

HW5 三次样条插值

王启骅 PB20020580

2022 年 10 月 27 日

1 实验结果分析

利用大 M 法，并用追赶法求解线性方程组得到的结果代入多项式方程 $S(x)$ 后解出表达式为图 1

```
求解得到多项式如下
S(x)=(-0.254241)x^3+(-6.864495)x^2+(-60.739813)x+(-175.848596), -9.000000<x<-8.000000
S(x)=(0.203803)x^3+(4.128545)x^2+(27.204504)x+(58.669582), -8.000000<x<-7.000000
S(x)=(0.252530)x^3+(5.151807)x^2+(34.367338)x+(75.382862), -7.000000<x<-6.000000
S(x)=(-0.244121)x^3+(-3.787903)x^2+(-19.270921)x+(-31.893655), -6.000000<x<-5.000000
S(x)=(0.095455)x^3+(1.305731)x^2+(6.197247)x+(10.553291), -5.000000<x<-4.000000
S(x)=(-0.082897)x^3+(-0.834490)x^2+(-2.363637)x+(-0.861221), -4.000000<x<-3.000000
S(x)=(-0.309266)x^3+(-2.871808)x^2+(-8.475589)x+(-6.973173), -3.000000<x<-2.000000
S(x)=(0.908460)x^3+(4.434548)x^2+(6.137123)x+(2.768635), -2.000000<x<-1.000000
S(x)=(-0.889575)x^3+(-0.959557)x^2+(0.743018)x+(0.970600), -1.000000<x<0.000000
S(x)=(0.203139)x^3+(-0.959557)x^2+(0.743018)x+(0.970600), 0.000000<x<1.000000
S(x)=(0.445017)x^3+(-1.685190)x^2+(1.468651)x+(0.728722), 1.000000<x<2.000000
S(x)=(-0.738108)x^3+(5.413561)x^2+(-12.728853)x+(10.193725), 2.000000<x<3.000000
S(x)=(0.747415)x^3+(-7.956150)x^2+(27.380281)x+(-29.915409), 3.000000<x<4.000000
S(x)=(-0.339953)x^3+(5.092273)x^2+(-24.813410)x+(39.676179), 4.000000<x<5.000000
S(x)=(-0.111902)x^3+(1.671511)x^2+(-7.709598)x+(11.169826), 5.000000<x<6.000000
S(x)=(0.079663)x^3+(-1.776662)x^2+(12.979437)x+(-30.208244), 6.000000<x<7.000000
S(x)=(0.163651)x^3+(-3.540425)x^2+(25.325781)x+(-59.016381), 7.000000<x<8.000000
S(x)=(-0.319969)x^3+(8.066459)x^2+(-67.529290)x+(188.597143), 8.000000<x<9.000000
S(x)=(0.354323)x^3+(-10.139426)x^2+(96.323667)x+(-302.961730), 9.000000<x<10.000000
S(x)=(-0.163425)x^3+(5.393014)x^2+(-59.000726)x+(214.786248), 10.000000<x<11.000000
```

图 1: $S(x)$

改变第 10 个压铁坐标后求解得到的 $S(x)$ 表达式为图 2

将两次结果画图得到图 3

根据计算结果和绘制出的曲线图可以得到，在第十个压铁的邻近两个区间中的函数以及趋势有明显的变化，而到了相隔一个区间的位置，两次 S 的差异已经明显缩小，趋势已经基本一致。到了相隔 3 个区间以上时，可以发现两次的拟合函数之间基本已经不存在差异。由此可见三次样条插值局部较为独立稳定。

改变第10个压铁点坐标后的多项式如下

$S(x) = (-0.254626)x^3 + (-6.874910)x^2 + (-60.833159)x + (-176.126320), -9.000000 < x < -8.000000$
 $S(x) = (0.205731)x^3 + (4.173675)x^2 + (27.555517)x + (59.576816), -8.000000 < x < -7.000000$
 $S(x) = (0.245201)x^3 + (5.002530)x^2 + (33.357500)x + (73.114776), -7.000000 < x < -6.000000$
 $S(x) = (-0.216734)x^3 + (-3.312299)x^2 + (-16.531474)x + (-26.663171), -6.000000 < x < -5.000000$
 $S(x) = (-0.006764)x^3 + (-0.162739)x^2 + (-0.783675)x + (-0.416839), -5.000000 < x < -4.000000$
 $S(x) = (0.298589)x^3 + (3.501488)x^2 + (13.873236)x + (19.125709), -4.000000 < x < -3.000000$
 $S(x) = (-1.732991)x^3 + (-14.782731)x^2 + (-40.979422)x + (-35.726949), -3.000000 < x < -2.000000$
 $S(x) = (6.221876)x^3 + (32.946473)x^2 + (54.478985)x + (27.911989), -2.000000 < x < -1.000000$
 $S(x) = (-11.690113)x^3 + (-20.789495)x^2 + (0.743018)x + (10.000000), -1.000000 < x < 0.000000$
 $S(x) = (11.003678)x^3 + (-20.789495)x^2 + (0.743018)x + (10.000000), 0.000000 < x < 1.000000$
 $S(x) = (-4.868399)x^3 + (26.826735)x^2 + (-46.873213)x + (25.872077), 1.000000 < x < 2.000000$
 $S(x) = (0.685617)x^3 + (-6.497362)x^2 + (19.774982)x + (-18.560053), 2.000000 < x < 3.000000$
 $S(x) = (0.365929)x^3 + (-3.620169)x^2 + (11.143402)x + (-9.928473), 3.000000 < x < 4.000000$
 $S(x) = (-0.237734)x^3 + (3.623794)x^2 + (-17.832449)x + (28.705995), 4.000000 < x < 5.000000$
 $S(x) = (-0.139292)x^3 + (2.147157)x^2 + (-10.449264)x + (16.400686), 5.000000 < x < 6.000000$
 $S(x) = (0.087002)x^3 + (-1.926127)x^2 + (13.990437)x + (-32.478717), 6.000000 < x < 7.000000$
 $S(x) = (0.161685)x^3 + (-3.494481)x^2 + (24.968915)x + (-58.095165), 7.000000 < x < 8.000000$
 $S(x) = (-0.319442)x^3 + (8.052584)x^2 + (-67.407603)x + (188.242216), 8.000000 < x < 9.000000$
 $S(x) = (0.354185)x^3 + (-10.135355)x^2 + (96.283843)x + (-302.832122), 9.000000 < x < 10.000000$
 $S(x) = (-0.163397)x^3 + (5.392100)x^2 + (-58.990701)x + (214.749692), 10.000000 < x < 11.000000$

图 2: $S(x)$

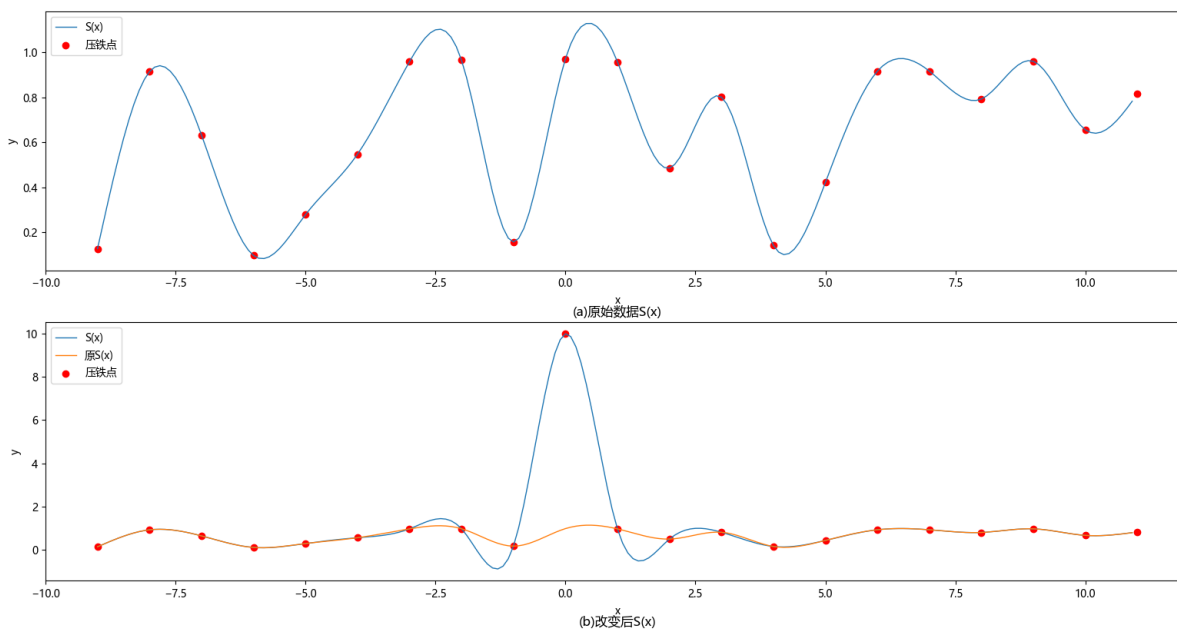


图 3: 压铁结果曲线