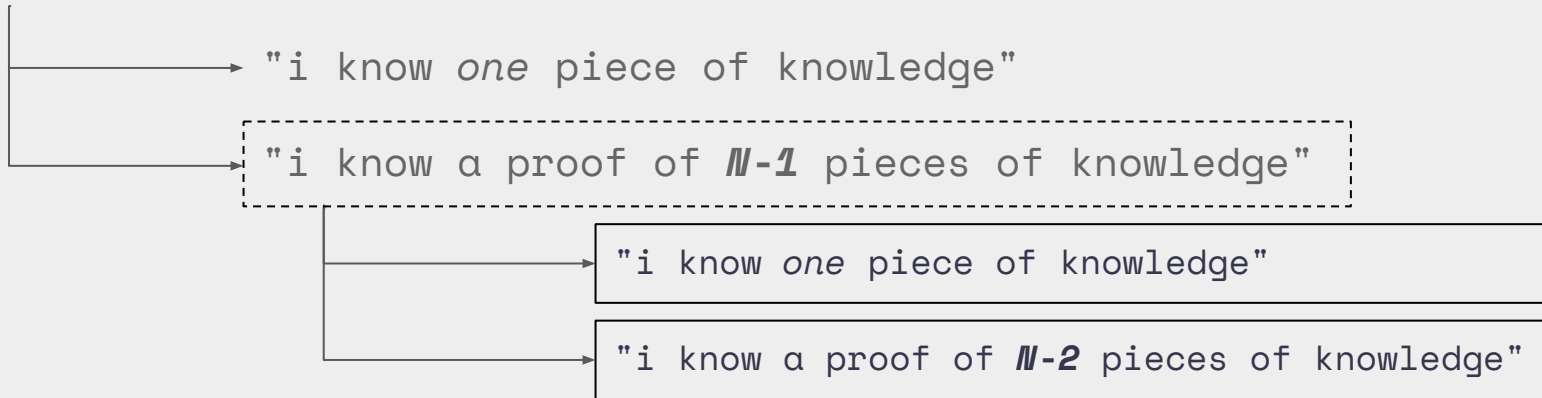
"i know *one* piece of knowledge"

"i know a proof of $\mathbb{N}-1$ pieces of knowledge"

compression: supercharging succinctness

for a low verifier cost, a prover can show

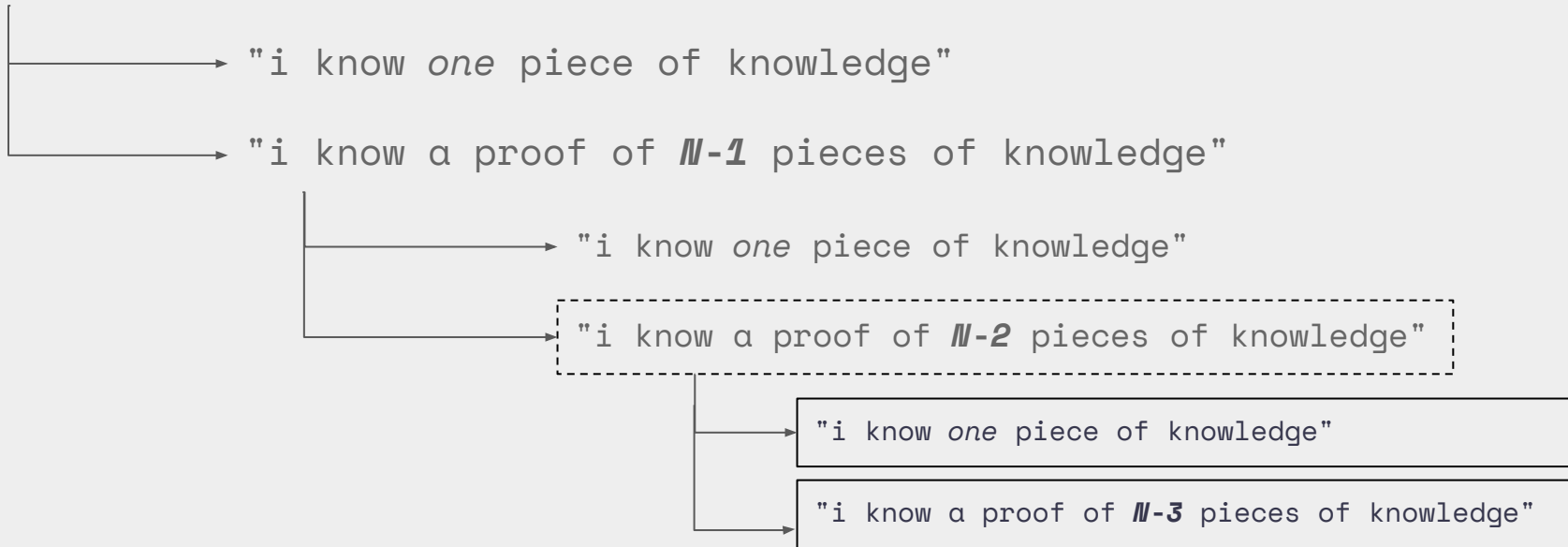
"i know n pieces of knowledge"



compression: supercharging succinctness

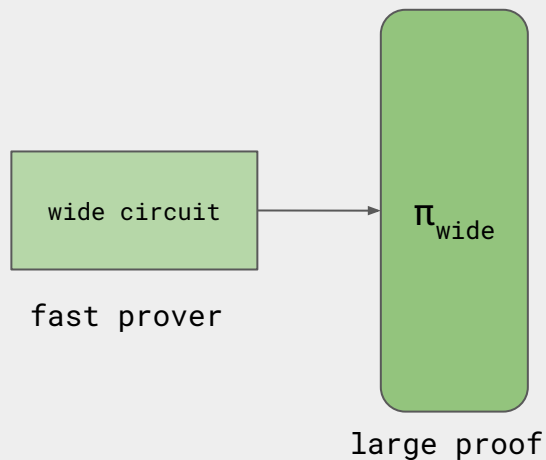
for a low verifier cost, a prover can show

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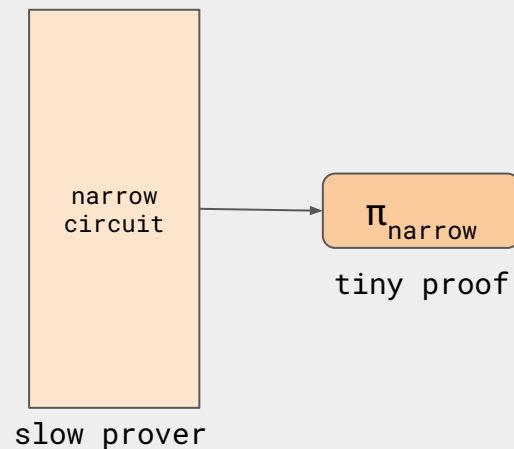
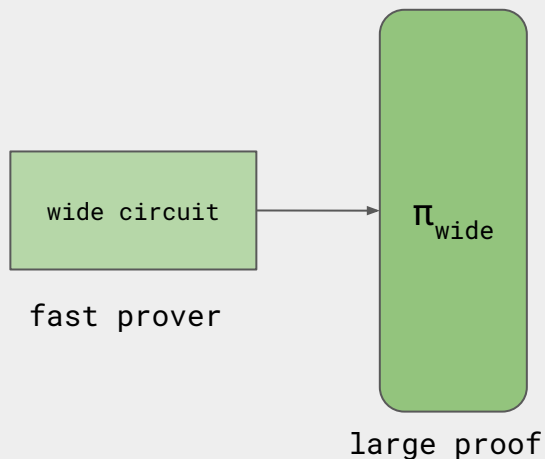
prover / verifier complexity tradeoffs 🍰

	fast prover	small proof / fast verifier
"wide" proof	✓	✗



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	fast prover	small proof / fast verifier
"wide" proof	✓	✗
"narrow" proof	✗	✓



prover / verifier complexity tradeoffs 🍰

	fast prover	small proof / fast verifier
"wide" proof	✓	✗
"narrow" proof	✗	✓



interoperability between proof systems

	fast prover	small proof / fast verifier
STARK	✓	✗
Groth16	✗	✓



compression: examples

a rollup of...



Signatures

Plumo: An Ultralight Blockchain Client



light client proofs

MINA

 polygon



Hermes

txes



Section 2

composability

(taking zk a step further)



composability

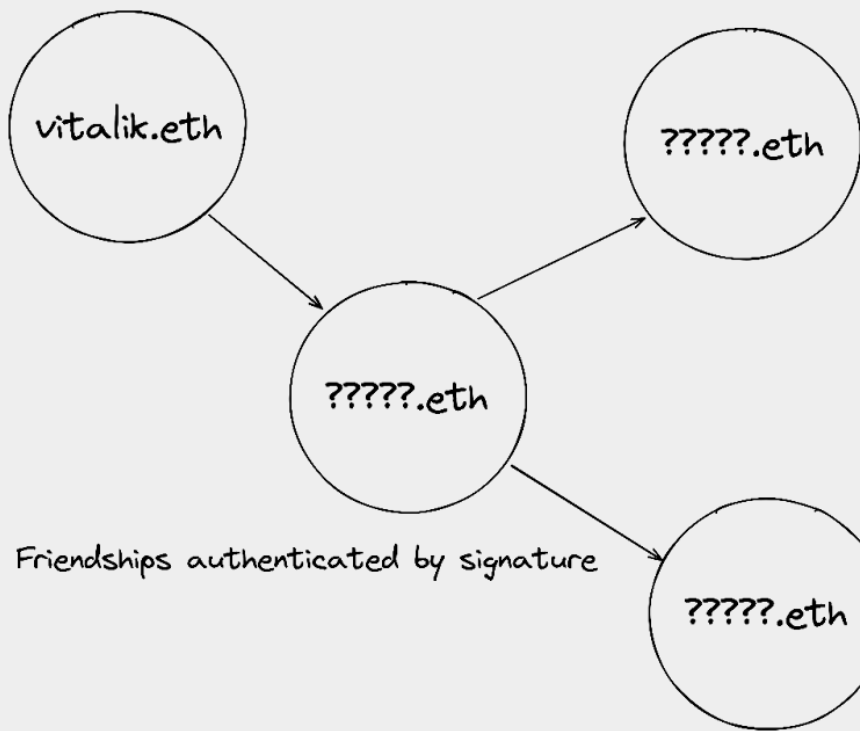
typically, zk proofs are thought of in the context:

*"a prover shows knowledge to a verifier, **without revealing the underlying fact.**"*

recursive zk proofs, in fact, unlock a stronger property:

*"a prover shows knowledge to a verifier, **without fully knowing the underlying facts themselves**"*

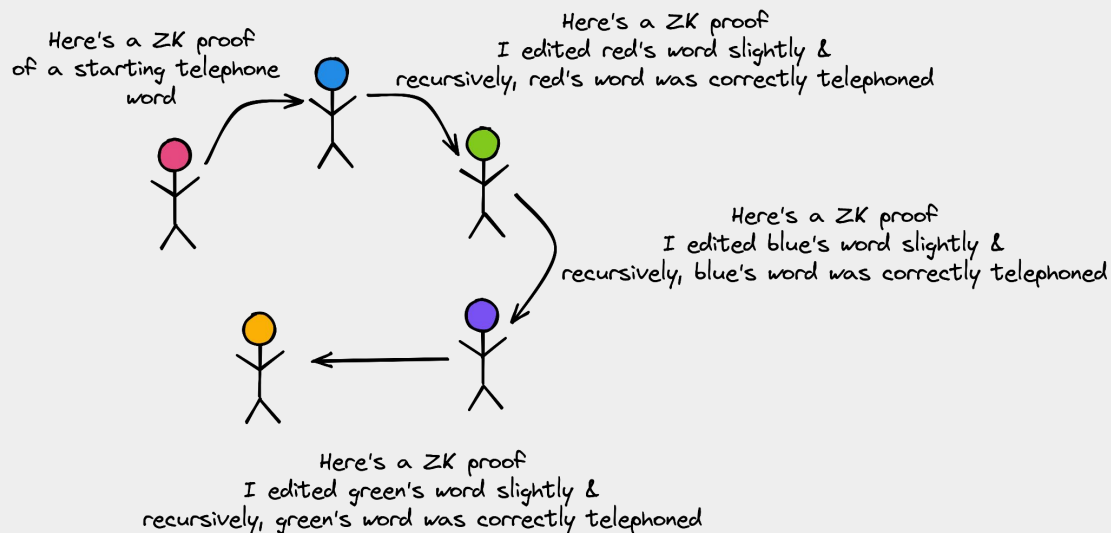
ETHdos: Erdős numbers on social graphs



I am 4 degrees away from vitalik.eth, but I do not know my path to him

composability: incomplete information games

- telephone/chinese whispers
- mafia
- private state channels





Section 3

implementations



schemes for recursive proof composition

relax requirements
on proof system



full recursion

needs:

succinct verifier

- Groth16
- FRI

atomic accumulation

succinct accumulator

- Inner Product
Argument (IPA)

split accumulation

succinct public accumulator

- Nova

schemes for recursive proof composition

needs:

succinct verifier

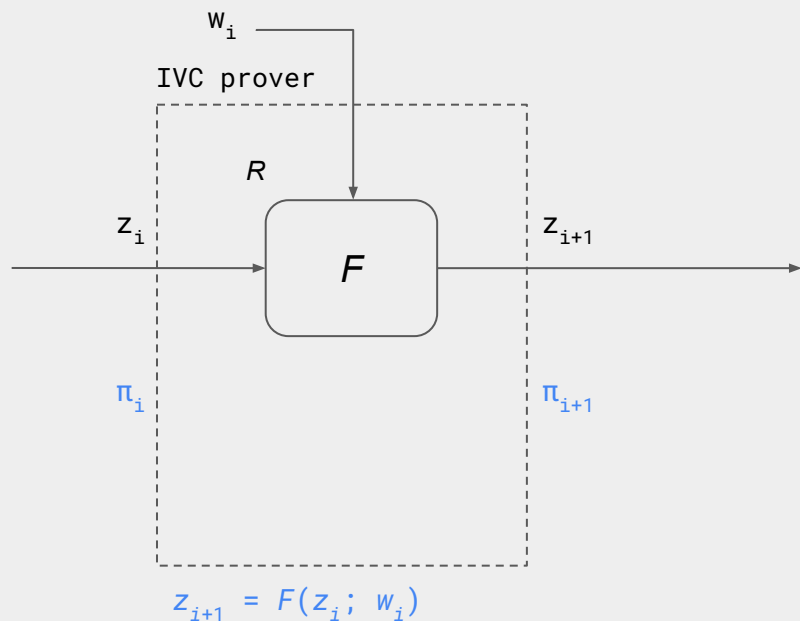
- Groth16
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full recursion

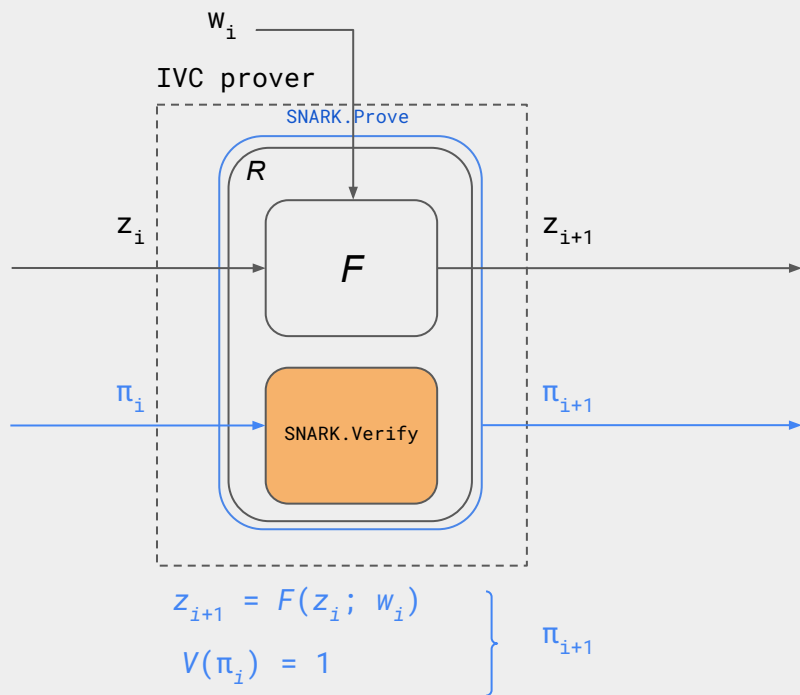
relax requirements
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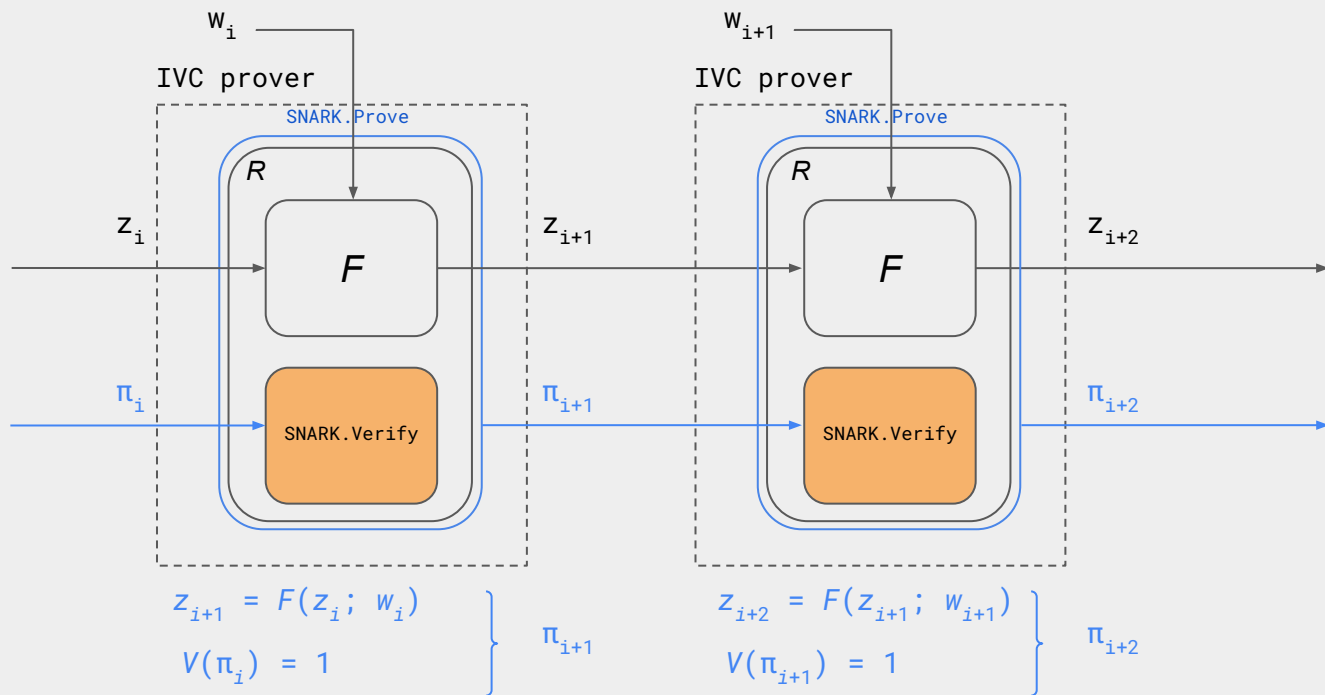
IVC from SNARKs with sublinear verification



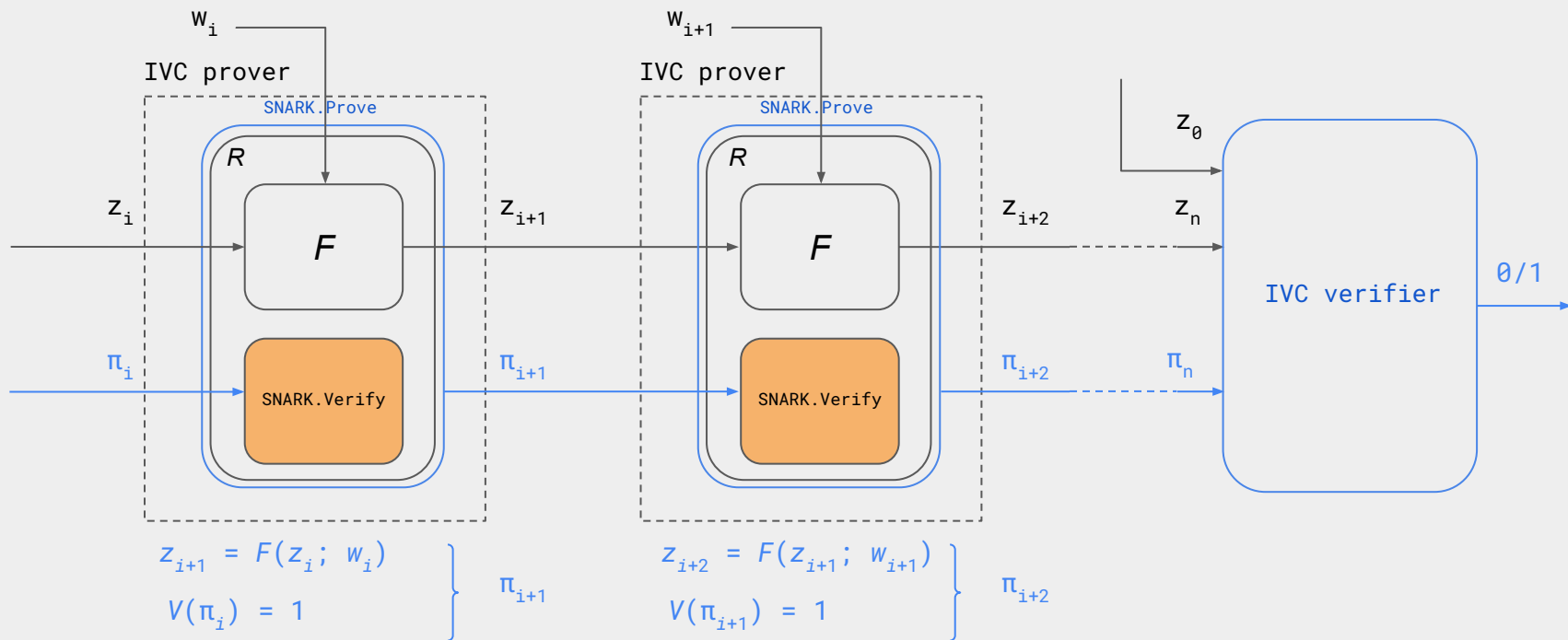
IVC from SNARKs with sublinear verification



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schemes for recursive proof composition

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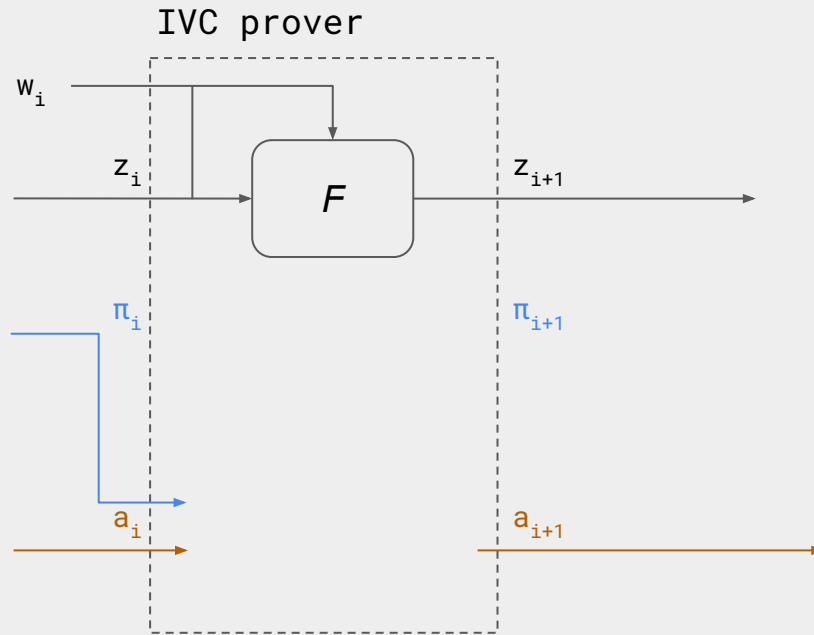
- Groth16
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atomic accumulation

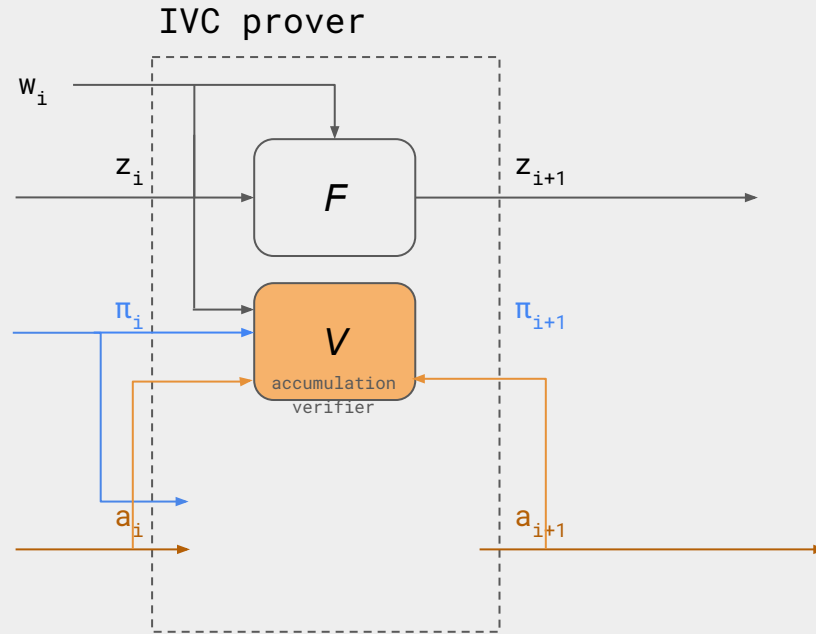
succinct accumulator

- Inner Product
Argument (IPA)

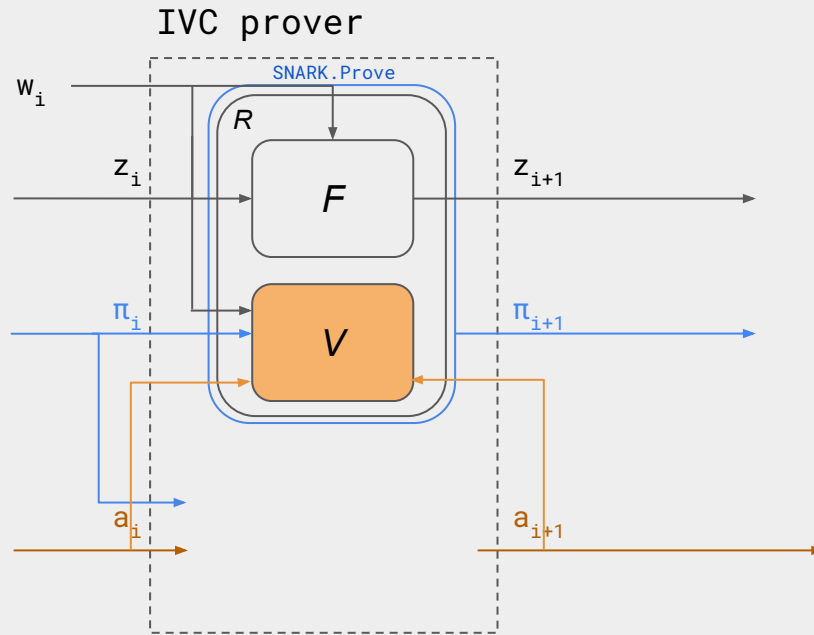
IVC from atomic accumulation



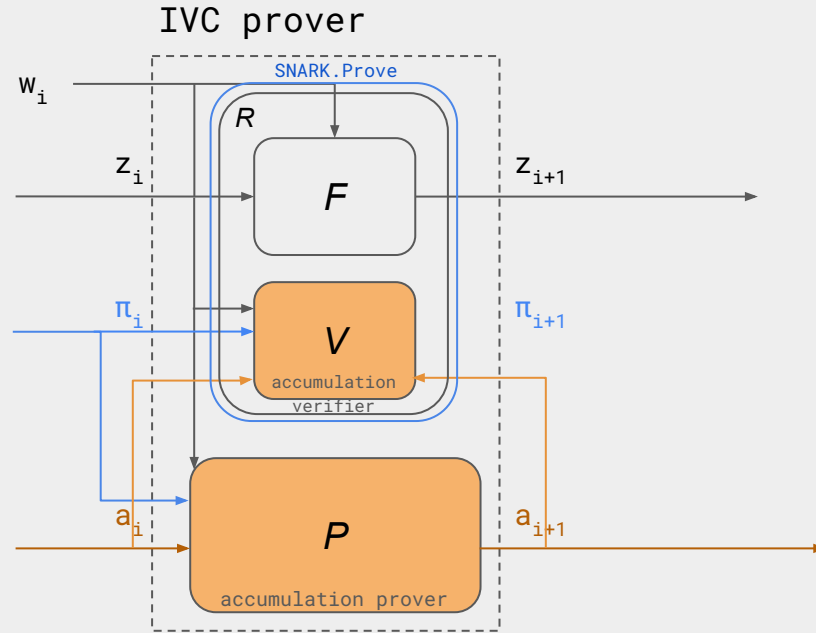
IVC from atomic accumulation



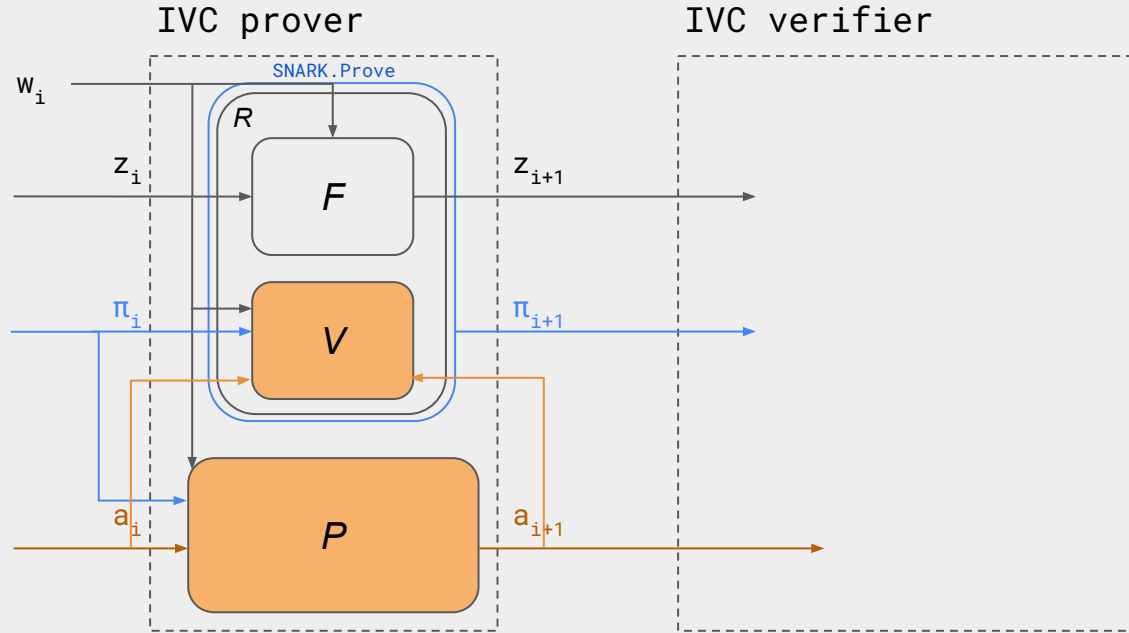
IVC from atomic accumulation



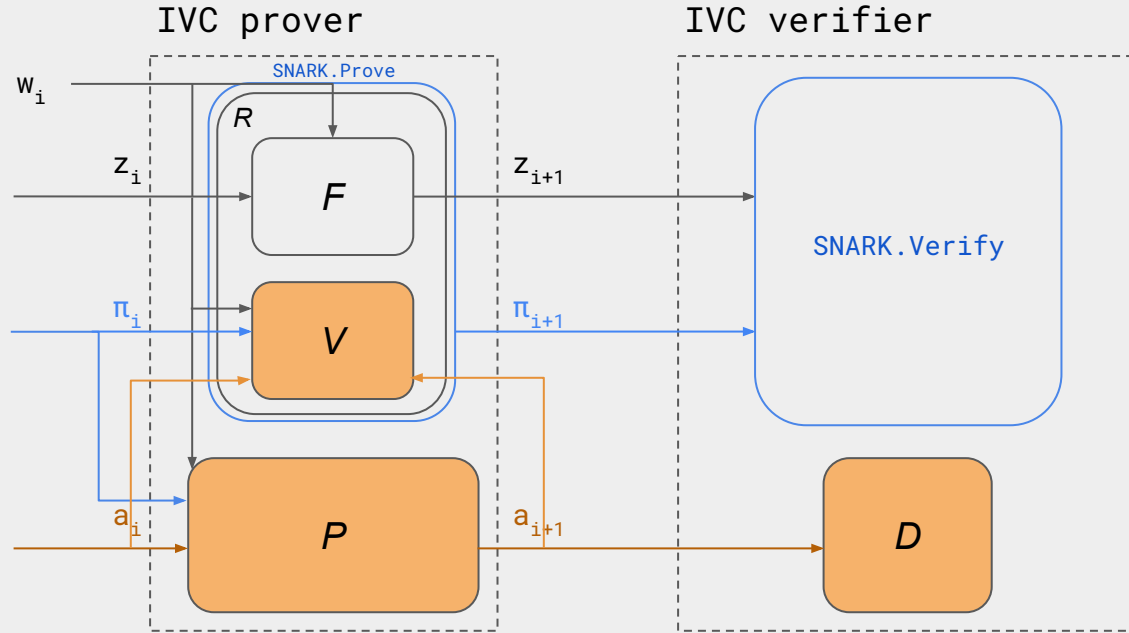
IVC from atomic accumulation



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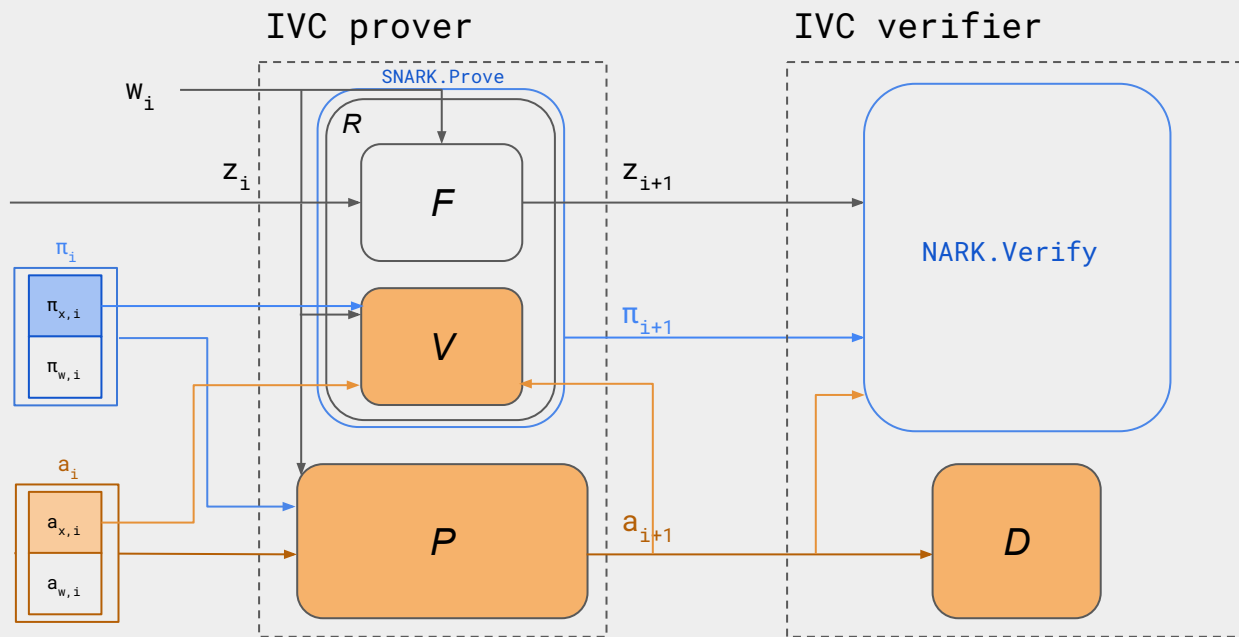
- Inner Product
Argument (IPA)

split accumulation

succinct public accumulator

- Nova

IVC from split accumulation [BCLMS20]



future of proving systems

- modular design → customisable proving stack
- recursion → composition across proof systems

modular proof systems: Halo 2

arithmetisation

encode values in the Lagrange basis;
constraints as polynomial identities



information-theoretic PIOP

vanishing argument;
multipoint opening argument



cryptographic compiler

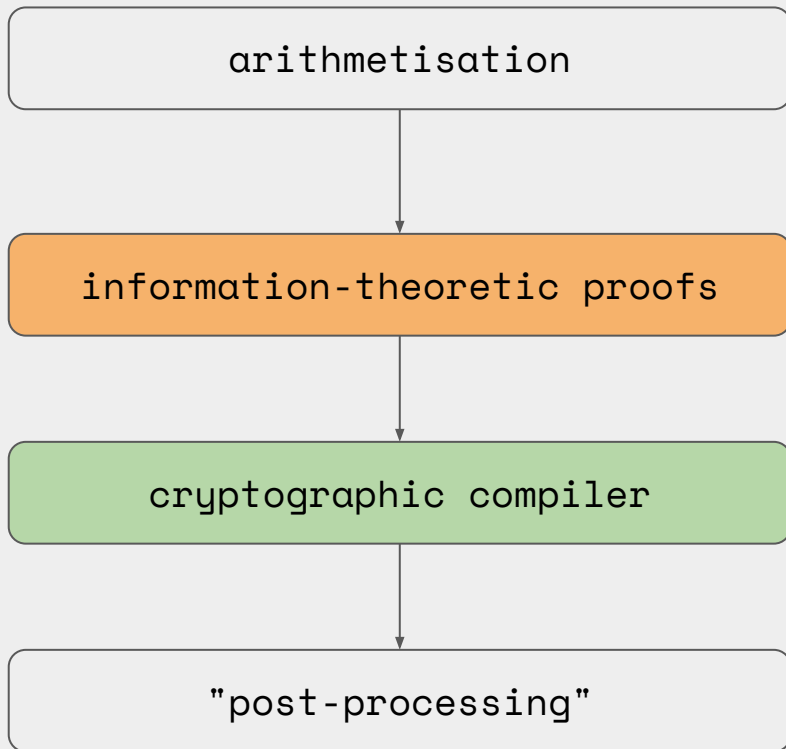
inner product argument;
Fiat-Shamir transform



"post-processing"

atomic accumulation

modular proof systems



turns a relation into a **constraint system** involving native operations over a finite field

provides soundness and zk guarantees even when prover and verifier are **computationally unbounded**

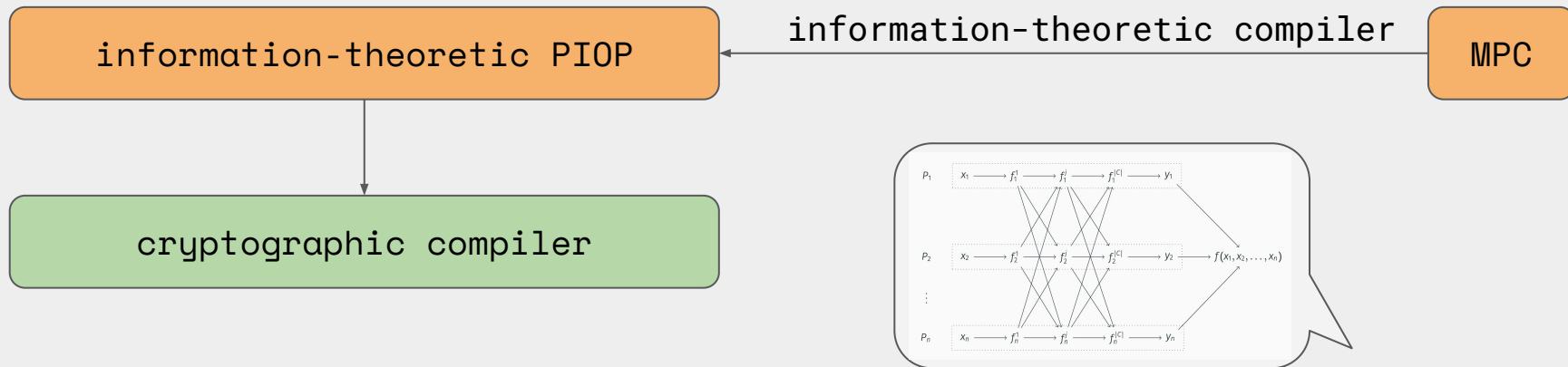
transforms proof system into concrete protocol involving **direct interaction** between prover and verifier

compositional schemes for the protocol (e.g. distributed proving, streaming prover, aggregation, accumulation, recursion)

types of proof composition

1. information-theoretic compilers

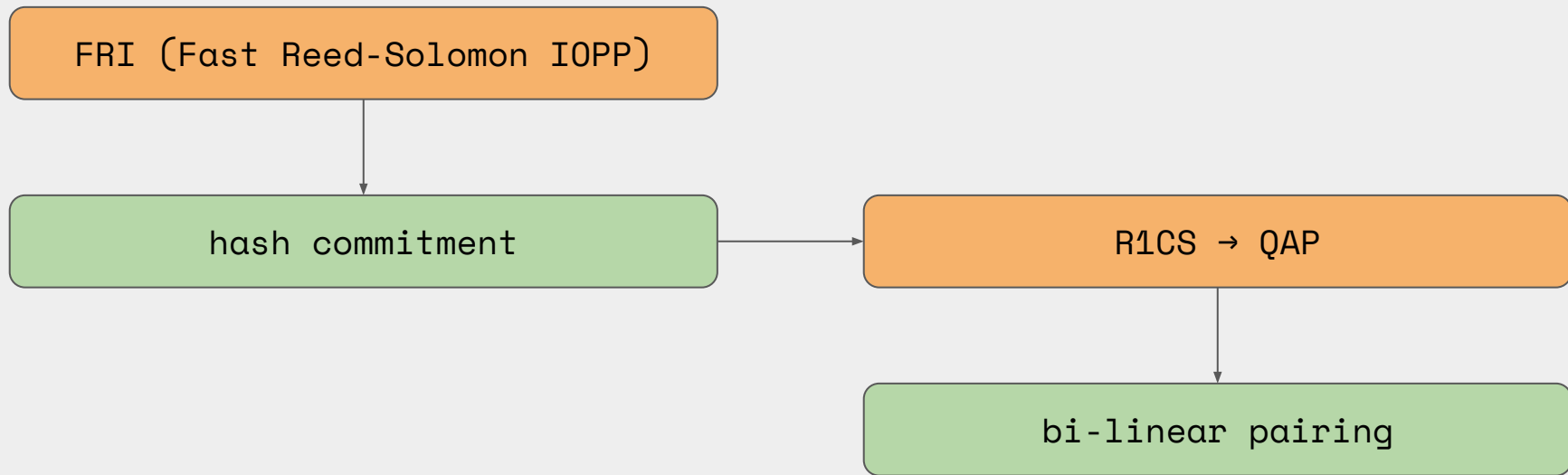
e.g. "MPC-in-the-head" [IKOS07]



types of proof composition

2. composing cryptographic compilers

e.g. STARK verifier in Groth16 prover



types of proof composition

3. tailor-made cryptographic compilers

e.g. GKR verifier in Groth16 prover [[BSB22](#)]

sum-check protocol

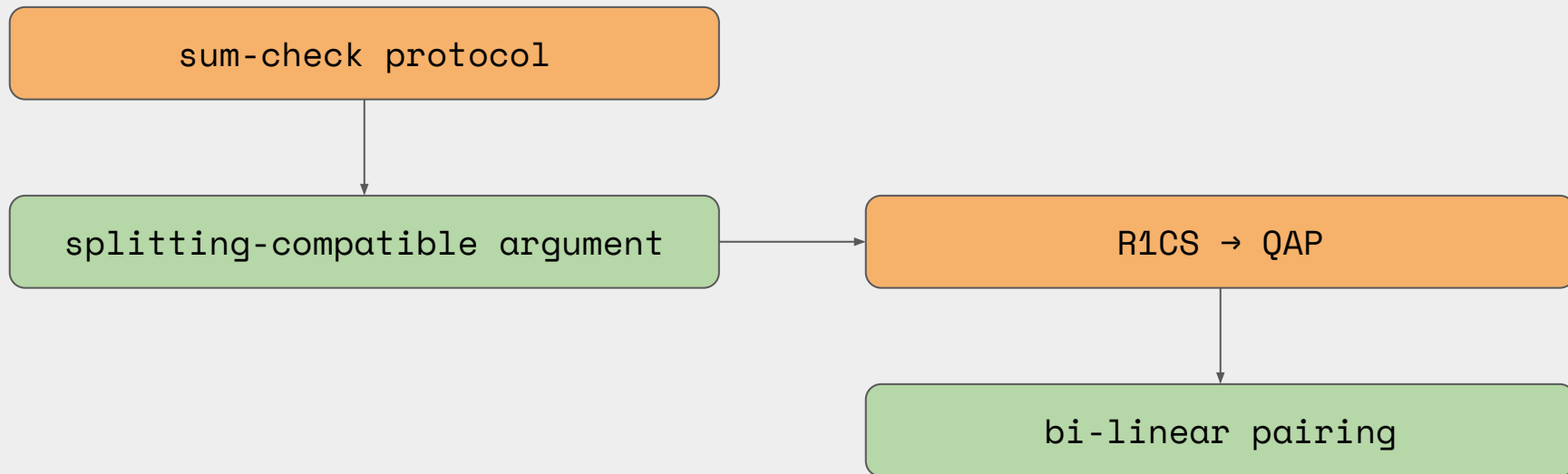
```
graph TD; A[sum-check protocol] --> B[Fiat-Shamir transform]
```

Fiat-Shamir transform

types of proof composition

3. tailor-made cryptographic compilers

e.g. GKR verifier in Groth16 prover [[BSB22](#)]



future of proving systems

- modular design → customisable proving stack
- recursion → composition across proof systems

can we systematise the composition of proof protocols?

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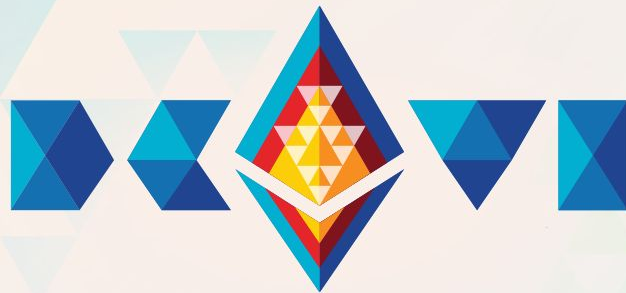
- better benchmarks for primitives (e.g. hashes, bigint, signatures, range proofs, ...)
- standardised criteria for comparing different compositions
- auditing / formal verification when composing proof systems



Section 4

recursion, aggregation, composition
task force





Thank you!

Ying Tong

Core Engineer, Electric Coin Company

yingtong@z.cash



@therealyingtong

Nalin

Person, 0xPARC

devcon@nibnalin.me



@nibnalin