Linear Data Chapter 1

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1. Let

$$\mathbf{A} = \begin{pmatrix} 15 & -14 & 13 & -12 & 11 \\ -10 & 9 & -8 & 7 & -6 \\ 5 & -4 & 3 & -2 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 15 & -14 & 13 & -12 & 0 \\ -10 & 9 & -8 & 7 & -6 \\ 5 & -4 & 3 & -2 & 1 \end{pmatrix}$$

   - Explicitly give (i.e., write down the numeral not just a formula) $A_{2,3}$.
     -8 is the entry in the second row, third column.
   - Is $\mathbf{A}$ a 5×3 matrix? Explain your answer.
     No, it is 3×5 because it has 3 rows and 5 columns.
   - Are $\mathbf{A}$ and $\mathbf{B}$ (mathematically) equal? Explain your answer.
     No, the $(1,5)$ entries are different.

2. Give an example of a data tensor with valence 3. (The example can be from the book or one you made up.)
   RGB image, sales of video games by console / review score / genre, measurements of a single quantity over time and 2D space, etc.

3. Given the following set

$$X = \{2,\, 4,\, 6,\, 8\} \quad \text{and} \quad Y = \{1,\, 2,\, 3\},$$

   explicitly give (e.g., write down the sets with numerical entries) of the outputs of the following requested set operations:

   (a) $X \cup Y$
   $\{1,\, 2,\, 3,\, 4,\, 6,\, 8\}$

   (b) $X \cap Y$
   $\{2\}$

   (c) $X \backslash Y$
   $\{4,\, 6,\, 8\}$

   (You don't need to write written explanations for the set problems.)

4. Given the function $f : X \to Y$ (with X and Y as above) defined as

$$f(2) = 2, \quad f(4) = 1, \quad f(6) = 3, \quad f(8) = 2,$$

answer the following questions. Justify your answers.

   (a) Is f injective?
   No, because 2 and 8 both map to 2.

   (b) Is f surjective?
   Yes, because each element of Y is an output of f.

   (c) Is f bijective?
   No, because it is not injective.

5. Using Python/Jupyter or Matlab/Matlab Live Script, perform the following:

   - Define a matrix
   $$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$
   - Define a (row) vector
   $$\vec{x} = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$$
   - Make the top row of $\mathbf{M}$ equal to $\vec{x}$.

   Hint: You only need 3 commands to perform the above tasks.
   Shortest Matlab commands (with or without semicolons)
   ```
   M=ones(2,4)
   x=zeros(1,4)
   M(1,:)=x
   ```
   It is also possible to hand code the entries of M and x, e.g.,
   ```
   A=[1 1 1 1; 1 1 1 1] or
   A=[1, 1, 1, 1; 1, 1, 1, 1]
   ```
   Shortest Python commands
   ```
   import numpy as np
   M=np.ones([2,4])
   x=np.zeros(4)
   M[0]=x
   ```
   It is also possible to hand code the entries of M and x, e.g.,
   ```
   M=np.array([[1,1,1,1],[1,1,1,1]])
   ```
   The last command could also be replaced with
   ```
   M[0,]=x or
   M[0,:]=x
   ```