

4-phase reshufflings with 2 ports

September 15, 2015

This document lists 2 port, 4-phase HSE reshufflings and their implementations in PRS. By symmetry, some reshufflings are redundant. Specifically, $[x_i \wedge y_i]$ is the same as $[y_i \wedge x_i]$, and $x_o \uparrow, y_o \uparrow$ is the same as $y_o \uparrow, x_o \uparrow$. In addition, when the ports are both active or both passive, their names are interchangeable, which makes other reshufflings redundant. Redundant reshufflings are not included. Some implementations require a state variable. We use u as our state variable.

1 Active X , Active Y

This combination has 30 orderings.

- 0) $x_o \uparrow; [x_i]; x_o \downarrow; [\neg x_i]; y_o \uparrow; [y_i]; y_o \downarrow; [\neg y_i]$
- 1) $x_o \uparrow; [x_i]; x_o \downarrow, y_o \uparrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 2) $x_o \uparrow; [x_i]; x_o \downarrow, y_o \uparrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 3) $x_o \uparrow; [x_i]; x_o \downarrow, y_o \uparrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 4) $x_o \uparrow; [x_i]; x_o \downarrow, y_o \uparrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 5) $x_o \uparrow; [x_i]; y_o \uparrow, x_o \downarrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 6) $x_o \uparrow; [x_i]; y_o \uparrow, x_o \downarrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 7) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; x_o \downarrow; [\neg x_i]; y_o \downarrow; [\neg y_i]$
- 8) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; x_o \downarrow, y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 9) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; x_o \downarrow; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 10) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; y_o \downarrow, x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 11) $x_o \uparrow; [x_i]; y_o \uparrow; [y_i]; y_o \downarrow; [\neg y_i]; x_o \downarrow; [\neg x_i]$
- 12) $x_o \uparrow, y_o \uparrow; [x_i]; x_o \downarrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 13) $x_o \uparrow, y_o \uparrow; [x_i]; x_o \downarrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 14) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow; [\neg x_i]; y_o \downarrow; [\neg y_i]$
- 15) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow, y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 16) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 17) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; y_o \downarrow, x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 18) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; y_o \downarrow; [\neg y_i]; x_o \downarrow; [\neg x_i]$
- 19) $x_o \uparrow, y_o \uparrow; [y_i]; y_o \downarrow; [x_i]; x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 20) $x_o \uparrow, y_o \uparrow; [y_i]; y_o \downarrow; [x_i \wedge \neg y_i]; x_o \downarrow; [\neg x_i]$
- 21) $x_o \uparrow, y_o \uparrow; [x_i]; x_o \downarrow; [\neg x_i \wedge y_i]; y_o \downarrow; [\neg y_i]$
- 22) $x_o \uparrow, y_o \uparrow; [x_i]; x_o \downarrow; [y_i]; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 23) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow; [\neg x_i]; y_o \downarrow; [\neg y_i]$
- 24) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow, y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 25) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; x_o \downarrow; y_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 26) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; y_o \downarrow, x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 27) $x_o \uparrow, y_o \uparrow; [x_i \wedge y_i]; y_o \downarrow; [\neg y_i]; x_o \downarrow; [\neg x_i]$
- 28) $x_o \uparrow, y_o \uparrow; [y_i]; y_o \downarrow; [x_i]; x_o \downarrow; [\neg x_i \wedge \neg y_i]$
- 29) $x_o \uparrow, y_o \uparrow; [y_i]; y_o \downarrow; [x_i \wedge \neg y_i]; x_o \downarrow; [\neg x_i]$

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$$\begin{array}{l} \neg y_i \rightarrow x_o \uparrow \\ y_o \rightarrow x_o \downarrow \end{array}$$

$$\begin{array}{l} x_o \rightarrow y_o \uparrow \\ \neg x_i \rightarrow y_o \downarrow \end{array}$$

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$$\begin{array}{l} u \rightarrow x_o \uparrow \\ \neg u \rightarrow x_o \downarrow \end{array}$$

$$\begin{array}{l} \neg x_i \wedge \neg y_i \rightarrow u \uparrow \\ x_i \wedge y_i \rightarrow u \downarrow \end{array}$$

$$\begin{array}{l} u \rightarrow y_o \uparrow \\ \neg u \rightarrow y_o \downarrow \end{array}$$

2 Passive X, Passive Y

This combination has 28 orderings.

- 1) $[x_i]; x_o \uparrow; [\neg x_i]; x_o \downarrow; [y_i]; y_o \uparrow; [\neg y_i]; y_o \downarrow$
- 2) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; x_o \downarrow; y_o \uparrow; [\neg y_i]; y_o \downarrow$
- 3) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; x_o \downarrow; y_o \uparrow; [\neg y_i]; y_o \downarrow$
- 4) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; y_o \uparrow; x_o \downarrow; [\neg y_i]; y_o \downarrow$
- 5) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; y_o \downarrow$
- 6) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; y_o \downarrow$
- 7) $[x_i]; x_o \uparrow; [\neg x_i \wedge y_i]; y_o \uparrow; [\neg y_i]; y_o \downarrow; x_o \downarrow$
- 8) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg x_i]; x_o \downarrow; [\neg y_i]; y_o \downarrow$
- 9) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
- 10) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
- 11) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg x_i \wedge \neg y_i]; y_o \downarrow; x_o \downarrow$
- 12) $[x_i]; x_o \uparrow; [y_i]; y_o \uparrow; [\neg y_i]; y_o \downarrow; [\neg x_i]; x_o \downarrow$
- 13) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; x_o \downarrow; y_o \uparrow; [\neg y_i]; y_o \downarrow$
- 14) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; x_o \downarrow; y_o \uparrow; [\neg y_i]; y_o \downarrow$
- 15) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; y_o \uparrow; x_o \downarrow; [\neg y_i]; y_o \downarrow$
- 16) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; y_o \downarrow$
- 17) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; y_o \uparrow; [\neg y_i]; x_o \downarrow; y_o \downarrow$
- 18) $[x_i \wedge y_i]; x_o \uparrow; [\neg x_i]; y_o \uparrow; [\neg y_i]; y_o \downarrow; x_o \downarrow$
- 19) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i]; x_o \downarrow; [\neg y_i]; y_o \downarrow$
- 20) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
- 21) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
- 22) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; y_o \downarrow; x_o \downarrow$
- 23) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg y_i]; y_o \downarrow; [\neg x_i]; x_o \downarrow$
- 24) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i]; x_o \downarrow; [\neg y_i]; y_o \downarrow$
- 25) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
- 26) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; x_o \downarrow; y_o \downarrow$
- 27) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg x_i \wedge \neg y_i]; y_o \downarrow; x_o \downarrow$
- 28) $[x_i \wedge y_i]; x_o \uparrow; y_o \uparrow; [\neg y_i]; y_o \downarrow; [\neg x_i]; x_o \downarrow$

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$$\begin{array}{ll} x_i \wedge \neg y_o \rightarrow x_o \uparrow & y_i \wedge x_o \rightarrow y_o \uparrow \\ \neg x_i \wedge y_o \rightarrow x_o \downarrow & \neg y_i \wedge \neg x_o \rightarrow y_o \downarrow \end{array}$$

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$$\begin{array}{ll} \neg u \rightarrow y_o \uparrow & x_i \wedge y_i \rightarrow u \downarrow \\ u \rightarrow y_o \downarrow & \neg x_i \wedge \neg y_i \rightarrow u \uparrow \end{array}$$

$$\begin{array}{l} \neg u \rightarrow x_o \uparrow \\ u \rightarrow x_o \downarrow \end{array}$$

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$$\begin{array}{l} p_i \rightarrow a_o \uparrow \\ \neg p_i \rightarrow a_o \downarrow \end{array}$$

$$\begin{array}{l} a_i \rightarrow p_o \uparrow \\ \neg a_i \rightarrow p_o \downarrow \end{array}$$

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$$\begin{array}{l} u \rightarrow a_o \uparrow \\ \neg u \rightarrow a_o \downarrow \end{array}$$

$$\begin{array}{l} \neg a_i \wedge p_i \rightarrow u \uparrow \\ a_i \wedge \neg p_i \rightarrow u \downarrow \end{array}$$

$$\begin{array}{l} u \rightarrow p_o \uparrow \\ u \rightarrow p_o \downarrow \end{array}$$