Linear Data
Lab 7

Content Creator: Emily J. King

Goals: Interpret vector addition and scalar multiplication algebraically and geometrically. Compute norm and distance. Examine how coding data as vectors and interpreting the output geometrically can be informative.

Section 2: Visualizing geometry

- Go to `https://www.geogebra.org/m/WShmQvQU`. This visualizes linear combinations of vectors $\vec{u}$ and $\vec{v}$ (in blue) in $\mathbb{R}^2$. Move the sliders for c_1 and c_2. Then

  $$\vec{w} = c_1\vec{u} + c_2\vec{v} \text{ (in red).}$$

  Move around $\vec{u}$ and $\vec{v}$ to find linear combinations of different vectors.

- Go to `https://www.geogebra.org/m/hfsc8dwg`. This computes the inner product and cosine similarity of $\vec{u}$ and $\vec{v}$. Set

  $$\vec{u} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

  - Make $\vec{v}$ shorter/longer but pointing in the exact same direction. What happens to the inner product and cosine similarity?
  - Make $\vec{v}$ shorter/longer but pointing in the exact opposite direction. What happens to the inner product and cosine similarity?
  - Slowly rotate $\vec{v}$ around in a circle. What happens to the cosine similarity?

Section 3: Playing with word embeddings

Go to Contexto. `https://contexto.me/`. This game is built on the GloVe word embedding that we learned in lecture last Monday. Your goal is to guess a word of the day. All of the possible words are rated by the closeness of their word embedding vectors. The higher the rank (smaller the rank number), the closer you are to solving it. Try a couple of moves.

Section 4: Geometry of linear transformations

Geogebra worksheets used:

Worksheet A: `https://www.geogebra.org/m/sqG26hQj`
This helps you see the impact of multiplying all of the points in the red square with side length one, lower left corner at the origin, and bottom/left sides aligned with the axes on the left by the given 2×2 matrix to output the orange set of points. `https://www.geogebra.org/m/sqG26hQj`
Type in the values of the matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and press enter. Then slide the n slider to the right to 1 to multiply the points in the square (on the left) by the matrix.

Worksheet B: `https://www.geogebra.org/m/n7kvdyff`
All of the points in the red 'F' are multiplied on the left by the given 2×2 matrix to output the blue set of points.

Matrices:

$$\mathbf{D1} = \begin{pmatrix} 0.5 & 0 \\ 0 & 3 \end{pmatrix}, \quad \mathbf{D2} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{D3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{D4} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} \cos(30) & -\sin(30) \\ \sin(30) & \cos(30) \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \quad \mathbf{A} = \text{random } 2 \times 2 \text{ matrix}$$

Exercises

1. Complete Section 1 in Matlab/Python.
2. Go to `https://www.geogebra.org/m/WShmQvQU`.
   (a) Set
   $$\vec{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$
   Move around the sliders for c_1 and c_2. Explain what you see.
   (b) Do you think there is a linear combination of $\vec{u}$ and $\vec{v}$ that equals
   $$\begin{pmatrix} 1 \\ -1 \end{pmatrix}?$$
   If so, give the (approximate) coefficients that you find. If not, explain why not.

(c) Set

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}.$$

Move around the sliders for c_1 and c_2. Explain what you see.

(d) Do you think there is a linear combination of $\vec{u}$ and $\vec{v}$ that equals

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}?$$

If so, give the (approximate) coefficients that you find. If not, explain why not.

3. Play at least 10 turns of Contexto. (Don't worry if you don't solve it.) Either take a screenshot of your top guess or write/type the word and score of your best guess and the word and score of your worst guess.

4. Complete Section 3 in Matlab/Python.