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On the disruptive power of small-teams research

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Abstract
Recent studies have shown that research by small teams is more likely to lead to disruptive results than research by large teams. Disruptive research challenges established paradigms. This paper offers a possible theory to explain this paradox. We argue that individuals in possession of research ideas with great disruptive potential have incentives to form small teams and compensate potential group weaknesses with a greater research effort rather than considering additional co-authors. Additional co-authors have the advantage of bringing more overall effort and expertise to the team, reducing technical difficulties, and increasing the chances of success and the potential value of the ideas. We show that individuals in possession of potentially disruptive research ideas prefer to keep teams as small as possible, because the resulting credits per co-author decrease as the value of the project is split among more co-authors.

Keywords Bibliometrics · Scientific impact · Disruption · Research teams' size

JEL Classification C72 · O31

Introduction

Research has become increasingly complex and interdisciplinary, with research collaborations having an increasingly larger number of co-authors (Gazni et al. 2012; Larivière et al. 2015; Persson et al. 2004; Wuchty et al. 2007). The size of research groups and the scientific impact in terms of citations have been shown to be highly correlated (Hsu and Huang 2011; Onodera and Yoshikane 2015; among others). However, authors are critical of the use of citations as an accurate measure of scientific impact due to its strategic use (Catalini et al. 2015), non-scientific aspects that play a role in the decision to cite (Bornmann and Daniel 2008), and the difficulty in capturing the difference between publications, collaborations, and citation practices across fields (Waltman 2016; Waltman and van Eck 2019).

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In a recent study, Wu et al. (2019) apply the network approach in Funk and Owen-Smith (2017), in order to challenge the use of citations as an accurate measure of scientific impact, and to offer a new perspective on this issue. They consider 24,174,022 research articles published from 1954–2014 and indexed in the Web of Science, and show that the median citations to articles increase with team size, but that the disruptiveness decreases with team size (In The Disruption Index and the Number of Co-authors section, we discuss in greater detail the negative relation between the number of co-authors and disruptiveness).[1] The results are also found to be robust for patents and software projects on GitHub, and across different scientific disciplines and researchers. These studies support the idea that research by small teams tends to make more disruptive contributions to science than research by large teams (Azoulay 2019). Simultaneously, the majority of scientists seem to prefer small and mid-sized grants, rather than large grants (Dimke et al. 2019). These findings have profound implications in terms of the organization and evaluation of research teams, and in terms of finance and prioritization of different research projects. In particular, Wu et al. (2019) leave open the question of why small teams are more likely to perform disruptive research.

This paper attempts to provide an answer to this question by proposing a simple theory to explain why research by smaller teams is more likely to lead to disruptive results than research by large teams. The argument is not so much based on the overlap of skills, talents, and experiences between the team members, which we consider important, but not sufficient to explain the paradox of small teams' greater ability to produce disruptive results. Instead, we argue that individuals in possession of potentially disruptive research ideas have incentives to form small teams and compensate potential group weaknesses with greater personal effort rather than considering additional co-authors. Additional co-authors have the advantage of bringing more overall effort and expertise to the team, reducing difficulties, increasing the chances of success, and potentially increasing the value of the project. However, individuals in possession of potentially disruptive research ideas prefer to keep teams small, because the resulting credits per co-author decrease as the value of the project is split among a larger number of co-authors.

We follow a game-theoretical approach that is based on the well-established "rent-seeking" literature on group contests (Nitzan 1991; Kolmar 2013; Flamand and Troumpounis 2015). According to this approach, several individuals provide joint effort to increase the chances of achieving a common objective.

In this paper, we characterize a research project along two dimensions: expected value and expected difficulty. We study how the optimal number of co-authors varies along these two dimensions. In our framework, the consideration of an additional co-author may add value and/or favor the successful completion of the research project. This aspect captures the co-authors' qualitative dimension. The quantitative dimension is captured by the effort that each co-author adds to the research project, i.e. the level of involvement in the project. This dimension might also capture free-riding effects. However, the consideration of an additional co-author may also reduce the credits awarded to the other co-authors, because the research project benefits must be split by a larger number of co-authors.

We found that in projects with lower expected value, i.e. projects that are expected to be less disruptive, and in projects with higher expected difficulty, i.e. projects that are

[1] In our context, disruptive research and disruptiveness refer to the capacity that research has to create new independent research lines and networks, and eventually to challenge, displace, and disrupt established paradigms.

expected to be relatively more difficult to achieve, individuals have larger incentives to increase the number of co-authors. The reduction in effort involved and the increase in the likelihood of success compensate the loss of the credits per co-author. By contrast, when the expected value of the research project is relatively high, individuals are less willing to involve other individuals, and are more willing to work alone or in smaller groups. The exception occurs when the difficulty of the project requires the consideration of additional co-authors.

Our results point out that small teams are more likely to lead to disruptive research (i.e. research with higher ex-ante expected value) than large teams because of strategic considerations and self-interest reasons inherent to the individuals. Researchers have expectations about the value and complexity of their ideas. Consequently, they define the optimal group size based on the balance between costs and benefits.

This paper is organized as follows. "Model setup" section presents the theoretical framework. "The disruption index and the number of co-authors" section focusses on the explanation of the index to measure disruptiveness and the empirical demonstration that small teams have frequently published disruptive papers. "Theoretical analysis" section presents the theoretical analysis and discusses the results. "Extensions to the baseline model" Section presents and discusses some extensions. "Conclusion" Section concludes. The proofs are relegated to an Appendix section.

The disruption index and the number of co-authors

This section provides an explanation of the recently proposed index for measuring disruptiveness, along with an empirical demonstration of the ability of small teams to publish disruptive research.

Besides counting the number of publications, it is standard practice in bibliometrics to use citation counts to measure the impact of publications on other publications (Tahamtan and Bornmann 2019). As demonstrated by Marx and Bornmann (2016) in an overview of various bibliometric approaches, the use of cited references data instead of times cited data in research evaluation allows interesting insights into research. For example, Uzzi et al. (2013) proposed the use of atypical combinations of cited references in scientific papers to identify papers including novel findings. Similar approaches have been proposed by Lee et al. (2015) and Wang et al. (2017) (see also Mairesse and Pezzoni 2018). All these approaches have in common the fact that unusual combinations are interpreted as "a proxy measure for recombination of knowledge" (Wagner et al. 2019) which might reflect creativity (see Tahamtan and Bornmann 2018b, for a critical reflection on the use of cited references data for measuring novelty): "the 'new' contributions, ones that bring original ideas, often involve incorporating ideas from different disciplines, research traditions, or frameworks" (Wagner et al. 2019).

In a recent paper, Funk and Owen-Smith (2017) propose a novel network approach to study how new inventions reshape the network of interlinked technologies. They propose a novel index to distinguish whether new inventions consolidate or destabilize the existing technology stream. They apply their approach to patents data. Building on this idea, Wu et al. (2019) consider citation data, and compare the cited references of a focal paper with the cited references of its citing papers to measure the disruptiveness of research; in our context, we call it the Disruption Index (DI). Azoulay (2019) explains the intuition behind the DI as follows: "when the papers that cite a given article also reference a substantial

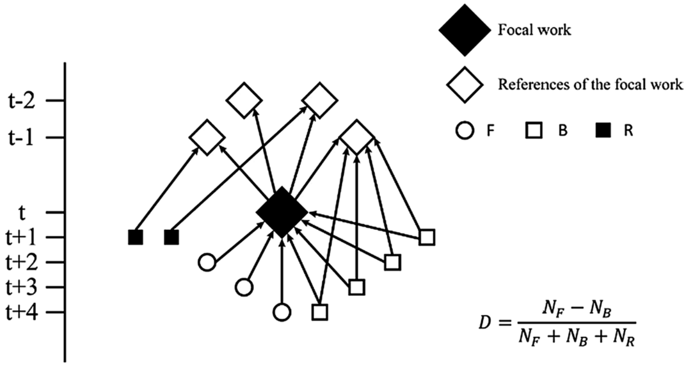

Fig. 1 Illustration of elements in citation networks (*source*: Wu and Wu 2019). *F* denotes the publications that cite the focal publication without citing any of its references, *B* denotes the publications that cite both the focal publication and any of its references, and *R* denotes the publications that cite any of the focal publication references without citing the focal publication. The original figure has been complemented by the formula for calculating disruptiveness (*D*). N_i denotes number of publications with the characteristic *i*

proportion of that article's references, then the article can be seen as consolidating its scientific domain. When the converse is true—that is, when future citations to the article do not also acknowledge the article's own intellectual forebears—the article can be seen as disrupting its domain" (p. 331).

Figure 1—adapted from Wu and Wu (2019)—reveals the calculation of the disruptiveness of a focal paper (*D*): N_F is the number of citing publications that cite the focal publication (but not citing any of the focal publication's cited references), N_B is the number of citing publications citing both the focal publication and any of its cited references, and N_R is the number of citing publications citing any of the focal publication's cited references (but not citing the focal publication itself). An extensive discussion of this approach can be found in Funk and Owen-Smith (2017). A critical discussion of the DI can be found in Wu and Wu (2019). Wu and Yan (2019) and Bu et al. (2019) have proposed other variants of the DI. Bu et al. 2019 propose an indicator, which considers whether the citing papers only refer to the focal paper or additionally to the publications cited in the focal paper. They call the new perspective provided by the indicator multi-dimensional. In the same vein, Figueiredo and Andrade (2019) consider a Bayesian approach in order to identify disruptive musicians using data from the All Music Guide.

We calculated the DI according to Wu et al. (2019) for papers available in the Max Planck Society's in-house database which is based on the Web of Science database (Clarivate Analytics). Since we could not calculate the DI for the entire database, we selected a random sample of papers according to the following three criteria. (1) We included only papers with the document type "article" published between 1980 and 2000. Thus, papers are comparable in view of this characteristic. (2) Since the DI results are more reliable, the more frequently cited references data are available for every publication (Bornmann and Tekles 2019b), we considered only articles with at least 20 cited references. (3) We used the same threshold on the citing side for the same reason (i.e. reliability of the data). In order to reduce the set of publications for the empirical analysis, we drew a random sample of 10,000 papers from the population of all articles published between 1980 and 2000. The population consists of 1,380,513 articles (considering the cited references' and citing papers' thresholds).

Table 1 Key characteristics of the random sample of papers (n = 10,000)

Statistic	Disruption index	Number of co-authors	Times cited
Median	− 0.003	4	50
Mean	− 0.005	4.7	86.109
Minimun	− 0.155	1	20
Maximun	0.272	459	8686
Standard deviation	0.013	12.569	172.867

Table 2 The Spearman rank-order correlation between the Disruption Index, number of co-authors, and times cited

	Disruption index	Number of co-authors	Times cited
Disruption index	1.00		
Number of co-authors	0.021	1.00	
Times cited	− 0.19	0.06	1.00

Table 1 shows the key characteristics of the considered set of papers in this study. The DI variable has a maximum value of 0.272, which is a relatively low value (see Bornmann and Tekles 2019a; Wu et al. 2019), and a minimum value of −0.155. The range of the number of co-authors in a paper is between 1 and 459. The Spearman rank-order correlation coefficients between the variables are reported in Table 2 (including the DI, number of co-authors, and times cited). Interpreted against the guidelines published by Cohen (1988), the correlation between citations and the DI is negative—on a low to medium level. Thus, both indicators appear to represent different dimensions. Similar results have been presented by Wu et al. (2019).

We found zero correlation between the number of co-authors and citations. This result is contrary to expectations, since the correlation is usually positive and substantial (see Tahamtan and Bornmann 2018a). The most likely reason for the unexpected result is our focus on papers with at least 20 citations, which reduced the heterogeneity of the times cited data. As expected, similar to the results of Wu et al. (2019), however, Table 2 also reveals a zero correlation between number of co-authors and DI. We repeated the analyses with papers with at least 1000 citations in our in-house database ($N = 6390$ papers). Papers with a high DI can be usually found among the most highly cited papers. As expected, based on the results of Wu et al. (2019), we found a negative correlation between number of co-authors and DI (on a small to medium level). Thus, one cannot expect more frequent disruptive research from large teams than from small teams.

Figure 2 illustrates the relationship between the DI and the number of co-authors. It is clearly visible that most of the papers with high DI values have only a few co-authors.

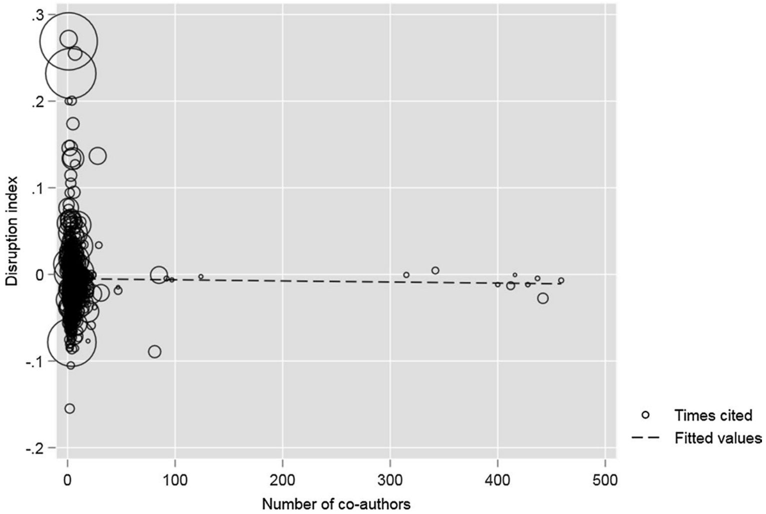

Fig. 2 The relationship between Disruption index, number of co-authors, and times cited

Model setup

In this section, we present the model in which: (1) individuals have expectations about the potential value of a given research project, where high expectations capture research projects that are ex-ante more likely to become disruptive in the future,[2,3] (2) individuals have expectations about the difficulty of the research project, and (3) individuals can choose the number of co-authors that participate in the project, i.e. the size of the project. Therefore, a research project is characterized along three dimensions: expected value, expected difficulty, and number of co-authors.

In this context, we want to understand how the optimal number of co-authors varies as a function of the expected value and difficulty.

Let $\overline{v}(n)$ denote the expected value of a research project with n co-authors, where $n \geqslant 1$. The natural assumption is to consider that the expected value of the project is non-decreasing in the number of co-authors n, i.e. $\partial\overline{v}(n)/\partial n \geqslant 0$, and concave, i.e. $\partial^2\overline{v}(n)/\partial n^2 \leqslant 0$ (Figure 3 provides an illustration). The concavity of $\overline{v}(n)$ captures the fact that increasing the number of co-authors may increase the expected value of a research project, but in a decreasing way (Bornmann and Osório 2019). In other words, each new co-author adds

[2] In our model, individuals do not have a preference for disruptive or incremental projects—it is exogenous to them—but they have a sense of the potential of a given project. In our model, we assume that disruptiveness is unpredictable, as it is often the case in reality, but we argue that the most disruptive projects are within the group of projects with higher expected value. This means that not all these projects will be disruptive and that many projects will be incremental. We follow the idea that high expected value leads to a high potential for disruptiveness. This might be a more subtle assumption than assessing the citation potential impact of research. The idea introduces a qualitative dimension that is more difficult to define in theoretical and empirical terms than only focusing on the quantitative dimension.

[3] Funk and Owen-Smith (2017) also consider the *mCD* index that attempts to capture ex-post, i.e. after publication, the magnitude of inventions. The idea behind the index is linked with the concept of ex-ante expected value in this paper. This idea may be empirically studied by investigating which projects have ended up being more or less disruptive.

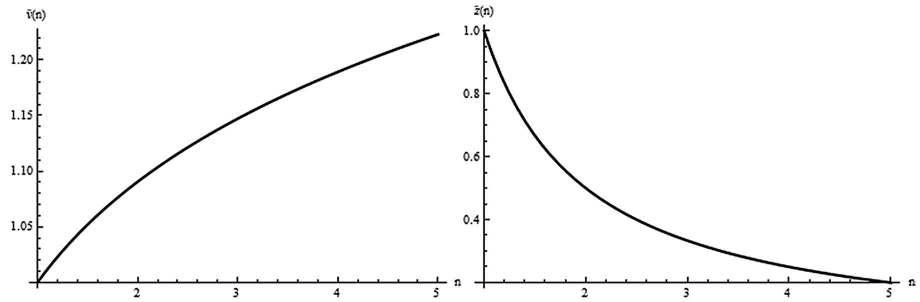

Fig. 3 Example of an increasing concave expected value function of the form $\bar{v}(n) = \bar{v}n^{\alpha}$ with $0 \leqslant \alpha \leqslant 1$ where $\alpha = 1/8$ in this illustration (left-hand side). Example of a decreasing convex expected difficulty function $\bar{z}(n) = \bar{z}/n^{\alpha}$ with $0 \leqslant \alpha \leqslant \infty$ where $\alpha = 1$ in this illustration (right-hand side). Other functional forms are possible

less value to the project than the previous one, and so on, i.e. the marginal contribution of each co-author to the total value of the project is decreasing.

Similarly, let $\bar{z}(n) \geqslant 0$ denote the expected difficulty of successful completion of the research project, which is non-increasing in the number of co-authors n, i.e. $\partial\bar{z}(n)/\partial n \leqslant 0$, and convex, i.e. $\partial^2\bar{z}(n)/\partial n^2 \geqslant 0$ (Figure 3 provides an illustration). The convexity of $\bar{z}(n)$ captures the fact that increasing the number of co-authors increases the likelihood of successful completion of the research project (because of the overlap in skills, talents, and experiences of co-authors), but in a decreasing way. In other words, each new co-author has less influence on the project's likelihood of success than the previous one, i.e. the marginal contribution of each new co-author to the success of the project is decreasing.

Therefore, the research project likelihood of success depends on the vector of efforts $X = \{x_1, \dots, x_n\}$ provided by each of the n co-authors and on the expected difficulty $\bar{z}(n)$, as follows:

$$p(X) = \frac{x_1 + \cdots + x_n}{x_1 + \cdots + x_n + \bar{z}(n)}, \tag{1}$$

where x_i denote the effort exerted by co-author $i \in \{1, \dots, n\}$ in the n co-authors research project. This approach follows the "rent-seeking" literature on group contests. In our context, however, the n co-authors compete for the successful completion of the joint research project, which is a more realistic assumption, and not against another group of researchers. Nonetheless, this possibility could be incorporated into our model.

Expression (1) implies that the completion of the project is uncertain and there is a risk of failure, i.e., with probability $1 - p(X)$ the completion of the research project fails and the expected value of the project equals zero $\bar{v}(n) = 0$.

Therefore, each co-author i chooses an effort level x_i that maximizes the individual expected utility, which depends on the likelihood of success, on the expected value of the project, on the individual share in the value of the project, and on the cost of effort. Hence, the objective function of each co-author i is given by:

$$u_i = p(X)s_i(X)\bar{v}(n) - x_i, \tag{2}$$

for all $i = 1, \dots, n$, where $s_i(X)$ denotes the share of co-author i in the value of the project, which depends on the individual effort and on the effort provided by the other co-authors, i.e. $s_i(X) = x_i/(x_1 + \cdots + x_n)$.[4]

Theoretical analysis

In order to study the incentives to add or remove co-authors from a research project, and to understand why small teams are associated with more disruptive research than large teams, we need to consider some functional form for $\overline{z}(n)$ and $\overline{v}(n)$. In this context, the properties of the function $\overline{z}(n)$ make the likelihood of success in expression (1) non-decreasing in the number of co-authors n, i.e., $\partial p(X)/\partial n \geqslant 0$, and concave, i.e., $\partial^2 p(X)/\partial n^2 \leqslant 0$, which is similar to the effect of n on $\overline{v}(n)$. Consequently, $\overline{z}(n)$ and $\overline{v}(n)$ will lead to similar effects on the utility function (2). For that reason, in order to simplify the analysis and to not duplicate these two effects, and in line with the properties of these functions (discussed in the previous section), we consider:

$$\overline{z}(n) = \overline{z}/n \text{ and } \overline{v}(n) = \overline{v}, \tag{3}$$

where $\overline{z}$ and $\overline{v}$ control the exogenous difficulty level and the value of the project, respectively.[5] Therefore, we can vary the strength of these two effects.

After replacing (3) into (1) and (2), we obtain the following symmetric Nash equilibrium solution and associated expected payoffs.

Proposition 1 *Given the number of co-authors n, the difficulty level $\overline{z}$ and the value of the project $\overline{v}$, in equilibrium, each co-author i provides the expected effort level:*

$$x_i^* = \frac{(n-1)\overline{v} - 2\overline{z} + \sqrt{(n-1)^2\overline{v}^2 + 4\overline{zv}}}{2n^2}, \tag{4}$$

for all $i = 1, \dots, n$, and obtains the expected utility:

$$u_i^* = \frac{(n^2+1)\overline{v} + 2\overline{z} - (n+1)\sqrt{(n-1)^2\overline{v}^2 + 4\overline{zv}}}{2n^2}, \tag{5}$$

for all $i = 1, \dots, n$.

[4] The "rent seeking" literature on sharing rules (Nitzan 1991; Flamand and Troumpounis 2015), often considers the general expression:

$$s_i(X) = \alpha \frac{1}{n} + (1-\alpha)\frac{x_i}{x_1 + \cdots + x_n},$$

for all $i = 1, \dots, n$, where $\alpha \in [0, 1]$ determines the importance given to the egalitarian sharing rule relative to the sharing rule based on individual effort. This approach can be extended to our context, with the additional extra parameter α. In our symmetric context, any approach along these lines delivers similar results. Since the case $\alpha = 0$ is richer and more realistic, we focus on this case.

[5] The expected value of the project is independent of the number of co-authors, as the concavity of the problem in n is already captured by $\overline{z}(n) = \overline{z}/n$. This simplification has no influence whatsoever on the results.

In order to have meaningful results, the effort provided by each co-author and the obtained expected utility must be positive. Otherwise, individuals have no incentives to provide effort and it might be more reasonable to not participate in the research project. The following result establishes the condition that guarantees that these regularities are satisfied. This same condition also guarantees equilibrium existence.

Proposition 2 *If $n \geqslant \bar{z}/\bar{v}$, then $x_i^* \geqslant 0$ and $u_i^* \geqslant 0$ for all $i = 1, \ldots, n$.*

This condition guarantees that equilibrium exists if the number of co-authors is sufficiently large and the difficulty of the project is not too high relative to the value of the project. For instance, the equilibrium always exists if the expected difficulty is low relative to the value of the project, i.e. $\bar{z}/\bar{v} \leqslant 1$, because in this case $n \geqslant 1$, which is always true. On the other hand, if the expected value is high relative to the difficulty of the project, then $\bar{z}/\bar{v} > 1$, and the number of co-authors must be larger than one for the equilibrium to exist. In fact, there is always an integer number n that guarantees that $n \geqslant \bar{z}/\bar{v}$.

Since individuals are rational and they can choose the number of co-authors, the following result establishes the optimal number of co-authors. This number maximizes the individuals' utility for a given expected difficulty and value of the project.

Proposition 3 *The optimal number of co-authors is given by $n^* = (\bar{v} + 4\bar{z})/(2\bar{v})$.*

Note that n^* is always larger than the condition of Proposition 2, which implies that we always have a well-behaved interior solution.

This result immediately leads to the following observation.

Corollary 1 *The optimal number of co-authors n^* increases with the expected difficulty of the project $\bar{z}$ and decreases with the expected value of the project $\bar{v}$.*

This corollary explains why disruptive research tends to be associated with small teams irrespective of the potential benefits of specialization and the multidisciplinary potential of large research teams. Intuitively, if a research project is expected to be difficult to achieve relative to the expected value, then the optimal number of co-authors will tend to be larger. By contrast, if a research project is expected to have a high value relative to the difficulty, then the optimal number of co-authors will tend to be smaller. In other words, ideas that are expected to have higher disruptive potential will be kept within small research groups.

In our simple model, the addition of co-authors increases the likelihood of success by increasing the aggregate effort and decreasing the complexity or difficulty to successfully achieve the research project. This produces an incentive to increase the number of co-authors. However, at the same time, the credits associated with the research project have to be split into a larger number of co-authors, and free-riding effects become more likely if the number of co-authors is large (or the Ringelmann effect—the tendency for individuals to decrease performance as the group size increases; Forsyth 2009). This situation produces an incentive to reduce the number of co-authors. At a certain point, for a given expected value and difficulty level, the addition of new co-authors does not compensate

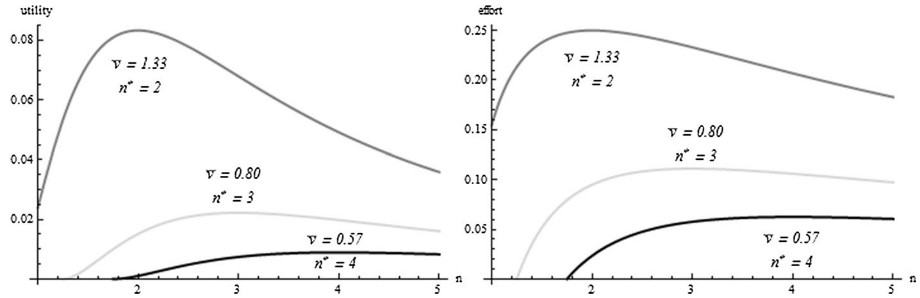

Fig. 4 Utility (left-hand side) and effort (right-hand side) per co-author as a function of n. The difficulty level is fixed at $\bar{z} = 1$. We can see that the optimal number of co-authors decreases (n^*) as the research project's value increases ($\bar{v}$). In addition, the higher the project's value, the higher the effort that each co-author is willing to provide

for the reduction in credits per co-author. In other words, the addition of new co-authors is rational only if the expected difficulty of the project is high relative to the expected value.[6]

Obviously, not all ideas with high expected value will become disruptive in the future. Some ideas (probably the majority) will become incremental; some ideas with low expected value may become disruptive in the future. Nonetheless, we argue that the ideas that are most likely to become disruptive in the future are within the group of ideas with high ex-ante expected value.

Figure 4 provides an illustration. It shows the utility (left-hand side) and effort (right-hand side) per co-author as a function of n. The difficulty level is fixed at $\bar{z} = 1$ in order to simplify the interpretation of the figure. The optimal number of co-authors is shown for different values of the research project, i.e. n^* equal to 2 , 3, and 4 for $\bar{v}$ equal to 1.33, 0.8 and 0.57, respectively, in accordance with Proposition 3, and satisfying the regularity condition of Proposition 2. Note that the optimal number of co-authors decreases as the value of the research project increases, as stated in Corollary 1. In addition, the higher the project value, the higher the effort provided by each co-author.

Extensions to the baseline model

In this section, we present some possible extensions of the baseline model and discuss their implications.

In order to study the co-authors' incentives to add or remove co-authors from a project, and to understand why small teams tend to be associated with more disruptive research than large teams, we have made several assumptions. The baseline model assumes that all co-authors are symmetric in terms of efficiency, skills, abilities, and credits. Consequently,

[6] Note that for a given expected value and difficulty level, n^* is the point that maximizes the individual effort. This observation supports the idea that individuals provide optimal effort levels when the group size is optimal or well-defined. If the group size is smaller than the optimal group size, i.e. $n < n^*$, individuals provide less than the optimal effort level because the difficulty of the project discourages the provision of effort. In this context, it would be optimal to consider more co-authors. If the group size is larger than the optimal group size, i.e. $n > n^*$, individuals tend to provide less than the optimal effort level because of free-riding behaviour. In this context, it would be optimal to consider less co-authors.

all co-authors have the same influence in the likelihood of success (1) and the objective function (2).

However, in reality, co-authors are frequently asymmetric and differentiated along multiple dimensions. For instance, one might consider differences between co-authors in terms of their contribution, skills and other qualitative features. Similarly, one might distinguish co-authors in terms of the obtained credits, roles and status within the research group. This issue is particularly relevant, for instance, in laboratory work and newly formed teams in which there may not exist a clear denominator in the assignment of credits.

Based on these and other similar preliminary considerations, we have captured several asymmetries between co-authors:[7]

Different effort efficiency The first approach introduces asymmetries in the quantitative dimension of the success likelihood in expression (1) by considering an effort efficiency or productivity parameter associated with each co-author:

$$p(X) = \frac{w_1 x_1 + \cdots + w_n x_n}{w_1 x_1 + \cdots + w_n x_n + \bar{z}(n)}, \tag{6}$$

where $w_i \geqslant 0$ denotes the efficiency of the effort exerted by co-author $i \in \{1, \dots, n\}$. In other words, if co-author i has a large w_i, the effort exerted by co-author i will have a larger impact in the success likelihood of the research project. On the other hand, if co-author i has a small w_i, the effort exerted by co-author i will have a smaller impact in the success likelihood of the project. Therefore, we can distinguish co-authors in terms of effort efficiency and productivity.

*Different effort costs*An alternative approach that is also linked with the co-authors, incentives to provide effort, and their productivity is to consider different effort costs in the objective function (2):

$$u_i = p(X) s_i(X) \bar{v}(n) - c_i x_i, \tag{7}$$

for $i = 1, \dots, n$, where $c_i \geqslant 0$. Therefore, the higher the value of c_i, the higher the co-author's i costs of effort, and consequently, the lower the incentives to provide effort in the project.

The effort costs may capture situations in which co-authors are involved in several projects. The greater the number of projects, the higher the amount of effort involved in new projects.

*Different skills/abilities*Another approach is to introduce asymmetries in the qualitative dimension of the likelihood of success in expression (1) by considering a skills/abilities parameter associated with each co-author. In this case, the assumption (3) becomes:

$$\bar{z}(n) = \bar{z} / \sum_{j=1}^{n} z_j, \tag{8}$$

where $z_i \geqslant 0$. Consequently, co-authors with larger z_i would lead to larger reductions in the difficulty of the project and larger increases in the likelihood of success. In this way, we can distinguish co-authors in terms of skills, abilities, and other characteristics.

[7] Note that once we depart from symmetry, it becomes difficult to have close form expressions, and the analysis must be done with the resource to numerical approximations.

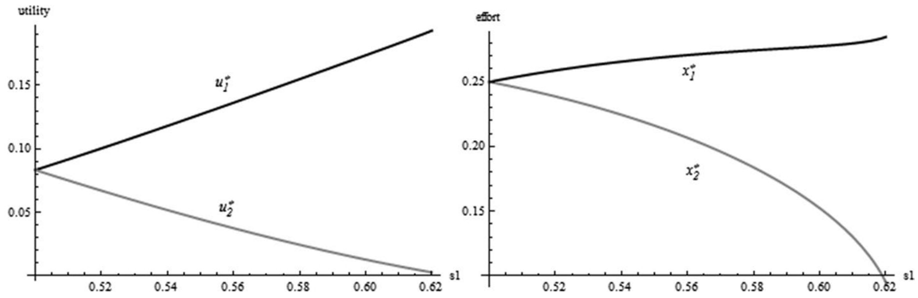

Fig. 5 Utility (left-hand side) and effort (right-hand side) of co-authors 1 and 2 as a function of s_1, for $\overline{v}(n) = 4/3$ and $\overline{z}(n) = 1/2$. In particular, as co-author's 1 share of the credits increases, co-author's 1 effort increases slightly, while co-author 2 effort decreases sharply

This approach has close form solution, and the equilibrium efforts and utilities remain symmetric as in Proposition 1. This is the case because the effect of z_i applies symmetrically to all co-authors.

*Different value contribution*An alternative approach that is also linked with the co-authors' qualities and characteristics is to consider that co-authors differ in their contributions for the expected value of the research project. In this case, we have:

$$\overline{v}(n) = \sum_{j=1}^{n} \overline{v}_j, \tag{9}$$

where $\overline{v}_i \geqslant 0$. Consequently, co-authors with larger $\overline{v}_i$ are expected to add more value to the research project, and for that reason would be preferred over the others.

Since the effect of v_i applies symmetrically to all co-authors, this type of approach also has close form solution as in Proposition 1.

Nonetheless, these qualitative considerations must be placed into their context, because better co-authors are also likely to demand higher returns, either in terms of credits or income. This aspect leads to interesting trade-offs.

Different credits - The baseline model assumes that the credits received by each co-author i depend on the effort provided as follows $s_i(X) = x_i/(x_1 + \cdots + x_n)$. Consequently, in a full symmetric setting, all co-authors obtain the same credits in equilibrium. However, we can consider the case in which co-authors have different shares in the project credits as follows:

$$s_i(X) = s_i x_i/(s_1 x_1 + \cdots + s_n x_n), \tag{10}$$

for $i = 1, \ldots, n$, where $0 \leqslant s_i \leqslant 1$ and $\sum_{j=1}^{n} s_j = 1$. The parameter s_i captures the co-author's i share in the own effort. This effort feedbacks into the actual share in the project credits, i.e., $s_i(X)$. Therefore, the higher the value of s_i, the higher the co-author's i actual share in the credits from the project, and higher the co-author i incentives to provide effort, and vice versa.

Table 3 **The strategic process of group formation:** Equilibrium efforts and utilities in a three co-authors' case with $z_1 = z_2 = 1, z_3 = 2.5$ and $\bar{z} = 1$, and $\bar{v}(n) = 4/3$

	Case 1		Case 2		Case 3		Case 4		Case 5	
	$s_1(X) = 0.50$		$s_1(X) = 0.33$		$s_1(X) = 0.50$		$s_1(X) = 0.75$		$s_1(X) = 0.55$	
	x_i^*	u_i^*	x_i^*	u_i^*	x_i^*	u_i^*	x_i^*	u_i^*	x_i^*	u_i^*
$i = 1$	0.250	0.083	0.256	0.089	0.298	0.152	0.279	0.178	0.306	0.189
$i = 2$	0.250	0.083	0.256	0.089	–	–	0.142	0.010	–	–
$i = 3$	–	–	0.256	0.089	0.298	0.152	–	–	0.286	0.118

Co-author 1 has the status of a project leader. For that reason, this person is present in all cases

Figure 5 shows in a two co-authors' case how equilibrium efforts and utilities vary as s_1 increases from 0.5 to 0.62.[8] As we increase s_1, the co-author's 1 incentives to provide effort increases slightly, while co-author's 2 incentives to provide effort fall sharply at a certain point. Consequently, movements away from the equal split disincentivize more than proportionally the unfavored co-author.[9]

Note that the actual share $s_1(X)$ (not shown in Fig. 5), which also takes into consideration the variations in co-authors' incentives to provide effort, increases from 0.5 to 0.83 when s_1 increases from 0.5 to 0.62.

The strategic process of group formation

Individuals with different skills and characteristics bring different ideas and solutions to a research project, but they are also likely to demand different returns in terms of credits and income. The decision to invite a new co-author to a project is a comparison between gains and losses, i.e. the utility before and after the addition of the new co-author. In our model, this is the main driving force in the process of group formation and in the determination of the optimal team size.

However, important trade-offs might emerge. For instance, a co-author may terminate a relation with another co-author if another co-author is available who would lead to a better cooperation. Similarly, two co-authors may not agree on the invitation of a new co-author. This is the case when one co-author benefits from the addition of a new team member, while the other co-author does not. This situation may lead to disagreement and bargaining between the team members, and possibly to an adjustment in the co-authors' share in the credits of the project.

The following hypothetical situation has the double propose of discussing some of the issues and trade-offs that emerge in multiple co-authors' team formation and illustrating numerically some of the extensions presented above. In particular, it considers simultaneously the *different skills/abilities* approach in expression (8) and the *different credits* approach in expression (10).

8 Note that for s_1 larger than 0.62 the numerical solution becomes unstable as co-author's 2 utility and effort converge sharply to zero.

9 Note that there is some strategic equivalence between the approaches in expressions (8) and (9), and between the approaches in expressions (6), (7) and (10). For instance, higher effort effciency, lower cost of effort, or higher credits share leads to an increase in the effort incentives, and vice versa.

The situation starts by considering a two co-authors' case in which both co-authors are symmetric and contribute equally to the success likelihood of the project, as in Expression (8), i.e. $z_1 = z_2 = 1$. In this case, each co-author provides an equilibrium effort of 0.250 and derives an equilibrium utility of 0.083 (case 1 in Table 3).

In order to be concrete, suppose that the research project is not possible without co-author 1. Co-author 1 owns the research idea, the economic resources, the tools or the laboratory facilities.

Subsequently, suppose that a third co-author with strong skills/abilities, i.e. $z_3 = 2.5$ (see Expression (8)), is available to join the project. In this case, it would be beneficial for both co-authors to invite the new co-author to the project. Consequently, each of the three co-authors would provide an equilibrium effort of 0.256 and derive an equilibrium utility of 0.089, which would be an improvement for everybody (case 2 in Table 3).

However, co-author 1 can improve the situation by removing co-author 2 and inviting co-author 3 to the project. In this case, each of the two co-authors would provide an equilibrium effort of 0.298 and derive an equilibrium utility of 0.152, which would be an improvement for co-author 1 (case 3 in Table 3).

However, co-author 2 can propose a joint collaboration with a larger share of credits for co-author 1, such that co-author 1 would improve over the utility of 0.152. For instance, the proposal $s_1(X) = 0.75$ and $s_2(X) = 0.25$, i.e., $s_1 = 0.605$ (see Expression (10)), would serve this object. In this case, the co-author's 1 equilibrium effort and utility are 0.279 and 0.178, respectively, and the co-author's 2 equilibrium effort and utility are 0.142 and 0.010, respectively (case 4 in Table 3).

Nonetheless, co-author 3 can propose an even better offer for co-author 1 , which co-author 2 cannot match without incurring in a negative utility. For instance, the proposal $s_1(X) = 0.55$ and $s_3(X) = 0.45$ (i.e. $s_1 = 0.534$) would serve this object. In this case, the co-author's 1 equilibrium effort and utility are 0.306 and 0.189, respectively, and the co-author's 2 equilibrium effort and utility are 0.286 and 0.118, respectively (case 5 in Table 3).

This hypothetical situation shows that the process of team formation and adding or removing researchers from a research project is not always peaceful and consensual, and it might be extremely strategic. One can imagine a great diversity of potential situations. In some cases there might be a leading author, while in other cases, co-authors may simultaneously decide on the addition or removal of co-authors. Many intermediate situations are also possible.

Conclusion

This paper offers a rationale for why disruptive research tends to be associated with small teams. Our results suggest that small teams' research is more likely to be disruptive than large teams' research because of strategic considerations and the individuals' self-interest. Once in possession of a research idea, researchers form expectations about the potential value and the difficulty to achieve this idea. Based on these expectations, they define the optimal group size of researchers who should be involved in the project. The ideas with greater expected value tend to be kept within smaller groups. Potential group weaknesses are compensated with additional effort, instead of additional co-authors.

In some situations, researchers' expectations may fail and some ideas that were expected to be disruptive may turn out to result in marginal or incremental contributions, and in other situations, researchers may come across some disruptive findings even if they were not specifically seeking them. Therefore, the obtained results should be seen in expected and aggregated terms: ideas and projects with higher ex-ante expected value are the ones that are more likely to become disruptive in the future (ex-post), and for that reason kept within small research groups. On the other hand, the ideas and projects with lower expected value (i.e. incremental projects) and/or higher complexity are more likely to be developed by larger groups.

The model presented and the results obtained might be convincing, but they are certainly not the last word on the small teams' disruptiveness paradox. Further research might complement our approach or may offer new perspectives. In this context, we would like to emphasize that the reality is far more complex and multidimensional than our analysis can capture. In connection with the incentives to produce disruptive or incremental research, an interesting topic of further research could be the consideration of aspects like risk aversion within the research groups and their strategic necessity to maintain stable funding streams. Another interesting avenue of further research might be the study of the links between research groups size and free-riding effects, the Matthew effect, or the idea of self-fulfilling prophecies in science (Merton 1948, 1968, 1988).

In the context of our model, it would be interesting to empirically test whether and how the number of co-authors changes over the course of a project, and how the change is correlated with the degree of disruptiveness and the distribution of skills, talents, and experiences within the group of co-authors.

Finally, we hope that our findings will help researchers and decision-makers to better understand the mechanisms behind the formation of research teams and to design optimal financing policies that directly incentivize disruptive or incremental research.

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Appendix A: Proofs

Proof of Proposition 1 After replacing (3) into (1) and (2), we obtain each co-author's i first-order condition from differentiating u_i with respect to x_i, which is given by:

$$-1 + (\frac{\bar{z}}{n} + \sum_{j \neq i}^{n} x_j)\bar{v}/(\frac{\bar{z}}{n} + \sum_{j=1}^{n} x_j)^2 = 0,$$

for all $i = 1, \ldots, n$. It is easy to show that the second order condition for a maximum is satisfied. Imposing symmetry among the n co-authors, i.e. $x_1 = \cdots = x_n$, we obtain the equilibrium effort given in expression (4). Note that there are two solutions, but one is meaningless because it implies a strictly negative level of effort. It is ignored for this reason. After replacing expression (4) into (2), we obtain the equilibrium expected utility given in expression (5). □

Proof of Proposition 2 After some algebra on inequalities $x_i^* \geqslant 0$ and $u_i^* \geqslant 0$ in expressions (4) and (5), respectively, we find that in both cases the condition $n \geqslant \bar{z}/\bar{v}$ is necessary and sufficient to guarantee these inequalities. □

Proof of Proposition 3 Since the problem is symmetric, the optimal number of co-authors is the value of n that maximizes u_i in Expression (5). The associated first order condition returns two extreme points, i.e. $n = \bar{z}/\bar{v}$ and $n = (\bar{v} + 4\bar{z})/(2\bar{v})$. The associated second order condition evaluated at these points returns $2\overline{vz}^2 > 0$ and $-4\overline{vz} < 0$, respectively. Therefore, the first extreme point corresponds to a minimum, while the second extreme point corresponds to a maximum. The first extreme point coincides with the condition in Proposition 2. Consequently, n^* is a well-behaved interior solution. □

Proof of Corollary 1 Simply take the first derivative of n^* with respect to $\bar{z}$ and $\bar{v}$ to obtain $2/\bar{v} > 0$ (increasing) and $-2\bar{z}/\bar{v}^2 < 0$ (decreasing), respectively. □

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