Multilevel amplitude shift keying in dispersion uncompensated optical systems

N. Avlonitis, E.M. Yeatman, M. Jones and A. Hadjifotiou

Abstract: The authors model and characterise the performance of 10 Gbit/s 4-ary amplitude shift keying (ASK) systems in dispersive environments at 1550 nm. Both non-return-to-zero (NRZ) and return-to-zero (RZ) formats are examined, with comparisons to on-off keying (OOK) in each case. Both single amplified links and cascaded fibre-amplifier links are modelled. While 4-ary ASK systems suffer a back-to-back sensitivity penalty of up to 7 dB with respect to OOK, they offer a significantly reduced dispersion sensitivity, particularly for RZ formats. This suggests advantages for M-ary coding in future systems employing optical time division multiplexing. Optimal level spacing for ASK is analysed and approximations for different noise regimes are shown to fit well to detailed calculations. Sensitivity to extinction ratio is examined; RZ systems are shown to have higher sensitivity than NRZ for both 4-ary and OOK. Finally, the authors show that frequency drift, or equivalently, a de-tuning in the centre frequency of the optical filter, is more severe for OOK than for 4-ary ASK, especially in the case of non-fully dispersion uncompensated systems.

1 Introduction

Researchers have for some time examined multilevel signalling as a way of increasing the capacity of optical communication systems. Four level signalling has an intrinsic signal-to-noise ratio (SNR) penalty due to the fragmentation of the main eye to three smaller eyes. This SNR degradation is demonstrated in [1, p. 145] by showing that the fundamental quantum noise limit is worse for 4-ary amplitude shift keying (ASK) compared with on-off keying (OOK). However, because of reduced spectral occupancy, we can expect *M*-ary ASK signals to be less sensitive to dispersion than OOK. Numerical simulations and experimental work by Walklin and Conradi [2] show that there are cases where multilevel signalling offers better performance than OOK, particularly in dispersive environments. Additionally, the reduced spectral width of multilevel signals offers the possibility of increased spectral efficiency, although in [3] it is shown that this cannot be realised without high-order optical filtering. Here, we extend previous results by comparing non-return-to-zero (NRZ) and return-to-zero (RZ) signalling formats and by examining the effect of level spacing, optical filter bandwidth and filter de-tuning in detail.

In this paper, we adopt models created for modelling OOK systems to model a 4-ary ASK system (Section 2.1). In Section 3.3, an extension to the model is presented to account for multiple fibre spans and therefore the evolution of the noise figure of the system. The performance evaluation of the systems analysed in this paper is based on bit error rate (BER) and Q calculations as described in Section 2.3. The problem of level spacing is considered in Section 2.4, where analytical equations relating the spacing with the optical extinction ratio and the various noise terms are presented. Next, we demonstrate the advantages of 4-ary ASK in single-span and multi-span systems. Numerical simulations of Section 3.1 show that multilevel signals have a wider range of system dispersion that can operate within a 1 dB dispersion penalty compared to OOK. The analysis includes results for NRZ coded pulses and RZ pulses with FWHM of 50 and 33% of the bit slot. The effect of the optical extinction ratio on the BER (and Q) is also considered.

In Section 3.2, we investigate how different parameters of an optical Fabry-Perot filter, bandwidth and filter de-tuning, affect the performance of OOK and 4-ary ASK. Models [4–6] similar to the one used in this paper use the concept of noise equivalent bandwidth, effectively assuming optical filters with wide bandwidth and rectangular transfer function. This neglects the effect of narrow filtering on the dispersed signal. The use of an arbitrary optical filter for the calculation of the BER, as in [7], results in a better estimation of the BER including the effects of inter-symbol interference (ISI). This is of particular interest in this paper as optical filters with relatively small bandwidths are used as opposed to previous work on the performance of multilevel ASK [2].

Finally, in Section 3.3, we demonstrate the improvement of sensitivity that multilevel signalling offers for 10 Gbit/s multiple stage systems with overall system dispersion levels exceeding 1500–1800 ps/nm, depending on the coding scheme.

2 Model

2.1 Channel model

The lightwave system model considered in this paper is shown in Fig. 1. The mathematical modelling is based on

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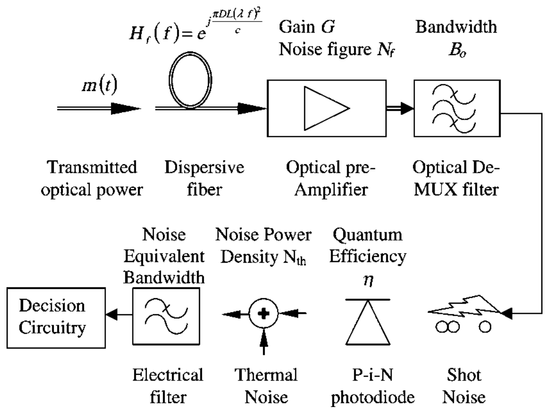

Fig. 1 *System model*

the evaluation of the derivatives of the moment generating function [1, p. 319]. The calculation of the BER is based on the assumption of Gaussian noise statistics for the decision variable [4–7].

The transmitted optical power waveform is created from a discrete sequence $I[n]$ by multiplying the spectrum of the sequence with the pulse waveform spectrum. Two pulse shapes are included in the analysis: raised cosine and Gaussian, representing the NRZ and RZ formats, respectively. The power spectra of the two pulse shapes, raised cosine and Gaussian, are described by (2) and (1)

$$G_{\mathrm{Gaus}}(f) = \mathrm{e}^{-(f^2/2f_\sigma^2)} \tag{1}$$

$$G_{\cos^2}(f) = \mathrm{sinc}\left(\frac{\pi f}{T}\right)\frac{\cos(B_{\mathrm{ex}}\pi f/T)}{1-(2B_{\mathrm{ex}}f/T)^2} \tag{2}$$

where f_σ is the $1/e$ spectral halfwidth of the pulse, T the bit period and B_{ex} the excess bandwidth factor or the normalised risetime.

The effect of attenuation and dispersion in the fibre is applied on the signal in the Fourier domain using the transfer function of the fibre

$$H_f(f) = \mathrm{e}^{j[\pi DL(\lambda f)^2/c]-(\alpha L/2)} \tag{3}$$

where D is the dispersion parameter, L is the fibre length, λ is the transmission wavelength (i.e. 1550 nm), c is the speed of light in vacuum and α is the attenuation coefficient. Next, we assume an optical pre-amplifier with flat amplifier gain spectrum, equal to G at the operating wavelength, and amplified spontaneous emission (ASE), with a spontaneous emission factor n_{sp}. The received optical power $P(t)$ is then given by [7]

$$\begin{aligned} P(t) &= 2\sqrt{Gc_1}c_2 s(t)\{n_c(t)\cos\phi_s(t) + n_s(t)\sin\phi_s(t)\} \\ &\quad + Gc_1c_2s^2(t) + c_2\{n_c^2(t) + n_s^2(t)\} \end{aligned} \tag{4}$$

In the above equation, $s(t)$ is the received amplitude of the optical field at the receiver. The coefficients c_1 and c_2 are the input and output coupling losses of the optical pre-amplifier, respectively. The noise process is described by two zero mean Gaussian distributed random variables, $n_c(t)$ and $n_s(t)$, corresponding to the in-phase and out-of-phase noise components of the bandlimited noise process, respectively. The power spectral density of the ASE noise, per polarisation, is given by

$$N_{\mathrm{ASE}} = (G-1)h\nu n_{\mathrm{sp}} \tag{5}$$

where $h\nu$ is the photon energy. The variance (power) of the random variables is therefore

$$\sigma^2 = P_{\mathrm{ASE}} = (G-1)h\nu n_{\mathrm{sp}}B_o \tag{6}$$

where B_o is the noise equivalent bandwidth of the optical filter.

Finally, the photodetection is incorporated into the model as a Poisson process with intensity

$$\lambda(t) = \frac{\eta}{h\nu}P(t) + \lambda_0 \tag{7}$$

where η is the quantum efficiency of the power conversion process and λ_0 is the dark current. From (7), the mean (8) and variance (9) of the detection current are derived

$$E[I_d] = R_sGc_1c_2s^2(t) * h_e(t) + 2c_2P_{\mathrm{ASE}}R_sH_e(0) + q\lambda_0H_e(0) \tag{8}$$

$$\begin{aligned} \sigma_{I_d}^2 &= qR_sc_1c_2s^2(t) * h_e^2(t) + 2qR_sc_2P_{\mathrm{ASE}}B_e \\ &\quad + 4R_s^2Gc_1c_2^2N_{\mathrm{ASE}} \\ &\quad \times \int_{-\infty}^{\infty} |\tilde{r}(f)\cdot H_o(f) * H_e^*(f)\mathrm{e}^{-j2\pi ft}\cdot H_o(f)|^2\mathrm{d}f \\ &\quad + q^2\lambda_0B_e + 2R_s^2c_2^2N_{\mathrm{ASE}}^2I_2 \end{aligned} \tag{9}$$

where R_s is the responsivity of the diode and is equal to $q\eta/h\nu$, $h_e(t)$ and $H_e(0)$ are the impulse response and the DC gain of the electrical filter, respectively. The asterisk $(*)$ corresponds to convolution. In (9), B_e is a measure of the noise equivalent bandwidth of the electrical filter

$$B_e = \int_{-\infty}^{\infty}|H_e(f)|^2\mathrm{d}f \tag{10}$$

The noise bandwidth of the ASE–ASE beat noise is measured through I_2 [8]

$$I_2 = \int_{-\infty}^{\infty}|H_e(f)|^2|H_o(f)|^2 * |H_o(f)|^2\,\mathrm{d}f \tag{11}$$

The contribution from the thermal noise σ_{th}^2 was also added as an additional term in (9).

The model presented above reduces to the one in [7], the exception being the inclusion of shot noise in this case. The model presented in [9], which also uses a Gaussian approximation, has also been considered and the same equations for the mean and variance have been derived in continuous form. However, the model presented in [9] places restrictions on allowed values of the optical filter bandwidth, as also noted by Walklin and Conradi [2], where it was adopted.

2.2 Multi-segment fibre links

The model presented in Section 2.1 can easily be extended to include a cascade of multiple spans of fibre. Assuming that there are k fibre spans, with amplification stages in between, then in each span, additional attenuation and coupling loss is compensated by amplifier gain, while additional dispersion and ASE power are added to the signal. By using the high input signal approximation, noise figure calculations, as explained in [10], can then track the ASE noise spectral density evolution. Note that this approach also takes into account the ASE–signal beat noise term, as seen by the receiver. Moreover, the use of fibre–amplifier stages rather than amplifier–fibre produces worse noise figures. However, with the adoption of this tactic, we can better model a realistic dispersive system with suppressed non-linear effects.

2.3 Q–BER evaluation

The performance of the system under consideration is measured using Q and BER measurements. In the general L level case, assuming Gaussian distributed noise and Grey encoding, between adjacent power levels, the average BER is given by

$$\mathrm{BER} = \frac{1}{L}\sum_{i=1}^{L-1} Q\left(\frac{I_{i+1} - \mathrm{thr}_i}{\sigma_{i+1}}\right) + Q\left(\frac{\mathrm{thr}_i - I_i}{\sigma_i}\right) \tag{12}$$

where L is the number of levels, I_i is the power associated with the ith symbol, thr_i is the threshold setting the decision boundary between the ith and the $(i+1)$th level, σ_i^2 is the variance related to the ith power level. Finally, the $Q(x)$ function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \mathrm{e}^{-(x^2/2)}\,\mathrm{d}x = 0.5\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right) \tag{13}$$

Since a truncated pseudorandom pulse sequence is used, the patterning effect of the sequence should be taken into account, as explained in [11]. Briefly, for the OOK case, eight possible patterns exist according to the values of the previous, current and next bit, whereas in the 4 level case, the number of possible patterns becomes 64. Otherwise, in order to avoid this pattern effect, DeBruijn sequences should be used, so that the output BER corresponds to the average over a selected bit interference range.

In addition to BER, the system Q_{BER} value is also used to evaluate the performance. In our case, we calculate first the BER as explained above, and then we find the equivalent Q_{BER} factor using

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\left(\frac{Q_{\mathrm{BER}}}{\sqrt{2}}\right) \tag{14}$$

The selection of Q_{BER} rather than the Q factor, as defined (in the binary case) by [12, eq. 4.5.10], is made because the definition of a Q factor in the 4-ary signalling case would be conceptually different to that in the OOK case, and therefore the comparison would not be valid.

2.4 Level and threshold setting

Utilising the Gaussian approximation, we can now derive the optimum level–threshold power settings for different noise scenarios. In our analysis, we assume that the dominant noise terms in (9) come from the signal–ASE beat noise and ASE–ASE noise terms plus the thermal noise from the electrical receiver. By using an approach similar to [13], the arguments of the two $Q(x)$ functions in (12) are set to be equal for each eye. From (8)

$$I_4 - I_3 = Gc_1c_2R_s(1-A)P_o \tag{15}$$

$$I_3 - I_2 = Gc_1c_2R_s(A-C)P_o \tag{16}$$

$$I_2 - I_1 = Gc_1c_2R_s(C-r)P_o \tag{17}$$

where A, C are the normalised level settings for levels 2 and 3 and r is the extinction ratio at the decision instant. Note that the ISI effect is effectively neglected here as the convolutional integrals are substituted by constant power levels.

Next, from (9) and by making use of (6)

$$\sigma_k^2 = 2(c_2R_sN_{\mathrm{ASE}})^2 I_2 + 4P_kGc_1c_2^2R_s^2N_{\mathrm{ASE}}B_{oe} + \sigma_{\mathrm{th}}^2 \tag{18}$$

In the equation above, k is an index to a corresponding power level. B_{oe} is a measure of the noise equivalent bandwidth of the combined optical and electrical filtering process, defined as

$$B_{oe} = \int_{-\infty}^{\infty} |H_e(f)|^2 |H_o(f)|^2\,\mathrm{d}f \tag{19}$$

Note here that by allowing the input signal to be at constant power, the ASE–signal beat noise is affected. This is because the ASE–signal beat noise depends on the spectral properties of the signal. Hence, assuming a CW signal results in the least possible modulation between ASE and signal, and therefore the worst case scenario, as the resultant noise spectrum is concentrated around the carrier.

The above analysis results in the following optimum power level settings

$$\begin{bmatrix} A_{\mathrm{opt}} \\ C_{\mathrm{opt}} \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ r \end{bmatrix} + K \tag{20}$$

where K is defined by

$$\begin{aligned} K = {} & \frac{1}{9}\sqrt{\frac{2N_{\mathrm{ASE}}I_2}{Gc_1P_oB_{oe}} + 4 + \frac{\sigma_{\mathrm{th}}^2}{Gc_1N_{\mathrm{ASE}}(Rc_2)^2P_oB_{oe}}} \\ & \times \sqrt{\frac{2N_{\mathrm{ASE}}I_2}{Gc_1P_oB_{oe}} + 4r + \frac{\sigma_{\mathrm{th}}^2}{Gc_1N_{\mathrm{ASE}}(Rc_2)^2P_oB_{oe}}} \\ & - \frac{2N_{\mathrm{ASE}}I_2}{9Gc_1P_oB_{oe}} - \frac{\sigma_{\mathrm{th}}^2}{Gc_1N_{\mathrm{ASE}}(Rc_2)^2P_oB_{oe}} \end{aligned} \tag{21}$$

For infinite extinction ratio ($r = 0$) and when spontaneous emission noise dominates, K from (21) goes to zero, and the threshold setting becomes quadratic: $C_{\mathrm{opt}} \to 1/9$ and $A_{\mathrm{opt}} \to 4/9$, or equivalently equally spaced in optical amplitude. For the same infinite extinction ratio and in the case of thermal noise dominance, σ_k reduces to σ_{th} and the spacing becomes equal in power, $C_{\mathrm{opt}} \to 1/3$, $A_{\mathrm{opt}} \to 2/3$. These results support the validity of our approximations since they coincide with the reported results [2, 13]. However, here we have extended the analysis to include the effects of thermal noise and coupling

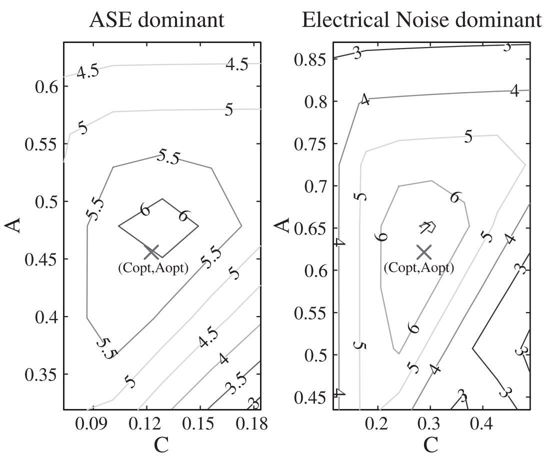

Fig. 2 *Q against level spacing. Contours correspond to calculations based on the model presented in this section. Crosses are the optimum levels*

losses on the optimal threshold levels. Application of (20) and (21) is of particular interest for systems where ASE and thermal noises are comparable, namely optical local area networks.

The plots shown in Fig. 2 present two different scenarios of level spacing, corresponding to SNR dominated by either ASE or thermal noise. Note that in Fig. 2, contour plots correspond to numerical results based on the model presented in the previous paragraphs, whereas crosses to the optimum levels as calculated by (20). The small differences between contour plots and calculated optimum spacing are due to the fact that ISI is not accounted for in (20). In the simulations that follow, levels are determined using (20), and decision thresholds are then optimised by minimising the BER.

3 Numerical results

3.1 Single stage

On the basis of analysis described in Section 2.1, we compare the performance of 4-ary ASK to the binary OOK case. The comparison includes the consideration of both 10 Gbit/s NRZ and RZ transmitted pulses, where NRZ pulses are modelled as raised cosines with a 0.24 width, and the RZ pulses as Gaussian pulses with either 0.33 or 0.5 FWHM width. For the comparison, we used an infinite extinction ratio ($r = 0$), an optical Fabry-Perot filter with 100 GHz cut-off frequency and finally an electrical filter with a third-order Bessel characteristic, in order to minimise additional dispersion, with a cut-off frequency at 0.65 bit rate. The input and output coupling losses to the optical pre-amplifier were taken as 1.5 dB and the gain as 25 dB. The noise figure of the optical pre-amplifier was taken to be 5.5 dB.

Fig. 3 shows the dependence of the receiver sensitivity on the amount of system dispersion, for a short span of 60 km and a BER of 10^{-9}. First, we observe that RZ coding produces better performance, both for OOK and 4-ary, for a small amount of dispersion. The superiority of the RZ coding is in accordance with the results of [14], even for the same optical and electrical filter characteristics. As the amount of system dispersion increases, the performance of the RZ format degrades faster than NRZ and this is mainly due to the spread of the energy of RZ pulses into adjacent bit slots. NRZ suffers less from this effect since the corresponding spectral width is less.

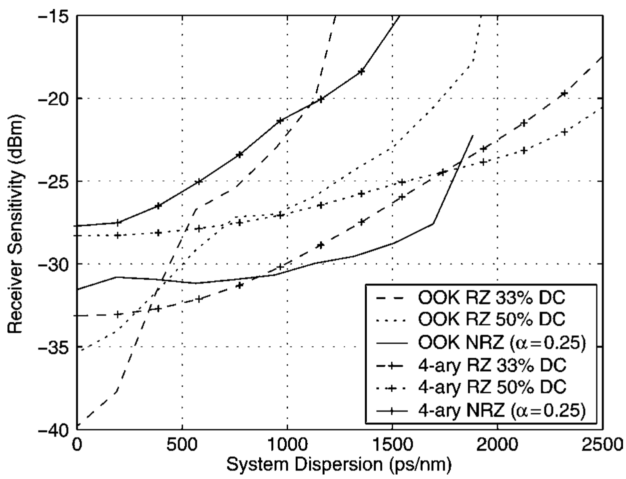

Fig. 3 *Receiver sensitivity against system dispersion, infinite extinction ratio*

Table 1: Zero dispersion receiver sensitivity and sensitivity penalty

	NRZ	RZ 50% DC	RZ 33% DC
OOK	−31.5 dBm	−35.4 dBm	−39.8 dBm
4-ary	−27.7 dBm	−28.3 dBm	−33.2 dBm
Power penalty	3.8 dB	6.6 dB	7.1 dB

The back-to-back sensitivities are shown in Table 1, from binary to quaternary. The fact that the penalty for RZ coding is larger than for NRZ can only be due to the cut-off frequency of the electrical filter, since the noise sources, thermal and ASE, are the same for OOK and 4-ary.

From Fig. 3, we can see that multilevel RZ pulses give sensitivities 1–3 dB, depending on the system dispersion level, worse than the binary NRZ case. The receiver sensitivity advantage of OOK decreases with system dispersion. Eventually at high dispersion levels, in excess of 1800 ps/nm, the multilevel RZ format outperforms binary signalling. This value corresponds to 105 km of standard single mode fibre ($D = 17$ ps/nm/km). Fig. 3 also shows that multilevel RZ coded signalling experiences a 2.5 and 4.5 dB penalty from 0 to 1600 ps/nm, for Gaussian pulses with FWHM of 0.33 and 0.5 times the bit slot, respectively. OOK signalling produces the same power penalty for a system dispersion range four times less (400 ps/nm).

Next, we compare OOK and 4-ary ASK in terms of susceptibility to change of system parameters. Usually, in practical systems, a power margin is allowed to guarantee a specific BER performance against small changes in the system. We examine the effect of this by allowing the average received power to be double the receiver sensitivity point at zero dispersion, or equivalently leave a 3 dB power margin and then determine the Q degradation with dispersion at constant power.

Fig. 4 shows clearly that a 3 dB power margin allows multilevel signalling to operate over a greater system dispersion range. Specifically, the maximum allowable system dispersion, at the Q equals 6 point, for OOK is around 300 and 1100 ps/nm for RZ and NRZ, respectively. The same figures for multilevel signalling are 1100, 1600 and 1910 ps/nm for RZ pulses with 33% DC, 50% DC and for NRZ pulses, respectively. The multilevel RZ format with FWHM pulse of 33% of the bit length gives

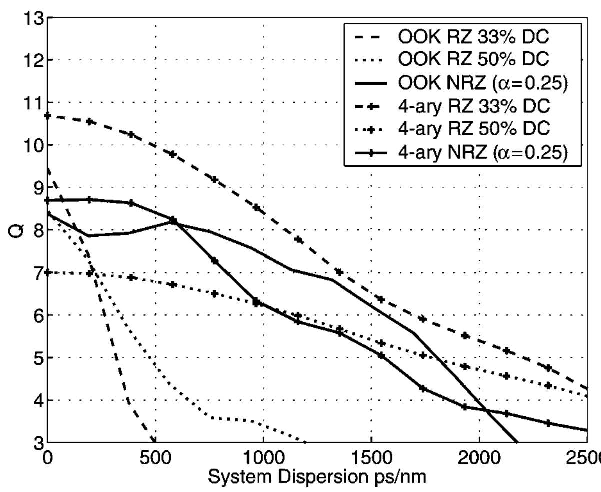

Fig. 4 *Q against system dispersion, comparison of various codes (RZ and NRZ) and modulation formats, OOK and 4-ary, for a 3 dB receiver sensitivity power margin*

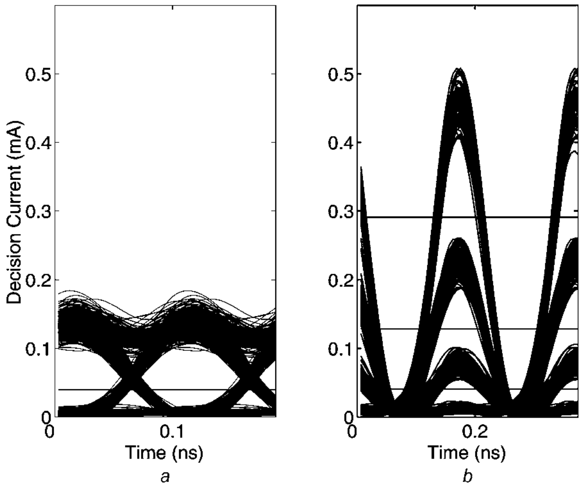

Fig. 5 *Eye diagrams for 10 Gbit/s systems, Gaussian pulses with 33% DC and system dispersion of 500 ps/nm*
a OOK
b 4-ary ASK (optimum level spacing)

the best performance, for a 3 dB margin, over the whole dispersion range, and in addition it enables the use of optical time division multiplexing (OTDM).

Examples of eye diagrams for 33% DC RZ modulation are shown in Figs. 5 and 6. These eyes correspond to a BER of 10^{-9}. Clearly, at low system dispersion levels, the receiver sensitivity is lower for OOK than in the 4-ary case. At a system dispersion of 1500 ps/nm, both the time jitter and eye closure in the OOK case necessitate the increase in power. The performance degradation is mainly due to ISI. In the 4-ary ASK case, all three eyes remain well structured as in the 500 ps/nm case. Note also that the thresholds, indicated as horizontal lines, remain virtually unchanged in the 4-ary case, as opposed to the OOK case where the strong ISI shifts the optimum threshold upwards.

Next, the effect of a finite optical extinction ratio, which in our case we define as the ratio r of minimum to maximum transmitted power, is examined. In Fig. 7, receiver sensitivity as a function of system dispersion is plotted for r

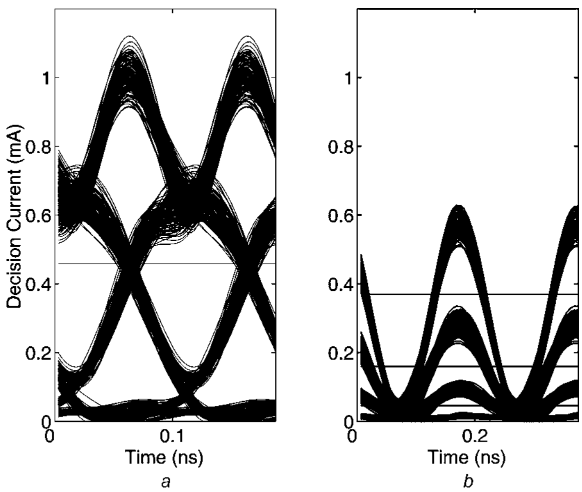

Fig. 6 *Eye diagrams for 10 Gbit/s systems, Gaussian pulses with 33% DC and system dispersion of 1500 ps/nm*
a OOK
b 4-ary ASK (optimum level spacing)

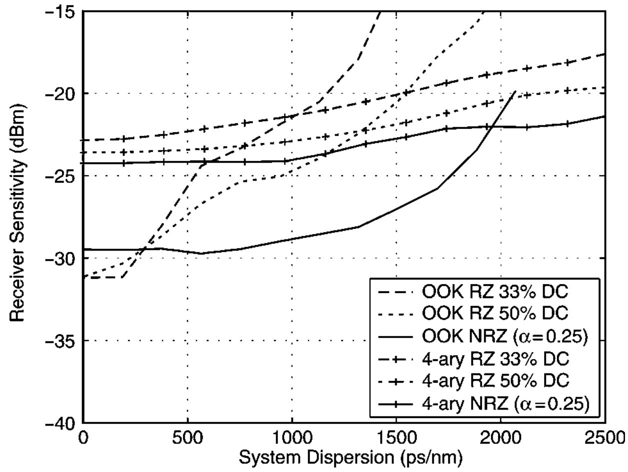

Fig. 7 *Receiver sensitivity against system dispersion, for $r = 10$ dB*

equal to 10 dB. The zero system dispersion penalty, from 20 dB extinction ratio to 10 dB, is shown in Table 2.

The values for the NRZ can be approximated by rough calculations for the eye closure penalty. A 10 dB extinction ratio produces a $20\log(1-0.1) \simeq -0.91$ dB eye closure, for the OOK case. For multilevel, the same figure gives the eye closure for each of the data eyes or a total of 2.7 dB penalty. From Table 2, we observe that multilevel signalling is affected more by an increase in the extinction ratio, as in the $r = 10$ dB graph the difference between OOK and 4-ary is greater than in the 20 dB case, except in the 33% RZ format where the difference is comparable. On the other hand, the performance is less affected by dispersion, giving a receiver sensitivity power degradation of only 2 dB from zero to 2000 ps/nm system dispersion, in 4-ary ASK. The same value for OOK is 10 dB for the best coding scheme, which is for NRZ pulses. An effect common to both OOK and 4-ary is that RZ is more strongly affected by reduced extinction ratio than NRZ. This is evident from Fig. 7, where the reduction in the performance of multilevel RZ coded signalling increases the difference between the two schemes, OOK and 4-ary, for low system dispersion.

Table 3 shows the cross points of the different systems for $r = 10$ and 20 dB. These cross points define the boundaries between the two systems. OOK systems are superior for links that require receiver sensitivity power levels lower than the aforementioned power values, otherwise 4-ary ASK is preferred. For RZ coding, a twofold increase in the maximum allowable system dispersion is accompanied by a 5–8 dB decrease in the receiver sensitivity, depending on the duty cycle used.

3.2 Optical filter considerations

Next, we examine the effect that the optical filter has on the two different modulation formats, OOK and 4-ary ASK. Figs. 8 and 9 show the receiver sensitivity against optical filter 3 dB bandwidth for 0 and 1000 ps/nm system

Table 2: Zero dispersion sensitivity penalty induced by changing the extinction ratio from 20 to 10 dB

	NRZ (dB)	RZ 50% DC (dB)	RZ 33% DC (dB)
OOK	1.6	3.7	7.2
4-ary	2.7	6.1	6.8

Table 3: Operating points for various extinction ratios

	Dispersion (ps/nm)			Power (dBm)		
	$r = 10$ dB	$r = 20$ dB	$r =$ Inf	$r = 10$ dB	$r = 20$ dB	$r =$ Inf
NRZ	1960	2000	N/A	−22.0	−20.0	N/A
RZ 50% DC	1335	715	710	−22.3	−27.3	−27.6
RZ 33% DC	1020	460	345	−21.3	−29.2	−32.8

dispersion, respectively, for an electrical cut-off frequency of 0.8 times the symbol rate. In Fig. 8, the trend is clear, increasing the optical filter bandwidth reduces the effect of ISI and as a result the performance improves. This improvement is greater for OOK and RZ coding, which have the largest bandwidth. On the other hand, the performance improvement of OOK with NRZ coding, and 4-ary signalling, with any format, reaches only 2 dB with an increase of the optical cut-off frequency from 10 to 100 GHz.

For 1000 ps/nm system dispersion, Fig. 9, the situation is reversed. Because OOK is more susceptible to dispersion, ISI is dominated by fibre rather than by optical filter dispersion, so the effect of increasing the optical bandwidth is mainly to increase the amount of ASE that is passed to the electrical receiver. On the other hand, in 4-ary ASK, although the performance now changes only by 1 dB, reduced by 1 dB from the 0 dispersion case, the receiver sensitivity still improves.

Another conclusion that can be drawn from the figures is that multilevel signalling reaches the optimum receiver sensitivity point for narrower optical filters than OOK, something that is particularly evident in the RZ case.

Next, we examined the effect of filter de-tuning on the receiver sensitivity. Figs. 10 and 11 show the numerical results for system dispersion levels of $DL = 0$ and $DL = 1000$ ps/nm, respectively. The zero dispersion case shows clearly that an initial increase in the filter de-tuning enhances the performance marginally, from 0.2 to 1 dB in the best case. This is because this de-tuning reduces the overall bandwidth of the two filtering processes, optical and electrical, whereas the effect on the signal, and therefore on ISI, is negligible. This situation is reversed in the 1000 ps/nm dispersion case. The already distorted signal is not enhanced in the OOK case, in contrast with the 4-ary ASK case where a small gain in the performance can still be achieved.

Commenting on the performance of different coding formats, RZ and NRZ, we note that relative performance of these follow the trends in Figs. 3 and 7. In the zero dispersion case, the different codes have the same performance at 40 GHz de-tuning. This is expected as the bit rate is 10 GHz and the filter width 50 GHz, so that the beneficial bandwidth coincides with the difference between the filter width and the de-tuning.

3.3 Multiple stages

Next, we compare OOK and four level signalling for systems with multiple amplification stages. The results of Fig. 12 are based on a system of 60 km amplifier spacing with amplifiers having a 5.5 dB noise figure, total coupling losses of 3 dB, fibre losses of 0.2 dB/km and amplifier gains equal to the total loss. The optical pre-amplifier is assumed to have the same parameters as in the previous section.

From Fig. 12, we can see that OOK is superior to multilevel signalling for low dispersion and short distances. On the other hand, the trade-off between allowable system dispersion range and receiver sensitivity is evident, since the system dispersion range for 4-ary ASK is 1.5 times the one for OOK for the same receiver sensitivity. Characteristically, the OOK RZ coded format with 33% DC requires a dramatic increase in power in order to sustain good performance. However, in all cases, the performance degradation for OOK becomes severe for system dispersion levels above 2100, 2000 and 1500 ps/nm for NRZ, RZ with 50% FWHM pulses and RZ with 0.33%, respectively. In four level systems, the sensitivity degradation is 5 dB even for transmissions corresponding to 180 km through SSMF fibre, and total system dispersion of 2000 ps/nm.

Comparing the coding formats, we found that the RZ coded Gaussian pulses with the 33% DC give the best performance for low system dispersion and the 50% DC for

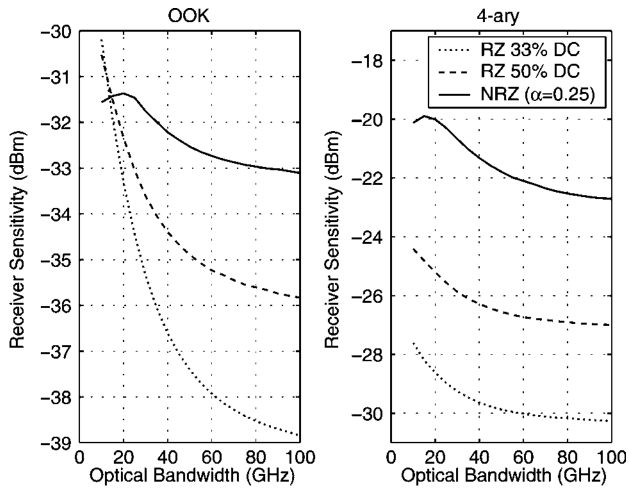

Fig. 8 *Receiver sensitivity against 3 dB bandwidth of Fabry-Perot optical filter, no system dispersion*

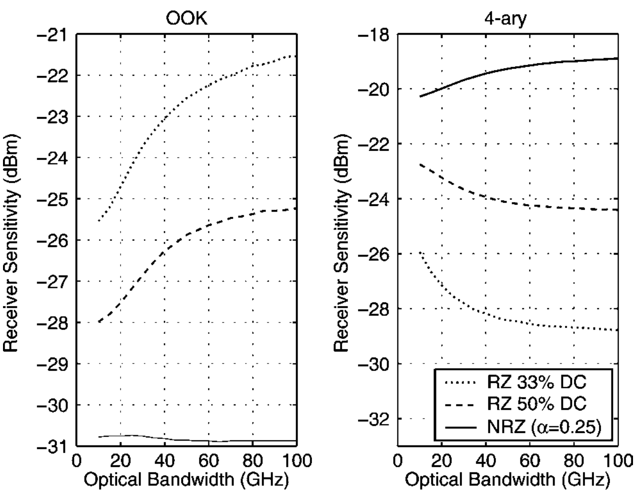

Fig. 9 *Receiver sensitivity against 3 dB bandwidth of Fabry-Perot optical filter, system dispersion is 1000 ps/nm*

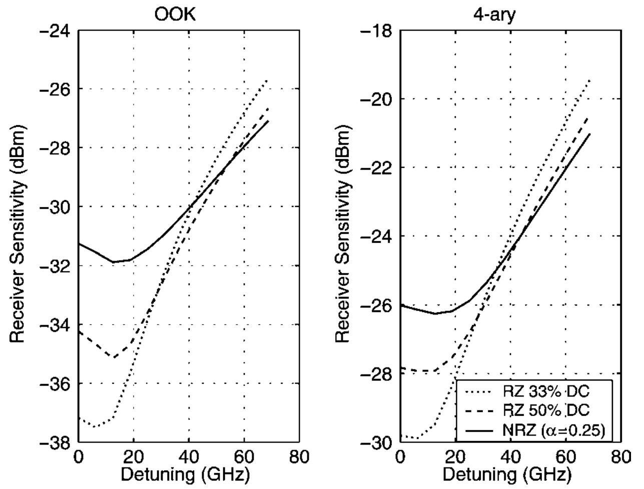

Fig. 10 *Receiver sensitivity against optical FP filter detuning (DL = 0 ps/nm, 50 GHz optical bandwidth)*

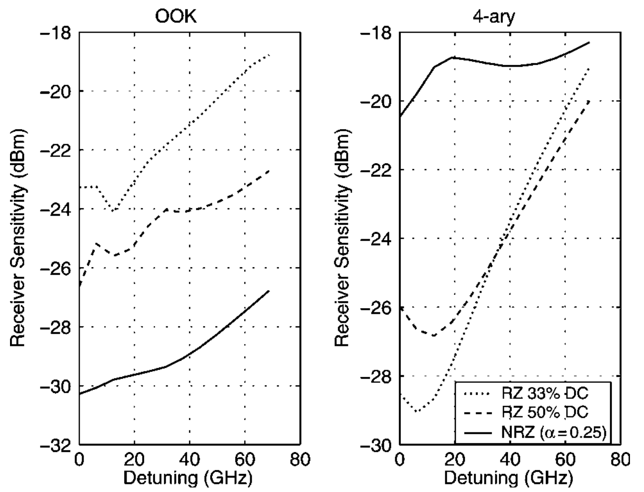

Fig. 11 *Receiver sensitivity against optical FP filter detuning (DL = 1000 ps/nm, 50 GHz optical bandwidth)*

longer distances or higher dispersion. For 4-ary ASK, the difference between the two RZ formats is 1–2 dB, whereas the NRZ format gives worse results by about 2–3 dB for low dispersion and 5–6 dB for system

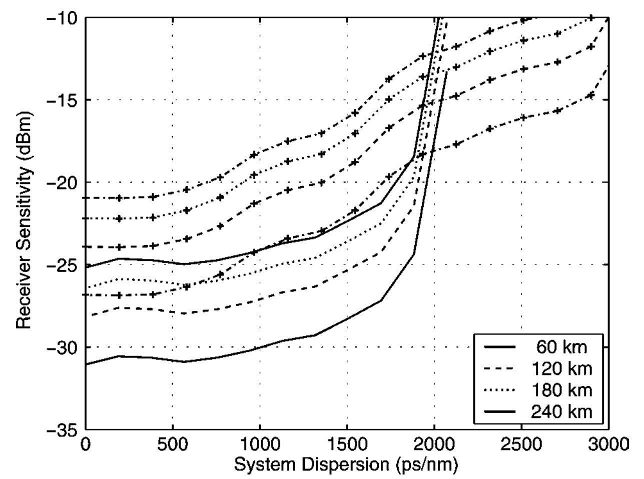

Fig. 12 *Sensitivity against system dispersion against distance, NRZ (raised cosine pulse, $\alpha = 0.25$), no marker for OOK, plus sign (+) for 4-ary*

dispersion over 1500 ps/nm. In OOK, the RZ coded pulses with 33% DC is superior for low dispersion, as for the 4-ary case; however at high dispersion levels, the NRZ outperforms the RZ format, with 50% DC, by 7 dB this time.

4 Conclusions

In this paper, we evaluated the performance of incoherent 4-ary ASK in dispersive environments and compared it with OOK, under a range of system dispersion levels and different coding schemes. We derived the optimum level spacing as a function of ASE, thermal noise and filter characteristics.

The numerical results confirm that multilevel signalling has an inherent receiver sensitivity penalty, because of division of the main eye into several eyes. However, this penalty is shown to decrease with the amount of system dispersion allowed in a link. Depending on the pulse coding format, for system dispersion levels beyond 1500–2000 ps/nm, or equivalent to about 105 km of SSMF, multilevel signalling exhibits better performance than OOK. NRZ and RZ encoded multilevel signals incur 10 and 5 dB sensitivity penalties, respectively, as the system dispersion is varied from 0 to 2000 ps/nm, whereas the same values for OOK transmission exceed 10 dB. The multilevel RZ format with 33% duty cycle showed the best performance, which could be of particular importance where the use of OTDM is proposed.

On the other hand, 4-ary ASK is more susceptible to the deterioration of OSNR. An extinction ratio of 20 dB marginally reduces the performance of 4-ary ASK, but a decrease to 10 dB gives dramatic results, as a consequence of the OSNR degradation.

For the first time, a model that can be used to examine the performance of multilevel ASK for arbitrary optical filtering was applied. The results show that increasing the optical bandwidth affects 4-ary ASK less than OOK in systems with large dispersion and low ASE noise. In addition, 4-ary ASK requires less optical filter bandwidth than OOK to reach the optimum performance. Moreover, a small filter de-tuning, in the range of the signal bandwidth, is tolerable in terms of system performance for 4-ary ASK even for a system dispersion of 1000 ps/nm. In this way, we have shown that 4-ary ASK can be used to enhance the capacity of systems which experience large frequency drifts, by incorporating more channels at edges of the dispersion map. In addition, the potential of 4-ary ASK in systems with off-center optical filtering could merit further examination.

In multi-segment fibre links, multilevel systems were found to outperform binary OOK for system dispersion levels in excess of 1500–2000 ps/nm (depending on the pulse format) for distances of around 200 km. For OOK, a 9–10 dB dispersion penalty is found for the same distance and system dispersion range, compared to the 2–3 dB penalty for 4-ary ASK. A trade-off between system dispersion and receiver sensitivity is evident in these cases.

In conclusion, it is difficult to neglect the back-to-back sensitivity penalty inherent in a system employing multilevel signalling. However, by allowing this penalty a benefit can be obtained in system dispersion tolerance, in addition to the inherent reduced spectral occupancy. Hence, multilevel signalling can provide a solution to dispersion limited fibre systems of single channel dispersion uncompensated links, or otherwise in multichannel links with incomplete residual dispersion compensation, for example at the edge of a dispersion compensated region.

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