

Find all functions $f : [0, +\infty) \rightarrow [0, +\infty)$ such that

$$f(f(x)) + 3f(x) - 270x = 133$$

For each real number \mathbf{a} , let $a : \mathbb{R} \rightarrow \mathbb{R}$ be the multiplication function by \mathbf{a} , that is $a(x) = \mathbf{a}x$. If $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, then $gf : \mathbb{R} \rightarrow \mathbb{R}$ denotes the function composition, $f + g : \mathbb{R} \rightarrow \mathbb{R}$ denotes the function $(f + g)(x) = f(x) + g(x)$ (note: this makes real-valued functions on real number a non-commutative ring). Then

$$\begin{aligned} f(f(x)) + \mathbf{3}f(x) - \mathbf{270}x &= (f^2 + 3f - 270)(x) \\ &= (f^2 - 15f + 18f - 270)(x) \\ &= ((f - 15)f + 18(f - 15))(x) \\ &= (f + 18)(f - 15)(x) \end{aligned}$$

The original equation becomes

$$(f + 18)(f - 15)(x) = 133$$