Linear Data Chapter 4

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1. If noise-canceling headphones estimate $\vec{s}$ to be the background sound, what do the headphones generate to cancel out the noise?
   $-\vec{s}$ since $\vec{s} + (-\vec{s}) = \vec{0}$.

2. Explicitly compute by hand (with work shown) the following Frobenius inner products

   (a) $\left\langle \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 4.56 & 3.12 & -1 \\ 10.9 & 0 & 5 \end{pmatrix} \right\rangle_{Fro}$
   You could either note that each term in the sum is a product with zero and thus the total inner product is zero or compute

   $$\left\langle \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 4.56 & 3.12 & -1 \\ 10.9 & 0 & 5 \end{pmatrix} \right\rangle_{Fro} = 0(4.56) + 0(3.12) + 0(-1) + 0(10.9) + 0(0) + 5(0)$$

   Vectorizing first and then computing the (Euclidean) inner product is also acceptable.

   (b) $\left\langle \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 5 & 2 \end{pmatrix} \right\rangle_{Fro}$

   $$\left\langle \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 5 & 2 \end{pmatrix} \right\rangle_{Fro} = 1(-1) + (-2)(1) + 0(5) + 1(2) = -1.$$

   Vectorizing first and then computing the (Euclidean) inner product is also acceptable.

3. Give an example of an application of hyperspectral imaging.
   Determining what species of plants are in a particular area. More generally, finding what materials are in a particular area (pavement, underground water, hidden tanks, etc.)

4. In 2017, the City of Fort Collins ran a study[1] to better understand bat populations in the city's parks and natural areas. The screenshot of a spectrogram of bat calls at a location in Fort Collins shown below appeared in that study.

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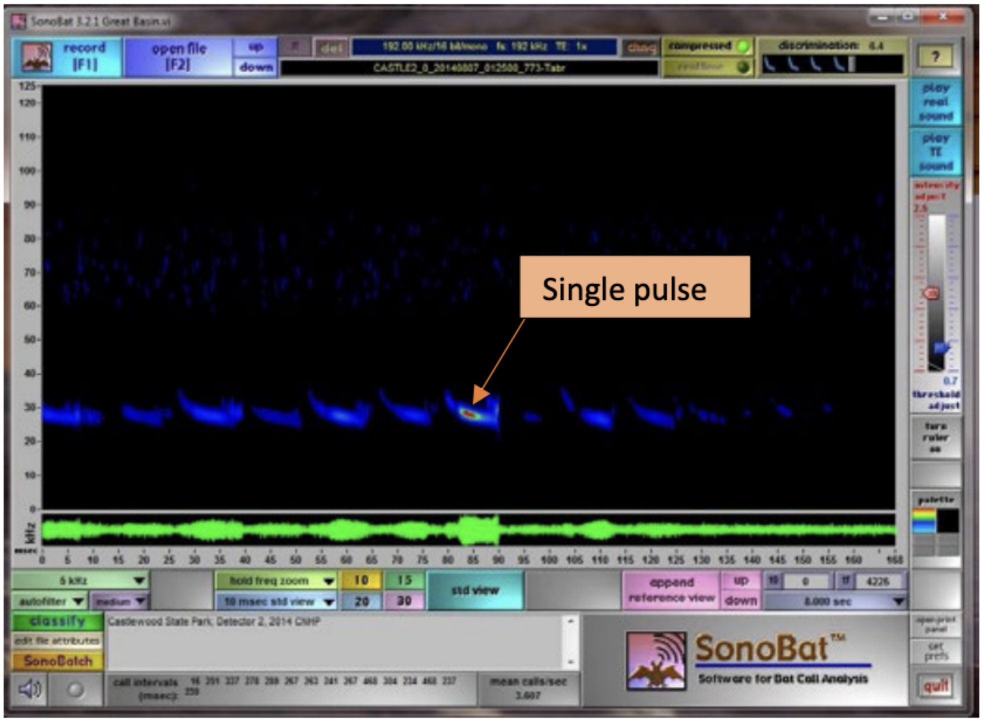
[1] Source: `https://www.fcgov.com/naturalareas/files/fort-collins-natural-area-bat-surveys-report_2017opt.pdf`, accessed 2024.02.18



- Explain what kind of information a spectrogram tells you about data. Discuss why that might be pertinent in the study of bats.
  A spectrogram gives information about approximately which frequencies are present at which times. Determining frequencies made by different bats at different times can help understand bats better.
- Explain how a spectrogram relates to one or more of the basic linear algebra operations (scalar multiplication, vector addition, linear combination, inner product).

  A spectrogram is a heat map showing linear combination coefficients approximating snapshots in time of the signal as linear combinations of sinusoids.
  (We will learn later than inner products are used to compute the coefficients of the inner products.)

5. Using Python/Jupyter or Matlab/Matlab Live Script, perform the following:

- Load two song vectors of the same length $\vec{x}$ and $\vec{y}$, either from the file provided during the lab or your own.
  Matlab, provided files:
  ```
  load('LinearDatasound.mat')
  ```
  Matlab, own files:
  ```
  [x,xFs] = audioread('youraudio.wav');
  L = 10e5+1;
  ```

```
x=x(1:L,:);
```
(and similar for `y`*)*

Python, provided files:
```
import numpy as np
npzfile=np.load('LinearDatasound.npz')
locals().update(npzfile)
```

Python, own files:
```
import numpy as np
import audio2numpy as a2n
x,xsr=a2n.audio_from_file("youraudio.mp3")
```
(and similar for `y`*)*

- Set $\vec{z}$ to be a linear combination of $\vec{x}$ and $\vec{y}$ where the coefficient for $\vec{x}$ makes it quieter and the coefficient for $\vec{y}$ makes it louder.
  Matlab:
  ```
  z = 0.5*x + 2*y;
  ```
  (Any coefficient strictly between 0 *and* 1 *for* $\vec{x}$ *and strictly greater than* 1 *for* $\vec{y}$ *would work.)*

  Python:
  ```
  z = 0.5*x + 2*y
  ```
  (Any coefficient strictly between 0 *and* 1 *for* $\vec{x}$ *and strictly greater than* 1 *for* $\vec{y}$ *would work.)*