Coq Tacticals and PVS Strategies: A Small Step Semantics

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- Coq's and PVS's common heritage: LCF theorem prover (1979).
- Small step semantics.

- Coq
  - Procedural,
  - Calculus of Inductive Constructions,
  - Intuitionnistic logic.
- PVS
  - Procedural,
  - Enriched Simple Types Theory,
  - Classical logic.

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- Common feature : proof language.

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- Independance w.r.t. implementation differences?

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$\leadsto$ What is the difference ?

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⇝ What is the difference ?

⇝ How do they operate ?

$$\dfrac{\dfrac{}{C, (A \Rightarrow B), A \vdash C} \quad \dfrac{\dfrac{}{B, A \vdash B, C} \quad A \vdash A, B, C}{(A \Rightarrow B), A \vdash B, C} \quad \dfrac{}{(A \Rightarrow B), A \vdash A, C}}{\dfrac{(A \Rightarrow (B \Rightarrow C)), (A \Rightarrow B), A \vdash C}{\vdash (A \Rightarrow (B \Rightarrow C)) \wedge (A \Rightarrow B) \wedge A \Rightarrow C}}$$

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+ goal numbering,

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+ proved and pendant goals

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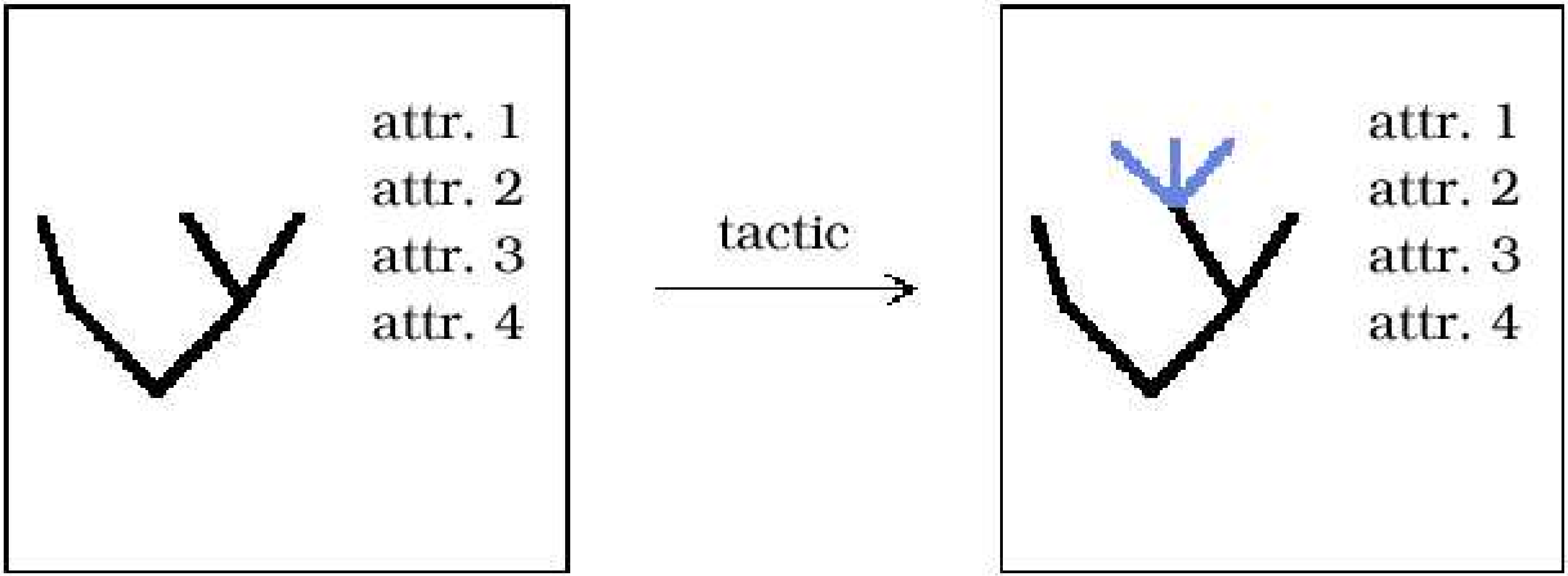
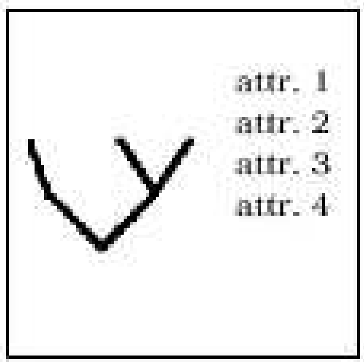
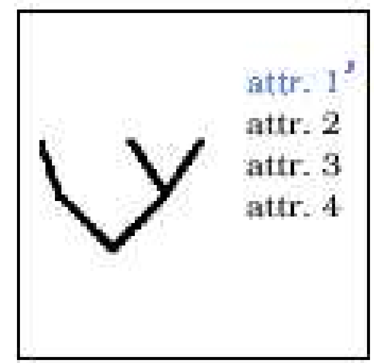
Object


attr. 1
attr. 2
attr. 3
attr. 4
tactic
attr. 1
attr. 2
attr. 3
attr. 4

TACTICALS – STRATEGIES

(if nil (fail) (split)) $\longrightarrow$ **(split)**





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$\rightsquigarrow$ Express the semantics for the tacticals, and provide some examples of simple tactics.

$\rightsquigarrow$ Expose the tacticals' small step semantics within a precise formal framework.

$\rightsquigarrow$ Implementation in Coq and PVS of new tacticals, towards a common basis.

SMALL STEP SEMANTICS

- Proof language $\mathcal{L}$:

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$$\text{term of } \mathcal{T}(\mathcal{L}_{\text{tactiques}}, \mathcal{L}_{\text{tacticals}})$$

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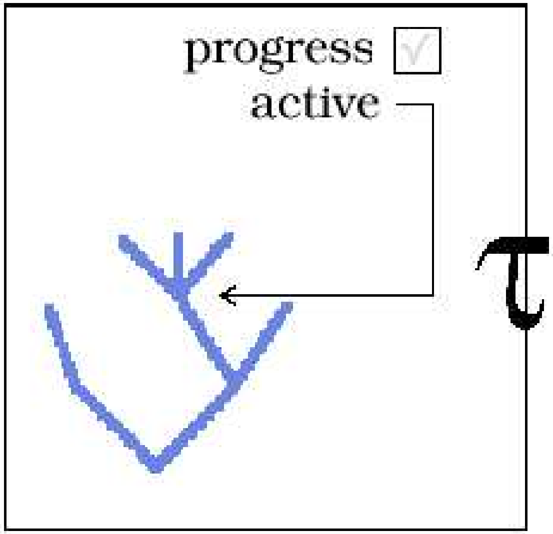

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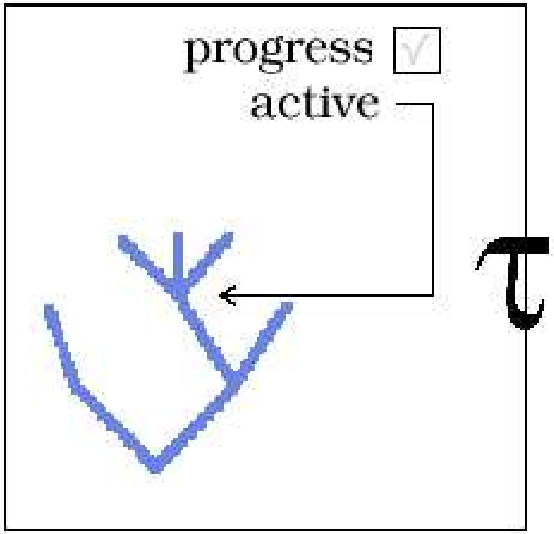

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  - tree of sequents,
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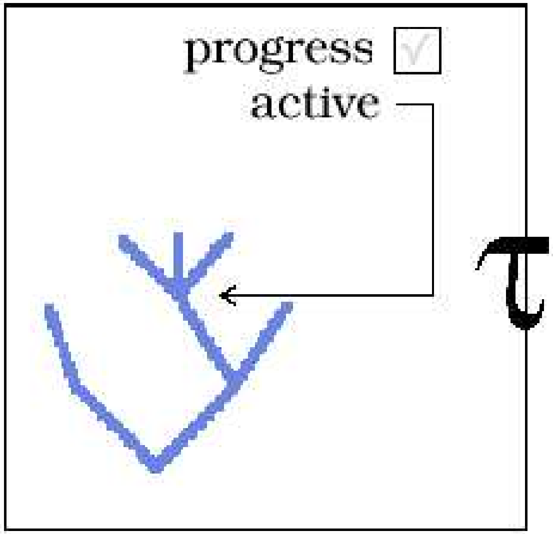
  - progression flag.
- Easy methods.
- Values peculiar to τ:
  $\perp_n$, $\top$, $\varnothing$.



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- Fonctional, total relation.

- τ. $\Gamma \vdash \Delta$ highlights the active sequent.

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$$\texttt{(split)}\%\tau.\ \ \Gamma \vdash A \wedge B\ = $$
$$\tau.\,\mathrm{addLeafs}\ (\Gamma \vdash A,\ \Gamma \vdash B).\,\mathrm{setProgress(true)}$$

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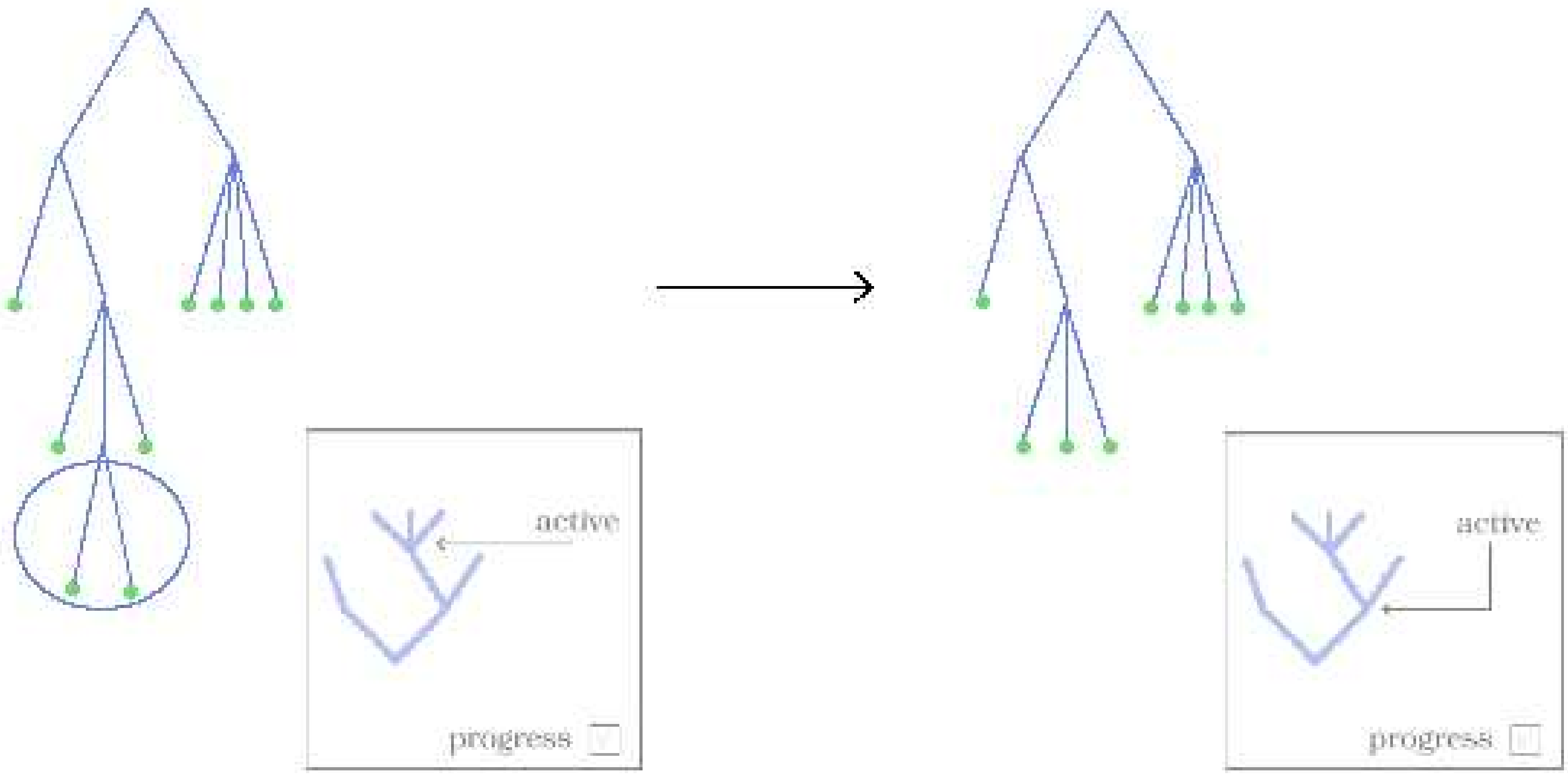
$$\textit{innermost}, \textit{leftmost}\ \&\ \textit{lazy}$$


active
progress
active
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$$(\texttt{if}\ \ t\ \ e_1\ \ e_2)\ /\,\tau \ \xrightarrow{\ \epsilon\ }\ e_2\,/\,\tau \quad \text{if}\ t = \texttt{nil}\ .$$

$$(\texttt{if}\ \ t\ \ e_1\ \ e_2)\ /\,\tau \ \xrightarrow{\ \epsilon\ }\ e_1\,/\,\tau \quad \text{if}\ t \neq \texttt{nil}\ .$$

$$v_1 \; ; e_2 \; / \; \tau \xrightarrow{\epsilon} e_2 \; / \; (v_1 \% \tau) \qquad \textbf{if } \forall n \geq 0 \; (v_1 \% \tau) \neq \bot_n$$
$$\textbf{and } \neg (v_1 \% \tau).\,\textbf{isActiveTreeProven()} \;\; .$$

$$v_1 \; ; e_2 \; / \; \tau \xrightarrow{\epsilon} v_1 \; / \; \tau \qquad \textbf{if } \exists n \geq 0 \; (v_1 \% \tau) = \bot_n$$
$$\textbf{or } (v_1 \% \tau).\,\textbf{isActiveTreeProven()} \;\; .$$

$$\text{First } [e_1|e_2|\ldots|e_n] \;/\; \tau \quad \xrightarrow{\epsilon} \quad \overline{\text{First}_{\mathcal{T}}}\ [e_1|e_2|\ldots|e_n] \;/\; \tau$$

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$$\overline{\text{First}_{\mathcal{T}}}\ [\,] \;/\; \tau\text{'} \quad \xrightarrow{\epsilon} \quad (\text{Fail } 0) \;/\; \tau \quad .$$

$$(\texttt{try}\ e_1\ e_2\ e_3)\ /\ \tau \xrightarrow{\epsilon} (\overline{\textbf{try}}_{\mathcal{T}}\ e_1\ e_2\ e_3)\ /\ \tau$$

$$(\overline{\textbf{try}}_{\mathcal{T}}\ v_1\ e_2\ e_3)\ /\ \tau\text{'} \xrightarrow{\epsilon} (\overline{\textbf{try}}_{\mathcal{T}}\ e_2)\ /\ (v_1\%\tau\text{'})\ \textbf{if}\ (v_1\%\tau\text{'}).\textbf{hasProgressed}$$
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$$(\overline{\textbf{try}}_{\mathcal{T}}\ v)\ /\ \tau\text{'} \xrightarrow{\epsilon} v\ /\ \tau\text{'} \quad \textbf{if}\ \forall n \geq 0\ (v\%\tau\text{'}) \neq \bot_n\ .$$

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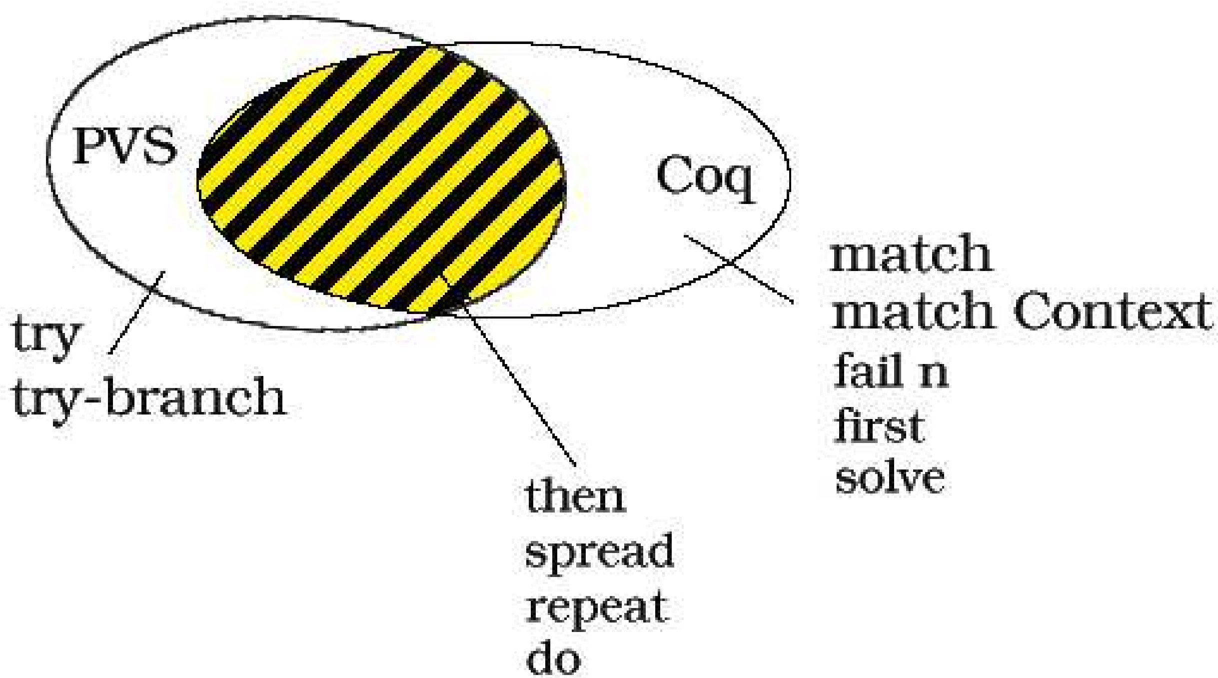
- Integrity through formalization.
- Allows for a comparison between the two languages.



CARRYING OUT

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$\rightsquigarrow$ Towards a coherent programming language

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$\leadsto$ Introduction of sophisticated tacticals.

- Enhancement of the tactical interoperability.

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- Real world testing conditions:

`http://research.nianet.org/fm-at-nia/Practicals/`

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- Continuation of the teams' cooperation.
- Source code maintenance and improvement.

- Gilles Dowek and César Muñoz.
- Alfons Geser, Assia Mahboubi.
- NIA and NASA formal method groups, LogiCal team.
- ENS Cachan, NIA and INRIA.