Simple PFC Model

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Model A

Model A systems are systems where the order parameter is not conserved, a common example is the Ising model of a spin system. Throughout the evolution of the system the total number of "up" spins does not need to stay constant. This system can be modelled by the following equations:

$$\phi^{n+1}(i,j) = \phi^n(i,j) + \frac{\Delta\bar{t}}{\Delta\bar{x}^2}\bar{\Delta}^2\phi^n n + 1(i,j) - \Delta\bar{t}\frac{\partial f(\phi^n(i,j))}{\partial\phi}$$

with $\bar{\Delta}^2$ is the discreet Laplacian which may be given by

$$\bar{\Delta}^2\phi^n(i,j) = \phi(i+1,j) + \phi(i-1,j) + \phi(i,j+1) + \phi(i,j-1) - 4\phi(i,j)$$

although there are other ways to represent the discreet Laplacian that in some cases may be better suited. Used periodic boundary conditions. The free energy functional may be expressed as a function with two minima, $f(\phi^n(i,j)) = a_1 + \frac{a_2}{2}(\phi^n(i,j))^2 + \frac{a_4}{4}(\phi^n(i,j))^4$ which has the following first order derivative

$$\frac{\partial\phi^n(i,j)}{\partial\phi} = a_2(\phi^n(i,j))^2 + a_4(\phi^n(i,j))^3$$

Notes on variables:

- $i, j = 0..N$ are indices for the ϕ array which represents the phase field of the system.
- N is the size of the array, here the system is assumed to have the same size in x and y directions.
- $n = 0..T$ is the time index of the sytem.
- T is the final time of the array.
- $\bar{t} = nM\Delta t$, M is the diffusion coefficient
- $\bar{x} = n\frac{\Delta x}{W_o}$, W_o is the interface width (?)
- $\Delta\bar{t} < \frac{1}{4}\Delta\bar{x}$, euler time marching algorithm stability restriction in 2D