

approximation-pattern

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Consider the problem of minimizing a function $f : D \rightarrow \mathbb{R}$

In many scenarios, it is hard to find an optimal, or it is even also hard to compute the value of f . The method below was inspired from the work of Isaac Vandermeulen, Roderich Groß, Andreas Kolling ?.

For a domain $X \subseteq D$ of the minimization problem, let $f_1 : X \rightarrow \mathbb{R}$ be the proxy function such that

$$(1) : f(x) = c(f_1(x)) + v(x) \quad \forall x \in X$$

Where c is a monotonically increasing function and $v : X \rightarrow \mathbb{R}$ is function on X .

Let x^* and x_1^* be the optimal values for f and f_1 in the domain $X \subseteq D$. Let $t_{\max} = \max_{x \in X} v(x)$ and $t_{\min} = \min_{x \in X} v(x)$ be the maximum value and minimum value of v over the domain X .¹

Consider 3 inequalities:

$$(A) : f(x_1^*) \leq f(x^*) + t_{\max} - t_{\min}$$

$$(B) : f(x_1^*) \leq c(f_1(x_1^*)) + t_{\max}$$

$$(C) : f(x^*) \geq c(f_1(x^*)) + t_{\min}$$

We have $f(x^*) \leq f(x_1^*)$ and $f_1(x_1^*) \leq f_1(x^*)$. Since c is a monotonically increasing function, so that $c(f_1(x_1^*)) \leq c(f_1(x^*))$.

By definition of t_{\max} , (B) holds,

$$f(x_1^*) \leq c(f_1(x_1^*)) + t_{\max} \leq c(f_1(x^*)) + t_{\max}$$

By definition of t_{\min} , (C) holds,

$$f(x^*) \geq c(f_1(x^*)) + t_{\min} = (c(f_1(x^*)) + t_{\max}) - (t_{\max} - t_{\min})$$

Hence,

$$(A) : f(x_1^*) \leq f(x^*) + (t_{\max} - t_{\min})$$

¹Here, we used the maximum value and minimum value for v since in the original work ?, the authors did not make it clear why x^* and x_1^* are independent from v and further more, their proof does not make a clear statement on the feasibility of the method to any problem but rather most problems.

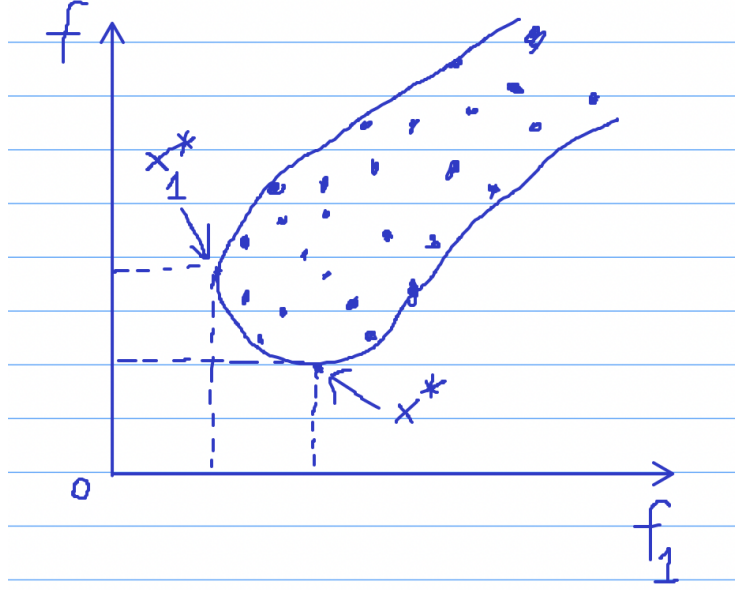


Figure 1: f_1 approximation

By choosing an appropriate proxy function f_1 , we can approximate the solution of f .

In the derivation, we have set t_{\max} and t_{\min} be the maximum and minimum value of v in the domain of X . However, the bound can be even better if we have some methods to approximate the maximum and minimum value of v in a subset of X that contains both x^* and x_1^*