Linear Data Chapter 12

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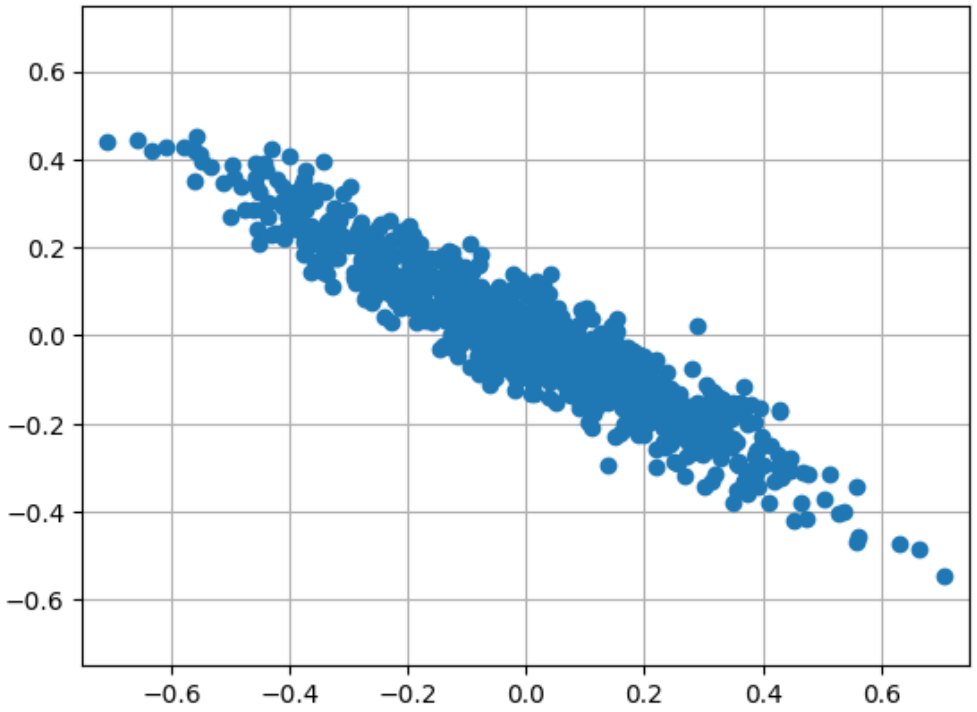
1. Consider the zero-centered data cloud pictured below.



   Let $\mathbf{X}$ be the 2×1000 zero-centered data matrix representing the data cloud. Which of the following are most likely to be the singular values of $\mathbf{X}$? You must justify your response with at least one complete sentence.

   A. $\sigma_1 = 5$, $\sigma_2 = 4$

   B. $\sigma_1 = 9$, $\sigma_2 = 1.5$

   C. $\sigma_1 = 8$, $\sigma_2 = -1$

2. Let $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top$ be the singular value decomposition of a $d \times n$ matrix $\mathbf{B}$, where $\mathbf{U}$ is a $d \times d$ orthogonal matrix with columns $\vec{u}_i$, $\mathbf{\Sigma}$ is diagonal $d \times n$ matrix with diagonal entries σ_i in non-increasing order, and $\mathbf{V}$ is a $n \times n$ orthogonal matrix with columns

$\vec{v}_i$. Assume the singular values are all distinct. What is a correct formula for the best rank 1 approximation of $\mathbf{B}$? You must justify your response with at least one complete sentence.

A. $\vec{u}_d \sigma_1 \vec{v}_n^\top$

B. $\vec{u}_1 \sigma_1 \vec{v}_1^\top$

C. $\mathbf{U\Sigma V}^\top$

3. Name at least one application of PCA from the textbook or some other source. In the latter case, list the source.

4. Assume that $\mathbf{A}$ is a 3×10 matrix and that $\mathbf{AA}^\top$ has the eigenvalues $1, 4, 25$.

   (a) Explicitly list with justification the eigenvalues of $\mathbf{A}^\top\mathbf{A}$ (including multiplicity).

   (b) Explicitly list with justification the singular values of $\mathbf{A}$.