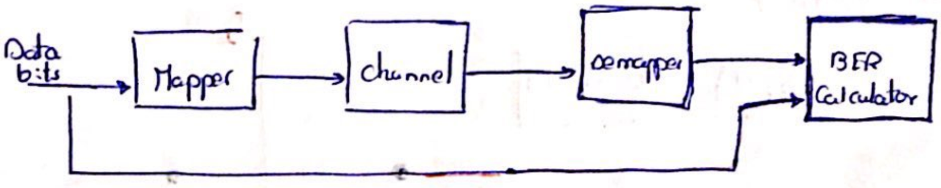
· Content

- project

1 project Details



1 Mapper

1 · phase shift keying $\boxed{PSK}$:

→ modulate phase to carry message

$$\cdot\ S_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (i-1)\frac{2\pi}{M}\right) \qquad 0 < t < T$$

$$= \sqrt{\frac{2E}{T}} \left[\cos (i-1)\frac{2\pi}{M} \underbrace{\cos 2\pi f_c t} - \sin (i-1)\frac{2\pi}{M} \underbrace{\sin(2\pi f_c t)} \right]$$

orthogonal basis

· to make normal

$$\cdot \int_0^T \cos^2 2\pi f_c t \, dt = \int_0^T \frac{1}{2}\left[1 + \cos 4\pi f_c t\right] dt$$

$$= \frac{T}{2}$$

Basis functions

$$\hookrightarrow \varphi_1(t) = \frac{\cos 2\pi f_c t}{\sqrt{\frac{T}{2}}} \quad , \quad \varphi_2(t) = \frac{\sin 2\pi f_c t}{\sqrt{\frac{T}{2}}}$$

$$\boxed{S_i(t) = \sqrt{E} \cos\left((i-1)\frac{2\pi}{M}\right) \varphi_1(t) - \sqrt{E} \sin\left((i-1)\frac{2\pi}{M}\right) \varphi_2(t)}$$

· signal representation for M-PSK

· There is another form → $S_i(t) = \sqrt{E} \cos\left((2i-1)\frac{\pi}{M}\right)\varphi_1(t) - \sqrt{E} \sin\left((2i-1)\frac{\pi}{M}\right)\varphi_2(t)$

second formula

• BPSK ← Binary phase shift keying

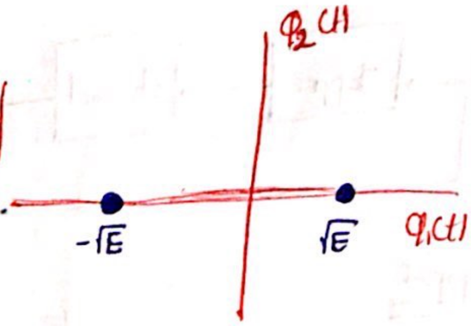
$M = 2 \begin{cases} 0 \\ 1 \end{cases}$

$$S_i(t) = \sqrt{E}\cos\left((i-1)\frac{2\pi}{2}\right)\varphi_1(t) - \underbrace{\sqrt{E}\sin\left((i-1)\frac{2\pi}{2}\right)\varphi_2(t)}_{0}$$

"$\sin(n\pi) = 0$"

$$S_i(t) = \sqrt{E}\cos\left((i-1)\pi\right)\varphi_1(t)$$

$\boxed{S_0(t) = -\sqrt{E}\,\varphi_1(t)}$ $\boxed{S_1(t) = \sqrt{E}\,\varphi_1(t)}$



• QPSK ← Quadrature phase shift keying

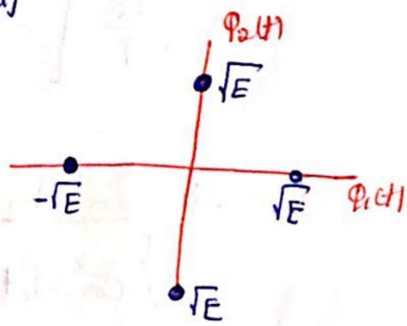
$M = 4 \begin{cases} 00 \\ 01 \\ 10 \\ 11 \end{cases}$

$$S_i(t) = \sqrt{E}\cos\left((i-1)\frac{2\pi}{4}\right)\varphi_1(t) - \sqrt{E}\sin\left((i-1)\frac{2\pi}{4}\right)\varphi_2(t)$$

$$S_i(t) = \sqrt{E}\cos\left((i-1)\frac{\pi}{2}\right)\varphi_1(t) - \sqrt{E}\sin\left((i-1)\frac{\pi}{2}\right)\varphi_2(t)$$

$S_0(t) = \sqrt{E}\,\varphi_2(t)$, $S_1(t) = \sqrt{E}\,\varphi_1(t)$

$S_2(t) = -\sqrt{E}\,\varphi_2(t)$, $S_3(t) = -\sqrt{E}\,\varphi_1(t)$

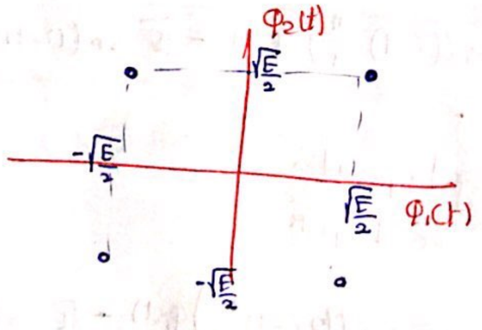


if we use second formula

$$S_i(t) = \sqrt{E}\cos\left((2i-1)\frac{\pi}{4}\right)\varphi_1(t) - \sqrt{E}\sin\left((2i-1)\frac{\pi}{4}\right)\varphi_2(t)$$

$S_0(t) = \sqrt{\frac{E}{2}}\,\varphi_1(t) + \sqrt{\frac{E}{2}}\,\varphi_2(t)$, $S_1(t) = \sqrt{\frac{E}{2}}\,\varphi_1(t) - \sqrt{\frac{E}{2}}\,\varphi_2(t)$

$S_2(t) = -\sqrt{\frac{E}{2}}\,\varphi_1(t) - \sqrt{\frac{E}{2}}\,\varphi_2(t)$, $S_3(t) = -\sqrt{\frac{E}{2}}\,\varphi_1(t) + \sqrt{\frac{E}{2}}\,\varphi_2(t)$



← just rotation

" doesn't effect on BER

depend on the distance bet points "

• 8psk

$M = 8 \rightarrow$ 000, 001, 010, 011, 100, 101, 110, 111

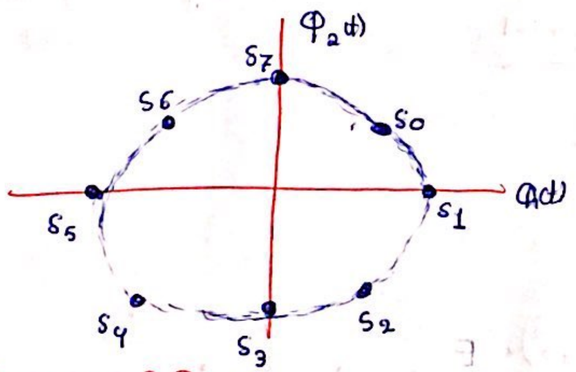
$$S_i(t) = \sqrt{E}\cos\left((i-1)\cdot\frac{2\pi}{8}\right)\varphi_1(t) - \sqrt{E}\sin\left((i-1)\cdot\frac{2\pi}{8}\right)\varphi_2(t)$$

$$S_i(t) = \sqrt{E}\cos\left((i-1)\frac{\pi}{4}\right)\varphi_1(t) - \sqrt{E}\sin\left((i-1)\cdot\frac{\pi}{4}\right)\varphi_2(t)$$

$S_0 = \sqrt{\frac{E}{2}}\,\varphi_1(t) + \sqrt{\frac{E}{2}}\,\varphi_2(t)$, $S_1(t) = \sqrt{E}$, $S_2(t) = \sqrt{\frac{E}{2}}\,\varphi_1(t) - \sqrt{\frac{E}{2}}\,\varphi_2(t)$

$S_3(t) = -\sqrt{E}\,\varphi_2(t)$, $S_4(t) = -\sqrt{\frac{E}{2}}\,\varphi_1(t) - \sqrt{\frac{E}{2}}\,\varphi_2(t)$

$S_5(t) = -\sqrt{E}\,\varphi_1(t)$, $S_6(t) = -\sqrt{\frac{E}{2}}\,\varphi_1(t) + \sqrt{\frac{E}{2}}\,\varphi_2(t)$, $S_7(t) = \sqrt{E}\,\varphi_2(t)$



2 • Amplitude shift keying (ASK):

→ modulate amplitude to carry message

$$s(t) = \sqrt{\frac{2E}{T}}\,a_i \cos\omega_c t \qquad 0 < t < T$$

$$S_i(t) = a_i\sqrt{E}\,\varphi_1(t) \qquad , a_i = \pm 1, \pm 3, \pm 5, \ldots$$

↳ 1 dimension.

3 • QAM ← quadrature amplitude modulation:

→ Hybrid bet ASK and psk

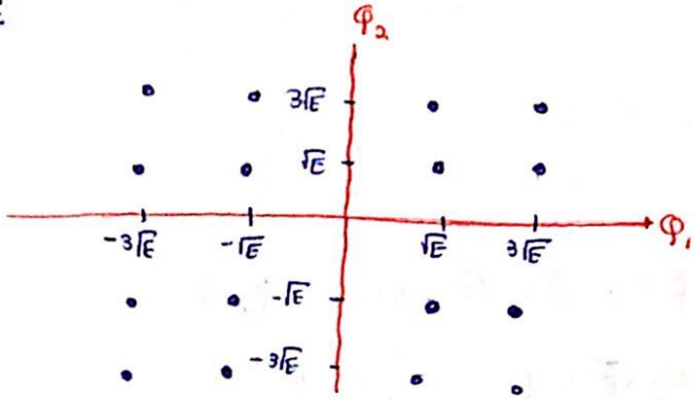
$$s_i(t) = \sqrt{\frac{2E}{T}}\,a_i \cos\omega_c t - \sqrt{\frac{2E}{T}}\,b_i \sin(\omega_c t) \qquad 0 < t < T$$

• $S_i(t) = \sqrt{E}\,a_i\,\varphi_1(t) - \sqrt{E}\,b_i\,\varphi_2(t)$, $a_i = b_i = \pm 1, \pm 3, \pm 5, \ldots$

↳ 2 dimension.

→ • Combining amplitude and phase modulation allows much higher bit rates.

- **16 QAM**



- E_b average energy per bit

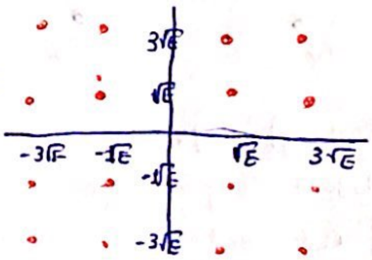
- $E_b = \dfrac{E_s}{\log_2 M}$ ← avg energy per symbol
- $E_s = \dfrac{\sum \text{energy of each symbol}}{M}$

} using Gray Coding
" one bit change "

- **Ex**

→ 16QAM

$$E_{s_{avg}} = \frac{\left[(1^2+1^2)\times 4 + (3^2+3^2)\times 4 + (1^2+3^2)\times 8\right]E}{16}$$



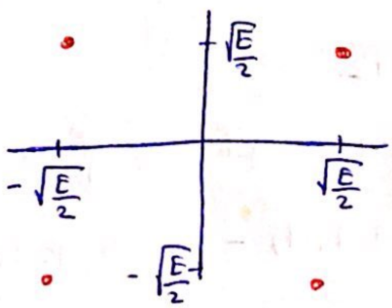
$M=16$

$$\boxed{E_{s_{avg}} = 10E}$$

$$E_b = \frac{E_s}{\log_2 M} = \frac{10E}{4} = 2.5E \qquad \boxed{E_b = 2.5E}$$

→ QPSK

$$E_s = \frac{\left[\left(\sqrt{\tfrac{1}{2}}\right)^2 + \left(\sqrt{\tfrac{1}{2}}\right)^2\right]E \times 4}{4}$$



$M=4$

$E_s = E$

$$E_b = \frac{E_s}{\log_2 M} = \frac{E}{2}$$

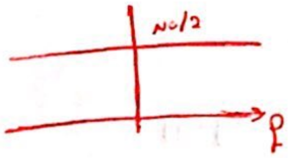
Note : IN PSK → $E_s = \dfrac{\sum \text{energy of each symbol}}{M}$ = energy of one symbol

2 The channel:

- The channel is an AWGN channel, In this model, the channel just adds noise to the transmitted signal. $\boxed{y = X + n}$

- In Matlab, we use the command "randn" to generate the AWGN noise and scale it by $\sqrt{\frac{No}{2}}$

- PSD of AWGN Noise



- [redacted] No?

For each SNR_{dB} in the range we get No by

SNR Range
1 : 10 dB

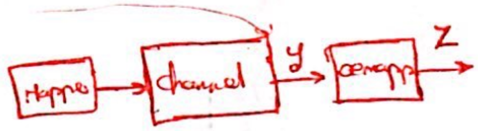
$$SNR = \frac{E_b}{N_o} \rightarrow$$ [redacted]

$$SNR = 10^{SNR_{dB}/10}$$

$$N_o = \frac{E_b}{10^{SNR_{dB}/10}}$$

3 Demapper:

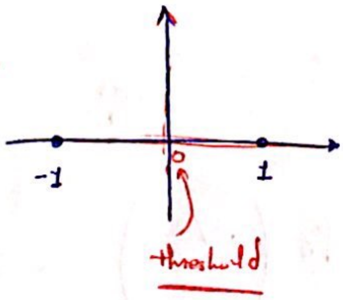
→ Take the output of the channel and decide on the symbol transmitted



EX.
↓

1 BPSK

y: output of channel
z: ~ ~ Demapper



if $y > 0$
- $Z = 1$

if $y < 0$
- $Z = 0$

Scanned with CamScanner

2 8psk

after channel

- $y = a + jb$

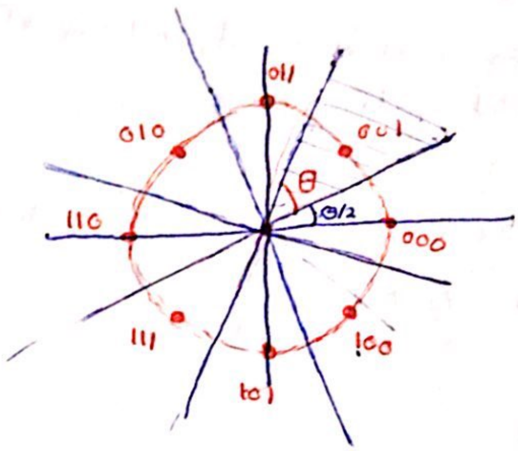
000 if $\frac{-\theta}{2} < \text{Arg}(y) < \frac{\theta}{2}$

001 if $\frac{\theta}{2} < \text{Arg}(y) < \frac{3\theta}{2}$

angle

⋮

100 if $-\frac{3\theta}{2} < \text{Arg}(y) < \frac{-\theta}{2}$



$\rightarrow \Theta = \frac{2\pi}{M} = \frac{2\pi}{8} = \frac{\pi}{4}$

- In MATLAB, we use Command " angle( ) " to get the angle of complex number

output of channel

4 "BER Calculation"

- using tight upper bound:

bit error rate

- $BER = \frac{SER}{\log_2 M}$ ← symbol error rate ← using Gray Coding

Ex:

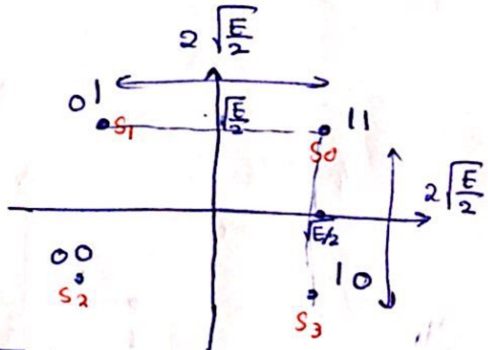
1 Qpsk

for S_0

neighbors $\rightarrow S_1, S_3$

$M = 4$

$$p(\text{error}|S_0) = \frac{1}{2}\, erfc\left(\frac{2\sqrt{\frac{E}{2}}/2}{\sqrt{N_0}}\right) + \frac{1}{2}\, erfc\left(\frac{2\sqrt{\frac{E}{2}}/2}{\sqrt{N_0}}\right) = erfc\left(\frac{\sqrt{E/2}}{\sqrt{N_0}}\right)$$



$\rightarrow SER = \sum p(\text{error}|S_i)\, p(S_i) = \frac{1}{4} \times 4 \times p(\text{error}|S_0)$

$\rightarrow \boxed{SER = p(\text{error}|S_0)}$

E or E_0 ← minimum unit to make the constellation points

$$BER = \frac{SER}{\log_2 M} = \frac{SER}{2} = \frac{1}{2}\, erfc\left(\frac{\sqrt{E/2}}{\sqrt{N_0}}\right)$$

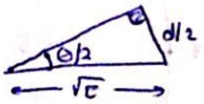
for psk $\rightarrow E = E_s \longrightarrow E_b = \frac{E_s}{\log_2 M} = \frac{E_s}{2} = \frac{E}{2} \longrightarrow \boxed{E = 2E_b}$

- $$BER_{Qpsk} = \frac{1}{2}\, erfc\left(\frac{\sqrt{2E_b/2}}{\sqrt{N_0}}\right) = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

total prob of error $= \sum p(\text{error}|S_i)\, p(S_i) = 4 \times \frac{1}{4} \times p(\text{error}|S_0)$

8psk

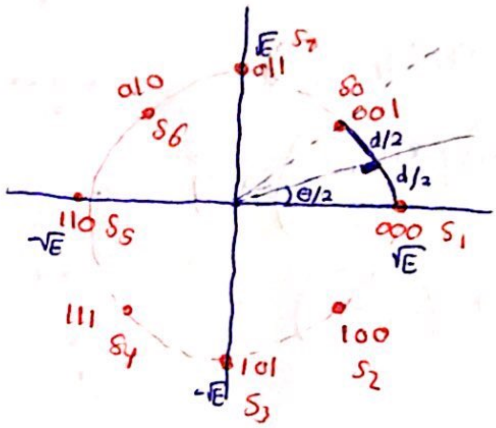
$$\theta = \frac{2\pi}{M}$$



$$\sin\frac{\theta}{2} = \frac{d/2}{\sqrt{E}}$$

$$\frac{d}{2} = \sqrt{E}\,\sin\frac{\theta}{2}$$

$$d = 2\sqrt{E}\,\sin\frac{\overset{\frac{2\pi}{M}}{\theta}}{2} = 2\sqrt{E}\,\sin\frac{\pi}{M}$$



For $S_0 \longrightarrow$ neighbours S_1 , S_7

$$P_e = \frac{1}{2}\,erfc\left(\frac{d/2}{\sqrt{N_0}}\right)$$

$$p(error|S_0) = \frac{1}{2}\,erfc\left(\frac{(\cancel{2}\sqrt{E}\sin\frac{\pi}{M})/\cancel{2}}{\sqrt{N_0}}\right) + \frac{1}{2}\,erfc\left(\frac{(2\sqrt{E}\sin\frac{\pi}{M})/2}{\sqrt{N_0}}\right)$$

$$p(error|S_0) = erfc\left(\frac{\sqrt{E}\,\sin\frac{\pi}{M}}{\sqrt{N_0}}\right)$$

$$SER = \sum p(error|S_i)\,\underbrace{p(S_i)}_{\frac{1}{8}}$$

$p(error|S_0) = p(error|S_{1,2,3,\dots 7})$

" same distance bet neighbours .

$$SER = \cancel{\frac{1}{8}} * \cancel{8} * erfc\left(\frac{\sqrt{E}\,\sin\frac{\pi}{M}}{\sqrt{N_0}}\right) = erfc\left(\frac{\sqrt{E}\,\sin\frac{\pi}{M}}{\sqrt{N_0}}\right)$$

for psk $\longrightarrow E = E_s \longrightarrow E_s = (\log_2 M)E_b$

$$E = E_b \log_2 M$$

$$\boxed{BER = \frac{SER}{\log_2 M} = \frac{1}{\log_2 M}\,erfc\left(\frac{\sqrt{E_b \log_2 M}\ \sin\frac{\pi}{M}}{\sqrt{N_0}}\right)}$$

general for M-ary psk

for 8psk $\longrightarrow M = 8$

$$\boxed{BER = \frac{1}{3}\,erfc\left(\frac{\sqrt{E_b * 3}\ \sin\left(\frac{\pi}{8}\right)}{\sqrt{N_0}}\right)}$$

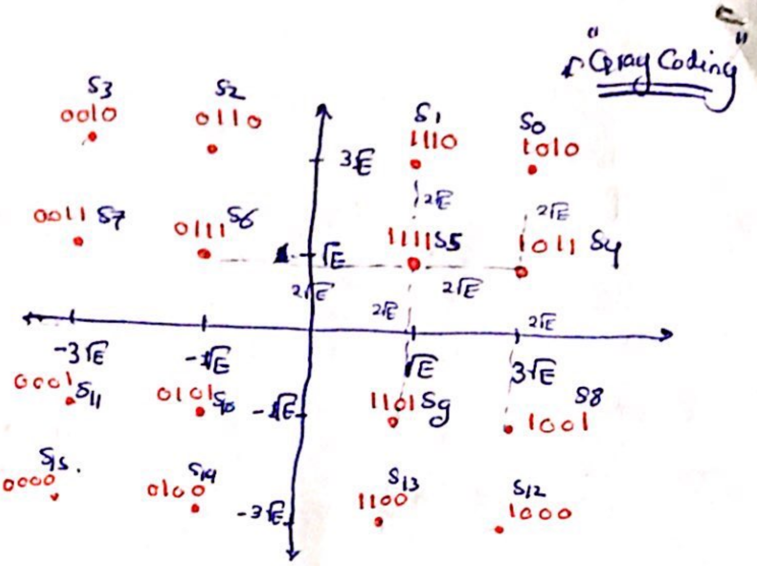
3 16 QAM

For S_5

→ neighbours S_1, S_4, S_6, S_9

$$p(\text{error}|S_5) = \frac{1}{2}\,\text{erfc}\left(\frac{2\sqrt{E}/2}{\sqrt{N_0}}\right) * 4$$

$$p(\text{error}|S_5) = 2\,\text{erfc}\left(\sqrt{\frac{E}{N_0}}\right)$$



• For S_4

→ neighbours S_0, S_5, S_8

$$p(\text{error}|S_4) = \frac{1}{2}\,\text{erfc}\left(\frac{2\sqrt{E}/2}{\sqrt{N_0}}\right) * 3$$

$$p(\text{error}|S_5) = \frac{3}{2}\,\text{erfc}\left(\sqrt{\frac{E}{N_0}}\right)$$

• For S_0

→ neighbours S_1, S_4

$$p(\text{error}|S_0) = \frac{1}{2}\,\text{erfc}\left(\frac{2\sqrt{E}/2}{\sqrt{N_0}}\right) * 2$$

$$p(\text{error}|S_0) = \text{erfc}\left(\sqrt{\frac{E}{N_0}}\right)$$

$S_5 \rightarrow$ like S_6, S_9, S_{10}

$S_4 \rightarrow$ like $S_1, S_2, S_7, S_8, S_{11}, S_{13}, S_{14}$

$S_0 \rightarrow$ like S_3, S_{12}, S_{15}

$$SER = \sum p(\text{error}|s_i)\, p(s_i)$$

$$= \frac{1}{16}\left[2*4\,\text{erfc}\left(\sqrt{\frac{E}{N_0}}\right) + \frac{3}{2}*8\,\text{erfc}\left(\sqrt{\frac{E}{N_0}}\right) + 4*\text{erfc}\left(\sqrt{\frac{E}{N_0}}\right)\right]$$

$$SER = \frac{3}{2}\,\text{erfc}\left(\sqrt{\frac{E}{N_0}}\right) = \frac{3}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2.5N_0}}\right)$$

in this section "page 4" we prove that $\rightarrow E_b = 2.5E$

$$\boxed{E = \frac{E_b}{2.5}}$$

$$BER = \frac{SER}{\log_2 M} \qquad M=16$$

$$\boxed{BER = \frac{3}{2} * \frac{1}{4}\,\text{erfc}\left(\sqrt{\frac{E_b}{2.5N_0}}\right)}$$

→ IN Matlab → we use the command erfc( ) to get the erfc function

• you can use function symerr for calculating BER also.