# Ranking of research output of universities on the basis of the multidimensional prestige of influential fields: Spanish universities as a case of study

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**Abstract** A university may be considered as having dimension-specific prestige in a scientific field (e.g., physics) when a particular bibliometric research performance indicator exceeds a threshold value. But a university has multidimensional prestige in a field of study only if it is influential with respect to a number of dimensions. The multidimensional prestige of influential fields at a given university takes into account that several prestige indicators should be used for a distinct analysis of the influence of a university in a particular field of study. After having identified the multidimensionally influential fields of study at a university their prestige scores can be aggregated to produce a summary measure of the multidimensional prestige of influential fields at this university, which satisfies numerous properties. Here we use this summary measure of multidimensional prestige to assess the comparative performance of Spanish Universities during the period 2006–2010.

**Keywords** Publication-based ranking · Spanish universities · Bibliometrics · Multidimensional prestige · Influential fields of study

### Introduction

The interest in the ranking of universities stems from the need to evaluate research output using to this aim some kind of objective metrics. For example, it may guide student choice of a university to pursue a graduate degree (Dridi et al. 2010).

The comparison of research output among universities has been raising an increasing amount of interest in the last few years (Liu and Cheng 2005; Buela-Casal et al. 2007; Aguillo et al. 2010; Torres-Salinas et al. 2011a; Shin et al. 2011), since the help it provides

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to allocate limited funds as fairly as possible. However funding agencies often make their decisions based on partial measures, resulting in unfair assessments of the research output of some of the studied universities (Billaut et al. 2010).

The number of papers produced in a year by each member of staff in an academic institution particularly in the EU and USA is regarded as an indication of their career success. Rankings based on publication in peer-reviewed journals are objective, and many faculty believe academic journals remain the fairest measure of the quality of our research (Dusansky and Vernon 1998). Since publication-based performance evaluations underlie the work of funding agencies, there are already mechanisms to ensure high levels of accuracy of these data (Dusansky and Vernon 1998).

In (Torres-Salinas et al. 2011a), it was presented a bidimensional quantitative-qualitative index to compare the research output of a group of universities in a given scientific field using different dimensions of analysis: (1) The quantitative dimension which shows the net production of a university in a given field during a period of time by using raw indicators that may be correlated with staff of the institution [i.e., NDOC, NCIT and H-index in (Torres-Salinas et al. 2011a)]; and (2) the qualitative dimension which can be seen as a measurement for academic excellence, focusing on the ratio of high-quality production on each university in a particular field during the same period of time, and is mostly independent of the size of the institution [i.e., %1Q, ACIT, and TOPCIT in (Torres-Salinas et al. 2011a)]. A combination of both dimensions provides a robust and objective way to compare research outputs.

Recently we can find different studies analyzing scientific fields or disciplines (Porter and Rafols 2009; Lillquist and Green 2010; Herrera et al. 2010) or using them to produce a global ranking (López-Illescas et al. 2011). A justification of this particular selection of scientific fields is given in (López-Illescas et al. 2011). This study of the Spanish academic system reveals that assessment of a university's research performance must take into account the disciplinary breadth of its publication activity and citation impact (López-Illescas et al. 2011).

In this paper we provide a new approach to the ranking of research production of universities over scientific fields based on a multivariate performance indicator space which integrates both quantitative and qualitative dimensions.

To this aim, we extend the one-dimensional measure developed by (Garcia et al. 2011a, b) to a multidimensional case following (Peichl and Pestel 2010) who proposes a class of economic measures of richness in Germany. Thus our approach identifies those fields of study at a university that are considered to be multidimensionally influential. Furthermore, the multidimensional prestige of influential fields is to be sensitive to changes in the score distribution of each dimension, which allows us to investigate inequality among multidimensionally influential fields.

For example, let  $U = \{s_1, s_2, ..., s_n\}$  be the set of fields of study at a given university of example. From (Torres-Salinas et al. 2011a) we have that research output and impact of field  $s_i$  at this university may be graded on the basis of the raw number of publications (NDOC), citations (NCIT), h-index, as well as relative measures of impact and visibility (e.g., JCR journal first quartile (%1Q), average citations (ACIT) and ratio of highly cited papers (TOPCIT)).

Therefore, regarding the number of dimensions (prestige indicators) to be used in a multidimensional setting in order to measure research output and impact of influential fields at a particular university, we may consider several indicators with different degrees of correlation among them, but which should be used for a distinct analysis of structural



changes at the score distribution of prestige in a given field of study, (Torres-Salinas et al. 2011a):

- NDOC: Basic indicator for total amount of raw production, it may depend on the number of researchers in the institution focused on the field of study, and how active they are.
- NCIT, ACIT, TOPCIT: According to (Bornmann and Daniel 2008), in bibliometrics the resonance, or impact, of a scientific work is measured via the number of citations. It can be assumed that the more important a work is for the further development of a field, the more frequently it is cited. That is, NCIT is a raw indicator of scientific relevance, and ACIT and TOPCIT indicate quality of the research output and ratio of very high-quality papers (Aksnes 2003; Aksnes and Sivertsen 2004), respectively.
- H-index: Probably the better known index in current bibliometrics, it has proven to be a robust measure of impact, (Hirsch 2005). By limiting its scope to the period of study, we avoid the seniority dependence the basic h-index usually presents.
- %1Q: The impact factor is widely considered a reliable measure of journal quality (Bornmann and Daniel 2008), so centering the analysis in the top quartile provides an indicator of top-quality papers. The ratio of citable papers that are top-quality serves as a relative size-independent indicator, %1Q.

In this paper, a field of study  $s_i$  at a given university is considered as having dimension-specific prestige when its score based on a given ranking model (e.g., either NDOC or %1Q) exceeds a threshold value. Then, we can define which fields  $s_i$  at a given university are considered to be prestigious in a multidimensional setting. Thus, a field of study at this university has multidimensional prestige only if it is an influential field with respect to a number of dimensions. Finally, after having identified the multidimensionally influential fields at a particular university, their prestige scores are aggregated to a summary measure of multidimensional prestige. The summary measure is not only sensitive to the number of dimensions but also takes into account changes in the ranking scores of influential fields of study at the university.

The setup of the paper is organized as follows: Section 2 defines the multidimensionally influential fields of study at a given university. The Sect. 3 introduces a summary measure of multidimensional prestige of influential fields, which satisfies numerous properties. Then in Sect. 4 we shall apply our approach to main universities in Spain in order to analyse the comparative multidimensional prestige of influential fields during the period 2006–2010. The data we employ is from (Torres-Salinas et al. 2011b). Section 5 concludes.

#### Multidimensionally influential fields of study at a given university

The number of fields of study at a given university is denoted with n as given above, and let  $d \ge 2$  be the number of dimensions in the multivariate indicator space.

Let **X** be the matrix of dimension-specific scores  $x_{ij}$  which denote the score of field of study  $s_i$  at the particular university, with  $1 \le i \le n$ , in ranking model corresponding to dimension j, with  $1 \le j \le d$ :

$$\mathbf{X} = \left[ x_{ij} \right]_{n \times d} \tag{1}$$



For each dimension j, there is a threshold  $z_j$  such that fields  $s_i$  at this university with score  $x_{ij}$  above threshold  $z_j$  are to be considered dimension-specific influential fields of study.

Let **z** be the  $1 \times d$  vector of dimension-specific thresholds. Using this vector it is possible identify whether field  $s_i$  is influential with respect to dimension j or not. Let  $\theta_{ij}$  be a function defined as:

$$\theta_{ij} = \begin{cases} 1 & \text{if } x_{ij} > z_j \\ 0 & \text{otherwise} \end{cases}$$
 (2)

Using function  $\theta_{ij}$  it is possible to construct a matrix  $\Theta^{0-1}$  which provides information about whether a field of study  $s_i$  at the given university is influential regarding dimension j or not:

$$\Theta^{0-1} = \left[\theta_{ij}\right]_{n \times d} \tag{3}$$

where each row vector  $\theta_i$  of  $\Theta^{0-1}$  gives us a vector of prestige counts which can be denoted as  $\mathbf{c} = (c_1, \dots, c_n)'$  whose elements  $c_i = \sum_{j=1}^d \theta_{ij}$  are equal to the number of dimensions in which field of study  $s_i$  is found to be prestigious.

We can now define which fields of study at a university are considered to be influential in a multidimensional sense: A field of study  $s_i$  at the given university is a multidimensionally influential field if it is prestigious for a number of dimensions which is greater than or equal to a certain integer k, with  $1 \le k \le d$ .

That is, a field  $s_i$  is multidimensionally influential if  $c_i \ge k$ , with  $c_i$  being the number of dimensions in which field of study  $s_i$  at the university was found to be influential.

For a given integer k, we can define a function  $\phi_i(\mathbf{z};k)$  which equals to one if field  $s_i$  is multidimensionally influential, and is zero otherwise:

$$\phi_i(\mathbf{z};k) = \begin{cases} 1 & \text{if } c_i \ge k \\ 0 & \text{otherwise} \end{cases}$$
 (4)

with **z** being the  $1 \times d$  vector of dimension-specific thresholds.

Therefore the subset of fields of study at the university which are multidimensionally influential is given by:

$$\Phi(\mathbf{z};k) = \{s_i, 1 \le i \le n | \phi_i(\mathbf{z};k) = 1\}$$

$$\tag{5}$$

For a given integer k, let w(k) be the number of multidimensionally influential fields at this university. From equation (5) it follows that w(k) is given by the cardinal of the subset  $\Phi(\mathbf{z}; k)$ :

$$w(k) = |\Phi(\mathbf{z}; k)| \tag{6}$$

where  $|\cdot|$  is the cardinality (size) of a set.

In case of k = 1, field of study  $s_i$  is multidimensionally influential when it is considered prestigious in only one single dimension (e.g., %1Q). But prestige in one single dimension may be something dangerous (Garcia et al. 2012b).

Second, in case of k = d, it is only considered as multidimensionally influential if it is prestigious for all dimensions under consideration. But this is a demanding requirement, especially if the number of dimensions d of the multivariate indicator space is large, which often identifies a very narrow slice of fields at the university under consideration.

In case of 1 < k < d we have an intermediate approach as proposed in (Alkire and Foster 2008).



## A summary measure of multidimensional prestige

Recall that the vector of prestige counts denoted as  $\mathbf{c}$  was defined such that  $\mathbf{c} = (c_1, \dots, c_n)'$ , where  $c_i = \sum_{j=1}^d \theta_{ij}$  is the number of dimensions in which field of study  $s_i$  is found to be prestigious, with  $\theta_{ij}$  being equal to one if field  $s_i$  is prestigious with respect to dimension j and zero otherwise as given in Eq. (2). Since a summary measure of the multidimensional prestige of influential fields at the university must take into account information on multidimensionally influential fields of study only, we must replace the elements of  $\mathbf{c}$  as follows:

$$c_i^k = \begin{cases} c_i & \text{if } c_i \ge k \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

From Eq. (7), we have that  $\mathbf{c}^{\mathbf{k}} = (c_1^k, \dots, c_i^k, \dots, c_n^k)'$  contains zeros for fields  $s_i$  not considered to be multidimensionally prestigious, that is, when a field of study  $s_i$  is not multidimensionally influential,  $c_i < k$ , its entry in  $\mathbf{c}^{\mathbf{k}}$  is zero.

Now we propose a number of constraints which an axiomatic measure of the multidimensional prestige of influential fields at a given university must satisfy. But first, following the approach given in (Garcia et al. 2011a), we define a summary measure MPIF of the multidimensional prestige of influential fields at the university as the normalized weighted sum of the field contribution to the overall prestige as follows:

**Definition 1** Given a configuration  $\mathbf{X} = \begin{bmatrix} x_{ij} \end{bmatrix}_{n \times d}$  of dimension-specific scores of size  $n \times d$ , and a  $1 \times d$  vector  $\mathbf{z} = (z_1, \dots, z_j, \dots, z_d)$  of dimension-specific thresholds, a summary measure of the overall prestige MPIF of multidimensionally influential fields at a given university is defined by a normalized weighted sum of field contributions to the overall prestige using weighting function f, as follows:

$$MPIF(\mathbf{X}, \mathbf{z}, k) = \frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} f\left(\frac{x_{ij}}{z_{j}}\right), \tag{8}$$

where the mathematical form of f depends on a set of axioms to be proposed.

Appendix 1 presents a set of axioms in order to define the exact form of a summary measure as that given in Definition 1 which shall have some desirable properties. To this aim we reformulate to the study of the multidimensional prestige of influential fields a number of constraints which were first used in an axiomatic approach to economic poverty measurement (Sen 1976; Takayama 1979; Peichl et al. 2008).

Next, following (Garcia et al. 2012b), a theorem states that five axioms given in Appendix 1 determine an axiomatic measure of multidimensional prestige of influential fields for a given domain-specific score configuration.

**Theorem 1** Let k be such that field of study  $s_i$  at a given university is multidimensionally influential if  $c_i \ge k$ , with  $c_i$  being the number of dimensions in which field  $s_i$  was found to be influential. Then, a summary measure of the multidimensional prestige of influential fields, given by a normalized weighted sum of domain-specific scores in the configuration  $\mathbf{X}$  of size  $n \times d$ , using a weighting function f as follows:

$$\frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} f\left(\frac{x_{ij}}{z_{j}}\right) \tag{9}$$



and such that satisfies Axioms 1 through 5 in Appendix 1, it can be defined as:

$$MPIF(\mathbf{X}, \mathbf{z}, k) = \frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} \left( 1 - \left( \frac{z_j}{x_{ij}} \right)^{\beta} \right)_{+} \cdot \phi_i(\mathbf{z}; k)$$
 (10)

with  $\beta > 0$  being a sensitivity parameter for the intensity of field prestige (for smaller values of  $\beta$  more weight is put on more intense prestige);  $(y)_+ = \max(y, 0)$ ; and where function  $\phi_i(\mathbf{z}; k)$  equals to one if field  $s_i$  is multidimensionally influential, and is zero otherwise.

*Proof* See Appendix 2.  $\square$ 

# Case of study: ranking of Spanish universities

Here we show the ranking of research output of Spanish universities during the period 2006–2010. To this aim we compute the multidimensional prestige of influential fields of study at each institution using a multivariate indicator space.

Dimensions of the multivariate indicator space

Six variables are candidates to be used in this analysis, (Torres-Salinas et al. 2011b): (1) Raw number of citable papers (articles, reviews, notes or letters) published in scientific journals (NDOC); (2) Number of citations received by all citable papers (NCIT); (3) H-Index (H); (4) Ratio of papers published in journals in the top JCR quartile  $\frac{100 \times N1Q}{NDOC}$  (%1Q); (5) Average number of citations received by all citable papers (ACIT); and (6) Ratio of papers that belong to the top 10 % most cited (TOPCIT). The data are available at http://www.rankinguniversidades.es.

Once the set of Spanish universities was chosen (see Table 1), along with a period of time (2006–2010), the research output of each university indexed in the Web of Science (SCI, SSCI and A&HCI) of the Web of Knowledge (Thomson-Reuters) was retrieved using the field "Address" as a filter and taking into account all the different names each university receives, (Torres-Salinas et al. 2011a). Next, the production of each one of the universities within different fields of study is extracted. The number n of fields at each university may be lesser than or equal to 12 ( $n \le 12$ ). Table 2 illustrates the 12 fields of study which were used in this analysis [see (Rankings ISI 2011) for further details].

A scientific work is considered to be part of a field if it was published in a journal indexed in one of the JCR journal categories in this particular field of study. In order to calculate the indicators related to journal Impact Factor, the editions of the JCRs for the period of time of interest should be used. The data were downloaded in September 2011.

Table 4 in (Torres-Salinas et al. 2011a, p. 778) shows correlation analysis among six bibliometric indicators (i.e., NDOC, NCIT, H, %1Q, ACIT, and TOPCIT) using data from the top 75 % Spanish universities in 2000–2009. In general, it turns out that the quantitative indicators (i.e., NDOT, NCIT, and H) are positively correlated as expected, and also, but to a lesser degree, there are correlations within the qualitative ones (i.e., %1Q, ACIT, and TOPCIT). The correlations between a quantitative indicator and a qualitative one are in general very low. Thus, following (Torres-Salinas et al. 2011a) we consider that this correlation is low enough to conclude that quantitative and qualitative indicators describe



**Table 1** Set of 56 Spanish universities which was chosen to perform the comparison of research output during the period 2006–2010

Spanish universities Alicante Aut. Barcelona Alcala Almeria Aut. Madrid Barcelona Burgos Cadiz Cantabria Card. Herrera CEU Carlos III Cartagena Castilla la Mancha Complutense Madrid Córdoba Coruña Deusto Europea de Madrid Extremadura Girona Granada Huelva Baleares Jaen Jaume I La Laguna La Rioja Las Palmas León Lleida Mondragón Oviedo Pais Vasco Polit. Cataluña Polit. Valencia Polit. Madrid Pontificia de Comillas Málaga Miguel Hernandez Murcia Navarra P. Navarra Pablo Olavide Pompeu Fabra Salamanca Ramón Llull Rey Juan Carlos Rovira i Virgil San Pablo-Ceu Santiago Compostela Sevilla **UNED** Valencia Valladolid Vigo Zaragoza

**Table 2** Scientific fields of study which were considered in the analysis of research output of each university

Scientific fie	lds
i	Name
1	Agricultural Sciences
2	Biological Sciences
3	Earth and Environmental Sciences
4	Economics and Business
5	Physics
6	Engineering
7	Mathematics
8	Medicine and Pharmacology
9	Other Social Sciences
10	Psychology and Education
11	Chemistry
12	Information and Communication Technologies

different aspects of information without loss of interpretability, as happens when using variables obtained from a principal component analysis.

From these results, we define the six dimensions of the multivariate indicator space as follows: (j = 1) NDOC; (j = 2) NCIT; (j = 3) H-index; (j = 4) %1Q; (j = 5) ACIT; and (j = 6) TOPCIT. Then we have that the number of dimensions in the multivariate indicator space is d = 6.

For each dimension of the multivariate indicator space we must define a threshold such that fields of study at a given university with ranking score above this threshold are to be considered dimension-specific influential fields. More precisely, given a dimension-specific threshold  $z_i$  as well as scores  $x_{ij}$  which denote the ranking score of field  $s_i$ 



corresponding to dimension j, we have that fields of study  $s_i$  with ranking score  $x_{ij}$  above threshold  $z_i$  are dimension-specific influential fields.

For example, thresholds  $z_j$ , with j=1,...,6, can be defined such that the top 30 % of the score distribution given by the corresponding ranking model (over all Spanish universities under consideration) are dimension-specific influential.

Recall that a field of study  $s_i$  at a given university is defined multidimensionally influential if it is prestigious with respect to a number of dimensions which is greater than or equal to a certain integer k, with  $1 \le k \le d$ . But in case of k = 1,  $s_i$  is multidimensionally prestigious when it is considered prestigious in only one dimension which can be something dangerous, (Garcia et al. 2012b). On the other hand, in case of k = d, it is only considered as multidimensionally influential if it is prestigious in all dimensions under consideration which is a demanding requirement and often identifies a very narrow slice of fields.

If we choose larger values for thresholds  $z_j$  and integer k (e.g., k=4 and thresholds  $z_j$  are such that the top 10 % of the score distribution given by the corresponding ranking model are prestigious), we have that the ranking of Spanish universities will be based on more elitist principles. By the contrary if the values of thresholds  $z_j$  and k decrease (e.g., k=2 and the top 40 % of the score distribution), it follows a more comprehensive analysis.

An intermediate approach corresponds to the situation in which, for example, k=2 and thresholds  $z_j$  are such that the top 30 % of the score distribution given by the corresponding ranking model are dimension-specific influential.

Multidimensional prestige of influential fields at the University of Granada

In this section, we illustrate the measurement of the multidimensional prestige of influential fields at the University of Granada.

Table 3 (second column) provides information on the one-dimensional score distributions of the six dimensions under consideration: (j = 1) NDOC; (j = 2) NCIT; (j = 3) H-index; (j = 4) %1Q; (j = 5) ACIT; and (j = 6) TOPCIT. Table 3 (first column) lists the 12 fields of study ordered as given in Table 2.

The multidimensional prestige MPIF ( $\mathbf{X}$ ,  $\mathbf{z}$ , k) was computed for k=2 and thresholds  $z_j$ , with  $j=1,\ldots,6$ , such that only the top 30 % of the score distribution given by the corresponding ranking model in each dimension are dimension-specific influential. In this case we have that  $z_1=306$ ;  $z_2=1,403$ ;  $z_3=16$ ;  $z_4=0.519$ ;  $z_5=4.882$ ; and  $z_6=0.122$ . The value of  $\beta$  in Eq. (10) is  $\beta=3$  following the results presented in (Garcia et al. 2011a, b).

For this same university, Table 3 (fourth column) lists prestige counts  $c_i = \sum_{j=1}^{d} \theta_{ij}$  which represent the number of dimensions in which field of study  $s_i$  is found to be influential, with  $\theta_{ij}$  being equal to one if field  $s_i$  is prestigious with respect to dimension j and zero otherwise as given in Eq. (2) (third column in Table 3).

Table 3 (fifth column) shows  $\phi_i(\mathbf{z}; k)$  values which equal to one if field of study  $s_i$  (at the University of Granada) is multidimensionally influential and is zero otherwise, as given in Eq. (4). Recall that we select k=2 for this example of application.

Table 4 lists the  $\theta_{ij}^{\beta}(k)$  values which are defined as:

$$\theta_{ij}^{\beta}(k) = \left(1 - \left(\frac{z_j}{x_{ij}}\right)^{\beta}\right)_{+} \cdot \phi_i(\mathbf{z}; k) \tag{11}$$



**Table 3** (First column) lists fields of study ordered as given in Table 2; (second column) NDOC, NCIT, H, %1Q, ACIT, and TOPCIT; (third column)  $\theta_{ij}$  equals to one if field  $s_i$  is prestigious with respect to dimension j and zero otherwise; (fourth column) lists prestige counts  $c_i = \sum_{j=1}^{d} \theta_{ij}$  that represents the number of dimensions in which field  $s_i$  is found to be influential; (fifth column) shows  $\phi_i(\mathbf{z}; k)$  values which equal to one if field  $s_i$  is multidimensionally influential and is zero otherwise

Univ	versity of C	Granada												
i	NDOC	NCIT	Н	%1Q	ACIT	TOPCIT	$\Theta^0$	-1					$c_i$	$\phi_i(z;k)$
1	174	821	14	0.724	4.718	0.167	0	0	0	1	0	1	2	1
2	958	5575	28	0.380	5.819	0.084	1	1	1	0	1	0	4	1
3	993	4567	23	0.539	4.599	0.106	1	1	1	1	0	0	4	1
4	103	255	8	0.136	2.476	0.175	0	0	0	0	0	1	1	0
5	834	11763	28	0.618	14.104	0.109	1	1	1	1	1	0	5	1
6	630	2699	22	0.614	4.284	0.121	1	1	1	1	0	0	4	1
7	777	1964	16	0.372	2.528	0.104	1	1	0	0	0	0	2	1
8	1412	8496	33	0.390	6.017	0.095	1	1	1	0	1	0	4	1
9	263	503	11	0.300	1.913	0.091	0	0	0	0	0	0	0	0
10	448	1477	16	0.225	3.297	0.123	1	1	0	0	0	1	3	1
11	1006	5595	26	0.584	5.562	0.079	1	1	1	1	1	0	5	1
12	502	2205	20	0.3430	4.392	0.191	1	1	1	0	0	1	4	1

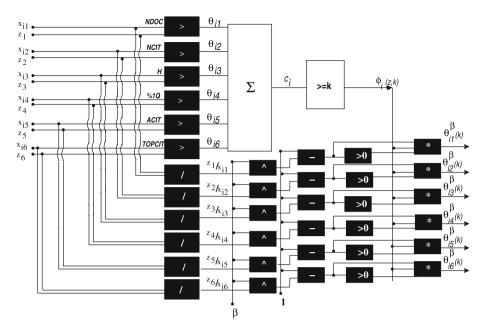
**Table 4** (First column) lists fields of study at the University of Granada ordered as given in Table 2; (second column) lists  $\theta_{ij}^{\beta}(k)$  elements given in Eq. (11)

i	$\theta_{ij}^{\beta}(k)$					
	j = 1	j = 2	j = 3	j = 4	j = 5	<i>j</i> = 6
1	0.0000	0.0000	0.0000	0.6316	0.0000	0.6101
2	0.9674	0.9841	0.8134	0.0000	0.4095	0.0000
3	0.9707	0.9710	0.6634	0.1072	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9506	0.9983	0.8134	0.4077	0.9585	0.0000
6	0.8854	0.8595	0.6153	0.3961	0.0000	0.0000
7	0.9389	0.6355	0.0000	0.0000	0.0000	0.0000
8	0.9898	0.9955	0.8860	0.0000	0.4659	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.6813	0.1429	0.0000	0.0000	0.0000	0.0242
11	0.9719	0.9842	0.7670	0.2981	0.3238	0.0000
12	0.7735	0.7424	0.4880	0.0000	0.0000	0.7394
$\frac{1}{n}\sum_{i=1}^{n}\theta_{ij}^{\beta}(k)$	0.6775	0.6094	0.4205	0.1534	0.1798	0.1145

Figure 1 illustrates the computation of the elements  $\theta_{ij}^{\beta}(k)$  for a field of study  $s_i$  at a given university.

Since, from Eq. (10), the summary measure MPIF(**X**, **z**, k) of multidimensional prestige of influential fields at a given university is equal to the sum of elements  $\theta_{ij}^{\beta}(k)$  divided by





**Fig. 1** Computation of  $\theta_{ii}^{\beta}(k)$  values for a field of study  $s_i$  at a given university

the value  $n \times d$ , it follows that MPIF ( $\mathbf{X}, \mathbf{z}, k$ ) = 0.3592 for the University of Granada. Again, Table 4 (first column) lists fields of study ordered as given in Table 2.

In addition to looking at the overall value of multidimensional prestige of influential fields at the University of Granada, we can provide information on how different dimensions of the multivariate indicator space contribute to the measure MPIF ( $\mathbf{X}$ ,  $\mathbf{z}$ , k) of multidimensional prestige. To this aim, we rewrite equation (10) as follows:

$$MPIF(\mathbf{X}, \mathbf{z}, k) = \frac{1}{d} \sum_{i=1}^{d} \frac{\sum_{i=1}^{n} \theta_{ij}^{\beta}(k)}{n} = \frac{1}{d} \sum_{i=1}^{d} \Pi_{j}^{\beta}(k)$$
 (12)

where  $\Pi_j^{\beta}(k) = \frac{1}{n} \sum_{i=1}^n \theta_{ij}^{\beta}(k)$  represents the contribution of each dimension j (multiplied by the number d of dimensions) to the measurement of multidimensional prestige of influential fields.

To the University of Granada, from Table 4 (bottom) we have that the contribution  $\Pi_j^\beta(k)$  of the NDOC dimension (j=1) is about 31.43 % of the multidimensional prestige, and taken together, the NDOC and NCIT dimensions make up about 59.7 % of the multidimensional prestige of influential fields of study at this university. Hence, the NDOC and NCIT dimensions play a dominant role to the measurement of the multidimensional prestige MPIF  $(\mathbf{X}, \mathbf{z}, k)$  for the University of Granada.

#### Results

In this section, we use the summary measure of multidimensional prestige MPIF to assess the comparative performance of selected Spanish universities during the period 2006–2010. Fifty-six main universities in Spain are considered in this experiment.



**Table 5** Ranking of Spanish universities during the period 2006–2010 according to the multidimensional prestige MPIF ( $\mathbf{X}$ ,  $\mathbf{z}$ , k) of influential fields, for different selections of k and thresholds  $z_j$ 

y         k=2         k=3         k=4           a         10%         20%         30%         40%         10%         20%         30%         40%         10%         10%         20%         30%         40%         10%	Kanking of Spamsh universities	ities												
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na de Madrid 3 5 5 4 4 4 5 4 5 4 4 5 4 4 3 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	Autónoma de Barcelona	2	2	2	2	2	2	2	2	4	2	2	2	2
ense de Madrid 5 3 6 3 5 3 6 6 2  de Compostela 7 8 7 7 6 8 7 9 9 8  vide Compostela 7 8 7 7 6 8 7 7 16  a la compostela 7 8 7 7 6 8 7 7 16  vica de Valencia 9 11 15 10 11 10 16 10 7  Virgili 12 12 9 8 12 12 8 8 15  co 10 9 13 21 9 11 13 14 15 10 11 14 17  La Mancha 40 18 14 13 33 18 13 14 26  Fabra 11 13 20 23 10 17 25 29 5  n Carlos 17 14 18 18 18 15 16 17 17 18 18 13 52  leares 20 17 18 18 18 15 16 17 17 18 18 13 52  La Mancha 40 18 14 13 20 23 10 17 25 29 5  leares 21 14 18 18 18 45 21 14 15 14 15 14 15 14  La Mancha 5 14 18 18 18 18 18 18 18 18 18 18 18 18 18	Autónoma de Madrid	3	5	5	4	4	5	4	4	3	4	3	5	4
care de Madrid         5         3         5         3         5         8         7         8         7         6         3         5         8         7         9         9         9         9         9         9         9         9         9         9         9         9         9         9         14         16         17         16         17         16         17         16         17         16         17         16         17         17         16         17         16         17         17         16         17         16         17         16         17         16         17         17         16         17         17         17         17         17         17         17         17         17         17         17         17         17         17         17         18 <th< td=""><td>Valencia</td><td>4</td><td>4</td><td>4</td><td>5</td><td>3</td><td>4</td><td>9</td><td>9</td><td>2</td><td>3</td><td>5</td><td>3</td><td>4</td></th<>	Valencia	4	4	4	5	3	4	9	9	2	3	5	3	4
de Compostela 7 8 7 7 6 8 7 7 16 14 14 15 16 16 17 16 18 17 17 16 18 17 17 16 18 17 17 16 18 17 17 16 18 17 17 16 18 17 17 18 18 18 18 18 18 18 18 18 18 18 18 18	Complutense de Madrid	5	3	9	3	5	3	5	5	∞	5	9	9	5
de Compostela         7         8         7         7         6         8         7         7         16           a         8         7         8         9         8         7         9         9         6           a         8         7         8         9         8         7         9         9         6           virgili         12         12         10         11         10         10         10         7         10         10         11         12         13         15         15         13         14         13         11         11         11         11         14         15         11         47         12         14         14         15         14         47         14 <td< td=""><td>Granada</td><td>9</td><td>9</td><td>3</td><td>9</td><td>7</td><td>9</td><td>3</td><td>3</td><td>14</td><td>9</td><td>4</td><td>4</td><td>9</td></td<>	Granada	9	9	3	9	7	9	3	3	14	9	4	4	9
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ica de Valencia         9         11         15         10         11         10         16         10         7           Virgili         12         12         9         8         12         12         8         15           voo         10         9         13         21         9         11         10         11         13           ico de Cataluña         13         16         12         15         14         15         11         14         15         14         17         14         17         14         14         15         14         15         14         14         15         14         15         14         15         14         15         14         15         14         15         14         15         14         15         14         1	Zaragoza	8	7	8	6	8	7	6	6	9	8	8	19	8
Virgilit         12         12         9         8         12         12         8         15           co         10         9         13         21         9         11         10         21         13           co         10         10         12         15         15         16         17         19         11         13         14         15         14         17         14         17         14         17         19         18         14         17         14         17         19         18         14         20         19         15         12         20         12           La Mancha         40         18         14         13         33         18         13         14         26         12         10         12 <t< td=""><td>Politécnica de Valencia</td><td>6</td><td>11</td><td>15</td><td>10</td><td>11</td><td>10</td><td>16</td><td>10</td><td>7</td><td>6</td><td>19</td><td>10</td><td>10</td></t<>	Politécnica de Valencia	6	11	15	10	11	10	16	10	7	6	19	10	10
too 10 9 13 21 9 11 10 21 13 14 15 15 15 15 15 15 15 15 15 15 15 15 15	Rovira i Virgili	12	12	6	8	12	12	∞	∞	15	16	6	8	11
16     10     10     12     15     9     11     12     54       ica de Cataluña     13     16     12     11     13     14     15     11     47       La Mancha     40     18     14     13     13     13     13     14     26       Fabra     11     13     20     23     10     17     25     29     5       n Carlos     17     21     19     15     24     19     18     13     52       eares     20     19     21     16     22     19     18     29       eares     32     20     17     14     18     45     21     14     15     42       23     24     19     18     23     22     19     16     20       4     18     18     45     21     14     15     42	País Vasco	10	6	13	21	6	11	10	21	13	11	12	15	12
tea de Cataluña 13 16 12 11 13 14 15 11 47  15 15 16 17 14 13 17 19 18  La Mancha 40 18 14 13 17 19 18  Fabra 11 13 20 23 10 17 25 29 5  n Carlos 17 21 19 15 24 19 18 13 52  La Mancha 50 19 15 24 19 18 13 52  La Mancha 60 19 15 10 17 18 25  La Mancha 74 19 18 18 45 21 14 15 42  La Mancha 75 19 10 10 17 14 15 17 17 17 17 17 17 17 17 17 17 17 17 17	Sevilla	16	10	10	12	15	6	11	12	54	15	10	17	12
La Mancha         40         18         17         14         13         17         19         18           La Mancha         40         18         14         13         33         18         13         12         20         12           Fabra         11         13         20         23         10         17         25         29         5           n Carlos         17         21         19         15         24         19         18         52           Leares         20         19         21         16         22         18         29           Leares         32         20         17         14         23         22         19         16         20           23         24         18         18         45         21         14         15         42           23         24         27         19         20         24         20         17         17	Politécnica de Cataluña	13	16	12	11	13	14	15	11	47	27	26	11	14
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11 13 20 23 10 17 25 29 5  17 21 19 15 24 19 18 13 52  20 19 21 16 21 16 22 18 29  32 20 17 14 23 22 19 16 20  21 14 18 18 45 21 14 15 42  23 24 27 19 20 20	Castilla-La Mancha	40	18	14	13	33	18	13	14	26	17	11	6	16
Carlos 17 21 19 15 24 19 18 13 52  20 19 21 16 21 16 22 18 29  eares 32 20 17 14 23 22 19 16 20  21 14 18 18 45 21 14 15 42  23 24 27 19 20 24 27 17 17	Pompeu Fabra	11	13	20	23	10	17	25	29	5	14	24	32	19
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32 20 17 14 23 22 19 16 20 21 14 18 18 45 21 14 15 42 23 24 22 10 20 24 20 17 17	Córdoba	20	19	21	16	21	16	22	18	29	13	21	12	20
21 14 18 18 45 21 14 15 42 23 24 22 19 20 24 20 17 17	Islas Baleares	32	20	17	14	23	22	19	16	20	24	14	18	20
71 71 00 10 01 60 10	Murcia	21	41	18	18	45	21	14	15	42	29	17	21	20
	Alicante	23	24	22	19	20	24	20	17	17	21	23	20	21



Table 5 continued
Ranking of Spanish universities

University	k = 2				k = 3				k = 4				Median
	10 %	20 %	30 %	40 %	10 %	20 %	30 %	40 %	10 %	20 %	30 %	40 %	
Cantabria	18	22	24	29	22	20	21	28	19	19	25	30	22
La Laguna	36	25	23	22	27	23	23	25	36	22	18	24	24
Salamanca	22	23	25	28	17	25	24	26	10	18	20	26	24
Alcalá de Henares	38	30	26	24	28	38	27	23	21	34	28	25	28
Jaume I de Castellón	14	26	32	31	18	28	30	30	11	26	27	28	28
Extremadura	43	34	27	25	38	31	26	22	32	41	22	22	29
Miguel Hernández	30	27	30	34	25	27	29	35	40	23	29	33	30
Navarra	19	28	36	38	16	56	35	37	6	20	32	34	30
Málaga	50	37	29	27	46	32	28	24	43	33	30	23	31
Girona	4	33	35	30	39	33	33	31	33	28	31	27	33
Lleida	48	39	33	32	43	40	32	32	39	31	33	29	33
Valladolid	99	32	31	33	99	29	31	33	99	99	35	31	33
Almería	37	35	38	37	29	34	37	36	22	30	34	36	36
Burgos	29	36	37	40	30	30	36	41	23	32	36	4	36
Cádiz	33	45	40	35	35	47	40	34	28	38	39	35	37
Politécnica de Madrid	53	31	28	26	50	37	34	27	48	50	51	38	38
Carlos III	39	52	39	39	32	46	41	38	25	36	40	42	39
Jaén	46	51	41	41	41	45	39	40	35	43	45	39	41
Pablo Olavide	31	29	34	36	47	41	38	43	44	47	49	41	41
Coruña. A	41	50	45	43	34	4	43	42	27	37	38	40	42
Europea de Madrid	25	40	50	53	37	35	45	50	31	40	43	52	42
Cardenal Herrera-Ceu	34	4	46	50	31	42	49	51	24	35	41	50	43
Huelva	45	46	43	42	40	49	47	39	34	42	4	37	43



 Table 5 continued

 Ranking of Spanish universities

Tomas of Themas and the second													
University	k = 2				k = 3				<i>k</i> = 4				Median
	10 %	20 %	30 %	40 %	10 %	20 %	30 %	40 %	10 %	20 %	30 %	40 %	
La Rioja	35	43	44	45	26	39	42	45	37	44	46	46	44
León	47	48	47	44	42	50	48	44	38	45	47	43	46
Palmas (Las)	51	47	49	46	48	43	4	46	45	48	37	45	46
Deusto	42	53	99	56	36	48	50	53	30	39	42	51	49
UNED	28	41	51	49	55	36	46	48	55	55	99	48	49
Mondragón	49	54	53	55	44	51	51	54	41	46	48	53	51
Politécnica de Cartagena	52	55	55	51	49	52	52	49	46	49	50	49	51
Pontificia de Comillas	26	38	48	52	51	53	53	55	49	51	52	54	52
Pública de Navarra	54	99	54	48	52	54	54	47	50	52	53	47	53
Ramón Llull	27	42	52	54	53	55	55	99	51	53	54	55	54
San Pablo-Ceu	55	49	42	47	54	99	99	52	53	54	55	99	54



Table 5 shows the ranking of the 56 Spanish universities according to the multidimensional prestige MPIF ( $\mathbf{X}$ ,  $\mathbf{z}$ , k) of influential fields of study, for different selections of k and thresholds  $z_j$  with  $j = 1, \ldots, 6$ . For our analysis the university with the best value of the multidimensional prestige of influential fields is assigned the rank # 1, the second best # 2, and so.

In order to produce the results given in Table 5, thresholds  $z_j$  with j = 1, ..., 6 were defined such that the top 10 % (or alternatively 20, 30, and 40 %) of the score distribution given by the corresponding journal ranking model (over all selected Spanish universities) are dimension-specific influential. For example, in case of the top 30% we have that  $z_1 = 306$ ;  $z_2 = 1,403$ ;  $z_3 = 16$ ;  $z_4 = 0.519$ ;  $z_5 = 4.882$ ; and  $z_6 = 0.122$ .

Regarding the value of k, here we follow an intermediate approach, and thus, a field  $s_i$  at a given university is defined multidimensionally influential if it is prestigious with respect to a number of dimensions which is greater than or equal to a certain integer k with 1 < k < 6. Thus the multidimensional prestige MPIF ( $\mathbf{X}, \mathbf{z}, k$ ) was computed for different values of k, with k = 2, 3, and 4.

Recall that if we choose larger values for thresholds  $z_j$  (e.g., only the top 20 % of the score distribution are dimension-specific influential), we have that the ranking of Spanish universities will be based on more elitist principles. In this case from Table 5 it follows that the top 10 Spanish universities were (for k=3 and thresholds  $z_j$  with  $j=1,\cdots,6$  such that the top 20 % of the score distribution given by the corresponding journal ranking model over all selected Spanish universities are dimension-specific influential): (1) Barcelona; (2) Autónoma de Barcelona; (3) Complutense de Madrid; (4) Valencia; (5) Autónoma de Madrid; (6) Granada; (7) Zaragoza; (8) Santiago de Compostela; (9) Sevilla; and (10) Politécnica de Valencia. This result is congruent with those from other academic ranking studies (ARWE 2011) which were applied to a small subset of Spanish universities.

By the contrary if the values of thresholds  $z_j$  decrease (e.g., the top 30 % of the score distribution are dimension-specific influential), it follows a more comprehensive analysis. In this case, from Table 5 it follows that the top 10 Spanish universities were (for k=2 and thresholds  $z_j$  with  $j=1,\ldots,6$  such that the top 30 % of the score distribution given by the corresponding journal ranking model over all selected Spanish universities are dimension-specific influential): (1) Barcelona; (2) Autónoma de Barcelona; (3) Granada; (4) Valencia; (5) Autónoma de Madrid; (6) Complutense de Madrid; (7) Santiago de Compostela; (8) Zaragoza; (9) Rovira i Virgili; and (10) Sevilla.

Looking for a general pattern of rankings across all the above selections for k and thresholds  $z_j$ , from Table 5 we have that the top 10 Spanish universities were (based on the Median rank): Barcelona; Autónoma de Barcelona; Autónoma de Madrid; Valencia; Complutense de Madrid; Granada; Santiago de Compostela; Zaragoza; Politécnica de Valencia; and Rovira i Virgili.

It should be pointed out that we have been able to report these results without assigning weights, since the various scores on different dimensions can be combined into a single score that reflects overall quality of a given university. Our ranking follows rigorous methodological criteria and thus may constitute an effective instrument for quality assessment of universities. The three main characteristics of our data were: (1) Internationally comparable data; (2) quantitative and qualitative indicators; and (3) open to verification.



#### Conclusions

Here we have presented a comparison of 56 Spanish universities based on the measurement of multidimensional prestige of influential fields of study (i.e., MPIF measure) during the period 2006–2010.

From the results showed in this paper, the top 10 Spanish universities during the period 2006–2010 were (based on the Median rank): Barcelona; Autónoma de Barcelona; Autónoma de Madrid; Valencia; Complutense de Madrid; Granada; Santiago de Compostela; Zaragoza; Politécnica de Valencia; and Rovira i Virgili.

In this paper we argue that this type of analysis, for example, may be relevant to the evaluation of research output using objective metrics in several quantitative and qualitative dimensions, which may guide student choice of a university to pursue a graduate degree or funding agencies to make their decisions regarding the allocation of limited funds.

An important methodological problem of the most commonly used global university rankings is their combination of multiple dimensions of university performance in a single aggregate indicator. These dimensions, which often relate to very different aspects of university performance are combined in a quite arbitrary fashion. This prevents a clear interpretation of the aggregate indicator.

A solution to this fundamental problem is to restrict a ranking to a single dimension of university performance that can be measured in an accurate and reliable way. This is the solution that the Leiden Ranking offers (http://www.leidenranking.com). The Leiden Ranking does not attempt to measure all relevant dimensions of university performance. Instead, the ranking restricts itself to the dimension of scientific performance. But the problem with this approach is clearly seen when applied to the ranking of universities. For instance, when using mean citation score in the Leiden-ranking, University of Granada ranks #14 position for the Spanish universities; whereas using number of publications (2005–2009) we have that University of Granada ranks #6 position. Thus, there exist significant differences between both rankings for University of Granada, when using the respective single dimension to perform the analysis.

Instead, the multidimensional prestige takes into account that several indicators should be used for a distinct analysis of structural changes at the score distribution of field prestige. We argue that the prestige of influential field of study at a given university should not only consider one indicator as a single dimension, but in addition take into account further dimensions.

In a multidimensional setting "prestige" relates to the recognition of the originality of research and its impact on the development of the same or related discipline areas from the viewpoint of several dimensions. That is, the perception of the recognition of the originality of research and its impact is not only restricted to the analysis of Impact Factor distribution, but different models, e.g, *H*-index, or citation impact, play an important role in the measurement of prestige.

After having identified the multidimensionally influential fields of study at a given university, their prestige scores can be aggregated to produce a summary measure of multidimensional prestige for this university which satisfies numerous properties (following an axiomatic approach).

Here we are using an axiomatic approach to derive a summary measure of the multidimensional prestige of influential fields at a given university. But, what does this mean? An axiomatic derivation consists of some terms, a number of axioms referring to those terms and partially describing their properties, and a rule or rules for deriving new propositions from already existing propositions.



There are a couple of main reasons why axiomatic systems are so useful: first, they're compact descriptions of the whole field of propositions derivable from the axioms, so large bodies of math can be compressed down into a very small compass; second, because they're so abstract, these systems let us derive all, and only, the results that follow from things having the formal properties specified by the axioms.

What are the limitations of the proposed approach? It is not rare that one would like to impose more axioms that are jointly compatible. It may also happen that the summary measure resulting from the original list of axioms is found to react very bad to some significant institutions. One must then formalize the characteristics of the particular institution and state an additional axiom that specifies how the criterion should behave in this situation, and finally determine the greatest subset of axioms from the original list that are compatible with the new axiom. Of course, compatibility may hold for several distinct such subsets.

In addition, other limitations have also been noted. For instance, the Thomson Reuters coverage of the literature in the humanities and social sciences is poor, and yet, the humanities and social sciences are important and often large part of the research profile of many universities. Hence, it would be of interest to introduce other document types (monographs for instance) that could permit a better coverage of certain fields such as social sciences and humanities, and develop methodologies that would adjust to these document types.

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# Appendix 1: Set of axioms

A first axiom states that a field of study at the given university which is not multidimensionally prestigious should not influence a summary measure of the overall prestige of multidimensionally influential fields.

**Axiom 1** Given two configurations of dimension-specific scores X and X' of the same size  $n \times d$  where the scores of multidimensionally influential fields at the university are the same in both cases, the summary measure of the multidimensional prestige of influential fields measured on either configuration should give the same value.

A second axiom can be justified on the idea that small changes in the configuration of dimension-specific scores for multidimensionally influential fields of study shall not lead to discontinuously large changes in the summary measure of multidimensional prestige.

**Axiom 2** The summary measure of the multidimensional prestige of influential fields at a given university should be a continuous function of dimension-specific scores for multidimensionally influential fields.

A third axiom states than an increment in some dimension-specific score (above the corresponding threshold  $z_j$ ) for a multidimensionally influential field of study shall increase the summary measure.

**Axiom 3** An index of multidimensional prestige of influential fields should increase whenever some dimension-specific score (above threshold  $z_j$  corresponding to that dimension) rises for a multidimensionally influential field of study.



Next another axiom states a property of subgroup decomposability. That is, the index has to be additively decomposable, i.e., the index of overall prestige is a weighted sum over several subgroups of fields of study in which the complete set U can be partitioned.

**Axiom 4** The overall prestige of multidimensionally influential fields can be decomposed into the weighted sum of subgroup-prestige indices.

And the following axiom requires that the summary measure of multidimensional prestige of influential fields shall increase after a progressive transfer (from a more influential field of study to a less prestigious one) of domain-specific scores above the corresponding threshold  $z_j$  between two multidimensionally influential fields at the university.

**Axiom 5** An overall prestige index should increase when a rank-preserving progressive transfer (above the corresponding domain-specific threshold) between two multidimensionally influential fields at a given university takes place.

## Appendix 2: Proof of Theorem 1

Here we follow the proof given in (Garcia et al. 2012a).

**Proof** Given a configuration X, let MPIF be a normalized weighted sum of the dimension-specific scores in X using weighting function f

$$MPIF = \frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} f\left(\frac{x_{ij}}{z_{j}}\right)$$
 (13)

where we have that f should be a continuous function for multidimensionally influential fields of study in order to satisfy Axiom 2, i.e., to verify that small changes in the configuration of dimension-specific scores (for multidimensionally influential fields at the university) shall not lead to discontinuously large changes in the summary measure MPIF.

But also it follows that weighting function f should be a strictly increasing function for multidimensionally influential fields of study at the university, since Axiom 3 states that an increment in some dimension-specific score (above the corresponding threshold  $z_j$ ) for a multidimensionally influential field shall increase the summary measure of multidimensional prestige MPIF.

From Axiom 1, a field of study which is not multidimensionally prestigious should not influence the overall prestige MPIF, i.e., MPIF is independent of the dimension-specific scores for fields of study at the given university which are not multidimensionally influential. Hence to fulfill Axiom 1 we have that

$$f\left(\frac{x_{ij}}{z_i}\right) = 0 \tag{14}$$

for all *i* such that  $\phi_i(\mathbf{z}; k) = 0$ ; where  $\phi_i(\mathbf{z}; k)$  equals to one if field  $s_i$  is multidimensionally prestigious and zero otherwise, as given in Eq. (4).

Now, from Axiom 4, the summary measure MPIF can be decomposed into the weighted sum of subgroup prestige indices. Thus it follows that the measure MPIF has to be additively decomposable.

Finally, following Axiom 5, the summary measure of multidimensional prestige MPIF should increase after a progressive transfer (from a more influential field of study to a less



prestigious one) of domain-specific scores above the corresponding threshold  $z_j$  between two multidimensionally influential fields at the university under consideration. Hence we have that weighting function f has to be concave for multidimensionally influential fields, and thus, the relative dimension-specific scores  $\frac{x_{ij}}{z_j}$  then have to be transformed by a function that is concave on  $(1, \infty)$  for multidimensionally influential fields of study.

For example, given a multidimensionally influential field  $s_i$ , we have that

$$f\left(\frac{x_{ij}}{z_j}\right) = \left(1 - \left(\frac{z_j}{x_{ij}}\right)^{\beta}\right) \cdot \phi_i(\mathbf{z}; k)$$

is concave for  $x_{ij} > z_j$  and  $\beta > 0$ .

To sum up, following Axiom 1 through Axiom 5, the summary measure MPIF

$$MPIF = \frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} f\left(\frac{x_{ij}}{z_{j}}\right)$$
 (15)

shall satisfy that  $f: R_+ \to [0, 1]$  is a strictly increasing and concave function on  $(1, \infty)$  for multidimensionally influential fields  $s_i$  at the given university.

Following (Peichl and Pestel 2010), if we define weighting function f as:

$$f\left(\frac{x_{ij}}{z_j}\right) = \left(1 - \left(\frac{z_j}{x_{ij}}\right)^{\beta}\right)_+ \cdot \phi_i(\mathbf{z}; k) \tag{16}$$

where  $(v)_+ = \max(v, 0)$ , we obtain a summary measure of the multidimensional prestige of influential fields, that resembles Eq. (10) satisfying Axiom 1 through Axiom 5, since f being defined as given in Eq. (16) it is a strictly increasing and concave function f:  $R_+ \rightarrow [0, 1]$  on  $(1, \infty)$  for multidimensionally influential fields  $s_i$ .

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