

Linear Data Chapter 5

Written by: Emily J. King

1. For the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

explicitly compute by hand (with work shown) the following.

- (a) $\mathbf{I}_2 \mathbf{A}$, where \mathbf{I}_2 is the 2×2 identity matrix.
- (b) \mathbf{A}^{-1}
- (c) \mathbf{A}^2
- (d) \mathbf{A}^{-2}
- (e) the diagonal of \mathbf{A}

2. Let

$$\mathbf{D} = \begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -10.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.8 \end{pmatrix}$$

Either explicitly compute \mathbf{D}^{-1} or explain why it doesn't exist.

3. Assume that \mathbf{B} , \mathbf{C} , \mathbf{E} are all 3×3 matrices such that

$$\mathbf{BC} = \begin{pmatrix} -6 & 4 & 4 \\ -1 & 0 & 3 \\ 3 & 2 & -7 \end{pmatrix}, \quad \mathbf{CB} = \begin{pmatrix} -5 & 4 & 4 \\ -1 & -2 & 4 \\ 1 & 6 & -6 \end{pmatrix}, \quad \mathbf{BE} = \begin{pmatrix} -2 & 2 & 4 \\ -2 & 1 & 3 \\ 2 & -5 & -7 \end{pmatrix}, \quad \mathbf{EC} = \begin{pmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ -2 & 0 & 4 \end{pmatrix}$$

Explicitly compute the following by hand. (I.e., write out the entries of the 3×3 matrix.)

- (a) [3 points] $\mathbf{B}(\mathbf{E} + \mathbf{C})$
- (b) $(\mathbf{E} + \mathbf{B})\mathbf{C}$
- (c) $\mathbf{E}^\top \mathbf{B}^\top$

4. Consider the matrices

$$\mathbf{G} = \begin{pmatrix} 1 & -3 \\ -1 & 2 \\ 0 & 5 \end{pmatrix} \quad \text{and} \quad \mathbf{H} = \begin{pmatrix} 1 & -3 & -1 \\ -1 & 2 & 0 \\ 0 & 5 & 5 \end{pmatrix},$$

noting that $H(:, 3) = 2H(:, 1) + H(:, 2)$.

- Is \mathbf{G} invertible? Explain your answer.
- Is \mathbf{H} invertible? Explain your answer.

5. Assume there is the following simplified grade book:

Name	Homework	Labs	Final Exam	Project
Avery	95	98	90	100
Blake	90	96	92	95
Carlos	83	79	79	90
Dax	55	30	65	60

Assume that the weights used to compute the final grades are homework 0.3, labs 0.2, the final 0.35, and the project 0.15.

- Write an explicit formula to compute Avery's final grade using a single inner product.
- Write an explicit formula to compute everyone's final grade simultaneously using a single matrix-vector product.
- Give an example of a linear algebra operation performed on the above data that is not informative to a professor who is trying to understand the students' performance in the course.

6. [3 points] Using Python/Jupyter or Matlab/Matlab Live Script, perform the following:

- Define \mathbf{M} be to a random 3×3 matrix.
- Test if $(\mathbf{M}^\top)^{-1}$ and $(\mathbf{M}^{-1})^\top$ are equal (up to floating point arithmetic).