# approximation-pattern

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Consider the problem of minimizing a function $f : D \to \mathbb{R}$

In many scenarios, it is hard to find an optimal, or it is even also hard to compute the value of $f$. The method below was inspired from the work of Isaac Vandermeulen, Roderich Groß, Andreas Kolling [**?** ].

For a domain $X \subseteq D$ of the minimization problem, let $f_1 : X \to \mathbb{R}$ be the proxy function such that

$$(1) : f(x) = c(f_1(x)) + v(x) \ \forall x \in x$$

Where $c$ is a monotonically increasing function and $v : X \to \mathbb{R}$ is function on $X$.

Let $x^*$ and $x_1^*$ be the optimal values for $f$ and $f_1$ in the domain $X \subseteq D$. Let $t_{\max} = \max_{x \in X} v(x)$ and $t_{\min} = \min_{x \in X} v(x)$ be the maximum value and minimum value of $v$ over the domain $X$. [1]

Consider 3 inequalities:

$$(A) : f(x_1^*) \leq f(x^*) + t_{\max} - t_{\min}$$
$$(B) : f(x_1^*) \leq c(f_1(x_1^*)) + t_{\max}$$
$$(C) : f(x^*) \geq c(f_1(x^*)) + t_{\min}$$

We have $f(x^*) \leq f(x_1^*)$ and $f_1(x_1^*) \leq f_1(x^*)$. Since $c$ is a monotonically increasing function, so that $c(f_1(x_1^*)) \leq c(f_1(x^*))$.

By definition of $t_{\max}$, $(B)$ holds,

$$f(x_1^*) \leq c(f_1(x_1^*)) + t_{\max} \leq c(f_1(x^*)) + t_{\max}$$

By definition of $t_{\min}$, $(C)$ holds,

$$f(x^*) \geq c(f_1(x^*)) + t_{\min} = (c(f_1(x^*)) + t_{\max}) - (t_{\max} - t_{\min})$$

Hence,

$$(A) : f(x_1^*) \leq f(x^*) + (t_{\max} - t_{\min})$$

[1] Here, we used the maximum value and minimum value for $v$ since in the original work [**?** ], the authors did not make it clear why $x^*$ and $x_1^*$ are independent from $v$ and further more, their proof does not make a clear statement on the feasibility of the method to any problem but rather most problems.

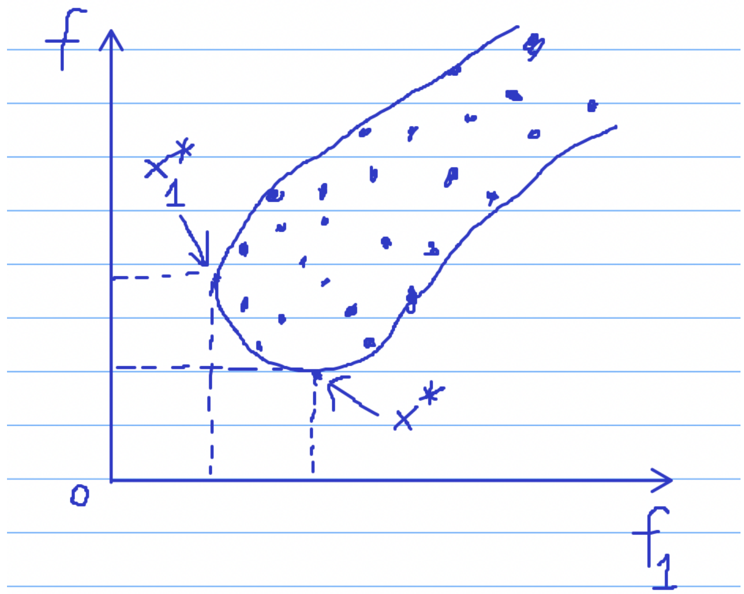


Figure 1: $f_1 approximation$

By choosing an appropriate proxy function $f_1$, we can approximate the solution of $f$.

In the derivation, we have set $t_{\max}$ and $t_{\min}$ be the maximum and minimum value of $v$ in the domain of $X$. However, the bound can be even better if we have some methods to approximate the maximum and minimum value of $v$ in a subset of $X$ that contains both $x^*$ and $x_1^*$