Linear Data Chapter 7

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1. Assume $\vec{x}, \vec{y} \in \mathbb{R}^{1001}$ and you know the following:

$$\|\vec{x}\| = 4 \quad \|\vec{y}\| = 2.$$

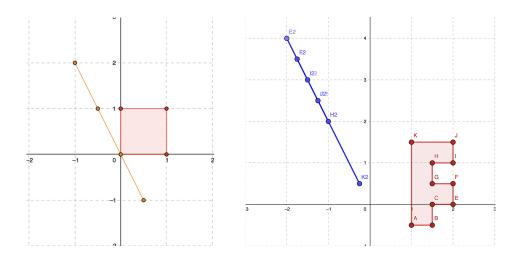
For each of the following, either explicitly compute the value or explain why there is insufficient information.

- (a) Explicitly compute $||-3\vec{y}||$. Explain your answer.
- (b) Explicitly compute $\left\| \frac{\vec{x}}{\|\vec{x}\|} \right\|$. Explain your answer.
- (c) Is it possible for $\|\vec{x} + \vec{y}\| = 6$? Explain your answer.
- 2. If $\vec{u}, \vec{v}, \vec{w}, \vec{z} \in \mathbb{R}^{300}$ are all word embeddings from GloVe and $\vec{u} \vec{v} \approx \vec{w} \vec{z}$, what might that mean semantically (i.e., about the meanings of the words)?
- 3. Using Python/Jupyter or Matlab/Matlab Live Script, perform the following:
 - Define

$$\vec{a} = \begin{pmatrix} 12.3 \\ 0.56 \\ -1.7 \\ 0.34 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 0.55 \\ 2.22 \\ -1.2 \\ 3.14 \end{pmatrix}$$

You don't need to force them to be column vectors in Python. 1D arrays suffice.

- Compute the (Euclidean) norm of \vec{a} .
- Compute the (Euclidean) distance between \vec{a} and \vec{b} .
- 4. Consider the Geogebra screenshots below showing the effects of multiplication by a certain 2×2 matrix **B**.



What must the determinant of ${\bf B}$ be? You must explain your answer to receive any credit.