Linear Data Chapter 6

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1. For each of the following subsets of $\mathbb{R}^3$, explain whether or not they are a subspace of $\mathbb{R}^3$.

   (a)
   $$U = \text{span}\left\{ \begin{pmatrix} 1.1 \\ -3.4 \\ 0.4 \end{pmatrix}, \begin{pmatrix} 0.65 \\ 0.23 \\ -0.44 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

   (b)
   $$V = \left\{ \begin{pmatrix} a \\ 0 \\ a^3 \end{pmatrix} \middle| \, a \in \mathbb{R} \right\}$$

   (c) $Z=$ the points in the z-axis

2. Assume that $f : \mathbb{R}^{100} \to \mathbb{R}^2$ is linear and that for certain $\vec{u}$, $\vec{v} \in \mathbb{R}^{100}$,
   $$f(\vec{u}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{and} \quad f(\vec{v}) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}.$$
   Explicitly compute with work the following:

   (a) $f(\vec{u} + \vec{v})$

   (b) $f(10\vec{v})$

3. Give an example of an application of a linear transformation to audio signals.

4. Assume that W is a vector space and $g, h : W \to \mathbb{R}$ are both linear maps. Show that the function
   $$k : W \to \mathbb{R}^2, \quad k(w) = \begin{pmatrix} g(w) \\ h(w) \end{pmatrix}$$
   is linear.