Linear Data Chapter 9

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1. Is
$$\left\{ \begin{pmatrix} 1.1 \\ -3.4 \\ 0.4 \end{pmatrix}, \begin{pmatrix} 0.65 \\ 0.23 \\ -0.44 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$$
a basis for $\mathbb{R}^3$? You must explain your answer to get any points.

2. Let
$$\vec{x} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{pmatrix}, \quad \vec{z} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad W = \text{span}\{\vec{x}, \vec{y}\}.$$
   (a) Show that $\{\vec{x}, \vec{y}\}$ is an orthonormal set.
   (b) Explicitly give with justification $\text{proj}_W \vec{z}$.
   (c) Give an explicit element of $W^\perp$ which is not the zero vector.

3. Let
$$\vec{v} = \begin{pmatrix} -3 \\ 3 \\ -3 \\ \vdots \\ -3 \\ 3 \end{pmatrix} \in \mathbb{R}^{100}$$
have alternating entries -3 and 3. Further let $\mathbf{D}$ have as columns the DCT-II basis for $\mathbb{R}^{100}$.
   (a) Which coefficient of $\mathbf{D}^\top \vec{v}$, i.e., the DCT of $\vec{v}$ / change of basis of $\vec{v}$ with respect to the columns of $\mathbf{D}$ should have the largest absolute value? Explain your answer.

Bonus I Give with justification the first entry of the DCT of $\vec{v}$.

4. Using Python/Jupyter or Matlab/Matlab Live Script, compute the following.

(a) Determine the dimension of the span of the following set of vectors:

$$\begin{pmatrix} 3 \\ 4 \\ -4 \\ 5 \end{pmatrix}, \begin{pmatrix} 8 \\ 0 \\ -12 \\ 12 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -4 \\ 1 \\ -2 \end{pmatrix},$$

(b) Generate an orthonormal basis for the span of the vectors above.

5. Let

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} 0 \\ 3/5 \\ -4/5 \end{pmatrix}, \quad \text{and} \quad \vec{w} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Define $V = \text{span}\{\vec{v}, \vec{u}\}$.

(a) Show that $\vec{v}$ and $\vec{u}$ are orthonormal.

(b) Do $\{\vec{u}, \vec{v}\}$ form a basis for V?

(c) Explicitly compute $\text{proj}_V \vec{w}$.

(d) Explicitly give a non-zero vector in $V^{\perp}$.