

4-phase reshufflings with 2 ports

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This document lists 2 port, 4-phase HSE reshufflings and their implementations in PRS. By symmetry, some reshufflings are redundant. Specifically, $[x_i \wedge y_i]$ is the same as $[y_i \wedge x_i]$, and $x_o \uparrow, y_o \uparrow$ is the same as $y_o \uparrow, x_o \uparrow$. In addition, when the ports are both active or both passive, their names are interchangeable, which makes other reshufflings redundant. Redundant reshufflings are not included. Some implementations require a state variable. We use u as our state variable.

1 Active X , Active Y

This combination has 30 orderings.

- 0) $x_o \uparrow; [x_i]; x_o \downarrow; [\neg x_i]; y_o \uparrow; [y_i]; y_o \downarrow; [\neg y_i]$
- 1) $x_o \uparrow; [x_i]; x_o \downarrow, y_o \uparrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 2) $x_o \uparrow; [x_i]; x_o \downarrow, y_o \uparrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 3) $x_o \uparrow; [x_i]; x_o \downarrow, y_o \uparrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 4) $x_o \uparrow; [x_i]; x_o \downarrow, y_o \uparrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 5) $x_o \uparrow; [x_i]; y_o \uparrow, x_o \downarrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 6) $x_o \uparrow; [x_i]; y_o \uparrow, x_o \downarrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 7) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; x_o \downarrow; [\neg x_i]; y_o \downarrow; [\neg y_i]$ ($X \uparrow; Y \uparrow; X \downarrow; Y \downarrow$)
- 8) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; x_o \downarrow, y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 9) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; x_o \downarrow; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 10) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; y_o \downarrow; x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 11) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; y_o \downarrow; [\neg y_i]; x_o \downarrow; [\neg x_i]$
- 12) $x_o \uparrow, y_o \uparrow; [x_i]; x_o \downarrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 13) $x_o \uparrow, y_o \uparrow; [x_i]; x_o \downarrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 14) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow; [\neg x_i]; y_o \downarrow; [\neg y_i]$
- 15) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow, y_o \downarrow; [\neg x_i \wedge \neg y_i]$ ($X \uparrow \parallel Y \uparrow; X \downarrow \parallel Y \downarrow$)
- 16) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 17) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; y_o \downarrow; x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 18) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; y_o \downarrow; [\neg y_i]; x_o \downarrow; [\neg x_i]$
- 19) $x_o \uparrow, y_o \uparrow; [y_i]; y_o \downarrow; [x_i]; x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 20) $x_o \uparrow, y_o \uparrow; [y_i]; y_o \downarrow; [x_i \wedge \neg y_i]; x_o \downarrow; [\neg x_i]$
- 21) $x_o \uparrow, y_o \uparrow; [x_i]; x_o \downarrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 22) $x_o \uparrow, y_o \uparrow; [x_i]; x_o \downarrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 23) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow; [\neg x_i]; y_o \downarrow; [\neg y_i]$
- 24) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow, y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 25) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 26) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; y_o \downarrow; x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 27) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; y_o \downarrow; [\neg y_i]; x_o \downarrow; [\neg x_i]$
- 28) $x_o \uparrow, y_o \uparrow; [y_i]; y_o \downarrow; [x_i]; x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 29) $x_o \uparrow, y_o \uparrow; [y_i]; y_o \downarrow; [x_i \wedge \neg y_i]; x_o \downarrow; [\neg x_i]$

$$\mathbf{7)} \quad x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; x_o \downarrow; [\neg x_i]; y_o \downarrow; [\neg y_i] \quad (X \uparrow; Y \uparrow; X \downarrow; Y \downarrow)$$

$$\begin{array}{ll} \neg y_i \rightarrow x_o \uparrow & x_o \rightarrow y_o \uparrow \\ y_o \rightarrow x_o \downarrow & \neg x_i \rightarrow y_o \downarrow \end{array}$$

$$\mathbf{15)} \quad x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow, y_o \downarrow; [\neg x_i \wedge \neg y_i] \quad (X \uparrow \parallel Y \uparrow; X \downarrow \parallel Y \downarrow)$$

$$\begin{array}{ll} u \rightarrow x_o \uparrow & \neg x_i \wedge \neg y_i \rightarrow u \uparrow \\ \neg u \rightarrow x_o \downarrow & x_i \wedge y_i \rightarrow u \downarrow \end{array}$$

$$\begin{array}{l} u \rightarrow y_o \uparrow \\ \neg u \rightarrow y_o \downarrow \end{array}$$

2 Passive X, Passive Y

This combination has 28 orderings.

- 1) $[x_i]; x_o \uparrow; [\neg x_i]; x_o \downarrow; [y_i]; y_o \uparrow; [\neg y_i]; y_o \downarrow$
 - 2) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; x_o \downarrow; y_o \uparrow; [\neg y_i]; y_o \downarrow$
 - 3) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; x_o \downarrow; y_o \uparrow; [\neg y_i]; y_o \downarrow$
 - 4) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; y_o \uparrow; x_o \downarrow; [\neg y_i]; y_o \downarrow$
 - 5) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; y_o \downarrow$
 - 6) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; y_o \downarrow$
 - 7) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; y_o \uparrow; [\neg y_i]; y_o \downarrow; x_o \downarrow$
 - 8) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg x_i]; x_o \downarrow; [\neg y_i]; y_o \downarrow$ ($X \uparrow; Y \uparrow; X \downarrow; Y \downarrow$)
 - 9) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
 - 10) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
 - 11) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg x_i \wedge \neg y_i]; y_o \downarrow; x_o \downarrow$
 - 12) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; [\neg x_i]; x_o \downarrow$
 - 13) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; x_o \downarrow; y_o \uparrow; [\neg y_i]; y_o \downarrow$
 - 14) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; x_o \downarrow; y_o \uparrow; [\neg y_i]; y_o \downarrow$
 - 15) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; y_o \uparrow; x_o \downarrow; [\neg y_i]; y_o \downarrow$
 - 16) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; y_o \downarrow$
 - 17) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; y_o \downarrow$
 - 18) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; y_o \uparrow; [\neg y_i]; y_o \downarrow; x_o \downarrow$
 - 19) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i]; x_o \downarrow; [\neg y_i]; y_o \downarrow$
 - 20) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$ ($X \uparrow \star Y \uparrow; X \downarrow \star Y \downarrow$)
 - 21) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
 - 22) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; y_o \downarrow; x_o \downarrow$
 - 23) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg y_i]; y_o \downarrow; [\neg x_i]; x_o \downarrow$
 - 24) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i]; x_o \downarrow; [\neg y_i]; y_o \downarrow$
 - 25) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
 - 26) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
 - 27) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; y_o \downarrow; x_o \downarrow$
 - 28) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg y_i]; y_o \downarrow; [\neg x_i]; x_o \downarrow$
- 8) $[xi]; x_o \uparrow; [yi]; y_o \uparrow; [\neg xi]; x_o \downarrow; [\neg yi]; y_o \downarrow$ ($X \uparrow; Y \uparrow; X \downarrow; Y \downarrow$)
- | | |
|--|---|
| $x_i \wedge \neg y_o \rightarrow x_o \uparrow$ | $y_i \wedge x_o \rightarrow y_o \uparrow$ |
| $\neg x_i \wedge y_o \rightarrow x_o \downarrow$ | $\neg y_i \wedge \neg x_o \rightarrow y_o \downarrow$ |
- 20) $[xi \wedge yi]; x_o \uparrow, y_o \uparrow; [\neg xi \wedge \neg yi]; x_o \downarrow, y_o \downarrow$ ($X \uparrow \star Y \uparrow; X \downarrow \star Y \downarrow$)
- | | |
|-----------------------------------|---|
| $\neg u \rightarrow y_o \uparrow$ | $x_i \wedge y_i \rightarrow u \downarrow$ |
| $u \rightarrow y_o \downarrow$ | $\neg x_i \wedge \neg y_i \rightarrow u \uparrow$ |
-
- | |
|-----------------------------------|
| $\neg u \rightarrow x_o \uparrow$ |
| $u \rightarrow x_o \downarrow$ |

$$\mathbf{16)} \quad a_o \uparrow; [pi]; p_o \uparrow; [ai]; a_o \downarrow; [\neg pi]; p_o \downarrow; [\neg ai] \quad (A \uparrow \parallel P \uparrow; A \downarrow \parallel P \downarrow)$$

$$\begin{array}{ll} \neg a_i \wedge \neg p_o \rightarrow a_o \uparrow & a_o \wedge p_i \rightarrow p_o \uparrow \\ a_i \wedge p_o \rightarrow a_o \downarrow & \neg a_o \wedge \neg p_i \rightarrow p_o \downarrow \end{array}$$

$$\mathbf{29)} \quad [pi]; a_o \uparrow; [ai]; p_o \uparrow; [\neg pi]; a_o \downarrow; [\neg ai]; p_o \downarrow \quad (P \uparrow \star A \uparrow; P \downarrow \star A \downarrow)$$

$$\begin{array}{ll} p_i \rightarrow a_o \uparrow & a_i \rightarrow p_o \uparrow \\ \neg p_i \rightarrow a_o \downarrow & \neg a_i \rightarrow p_o \downarrow \end{array}$$

$$\mathbf{36)} \quad [pi]; a_o \uparrow; p_o \uparrow; [ai \wedge \neg pi]; a_o \downarrow; p_o \downarrow; [\neg ai] \quad (P \uparrow; A \uparrow; P \downarrow; A \downarrow)$$

$$\begin{array}{ll} u \rightarrow a_o \uparrow & \neg a_i \wedge p_i \rightarrow u \uparrow \\ \neg u \rightarrow a_o \downarrow & a_i \wedge \neg p_i \rightarrow u \downarrow \end{array}$$

$$\begin{array}{l} u \rightarrow p_o \uparrow \\ u \rightarrow p_o \downarrow \end{array}$$

$$\mathbf{44)} \quad [pi]; a_o \uparrow; p_o \uparrow; [ai \wedge \neg pi]; a_o \downarrow; p_o \downarrow; [\neg ai] \quad (\mathbf{WCHB})$$

$$\begin{array}{ll} \neg a_i \wedge p_i \rightarrow a_o \uparrow & a_o \rightarrow p_o \uparrow \\ a_i \wedge \neg p_i \rightarrow a_o \downarrow & \neg a_o \rightarrow p_o \downarrow \end{array}$$

This is known as the weak-condition logic half buffer (WCHB).

$$\mathbf{41)} \quad [pi]; a_o \uparrow; p_o \uparrow; [ai]; a_o \downarrow; [\neg pi]; p_o \downarrow; [\neg ai] \quad (\mathbf{PCHB})$$

$$\begin{array}{ll} \neg a_i \wedge p_i \wedge \neg p_o \rightarrow a_o \uparrow & a_o \rightarrow p_o \uparrow \\ p_o \wedge a_i \rightarrow a_o \downarrow & \neg a_o \wedge \neg p_i \rightarrow p_o \downarrow \end{array}$$

This is known as the pre-charge half buffer (PCHB).