Linear Data Chapter 8

Written by: Emily J. King

1. Assume $\vec{x}, \vec{y} \in \mathbb{R}^{1001}$ and you know the following:

$$\|\vec{x}\| = 4, \quad \vec{y} = 0.5\vec{x}.$$

For each of the following, explicitly compute the value.

(a) Explicitly compute $\|\vec{y}\|$. Explain your answer.
The norm is absolutely homogenous:

$$\|\vec{y}\| = |0.5| \, \|\vec{x}\| = 0.5(4) = 2$$

(b) Explicitly compute the cosine similarity of $\vec{x}$ and $\vec{y}$. Explain your answer.
$\vec{x}$ and $\vec{y}$ are pointing in the exact same direction since $\vec{y}$ is a positive scalar multiple of $\vec{x}$. Thus, the cosine similarity is equal to $1 = \cos(0)$.

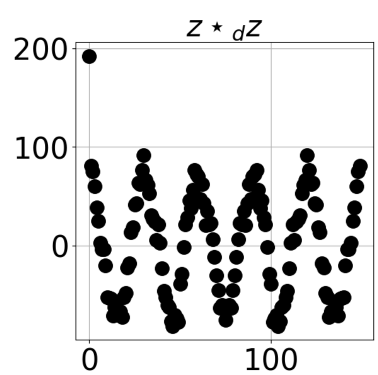
(c) Explicitly compute $\langle \vec{x}, \vec{y} \rangle$. Explain your answer.
[METHOD ONE]

$$\langle \vec{x}, \vec{y} \rangle = \langle \vec{x}, 0.5\vec{x} \rangle = 0.5 \langle \vec{x}, \vec{x} \rangle = 0.5 \|\vec{x}\|^2 = 0.5(16) = 8.$$

[METHOD TWO]

$$\langle \vec{x}, \vec{y} \rangle = \|\vec{x}\| \, \|\vec{y}\| \cos(\theta_{\vec{x},\vec{y}}) = 4(2)(1) = 8.$$

2. The following plot is the autocorrelation of a vector $\vec{z} \in \mathbb{R}^{150}$.



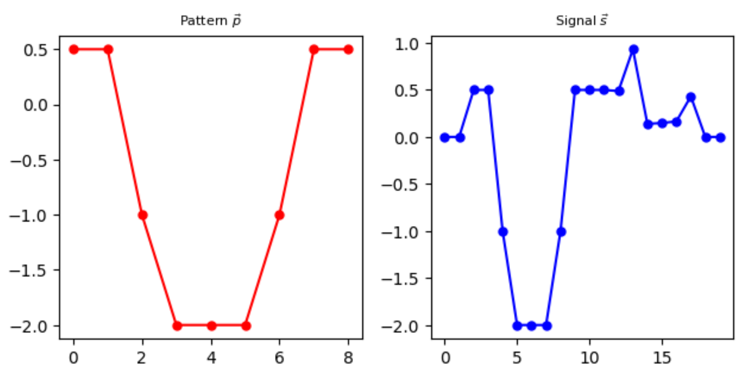
Given that information, write in complete sentences as many properties of $\vec{z}$ as you can.

About every 30 points there is a local maximum in the autocorrelation, suggesting that z is approximately periodic with a period of 30 points (alternatively: has hidden periodic structure with a period of 30 points). The value of an autocorrelation at zero is always at least as big as all other points because a vector matches itself best when it isn't shifted at all; if a vector were perfectly periodic, then there would be other autocorrelation values which are equally as high as the value at 0. This is not the case here, suggesting that z is a vector with approximately a period of 30 points with a high amount of added noise. 6 points for having some periodic structure, 1 additional point for period = 30.

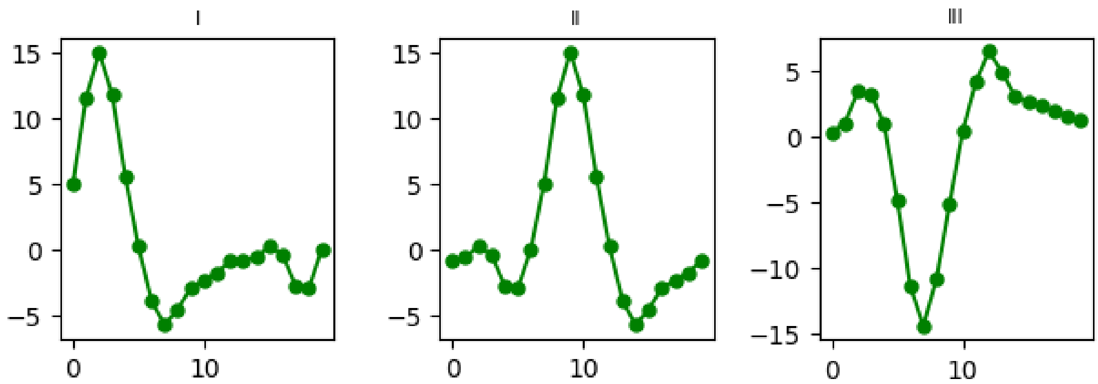
3. Explain the different goals of cross-correlation/convolution of an image with a Gaussian kernel versus a Sobel filter.

   Cross-correlation/convolution of an image with a Gaussian filter is done to smooth and in part to denoise. Typically the filter is chosen small enough that key features aren't blurred out, but one may also choose a larger filter to intentionally blur an image. Cross-correlation with Sobel filters is performed to detect vertical and horizontal edges in images by approximating horizontal, respectively, vertical derivatives. Trigonometry may be used to detect edges at other orientations.

4. Consider the pattern vector $\vec{p}$ and signal vector $\vec{s}$ pictured below



What is most likely the cross-correlation $\vec{p} \star \vec{s}$? You MUST justify your answer to receive ANY points.



$\vec{s}$ contains an exact copy of $\vec{p}$ two shifts to the right; so the only possible answer is I, which has a local (actually, global) maximum two positions to the right.