Linear Data Chapter 3

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1. Consider the following vectors

$$\vec{x} = \begin{pmatrix} 0.1 \\ 3.2 \\ 5.4 \\ 6.0 \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad \vec{z} = \begin{pmatrix} 3 \\ 3 \\ 0.1 \end{pmatrix}$$

For each of exercises a–e, either compute the desired quantity by hand with work shown or explain why the desired quantity is not defined.

(a) $10\vec{x}$

$$10 \begin{pmatrix} 0.1 \\ 3.2 \\ 5.4 \\ 6.0 \end{pmatrix} = \begin{pmatrix} 10(0.1) \\ 10(3.2) \\ 10(5.4) \\ 10(6.0) \end{pmatrix} = \begin{pmatrix} 1 \\ 32 \\ 54 \\ 60 \end{pmatrix}$$

(b) $10\vec{x} - 2\vec{y}$

Using the answer from part a,

$$10\vec{x} - 2\vec{y} = \begin{pmatrix} 1 \\ 32 \\ 54 \\ 60 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ (-1)(-2) \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 32 \\ 54 \\ 60 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 32 \\ 56 \\ 60 \end{pmatrix}$$

(c) $\vec{y} + \vec{z}$

Not defined since the vectors have a different number of entries.

(d) $\langle \vec{x}, \vec{y} \rangle$

$$0.1(0) + 3.2(0) + 5.4(-1) + 6.0(0) = -5.4$$

(e) $\langle \vec{x}, \vec{z} \rangle$

Not defined since the vectors have a different number of entries.

2. Give an example of a linear combination which is not feasible for a particular data problem.
   Many possibilities. One of the examples from lecture is that adding rows of a bus schedule is not informative for a problem involving bus schedules, thus is not feasible. Another example from the lecture is using a negative coefficient in a linear combination of vectors of recipe ingredients in any problem about creating real food.

3. Apply the IVORY Heuristic to a grade book (grid of numbers with rows corresponding to students and columns corresponding to graded assignments).
   One example

   Issue *Determine the average grades for each assignment.*

   Vector *Rows of the grade book.*

   Operators *Adding and scaling the rows.*

   Recipe *Take the average of the rows.*

   Yes *the sample recipe is informative, i.e., it yields the average grades for each assignment.*

   Another example

   Issue *Determine each student's final grade.*

   Vector *Columns of the grade book.*

   Operators *Adding and scaling the rows.*

   Recipe *Take the weighted average of the columns with the weights coming from the weighting of the different assignments.*

   Yes *the sample recipe is informative, i.e., it yields the final grades for each student.*

   But there is flexibility, depending on the issue decided and the vectors chosen.

4. Consider the following figure[1]

[1]Source: Big Bend's Aerosol and Extinction Budgets during BRAVO, `https://vista.cira.colostate.edu/Improve/wp-content/uploads/2016/05/3_BigBendBxtBudge1.pdf`, Accessed: 02/03/2024

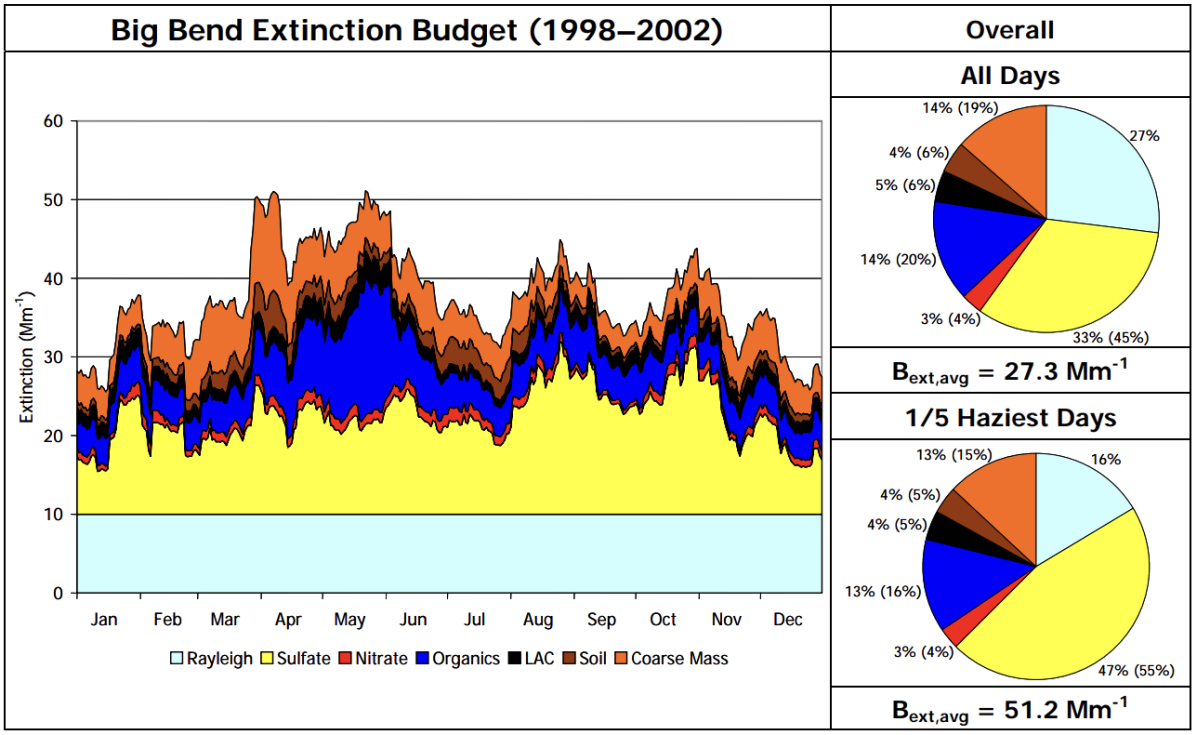

Figure 3-12. Big Bend National Park five-year light extinction budget. All days that fall on the same day of the year were averaged together, then the data were smoothed using a 15-day moving average.

from a report by Colorado State University's Cooperative Institute for Research in the Atmosphere (CIRA) on research to understand aerosols in the atmosphere at Big Bend National Park.

(a) Explain which of the the following linear algebra operations (scalar multiplication, vector addition, linear combination, inner product) was definitely used to generate the figure. You do not need any further information than what is shown in the figure.
Inner products are used to compute the moving average.

(b) Explain what benefit there is to using that linear algebra operation the way the researchers did.
Moving averages smooth out data to get rid of small fluctuations.